Analysis of modified railway passenger truck designs to improve lateral stability/curving behaviour compatibility

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Modified railway passenger truck designs are considered to improve the compatibility between dynamic stability and the ability of the vehicle to steer around curves. A comparative study on the non-linear steady state curving behaviour of some unconventional truck designs is carried out. The study reveals that modified truck designs with primary yaw dampers improve the performance. A particular design of railway truck in which the rear axle is an independently rotating wheelset and in which the primary suspension in the leading axle is different from the trailing axle—known as the unsymmetric wheelset and suspension truck—has the potential to achieve overall superior performance compared to other truck models considered in this paper, for design speeds of around 80 m/s. For the lower design speed of around 60 m/s, a conventional truck provided with primary yaw damper achieves adequate performance and is recommended for its simpler design features.

Key words: railway trucks, suspension design, independently rotating wheelset, yaw damper

1 INTRODUCTION

There has been an increased research effort by different organizations around the world to design railway trucks with improved performance both on tangent track and on curves. The trucks should be capable of high-speed operation and they should achieve acceptable performance in curves in terms of reduced wear of wheels and rail. In the design of conventional railway vehicles with rigid axles there is a conflict between dynamic stability at high speeds and the ability of the vehicle to steer around curves. A number of researchers have studied the performance of alternative truck configurations in order to increase the compatibility in performance. For example a radial truck with axle interconnections helps to achieve design critical speeds at lower values of primary longitudinal stiffness. Lower values of primary longitudinal forces lessen the curving forces and helps in aligning the wheelsets in curves. Horak et al. obtained optimum suspension configurations for best possible trade-off between stability and curving behaviour for a radial truck (1). It was concluded that only a marginal advantage will be achieved by choosing a radial design over a well designed conventional truck for main line operations. However in urban transit applications, where sharp curves are common, radial truck would achieve lower flange force.

Dynamics of a railway truck with independently rotating wheelset (IRW) has received wide attention in recent years due to the fact that hunting is eliminated in a truck equipped with IRW. Several possible guidance systems that have been suggested are reviewed by Dukkipati et al. (2) Eickhoff investigated the potential benefits and areas of applicability of railway vehicles with IRW (3). He concluded that tolerances on the manufacture and maintenance of such vehicles would have to be strict to avoid problems of offset running and the consequent deterioration in ride. Elkins, in his study of freight truck with IRW, concluded that a freight truck with IRW in the trailing axle achieves overall superior performance compared to a conventional truck or a freight truck with both axles having IRW (4). Suda and Anderson have studied the dynamic behaviour of a longitudinally unsymmetric suspension truck in curve negotiation (5). A comparative study of the lateral stability and steady state curving behaviour of the unconventional truck models was done using linear models by Narayana Swamy et al. (6). The primary suspension parameters for each configuration were optimized to have the best possible trade-off between stability and off-flange curving performance. It was found that an unsymmetric wheelset truck possesses better compatibility between stability and off-flange curving performance compared to other truck models. More recently Suda examined the high-speed stability and curving performance of the railway truck using linear models (7). In these investigations he compared the unsymmetric suspension truck performance with a conventional truck. He concluded that any potential benefits of the unsymmetric truck could be realized in a truck equipped with both unsymmetric suspension and unsymmetric wheelset and provided with a primary yaw damper and a secondary yaw damper.

In the present investigation, the steady state curving behaviour of some unconventional railway trucks are evaluated incorporating non-linear wheel–rail profiles and creep force saturation. The unconventional truck designs are modified designs unlike the trucks in revenue service which have rigid axles and symmetric plan view primary suspension. A radial truck, a truck provided with primary yaw dampers, an unsymmetric truck with suspension asymmetry and a truck provided with independently rotating wheelset or a combination of these designs are referred to as unconventional truck models in this paper. A comparative performance evaluation of these truck designs is carried out with a view to arriving at a configuration that achieves compatibility.
between dynamic stability and curving behaviour. Configurations of railway trucks considered for performance evaluation in this paper are: (a) conventional truck; (b) radial truck; (c) unsymmetric suspension truck; (d) unsymmetric wheelset and suspension truck. The truck configurations are shown schematically in Fig. 1.

