Residual stresses in Al$_2$O$_3$-ZrO$_2$ laminar system

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Abstract. The aim of the paper is to present an analytical model of thermal stresses in a laminar system, and consequently to verify the model validity by comparing calculated thermal stresses with measured ones by indentation method. Analytical models of thermal stresses acting in anisotropic and isotropic laminar plane systems are presented, and consequently applied to the thermal-stress-strengthened Al$_2$O$_3$-ZrO$_2$ laminar ceramics.

Introduction

Originating during a cooling process of the initial temperature, $T_1$, and the final temperature, $T_2$, thermal stresses are an important phenomenon observed in a system characterized by the difference in thermal expansion coefficients between individual components of a system [1]-[5], influencing mechanical properties [6]-[9], superconductivity [10,11], and last but not least, are a reason of crack formation [12]-[18]. In contrast to this negative influence, thermal stresses positive influence fracture toughness, representing a resistance against tensile mechanical loading, as considered in design of mechanical properties of ceramic materials, exhibiting low fracture toughness at tensile mechanical loading. Considering laminar ceramic composites of different thermal expansion coefficients of individual layers, a layer of low fracture toughness and, as needed, of lower thermal expansion coefficient than that of a neighbouring layer, is thus acted by compressive thermal stresses, from which an improvement of low fracture toughness of a ceramic system is resulted. With regard to bending of a laminar system, compressive thermal stresses are also required to be induced in a layer acted by tensile mechanical loading.

Analytical model

Consisted of the anisotropic layers A and B with the different thermal expansion coefficients $\alpha_A$ and $\alpha_B$ along the axis $x_i$ ($i=2,3$), with the thickness $t_A$ and $t_B$, low in comparison with the plate size, the laminar system shown in Fig.1, cooling down from the initial temperature $T_i$ to the final temperature $T_2$ of a cooling process, is accordingly acted by the normal stresses $\sigma_{iA}$ and $\sigma_{iB}$, along the axis $x_i$, respectively, assuming the normal stress $\sigma_{1A}=\sigma_{1B}=0$ along the axis $x_1$.

Fig.1. Laminar system of the layers A and B of the thickness, $t_A$ and $t_B$, respectively, and the Cartesian system ($Ox_1x_2x_3$), where the normal stress, $\sigma_{1}=\sigma_{1A}=\sigma_{1B}=0$. 


In contrast to the analytical model assuming the \( x_i \)-dependence of the strain \( \varepsilon_{ii} \) [17], the normal strains of the layers A and B, \( \varepsilon_{22Q} \) and \( \varepsilon_{33Q} \) (\( Q=A,B \)), respectively, as a sum of elastic and thermal components, have the forms

\[
\varepsilon_{22Q} = s_{22Q}\sigma_{22Q} + s_{23Q}\sigma_{33Q} + \alpha_{2Q}(T_1 - T_2),
\]

\[
\varepsilon_{33Q} = s_{23Q}\sigma_{22Q} + s_{33Q}\sigma_{33Q} + \alpha_{3Q}(T_1 - T_2),
\]

where the elastic coefficient for an anisotropic continuum \( s_{ij} = s_{ji} \), dependent on a position of crystalline lattice axes, is presented in [5]. Considering a perfect A-B bonding, represented by the strain condition determined as

\[
\varepsilon_{iiA} = \varepsilon_{iiB}, \quad i=2,3,
\]

and regarding the force balance condition, related to laminar system with \((n+1)\) layers A and \( n \) layers B, derived as

\[
(n+1) t_A\sigma_{iiA} + nt_B\sigma_{iiB} = 0, \quad i=2,3,
\]

the normal stresses \( \sigma_{iiA} \) and \( \sigma_{iiB} \) acting along the axis \( x_i \) \( (i=2,3) \) in the anisotropic layers A and B, respectively, have the forms

\[
\sigma_{iiA} = -xc_i n(T_1 - T_2), \quad i=2,3, \quad \sigma_{iiB} = c_i (n+1)(T_1 - T_2), \quad i=2,3, \quad x = \frac{t_B}{t_A},
\]

where the coefficients \( c_i \) and consequently \( c_{ij} = c_{ji} \) \( (i,j=2,3) \) are derived as

\[
c_{2+i} = \frac{c_{3-i3-i}(\alpha_{2+iB} - \alpha_{2+iA}) - c_{23}(\alpha_{3-iB} - \alpha_{3-iA})}{c_{22}c_{33} - c_{23}^2}, \quad i=0,1,
\]

\[
c_{ij} = c_{ji} = s_{ijA}nx + s_{ijB}(n+1), \quad i,j=2,3.
\]

