Another View of Investment: 40 Years Later

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September 2001

Abstract

Does it matter whether productivity growth is embodied in new machines or whether it is disembodied and lifts the productivity of all equipment? Phelps (1962) argued that the composition of the sources of growth is irrelevant in the long run. In this paper we reconsider Phelps' result and take our analysis a step further. We consider the relevance of embodied technological change for the long run equilibrium properties as well as for the short-run transitional dynamics. We do so in a general vintage capital framework that nests Phelps' (1962) model and use it to address the importance of embodied technological change in explaining the current investment driven business cycle phenomena.

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1 Introduction

Does it matter if the growth of productivity is driven by disembodied technological change that raises the productivity of all factors of production, including all capital in place, or if it is driven by continuous improvements in the quality of new capital goods? This is the central question of the embodiment issue that has been widely debated among macroeconomists. The answer to the question must distinguish between the long-run and short run.

Long run considerations focus on the dependence of steady state values on the composition of the sources of economic growth. This research agenda was set by Phelps (1962) who showed, among other things, that the elasticity of the steady state level of output with respect to the savings rate does not depend on the composition of embodied and disembodied technological progress. (For the debate that followed see also Matthews (1964), Phelps and Yaari (1964), Levhari and Sheshinski (1967), and Fisher, Levhari and Sheshinski (1969).) Following Phelps (1962), Denison (1964) developed the empirical case for the unimportance of the embodiment question, and the literature on the long-run consequences of embodied technical progress subsided[1].

Short run considerations focus on the business cycle implications of embodied versus disembodied technological change. Recent works on this subject by Greenwood, Hercowitz, and Krusell (2000), Gilchrist and Williams (2000), and DeJong, Ingram, and Whiteman (2000), study the transitional dynamics of various vintage capital models. Our main focus is also on the short run implications of embodied technological change. In particular, we are interested in whether investment driven economic expansions stimulated by significant innovations and quality improvements in capital goods, tend to produce subsequent economic slowdowns.

This question is of particular relevance for current economic developments. As Tevlin and Whelan (2000) have documented, the economic expansion of the 1990's was mainly driven by an economy-wide investment boom, particularly in computers and software. To illustrate this point we plot the contribution of fixed private nonresidential investment to overall GDP growth. Apart from the significant drag that investment has on GDP growth in the first and second quarters of 2001, the most remarkable obser-

[1] In more recent work, Boucekinne et al. (1999) suggest that the composition of the sources of growth may affect long run properties of an economy if technological change is endogenous.

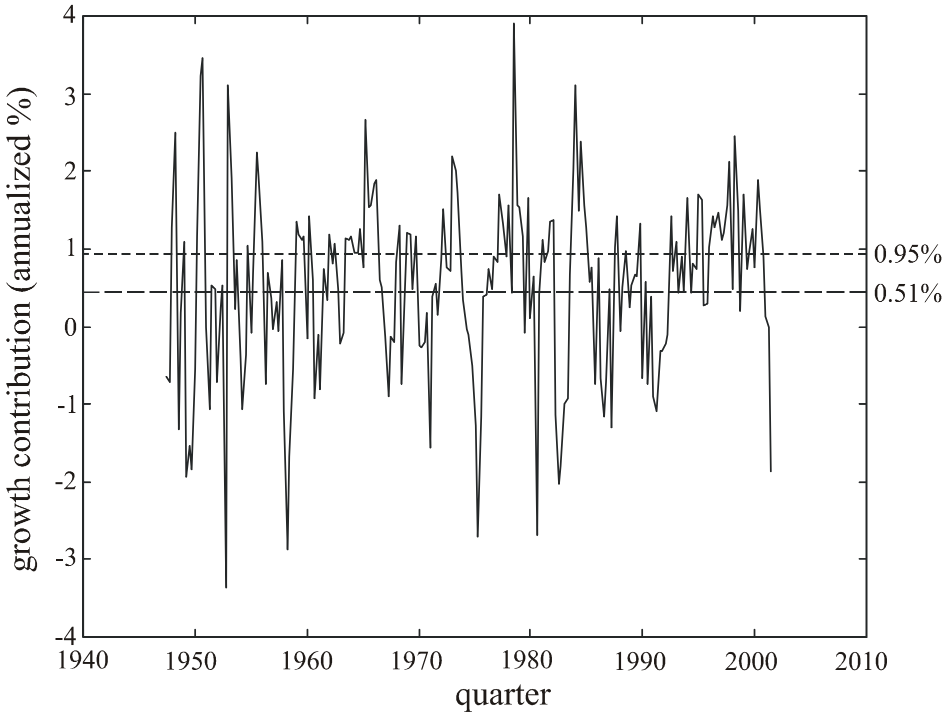


Figure 1: Contribution of investment to GDP growth

vation about the recent past is the unprecedented contribution of investment to the 1990's expansion. In particular, the average growth contribution of investment to GDP in the postwar period is about 0.51%, while that in the period 1992-2000 is 0.95%. Furthermore, this is not due to a particular outlier in the 1990's, but instead to an unprecedented sustained increase in the importance of investment in GDP growth.