2 CONFIGURATIONS OF RAILWAY TRUCKS

2.1 Conventional truck
In order to improve the dynamic stability, conventionally stiff primary suspension is used which deteriorates the curving performance of the railway vehicle. In the case of a conventional truck, one way to improve the trade-off is by providing primary yaw dampers as shown in Fig. 2. The yaw dampers are fitted to the front axle of the leading truck and rear axle of the trailing truck. If the primary yaw dampers are used, the longitudinal restraint is removed for low velocity (steering) motions; for higher frequency motions the damper resistance increases and the higher restraint helps to achieve desired critical speeds. Yaw dampers provided in the primary suspension helped achieve a better curving/stability trade-off for the British Rail Class 37 locomotive as was reported by Pennington et al. (8).

The conventional passenger railway truck configuration is referred to as truck C in this paper. The conventional truck fitted with yaw dampers is referred to as truck CD.

2.2 Radial truck
In a radial truck the wheelsets are interconnected by cross-bracing which provides the inter-axle shear stiffness that helps to achieve higher critical speed compared to the conventional truck model. This configuration is referred to as truck R in this paper.

The dynamics of a flexible track are strongly affected by inter-wheelset forces either through the truck frame as in the case of a conventional truck or inter-connection elements as in the case of a radial truck. Any form of elastic inter-wheelset structure can be represented by the generalized bending stiffness $K_b$ and generalized shear stiffness $K_s$, as explained in reference (6).

Neglecting the dynamic effects due to truck frame inertia and primary suspension damping, the conventional stiffnesses $K_{ps}$ and $K_{py}$ and self-steering radial truck stiffnesses $k_p$ and $k_y$ can be transformed into equivalent static stiffnesses $K_b$ and $K_s$ as given below:

$$K_b = K_{ps} d_p^2 + k_b$$

$$K_s = \frac{K_{py} K_{ps} d_p^2}{b^2 K_{py} + d_p^2 K_{ps}} + k_s$$

In the case of a conventional truck with elastic connections at the primary suspension, $k_p = 0$ and $k_y = 0$. Then, by substituting equation (1) into equation (2) and setting the criterion that $K_{py} \geq 0$, it can be shown that for the conventional truck $K_s \leq K_y/b^2$ and for the radial truck $K_s$ can be greater than $K_y/b^2$. In the above equations, $b$ is half the wheelbase and $d_p$ is half the lateral spacing of the primary suspension as given in the appendix.

2.3 Unsymmetric suspension truck
Another modified design considered for performance evaluation is a truck with an asymmetry in the primary suspension. A longitudinal asymmetry is introduced in
The stiffness matrix for the four degree of freedom unsymmetric suspension truck model can then be derived as (7)

\[
K = \begin{bmatrix}
k_a & -(b - a)k_a & -k_s \\
-k_a & k_b + (b - a)^2k_a & -(b - a)k_s \\
-k_s & (b - a)k_s & k_s
\end{bmatrix}
\]

The condition for perfect steering derived for the US truck is then given by (6)

\[
a_{wp} = \frac{k_b \alpha}{f_{33} a\lambda}
\]

2.4 Unsymmetric wheelset and suspension truck

The fourth configuration is the unsymmetric wheelset and suspension truck with primary yaw damper which is a USD truck in which trailing axle has an independently rotating wheelset (IRW). This configuration is referred to as a UWSD truck. For the truck to have bi-directional operation, such a truck should be provided with switched dampers and electromagnetic clutches on the axles (7).

3 SCOPE OF THE PAPER

The purpose of the present investigation is to analyse the stability and steady-state curving performance of the truck designs. For the lateral stability analyses, critical speed is typically found by determining the speed at which the damping ratio becomes negative. To analyse the steady-state curving behaviour, non-linear wheel-rail profile characteristics and non-linear creep forces are considered. The steady-state curving program solves the following non-linear equation:

\[
K Y = F(Y)
\]

where \(K\) is the suspension stiffness matrix and \(F(y)\) are the external forces and moments acting on the vehicle.