Considering \( \alpha_Q = \alpha_Q \), \( s_{ijQ} = 1/E_Q \) \( (i=2,3; \ Q=A,B) \), \( s_{ijQ} = -\nu_Q/E_Q \) \( (i\neq j; \ i,j=2,3) \), where \( E \) and \( \nu \) are the Young’s modulus and the Poisson’s number, respectively, the normal stress \( \sigma_Q = \sigma_{iiQ} \), acting along the axis \( x_i \) \( (i=2,3) \) in the isotropic layer \( (Q=A,B) \), are transformed to the forms derived by Chartier et al. [6]

\[
\sigma_A = -\frac{E_A E_B}{xnE_B(1-\nu_A)} \frac{(\alpha_B - \alpha_A)(T_1 - T_2)}{(n+1)(1-\nu_B)},
\]

\[
\sigma_B = \frac{xnE_A E_B}{xnE_B(1-\nu_A)} \frac{(\alpha_B - \alpha_A)(T_1 - T_2)}{(n+1)(1-\nu_B)}.
\]

**Indentation model**

The relation between the indentation load, fracture toughness and the length of the indentation cracks can be described by the equation:
\[ K_{IC} = \chi \left( \frac{P}{c_0^{3/2}} \right) \]  

where \( K_{IC} \) = toughness of the stress-free material, \( \chi \) = dimensionless constant, \( P \) = indentation load and \( c_0 \) = crack length.

For the parameter \( \chi \) a value 0.089 was found by the regression analysis of the value \( P \) and \( c_0 \) and considering a value for the monolithic alumina \( K_{IC} = 3.61 \text{ MPam}^{1/2} \) measured by Chevron notch technique (18).

In the case of the residual stress in the material, the Eq.(10) becomes

\[ K_{IC} = \chi \left( \frac{P}{c_1^{3/2}} \right) + Y\sigma_{res}(c_1)^{1/2} \]  

where \( c_1 \) = crack length in the material with residual stress, \( Y = 1.29 \) and \( \sigma_{res} \) = residual stress.

**Al₂O₃-ZrO₂ laminar system**

Considering the analytical model by Chartier et al. (see Eqs.(8), (9)) [6], Fig.2 shows the compressive and tensile stresses \( \sigma_A < 0 \) and \( \sigma_B > 0 \) in the layers A and B of the chemical composition Al₂O₃ and Al₂O₃(60%)+ZrO₂(40%), respectively, as functions of the parameter \( x = t_B/t_A \) for the 9-layered laminar system \( (n=4) \), for parameters listed in Tab.1, and for the initial and final temperature \( T_1 = 1200°C \) and \( T_2 = 20°C \) of a cooling process, respectively.

![Fig.2. The compressive and tensile stresses \( \sigma_A < 0 \) and \( \sigma_B > 0 \) in the layers A and B as functions of \( x = t_B/t_A \).](image)

**Tab.1.** Parameters of the 9-layered laminar system.

<table>
<thead>
<tr>
<th>Layer</th>
<th>E [GPa]</th>
<th>( \mu )</th>
<th>( \alpha ) [( 10^{-6}\text{K}^{-1} )]</th>
<th>F [N]</th>
<th>HV</th>
<th>( \perp 2c ) [( \mu m )]</th>
<th>K_{ICmon}/K_{ICin} [\text{MPam}^{1/2}]</th>
<th>t [( \mu m )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al₂O₃ A</td>
<td>385</td>
<td>0.225</td>
<td>9</td>
<td>48</td>
<td>-</td>
<td>160</td>
<td>3.5/5.54</td>
<td>193</td>
</tr>
<tr>
<td>Al₂O₃(60%) + ZrO₂(40%) B</td>
<td>311</td>
<td>0.239</td>
<td>10.08</td>
<td>50</td>
<td>1337</td>
<td>-/-</td>
<td></td>
<td>529</td>
</tr>
</tbody>
</table>

Using the Eq (11), for the residual stress in the Al₂O₃ layer of the Al₂O₃/Al₂O₃(60%)+ZrO₂(40%) composite a value of \( \sigma_{Al₂O₃} = - 214.2 \text{ MPa} \) was obtained.

Considering Eq.(10) and the \( \sigma_B - x \) dependence for \( x = t_B/t_A = 2.74 \) in Fig.2, the residual stress in the Al₂O₃ layer is \( \sigma_A = - 390 \text{ MPa} \), which is significantly higher as the measured one but is a good agreement with the result of the FE calculation, [19].
Conclusions

• An analytical model of thermal stresses in anisotropic laminar system has been derived and transformed to that for an isotropic laminar system derived by Chartier et al. [6];
• the residual stress in the Al₂O₃ layer of the Al₂O₃/Al₂O₃(60%)+ZrO₂(40%) composite was characterized using indentation technique and a compressive stress of \( \sigma_{\text{Al}_2\text{O}_3} = -214.2 \text{ MPa} \) was obtained.
• the difference between the predicted and measured results can be explained by the boundary condition assumed within the analytical model.

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