On the downside, recent newspaper accounts have now claimed that the 2001 U.S. economic slowdown may be different than previous slowdowns and recessions. It is claimed that the culprit for the unexpected severity of the investment bust is the sustained investment boom of the 1990es[2]. According

[2]For example, The Economist of June 30, 2001 suggests (in "Another Shot from Dr. Feelgood: Will the Fed's latest cut in the interest rateget the economy moving again?", pp.26-27):

"In contrast to the post-war norm the expansion was not "murdered" by the Federal Reserve. The contraction started with an invstment bust, as firms that had radically to this view, the current bust is indeed the result of an excessive investment boom fueled by overoptimistic expectations generated by the information technology "revolution". Another older view, advocated as early as the beginning of the last century, suggests that major aggregative technological innovations like the steam engine, electricity, or more recently information technology, give rise to a burst of new investment, inevitably followed by low investment activity once the initial phase of plant and equipment replacement and modernization is completed. Indeed further down the line as these new plants and equipment have to be scrapped, damped investment echoes may be observed (see Robertson, 1915), giving rise to oscillatory transition dynamics of output and employment. A key point is that the investment boom and subsequent slowdown in economic activity need not entirely arise from mistakes of optimism (dotcom bankruptcies notwithstanding), but may represent the normal course of planned modernizations and replacement investments. No doubt technological innovations that are pervasive enough to stimulate substantial investments across a wide range of industries are rare events, and probably hard to systematically observe in high frequency data. Therefore there may be some validity to the suggestion that the current slowdown differs from the recent others that have been caused and driven by declines in consumption expenditures.

The purpose of this paper is to study economic propagation mechanisms of vintage models for economies that experience various embodied or disembodied technological innovations major enough to give rise to bursts of aggregate investment activity. While technological innovations may indeed provide the rationale for sudden bursts of investment, a critical element of our analysis relies on the structure of depreciation and economic obsolescence that departs from the standard radioactive exponential form. Whether scrapping old equipment is endogenous or not, to the extent that scrapping must occur due to physical and economic obsolescence, investment booms will be followed by slowdowns in economic activity and subsequent echo effects. The induced transition dynamics in output, consumption and employment may then be quite complex. The initial phases of technological innovations may also see accelerated replacement and adoption waves, especially if there is some persistence in the rate of innovations. We will see that if technological

over-invested during the boom years of the late 1990es suddenly cut back. This collapse of capital spending lies behind the recession in American industry." See also Robert Reich, New York Times, "How long can consumers keep spending", September 2, 2001, Week in Review Section, p. 9.

change is of the embodied kind investment booms will give rise to subsequent busts, generating a hump shaped response in output levels.

We introduce a vintage capital model that specifically allows for the scrappage of capital goods and thus for echo effects. We show that in our economy Phelps'(1962) long-run result on the independence of the elasticity of output with respect to the savings rate does not hold if labor supply is endogenous. We then show that the short run dynamics of the economy are very different when driven by embodied technology shocks rather than by disembodied productivity shocks that are conventionally considered in the business cycle literature. In particular, we will see that echo effects are much more profound in investment driven expansions caused by embodied technological change than in those driven by standard Hicks neutral productivity shocks.

Our plan of action is as follows. In order to illustrate the basic mechanism for short-term propagation mechanisms, in Section 2 we give an illustrative example of a simple vintage capital model that generates echo effects. We then proceed with the introduction of our more general model in Section 3. Section 4 is devoted to the steady state properties of our model economy, where we duplicate Phelps'(1962) result in the context of our framework but show how it fails to hold when labor supply is endogenous. In Section 5 we address the question how embodied and disembodied technological change affect the transitional dynamics of the economy. We provide an analysis of impulse responses to technological shocks under various parametrizations to illustrate the propagation mechanisms and the main intuition behind our results. In particular we illustrate how in our model disembodied shocks generate hump-shaped impulse responses in output while disembodied shocks do not. Finally, in Section 6, we conclude and a technical appendix follows.