Non-linear forces such as flanging forces, creep forces as functions of wheel load and maximum creep force as a function of the wheel-rail adhesion limit are included in the model. The analytical technique is a multi-step iteration on the simultaneous equations of motion for which the IMSL subroutine NEQNF is used. To predict the wheel-rail geometric parameters like contact angle, roll angle, rolling radius for any particular value of lateral displacement, the wheel-rail characterization program written by Cooperrider and Heller (9) was used.

4 STABILITY PROPERTIES

A linearized model is used to study the stability characteristics of various truck designs. For the stability analyses a full car body model has been considered. This consists of 17 degrees of freedom, namely lateral
and yaw motion for each wheelset accounting for eight degrees of freedom and the lateral, yaw and roll motions for two truck frames and one car body contributing to nine degrees of freedom. The model of a single truck is shown schematically in Fig. 4. Values of vehicle parameters used are obtained from the paper by Horak et al. (1) and are tabulated in Appendix 1. The equations of motion for the basic four-degree-of-freedom truck model was given in reference (6) and was extended to include motions of the front and rear truck and the car body.

4.1 Conventional truck

From the eigenvalue analyses, critical speed as a function of bending stiffness $K_b$ is plotted in Fig. 5a for the conventional truck for different conicities. From the figure, the bending stiffness to achieve a critical speed of 80 m/s is found to be $K_b = 5 \times 10^6$ N m/rad for the conicity of $\lambda = 0.15$. If the design critical speed is 60 m/s, the value of bending stiffness required is $K_b = 2.5 \times 10^6$ N m/rad. Figure 5b shows the variations for $\lambda = 0.15$ for different Kalker creep coefficients. For 75 per cent and 100 per cent Kalker values, there is a decrease in the critical speeds for values of $K_b$ less than $1 \times 10^7$ N m/rad. For the analysis, 50 per cent Kalker values are chosen as nominal values of creep coefficients.

In the case of the conventional truck, yaw dampers are provided in the leading axle of the front truck and the trailing axle of the rear truck. For the case of the truck provided with yaw damper, the effects of varying yaw damper coefficient $C_{dp}$ (N m s/rad) and the series damper stiffness $K_{dp}$ (N m/rad) on the critical speed is shown in Fig. 5c. The damper series stiffness comes from the damper end mountings and bulk modulus of the damper working medium. From the plot it is seen that for low values of damper coefficient, the increase value of stiffness $K_{dp}$ does not increase the critical speed. Accordingly, values of $C_{dp} = 2 \times 10^5$ N m s/rad and $K_{dp} = 2 \times 10^6$ N m/rad are chosen. Figure 5d shows the variation of critical speed for different $K_b$ for the case of the conventional truck with primary yaw damper from which $K_b = 1 \times 10^6$ N m/rad is chosen to achieve a critical speed of 80 m/s. For the critical speed of 60 m/s, the value of $K_b = 3 \times 10^5$ N m/rad is chosen. Figure 5e shows the variations for $\lambda = 0.15$ for different Kalker creep coefficients. Figure 5f shows the improvement of the dynamic stability because of the primary yaw damper.

4.2 Radial truck

The performance of the radial truck is also considered for comparison with other truck models. The variation of critical velocity for different $K_b$ is shown in Fig. 5g. The radial truck shows improved stability performance compared to the conventional truck due to the influence of inter-axle shear stiffness $K_s$. From the plot for the case $\lambda = 0.15$, $K_b$ is chosen to be $1.75 \times 10^6$ N m/rad for the case of 80 m/s critical speed and $1.25 \times 10^6$ N m/rad for critical speed of 60 m/s.