2 A Simple Illustrative Vintage Model

We start by presenting a simplified vintage capital model for expository purposes. In the simplified model, our economy consists of a single representative agent, supplying a fixed quantity of labor L. Output is produced by allocating labor to each vintage of capital. Each vintage of capital and the labor allocated to it then produces output according to a constant returns to scale production function. The agent optimizes the the discounted utility of future consumption:

$$Max_{\{k\}} \sum_{t=0}^{\infty} \beta^t U \left(\sum_{\tau=0}^{T} f\left(k_{t-\tau}, A_{t-\tau} l_{t-\tau}^t\right) - k_{t+1} \right)$$

where $\beta < 1$ is the discount factor, A is embodied technological progress that makes newer vintages more productive, $k_{t-\tau}$ is capital of vintage $t-\tau$ used at time t, $l_{t-\tau}^t$ is labor at t allocated to $k_{t-\tau}$,and f is the constant returns to scale neoclassical production function for each vintage. We take the scrappage time for vintages as exogenous: vintages last $T+1$ periods.[3] The labor market constraint requires that labor supply exceeds labor demand, such that $L \geq \sum_{\tau=0}^{T} l_{t-\tau}^t$.

The Euler equation for this problem, assuming interiority, is given by

$$U'(c_t) = \sum_{s=0}^{T} \beta^{s+1} U'(c_{t+1+s}) f_k\left(k_{t+1}, A_{t+1} l_{t+1}^{t+1+s}\right)$$

This equation simply states that the marginal utility of forgone consumption today equals the discounted value of the future marginal products, measured in utility, of a unit investment in today's vintage capital. Note that at a steady state, or if utility is linear so that U' cancels on both sides of the Euler equation above, we have the discounted sum of marginal products of a unit investment today equal to unity. Thus if there is an increase in the marginal product of the latest vintage, there will be a burst of investment in the newest vintage (non-negativity constraints on consumption allowing) to "re-equate" this composite discounted marginal product to the discount rate. Subsequent investment will drop, but when the initial vintage with a spike is finally scrapped, another smaller burst of investment will take place to maintain the equality of the discounted composite marginal product to the discount rate, resulting in damped echoes. These investment dynamics,

[3] This again is for simplicity. If f was a CES production function with an elasticity of substitution less than unity, then older depreciated vintages with that are less efficient would not be allocated any labor. By contrast if f has an elasticity of substitution greater than unity or is Cobb-Douglas, all existing vintages would be allocated some labor, because the production function f satisfies Inada conditions in this case. Since our analysis and results only require a finite truncation, whether fixed and exogenous or variable and endogenous, we adopt the simpler exogenous specification. (See also Benhabib and Rustichini (1991).) For the case where T may be endogenous see Benhabib and Rustichini (1993). Note also that for notational simplicity we suppress any ongoing depreciation that may occur before scrappage.

depending on the nature of the technology shock, will have implications for the dynamics of employment, consumption and output. In particular, output may exhibit a hump-shaped impulse response to technological innovations with a peak beyond T, the lifetime of equipment, and employment may also follow the oscillatory pattern induced by investment expenditures.

The model with linear utility however, is likely to generate echo effects that are too pronounced. Nonlinear utility dampens echoes by introducing consumption smoothing, while at the same time preserving the impact effects of a technological innovation on investment[4]. Therefore, we generalize the model to include non-linear utility and we specify various types of shocks to more thoroughly explore the propagation mechanisms induced by technological innovations that simultaneously affect a large number of industries.

3 The vintage capital model with technology shocks

We are interested in the short run dynamic consequences of shocks to embodied and disembodied technological change in an economy in which capital goods are scrapped and give rise to echo effects. We construct a stochastic general equilibrium model with vintage capital in which (*i*) there are stochastic shocks to embodied as well as disembodied technological change, (*ii*) there is exogenous scrappage of capital goods, and (*iii*) we allow (as an extra) for capital goods of different vintages to be complimentary.

We introduce the model in three steps. The final model with which we study transitional dynamics is a reduced form of an underlying standards vintage capital model. Therefore we first introduce the underlying vintage capital model, then derive the reduced form on which our results are based.

[4]A closed form solution with non-linear utility for a slightly different but related vintage model exists if the product of the intertemporal elasticity of substitution σ^{-1} in the utility function $(1-\sigma)^{-1}c^{(1-\sigma)}$ and the elastisticity of substitution in a CES production function is set to unity (See Benhabib and Rustichini(1994)). This generalizes the closed form solutions, with or without vintages, for standard log utility-Cobb-Douglas production function combination. The solution reduces, as it does in the case with linear utility, to a constant savings rate yielding a linear difference equation in vintages with all roots complex or negative, except in this case of non-linear utility, it also has a dominant Frobenious root which is positive and less than unity, and which controls the asymptotic dynamics. (See Sato (1970).) Thus the curvature of utility calibrates propagation dynamics.