4.3 Unsymmetric suspension truck with damper

In this section the stability characteristics of unsymmetric suspension trucks are considered. The plot in Fig. 6a shows the variation of critical velocity for different damper parameters. It is seen that a value of

![Fig. 4 Linear truck model](image_url)
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**Fig. 5b** Effect of creep coefficient on critical velocity

**Fig. 5c** Influence of damper parameters for CD truck

**Fig. 5d** Critical velocity for CD truck

**Fig. 5e** Effect of creep coefficient for CD truck

**Fig. 5f** Stability improvement due to yaw damper

**Fig. 5g** Critical velocity for radial truck

\[ C_{\text{op}} = 2 \times 10^5 \text{ N m s/rad} \] and \[ K_{\text{op}} = 2 \times 10^6 \text{ N m/rad} \] gives acceptable stability performance. Primary lateral suspension affects the performance of the unsymmetric suspension truck as seen from Fig. 6b and for this truck \[ K_{pr} = 5 \times 10^5 \text{ N/m} \] is chosen. Figure 6c shows the variation of critical velocity for different values of bending stiffness. The value of bending stiffness is chosen to be \[ K_b = 3.9 \times 10^5 \text{ N m/rad} \] and a value of asymmetric index ratio \( a_r = a_j/b \) of 0.82 is needed to achieve a critical speed of 80 m/s.

One more configuration taken up for performance evaluation is the unsymmetric suspension truck having IRW in the trailing axle, referred to as a UWSD truck. The same values of damper parameters as given for the USD truck are chosen. The variation of critical speed for different values of \( K_b \) and \( \lambda \) is shown in Fig. 6d. A
Fig. 6a Influence of damper parameters for USD truck

Fig. 6b Influence of primary lateral stiffness for USD truck

Fig. 6d Critical velocity for UWSD truck

Fig. 6e Comparison of stability performance for US trucks

Comparison of the variation for the cases of US, USD and UWSD tracks is shown in Fig. 6e. The improvement of stability characteristics due to the addition of the primary yaw damper can be easily seen. Also it is observed that if IRW is provided in the trailing axle as in UWSD trucks, dynamic stability is improved for values of $K_a$ greater than $3 \times 10^6$ N m/rad. Values of $K_a$ chosen for two different critical speeds of 60 m/s and 80 m/s are shown in the table for the different configurations of trucks considered in this paper.

5 CURVING ANALYSIS

For the steady-state curving analysis, non-linear wheel-rail geometry and non-linear creep force saturation are considered. The analytical formulation is shown in Appendix 2 (see also Fig. 7).

5.1 Wear index

One objective for the present investigation is to reduce wheel and rail wear in curves. The wear index suggested by Elkins and Allen is based on the premise that rates of wheel and rail wear can be related to the forces and creepages between the wheels and rail (12). Mathematically, the work done in the contact path is equal to the
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Fig. 7 Free-body diagram for wheelset

\[ W = F_e \cdot \xi \]  \hspace{1cm} (10)

The wear index is not necessarily indicative of the actual wear taking place at the wheel-rail contact patch. But a comparison of wear index for different truck configurations could be done as a means of measuring the curving performance.

5.2 Results of the simulation

For the analyses, the CN-A wheel profile and new rail profile that give an effective conicity of around 0.15 are considered. The two independent forcing inputs that describe the curving condition to which the vehicle is exposed are: track curvature \( 1/R \), where \( R \) is the radius of the curve; and the cant deficiency, which accounts for the lateral unbalance. In North America, track curvature is often expressed in terms of degree curve \( D \), which is defined as the angle subtended at the centre of the curve due to a 100 ft (30.48 m) chord (6).