3.1 Microfoundations

In our model economy a representative household maximizes the expected present discounted value of the stream of utilities. Let C_t denote consumption at time t and L_t be the fraction of time the household spends on working. Given the stochastic processes that drive exogenous technological change, the household maximizes

$$E_t\left[\frac{1}{1-\sigma}\sum_{s=0}^{\infty}\beta^s\left[C_t^{\phi}\left(1-L_t\right)^{1-\phi}\right]^{1-\sigma}\right]$$

subject to the resource constraint

$$C_t = Y_t - k_{t+1},$$

the production function

$$Y_t = \left[\sum_{\tau=0}^{T}\left\{\left(\left(1-\delta\right)^{\tau}k_{t-\tau}\right)^{\alpha}\left(X_t A_{t-\tau} l_{t-\tau}^{t}\right)^{1-\alpha}\right\}^{\theta}\right]^{1/\theta} \quad \text{where } \theta < 1 \text{ and } 0 < \alpha < 1,$$

and the labor market equilibrium condition

$$L_t = \sum_{\tau=0}^{T} l_{t-\tau}^{t}, \text{ and } 0 \leq L_t \leq 1.$$

In this model, technological change is driven by two different sources. The first is standard disembodied technological progress, represented by X_t, and applies identically to the productivity of all capital vintages in place. The second is embodied technological change, represented by A_t, which only applies to its respective vintage.

This setup is similar to the simple model of the previous section, but differs from it in three important respects. First of all, it has an elastic labor supply. Second, it contains physical deprecation of capital, given by the exogenous rate δ. Finally and most importantly, the production function has a two level representation. The first level is a CES-aggregate of vintage specific outputs. When $\theta = 1$, we obtain the standard case in which output is the sum of vintage specific output levels and output produced using the various vintages are perfect substitutes. When $\theta < 1$, output produced using various vintages is complementary. For example we could think about this as computers being complementary to other equipment already in place. The second level of the vintage production is Cobb-Douglas, which simplifies the representation and analysis of the model. The next two subsections are devoted to an exposition of these features.

3.2 Units of measurement of capital good

In the above representation we have assumed that the price of capital goods is equal to the price of consumption goods. Essentially k_t is measured in terms of consumption goods. The Cobb-Douglas specification allows us further simplification. Note that when we define $Q_t = A_t^{\frac{1-\alpha}{\alpha}}$ we have

$$k_t' = Q_t k_t$$

We can write the resource constraint as

$$C_t = Y_t - \frac{1}{Q_{t+1}} k_{t+1}'$$

while the production function has the representation

$$\begin{aligned} Y_t &= \left[\sum_{\tau=0}^{T} \left\{ \left((1-\delta)^{\tau} Q_{t-\tau} k_{t-\tau} \right)^{\alpha} \left(X_t l_{t-\tau}^{t} \right)^{1-\alpha} \right\}^{\theta} \right]^{1/\theta} \\ &= \left[\sum_{\tau=0}^{T} \left\{ \left((1-\delta)^{\tau} k_{t-\tau}' \right)^{\alpha} \left(X_t l_{t-\tau}^{t} \right)^{1-\alpha} \right\}^{\theta} \right]^{1/\theta} \end{aligned}$$

Essentially, Q_t is the number of quality units of the capital good embodied in each unit of the consumption good invested at time t. Therefore, k_t' is the amount of investment measured in constant quality units of the capital good. Here Q_t is the relative price of the consumption good in terms of constant quality units of the capital good. It is the relative investment price.

Since the representation with Q_t is easier to handle than that with A_t, in the rest of this paper we will use the representation where investment is measured in constant quality units. For notational convenience, we will drop the apostrophe and let k_t denote investment in constant quality units from now on. This representation coincides with that chosen by Greenwood, Hercowitz, and Krusell (1997), who call Q_t 'investment specific technological change'.

3.3 Labor allocation and reduced form production function

Given this new definition of investment, the production function is

$$Y_t = \left[\sum_{\tau=0}^{T} \left\{ \left((1-\delta)^{\tau} k_{t-\tau} \right)^{\alpha} \left(X_t l_{t-\tau}^{t} \right)^{1-\alpha} \right\}^{\theta} \right]^{1/\theta} \tag{1}$$

The optimal labor allocation problem in this economy is to choose $\left\{ l_{t-\tau}^{t} \right\}_{\tau=0}^{T}$ in order to maximize (1) subject to the restriction that $L_t = \sum_{\tau=0}^{T} l_{t-\tau}^{t}$. This implies equating the marginal products of labor across the different vintages. This problem has a specific closed form solution, which, after a bit of algebra, can be shown to be

$$l_{t-\tau}^{t} = \left(\frac{\left\{ \left((1-\delta)^{\tau} k_{t-\tau} \right)^{\alpha} \left(X_t l_{t-\tau}^{t} \right)^{1-\alpha} \right\}}{Y_t} \right)^{\theta} L_t$$

Substituting this optimal labor assignment in the production function (1), we obtain that the reduced form production function:

$$Y_t = \left(\left[\sum_{\tau=0}^{T} \left((1-\delta)^{\tau} k_{t-\tau} \right)^{\frac{\alpha\theta}{1-(1-\alpha)\theta}} \right]^{\frac{1-(1-\alpha)\theta}{\alpha\theta}} \right)^{\alpha} \left(X_t L_t \right)^{1-\alpha}$$

If we define $\gamma = \frac{\alpha\theta}{1-(1-\alpha)\theta} < 1$, we can define a Jelly capital stock, similar to that which is conventionally derived in the vintage capital literature (see Solow (1960)):

$$J_t = \left[\sum_{\tau=0}^{T} \left((1-\delta)^{\tau} k_{t-\tau} \right)^{\gamma} \right]^{1/\gamma}$$

Then the reduced form production function is

$$Y_t = J_t^{\alpha} \left(X_t L_t \right)^{1-\alpha}$$

Note that $\theta = 0$ implies that $\gamma = 0$, which generalizes the results on the existence of a capital aggregate in Fisher (1965) to the case in which total output is a CES aggregate of vintage specific Cobb-Douglas production functions.

3.4 Reduced form representation

Unlike the vintage models studied in the 60es, the savings rate in our model is endogenous. In order address the long run implications derived by Phelps (1962), we introduce an investment tax credit, τ_I, which is financed with a non-distortionary lump sum tax, denoted by τ_t. Using the transformations of the model derived above and adding the investment tax credit, in the reduced form representation of our model a representative household maximizes the expected present discounted value of its stream of utility levels, i.e.

$$E_t \left[\frac{1}{1-\sigma} \sum_{s=0}^{\infty} \beta^s \left[C_t^{\phi} (1-L_t)^{1-\phi} \right]^{1-\sigma} \right]$$

subject to the revised resource constraint

$$C_t = Y_t - \frac{1-\tau_I}{Q_{t+1}} k_{t+1} - \tau_t,$$

the reduced form production function

$$Y_t = J_t^{\alpha} (X_t L_t)^{1-\alpha} = F(J_t, X_t L_t),$$

the Jelly capital identity

$$J_t = \left[\sum_{\tau=0}^{T} \left((1-\delta)^{\tau} k_{t-\tau} \right)^{\gamma} \right]^{1/\gamma},$$

and the stochastic processes that drive technological change. The government is assumed to run a balanced budget such that at each point in time $\tau_t = \tau_I k_{t+1}/Q_{t+1}$.

For the stochastic processes for X_t and Q_t we choose log-trend stationary processes of the form

$$\ln Q_t = \ln(1+q)\, t + \ln Q_t^{(t)} \tag{2}$$

where

$$\ln Q_t^{(t)} = \rho_Q \ln Q_{t-1}^{(t)} + \varepsilon_{Q,t}$$

and, similarly

$$\ln X_t = \ln(1+g)\, t + \ln X_t^{(t)} \tag{3}$$

where

$$\ln X_t^{(t)} = \rho_X \ln X_{t-1}^{(t)} + \varepsilon_{X,t}$$

and $\varepsilon_{X,t}\tilde{}N(0,\sigma_X)$ and $\varepsilon_{Q,t}\tilde{}N(0,\sigma_Q)$. We choose these processes as trend stationary, purely for expository purposes because the resulting impulse responses are more easily interpretable. We have, however, generated unreported results, in which $\ln X_t^{(t)}$ and $\ln Q_t^{(t)}$ followed ARIMA(1,1,0) processes in deviation of a deterministic trend, as is suggested to be the appropriate specification by Lippi and Reichlin (1994). The results using this specification were, in deviation of the trend, qualitatively very similar to the ones that we will present below and are therefore omitted.

3.5 Optimality conditions

The optimality conditions of this model are fairly straightforward variations of the standard RBC model, as discussed for example in King, Plosser, and Rebelo (1988). The optimal labor supply is determined by the intratemporal optimality condition. This condition implies that at any point in time the representative agent equates the marginal disutility of work to the marginal utility of consumption that can be obtained by working. In our model this implies

$$\frac{C_t}{1-L_t} = \frac{\phi}{1-\phi} X_t F_L(J_t, X_t L_t)$$

The inter-temporal optimality condition implies that the representative agent chooses the investment level in order to equate the marginal utility of current consumption to the expected present discounted value of the returns to the current investment. The corresponding Euler equation for investment is:

$$\begin{aligned} & (1-\tau_I)\left[C_t^{\phi-\frac{1}{1-\sigma}}(1-L_t)^{1-\phi}\right]^{1-\sigma} \\ = \; & Q_{t+1}E_t\left[\sum_{s=1}^{T+1}\beta^s\left[C_{t+s}^{\phi-\frac{1}{1-\sigma}}(1-L_{t+s})^{1-\phi}\right]^{1-\sigma} F_J(J_{t+s}, X_{t+s}L_{t+s})\frac{\partial J_{t+s}}{\partial k_{t+1}}\right] \end{aligned}$$

where we assume that Q_{t+1} is known at time t. The major difference between this Euler equation and those obtained using capital aggregates based on the perpetual inventory method is that this Euler equation is not stated in terms of a co-state variable, but instead in terms of a finite sum of returns to capital. It implies that instead of the level of the Jelly capital aggregate