The use of primary yaw dampers in the conventional truck and the unsymmetric suspension truck enhances the curving performance since the acceptable stability is obtained for low values of primary bending stiffness. The variation of angle of attack for the case of the 80 m/s critical speed is shown in Fig. 8a. For the conventional truck the angle of attack rises rapidly as the curvature increases. As seen from the plot, the conventional truck will not be able to negotiate curves greater than ten degrees. For the trucks with yaw dampers the angle of attack is found to be less than one mrad throughout the ranges of curvature. The USD truck achieves near perfect steering. The variation of wear index is given in Fig. 8b. The wear index for the conventional truck increases sharply as the curvature increases. The wear index for CD and USD trucks are almost identical. The wear index for the UWSD truck is seen to be less than for other configurations. This is due to the fact that in the UWSD truck, the trailing axle is an IRW. Longitudinal creepage is assumed to be zero for IRW (this assumption is valid for single point contact) and hence the contribution it will make to the work index is nearly zero. Thus it is seen that the best compatibility between dynamic stability and steering behaviour is achieved in the case of the UWSD truck. Another potential advantage of using modified designs of trucks is the reduction of wheelset drag forces during curve negotiation which reduces consumption of energy. The plots in Figs 8c and 8d show that drag forces for trucks with dampers are comparatively less. Especially since the UWSD truck has IRW in the trailing axle, the trailing wheelset drag is almost zero in comparison to other truck models. In all these plots, radial truck performance is also indicated and it is seen that trucks with dampers achieve performance levels superior to the radial truck.

The comparisons of curving results optimized for the critical speed of 60 m/s for different truck configurations are shown in Figs 9a to 9d. The results also include the case of unsymmetric suspension (US) truck without yaw damper. The superior performance of trucks provided with primary yaw dampers can be seen. For a design speed of 60 m/s, it is seen that the CD truck will perform as well as the USD or UWSD trucks. As seen from Fig. 9d the drag force is less for the UWSD truck compared to the USD and CD trucks which can be the only advantage of opting for this more complicated design. The results of wear index for ten-degree curves (175 m radius) and 20-degree curves (87.5 m radius) are given in Table 1. For trucks designed for the critical speeds of 60 m/s, it is seen that the CD truck will perform as well as the USD or UWSD trucks.

### Table 1 Wear Index for different truck configurations

<table>
<thead>
<tr>
<th>Truck type</th>
<th>( K_p ) (N m/rad)</th>
<th>( K_b ) (N m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.5 \times 10^6</td>
<td>10</td>
</tr>
<tr>
<td>R</td>
<td>1.25 \times 10^6</td>
<td>10</td>
</tr>
<tr>
<td>US</td>
<td>2.05 \times 10^6</td>
<td>10</td>
</tr>
<tr>
<td>CD</td>
<td>3 \times 10^4</td>
<td>10</td>
</tr>
<tr>
<td>USD</td>
<td>6.43 \times 10^4</td>
<td>10</td>
</tr>
<tr>
<td>UWSD</td>
<td>2.25 \times 10^3</td>
<td>10</td>
</tr>
</tbody>
</table>

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speed of 80 m/s, for negotiating a 20-degree curve, the wear index predicted for the radial truck is 1329 N m/m. The corresponding value for the CD truck is 495 N m/m, for the USD truck is 555 N m/m and for the UWSD truck is 108 N m/m. Comparing the results for the case of a 60 m/s critical velocity, to negotiate 10-degree curves, the predicted wear index for the C truck is 629 N m/m, for the R truck is 200 N m/m, for the CD truck is 129 N m/m, for the USD truck is 88 N m/m and for the UWSD truck is 68 N m/m.

The comparison shows that the best compatibility between stability and the curving behaviour is achieved in the case of the UWSD truck for the design speed of 80 m/s. For the design speed of 60 m/s, adequate performance is obtained using the CD truck configuration. Thus an optimum design can be arrived at depending on the speed requirements and radii of curves found in a particular rail route. For moderate speeds in the region of 50–60 m/s, the CD truck gives adequate stability and the predicted wear index for the wheel is much less compared to the conventional and radial trucks. This design is also less complicated compared to USD or UWSD trucks precluding the use of electromagnetic clutches and switching requirement for the damper. For higher speeds and more curvaceous routes, the UWSD truck is potentially the ideal configuration since it predicts the lowest wear for the wheels and wheelset drag forces are at a minimum.