J_t, the relevant state variable in this model is a sequence of past investment levels $\{k_{t-\tau}\}_{\tau=0}^{T}$.

4 Steady state and long run implications

To study the perfect foresight steady state and transitional dynamics of this model, we have to transform the variables such that they are constant in the perfect foresight steady state and are stationary around it. The common trend that drives consumption and output in this model is

$$P_t = G^t, \text{ where } G = (1+q)\,(1+q)^{\frac{\alpha}{1-\alpha}}$$

Since the second level production function is Cobb-Douglas, we can use

$$Y_t = J_t^{\alpha}\,(X_t L_t)^{1-\alpha} = \left(J_t \left/ \left((1+q)^t\right)\right.\right)^{\alpha} \left(P_t X_t^{(t)} L_t\right)^{1-\alpha}$$

The transformed variables in this model then are

$$\widetilde{Y}_t = \frac{Y_t}{P_t},\ \widetilde{C}_t = \frac{C_t}{P_t},\ \widetilde{k}_t = \frac{k_t}{(1+q)^{t-1} P_{t-1}},\ \text{and } \widetilde{J}_t = \frac{J_t}{(1+q)^t P_t}$$

where, contrary to standard growth models, investment grows at a faster rate than output and consumption because of the steady decline in its relative price, represented by $(1+q)^t$ in the correction of investment and the capital stock.

In terms of these transformed variables, we can rewrite

$$\widetilde{C}_t = \widetilde{Y}_t - \frac{1}{1+q}\frac{1}{Q_{t+1}^{(t)}}\widetilde{k}_{t+1} \tag{4}$$

$$\widetilde{Y}_t = F\left(\widetilde{J}_t, X_t^{(t)} L_t\right) \tag{5}$$

$$\widetilde{J}_t = \left[\sum_{\tau=0}^{T}\left((1-\delta)^{\tau}\left[\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{1+g}\right)\right]^{\tau+1} k_{t-\tau}\right)^{\gamma}\right]^{1/\gamma} \tag{6}$$

The intratemporal optimality condition reduces to

$$\frac{\widetilde{C}_t}{1-L_t} = \frac{\phi}{1-\phi} X_t^{(t)} F_L\left(\widetilde{J}_t, X_t^{(t)} L_t; Z_t\right) \tag{7}$$

and for the intertemporal optimality condition we obtain

$$1 = \frac{Q_{t+1}^{(t)}}{1-\tau_I}(1+q)\,E_t\left[\sum_{s=1}^{T+1}\beta^s\left[\left(\frac{\tilde{C}_{t+s}}{\tilde{C}_t}\right)^{\phi-\frac{1}{1-\sigma}}\left(\frac{1-L_{t+s}}{1-L_t}\right)^{1-\phi}\right]^{1-\sigma}\times\right. \tag{8}$$
$$\left.(G)^{\phi(1-\sigma)s}F_J\left(\tilde{J}_{t+s}, X_{t+s}^{(t)}L_{t+s}; Z_{t+s}\right)\frac{\partial \tilde{J}_{t+s}}{\partial \tilde{k}_{t+1}}\right]$$

where

$$\frac{\partial \tilde{J}_{t+s}}{\partial \tilde{k}_{t+1}} = \left((1-\delta)^{s-1}\left[\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{1+g}\right)\right]^s\right)^\gamma\left(\frac{\widetilde{J}_{t+s}}{\widetilde{k}_{t+1}}\right)^{1-\gamma}$$

The perfect foresight steady state of this model is a stationary point of the dynamic system implied by the transformed optimality conditions under the assumption that the variances of the shocks are zero, i.e. $\sigma_Q^2 = \sigma_X^2 = 0$. It is a quintuple

$$\{Y^*, C^*, k^*, J^*, L^*\}$$

which satisfies

$$C^* = Y^* - \frac{1}{1+q}k^* \tag{9}$$

$$Y^* = F(J^*, L^*) \tag{10}$$

$$C^* = \frac{\phi}{1-\phi}F_L(J^*, L^*)(1-L^*) \tag{11}$$

$$J^* = \left[\sum_{\tau=0}^{T}\left((1-\delta)^\tau\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^\gamma\right]^{\frac{1}{\gamma}}k^* \tag{12}$$

$$1 = \frac{1+q}{1-\tau_I}\left[\sum_{\tau=0}^{T}\left((1-\delta)^\tau\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^\gamma\right]^{\frac{1-\gamma}{\gamma}}\times \tag{13}$$
$$\sum_{s=1}^{T+1}\left(\beta(1+g)^{\phi(1-\sigma)-\gamma}(1+q)^{\frac{\alpha}{1-\alpha}\phi(1-\sigma)-\frac{\gamma}{1-\alpha}}\right)^s(1-\delta)^{\gamma(s-1)}F_J(J^*, L^*; 1)$$

It is fairly straightforward to show that this steady state exists and is unique, because the above equations can be solved sequentially. The details of this calculation are left for section A.1 of Appendix A.

4.1 The old 'New View of Investment' revisited

A central issue in the literature on vintage capital, initially addressed by Phelps (1962) is whether the effect of changes in the savings rate on the steady state level of output depends on the composition of embodied and disembodied technological progress. The growth rate of output on the perfect foresight steady state balanced growth path is

$$G = (1+g)(1+q)^{\frac{\alpha}{1-\alpha}} \tag{14}$$

where g represents disembodied technological change and q embodied technological change. The precise question is whether the elasticity of the steady state level of output with respect to the savings rate depends on the composition of G, that is whether it is independent of variations of g and q that keep G constant. In a very similar model which assumes an inelastic labor supply and an exogenous savings rate, Phelps (1962) shows that this elasticity is independent on the composition of growth.

The issue of studying an effect of an exogenous change in the savings rate is not directly appropriate for our model, so we address the issue in two steps. In the first step we consider the effect of a change in the investment tax credit on the savings rate. In the second step we follow Phelps (1962) and address the effect of a change in the savings rate on the level of steady state output. The mathematical details of these two steps are given in section A.2 of Appendix A.

As shown in section A.2, the steady state savings rate in this model equals $s^* = \widetilde{s}^* / (1-\tau_I)$, where

$$\widetilde{s}^* = \alpha \frac{\sum_{\tau=0}^{T} \left((1-\delta)^{\tau} \left(\frac{1}{G(1+q)} \right)^{\tau+1} \right)^{\gamma} \left(\beta G^{\phi(1-\sigma)} \right)^{\tau+1}}{\left[\sum_{\tau=0}^{T} \left((1-\delta)^{\tau} \left(\frac{1}{G(1+q)} \right)^{\tau+1} \right)^{\gamma} \right]}$$

and where $\beta G^{\phi(1-\sigma)} < 1$ is a necessary condition for the boundedness of the representative household's objective function. The elasticity of the steady state savings rate with respect to the investment tax credit equals $\tau_I / (1-\tau_I)$ and does not depend on the composition of G in q and g. In fact, it does not even depend on the growth rate, G. What does depend on the composition of G however is the level of the savings rate. The bigger the share of productivity growth that occurs through embodied technological change, the higher is the savings rate, simply because in that case investment becomes more important as a source of growth relative to the case where it simply adds to the existing capital stock.

The level of steady state output as a function of the savings rate is given by

$$Y^* = (s^*)^{\frac{\alpha}{1-\alpha}} \left(\left[\sum_{\tau=0}^{T} \left((1-\delta)^{\tau} \left(\frac{1}{G(1+q)} \right)^{\tau+1} \right)^{\gamma} \right]^{\frac{1}{\gamma}} \right)^{\frac{\alpha}{1-\alpha}} \times \left(\frac{\phi(1-\alpha)}{\phi(1-\alpha)+(1-\phi)(1-s^*)} \right)$$

This differs in an important way from Phelps (1962). Because we consider an endogenous labor supply, the elasticity of this expression with respect to the savings rate equals

$$\left[\frac{\alpha}{1-\alpha} + \frac{(1-\phi)\, s^*}{\phi(1-\alpha)+(1-\phi)(1-s^*)} \right]$$

and does depend on the endogenous steady state savings rate s^*. Since the steady state level of the savings rate depends on the composition of the sources of growth, the elasticity also depends on the composition of growth, through the effect of the this composition on the steady state labor supply. Note that this result hinges on two things. The first is that we have made the labor supply elastic, i.e. if $\phi = 1$ and $L_t = 1$ for all t, then this elasticity does not depend on the steady state savings rate, and we recover exactly the Phelps result that the elasticity is given by the ratio of factor shares. The second is that we do not take the steady state savings rate as exogenously given, but allow it to be determined by the composition of the sources of growth.