The variation of angle of attack with friction coefficient is shown in Fig. 10a for optimized values of suspension parameters considered for the design speed of 80 m/s. The vehicle is negotiating a ten-degree curve. The angle of attack decreases with increasing coefficient of friction for the case of the conventional truck and radial truck. For trucks with dampers, the angle of attack is much less and is not affected by variation in friction coefficient. The variation of total wear index for a truck with friction coefficient is shown in Fig. 10b. For the trucks provided with dampers, the wear index remains constant for all μ. A substantial reduction in
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Fig. 9a Variation of predicted angle of attack, critical speed = 60 m/s

Fig. 9b Variation of predicted wear index, critical speed = 60 m/s

Fig. 9c Leading wheelset drag, critical speed = 60 m/s

Fig. 9d Trailing wheelset drag, critical speed = 60 m/s

Fig. 10a Variation of angle of attack with friction coefficient

Fig. 10b Variation of wear index with friction coefficient

the wear index for the case of trucks with dampers can be seen compared to the conventional and radial truck designs.

6 CONCLUSION

Modified truck designs are considered to improve the compatibility between dynamic stability and curving
The performance is improved if the conventional and unsymmetric suspension trucks are provided with yaw dampers in the primary suspension. Non-linear curving analyses revealed that for the optimized values of suspension parameters obtained in the stability analyses, the UWSD truck exhibits reduced wear index and reduced wheelset drag compared to other truck designs. Out of the different configurations considered, the UWSD truck achieves acceptable performance in terms of lowest wear of wheel for a design speed of around 80 m/s. For a lower speed of around 60 m/s, the conventional truck with the primary yaw damper (CD truck) would give adequate performance.

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REFERENCES


APPENDIX 1

Notation and vehicle parameters

Wheel rail parameters

- Wheelset roll coefficient
- Centered contact angle
- Contact angle difference
- Lateral creep force coefficient
- Lateral/spin creep coefficient
- Spin creep coefficient
- Longitudinal creep force coefficient

Wheel parameters

- Half of track gauge
- Load per wheelset
- Conicity
- Wheelset mass
- Wheelset yaw moment of inertia
- Wheelset moment of inertia about zz-axis
- Centered wheel rolling radius

Truck parameters

- ½ prim. spring lat. spacing
- Half of wheelbase
- Height of truck c.g. above axle centre
- Height from truck c.g. to sec. lat. spring
- Distance from truck c.g. to sec. spring
- Truck frame mass
- Truck yaw moment of inertia

Car body parameters

- Half sec. spring lat. spacing
- Height of car body c.g. to sec. lat. springs
- Mass of car body

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Primary suspension parameters

\( K_b \) Generalized bending stiffness \( \text{variable N m/rad} \)

\( K_s \) Generalized Shear Stiffness \( \text{variable N/m} \)

\( K_{pr} \) Conventional primary longitudinal suspension \( 3.56 \times 10^6 \text{ N/m} \)

\( k_p \) Conventional primary lateral suspension \( 3.02 \times 10^5 \text{ N/m} \)

\( k_y \) Inter-axle bending stiffness for radial truck \( 1 \times 10^7 \text{ N/m} \)

\( k_{sy} \) Inter-axle shear stiffness for radial truck \( \text{variable N m/rad} \)

Secondary suspension parameters

\( k_{sy} \) Yaw stiffness \( 1.34 \times 10^6 \text{ N m/rad} \)

\( k_{sy} \) Vertical stiffness \( 3.502 \times 10^5 \text{ N/m} \)

\( k_{sx} \) Lateral stiffness \( 3.24 \times 10^6 \text{ N/m} \)

APPENDIX 2

Analytical formulation

The development of the analytical method for the non-linear model is based on the method given in the paper by Horak and Wormley (10). To predict the wheel-rail forces in curving, non-linear wheel-rail geometry and non-linear creep forces have been taken into consideration. Figure 7 shows the free body diagram of a single wheelset. In the figure, \( \delta_1 \) and \( \delta_2 \) are the contact angles at the left and right wheels and \( \phi \) is the roll angle of the wheelset. The equations of motion for wheelset lateral and yaw can be written from the free body diagram. The external forces and moments acting on the wheelset are:

\[ F\text{,ysusp}, M\text{,ysusp}: \text{Primary suspension force and moment;} \]

\[ F_{rz}, M_{rz}, F_{ry}, F_{ly}, M_{ly}, F_{by}: \text{Creep forces and moments;} \]

\( N_r, N_l: \text{normal wheel–rail forces.} \)

The wheelset force balance for the steady state analysis lead to the following equation:

Lateral:

\[ F_{ly} + F_{by} + N_r \sin(\delta_2 - \phi) - N_l \sin(\xi_1 + \phi) + F\text{,ysusp} = 0 \]  \( (11) \)

Yaw:

\[ a(F_{rz} - F_{lz}) + a(F_{ry} - F_{ly}) \psi + (N_r \sin(\delta_1 - \phi) + N_l \sin(\xi_1 + \phi))a\psi + M_{lz} + M_{rz} + M\text{,ysusp} = 0 \]  \( (12) \)

The creep forces in the above equations are determined using Kalker’s linear creep theory. The longitudinal, lateral and spin creepages in the contact planes for the steady-state conditions are derived following the development given by Garg and Dukkipati (11). They are:

Longitudinal:

\[ \xi_{lx} = \frac{r_l}{r_o} + \frac{r_l \beta}{V} - 1 - \frac{a}{R} \]  \( (13) \)

\[ \xi_{rz} = \frac{r_r}{r_o} + \frac{r_r \beta}{V} - 1 + \frac{a}{R} \]  \( (14) \)

\[ \xi_{sy} = \frac{\psi}{\cos \delta_1} \]  \( (15) \)

\[ \xi_{ry} = \frac{\psi}{\cos \delta_r} \]  \( (16) \)

Spin:

\[ \xi_{ls} = \frac{\sin \delta_1 + \beta \sin \delta_r + \cos \delta_1}{r_o} \]  \( (17) \)

\[ \xi_{rs} = - \frac{\sin \delta_1 - \beta \sin \delta_r + \cos \delta_1}{r_o} \]  \( (18) \)

In the above equations, \( \xi \) is the creepage, \( \psi \) is the angle of attack, \( \beta \) is the differential spin of the left and right wheels, \( R \) is the radius of the curve, \( r_l \) and \( r_r \) are the left and right wheel rolling radii. The creep forces are non-linear functions of the creepages, normal wheel loads and wheel–rail geometry, mainly due to the dry friction saturation limit equal to \( \mu N \), where \( \mu \) is the coefficient of friction and \( N \) is the normal wheel load. A heuristic non-linear creep model which combines Kalker’s linear creep theory with a creep force saturation representation is used (10). This is essential for adequate representation of the curving behaviour of the rail truck. The contact plane creep forces and moments are then transformed to the track coordinate by the following equations.

Longitudinal:

\[ F_{lx} = F’_{lx} \quad F_{rx} = F’_{rx} \]  \( (19) \)

Lateral:

\[ F_{by} = F’_{by} \cos(\delta_1 + \phi) \quad F_{ry} = F’_{ry} \cos(\delta_1 - \phi) \]  \( (20) \)

Vertical:

\[ F_{lz} = F’_{lz} \sin(\delta_1 + \phi) \quad F_{rz} = F’_{rz} \sin(\delta_1 - \phi) \]  \( (21) \)

Once these forces are calculated, the program uses them in equation (9) and the iterations are done until steady-state equilibrium is achieved.

To simulate the case of IRW for the trailing axle of the unsymmetric wheelset truck, the longitudinal creep force is limited to \( F_l \) as given by the following equation:

\[ F_{lim} = \text{sign}(F_l)\mu_{free} N \]  \( (22) \)

In this case the value of longitudinal creep coefficient is taken as \( 10^{-4} \).