Hence, where Phelps' (1962) result suggests that the elasticity of the steady state level of output with respect to an investment tax credit will not depend on the composition of the sources of growth, our result suggests it does. In particular, the more important is embodied technological change, the higher is the savings rate and the higher the elasticity of output with respect to the savings rate. Consequently, an investment tax credit is more effective when the relative importance of embodied technological change is high.

We have now shown that the composition of growth matters in the long-run, we will proceed by considering whether it also matters in the short run.

Table 1: Parameter calibration

parameter	value	varied in ...
σ	2.00	
β	0.95	
ϕ	0.50	
α	0.33	
δ	0.05	
γ	1.00	Figure 5
ρ_Q	0.90	
ρ_X	0.90	
g	0.021	Figure 6
q	0	Figure 6
T	50	Figure 4

5 Transitional dynamics and short run implications

What are the short-run effects of embodied technological change? How do they differ from more commonly studied disembodied productivity shocks? In order to address this question, we will approximate the transitional dynamics of our model economy around its perfect foresight steady state. The details of our approximation method are described in section A.3 of Appendix A.

We study the transitional dynamics induced by the various shocks through the impulse response functions that the shocks generate for the log-linear approximation to the solution of our model. For this purpose we must specify a benchmark calibration of the parameters of our model. Table 1 summarizes this benchmark calibration. In particular, we choose our Jelly aggregate in the standard additive representation ($\gamma = 1$) with no complementarities, and taxes are set to zero.

The recent literature (Greenwood, Hercowitz and Krusell (1997), Greenwood and Jovanovic (1998)) has stressed the improvements in the quality of capital goods embodied in vintages, as reflected in the data by the falling relative price of capital goods, as critical for the proper accounting of productivity growth. The relative price of fixed private non-residential investment

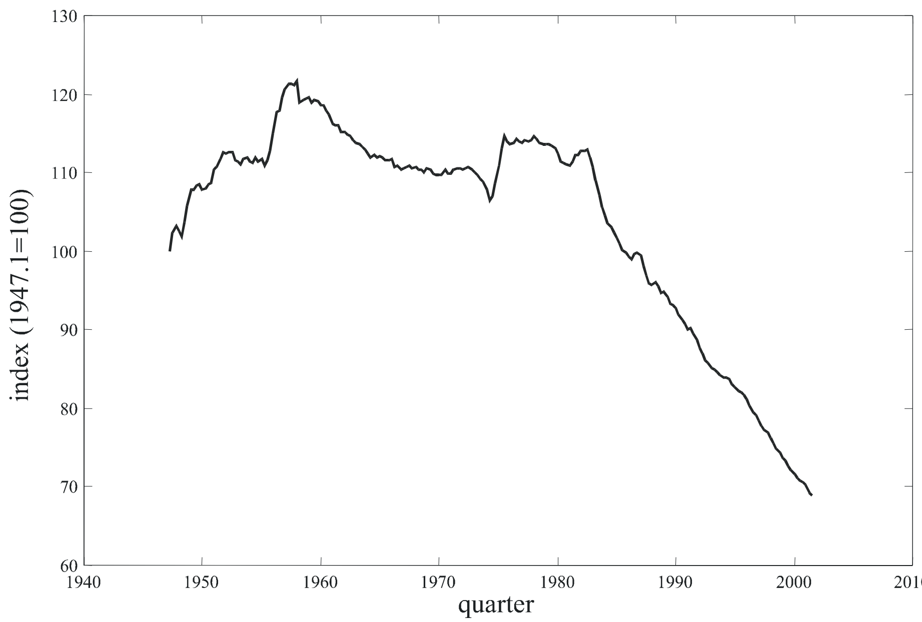


Figure 2: Relative price of investment

relative to the PCE deflator is plotted in figure 2. This is basically $1/Q_t$ as measured by the Bureau of Economic Analysis. The steep decline since the mid-80's is due to the rapid quality improvements of computers and their increased importance for the capital stock. The BEA's price indices are subject to a lot of criticism however, because many studies, most notably Gordon (1990), have shown that the index is probably improperly quality adjusted. Greenwood, Hercowitz and Krusell (1997) use Gordon's (1990) results to argue that even before 1985 the properly quality adjusted relative price of equipment already declined at an annual rate of about 3.5%.

However the relative investment price is measured, all studies suggest that shocks to Q_t are a potentially important source of economic fluctuations, and during the last decade even more than before. We will therefore first focus on embodied technological progress that improves the quality of capital goods. Newer equipment is more productive than older equipment, in the sense that a forgone unit of consumption on average yields more new capital units than in the past, and therefore, the relative price of capital, Q_t, tends to decline over time. The stochastic process for Q_t is described by the equation (2).

The solid line in figure 3 depicts the impulse response to an initial shock

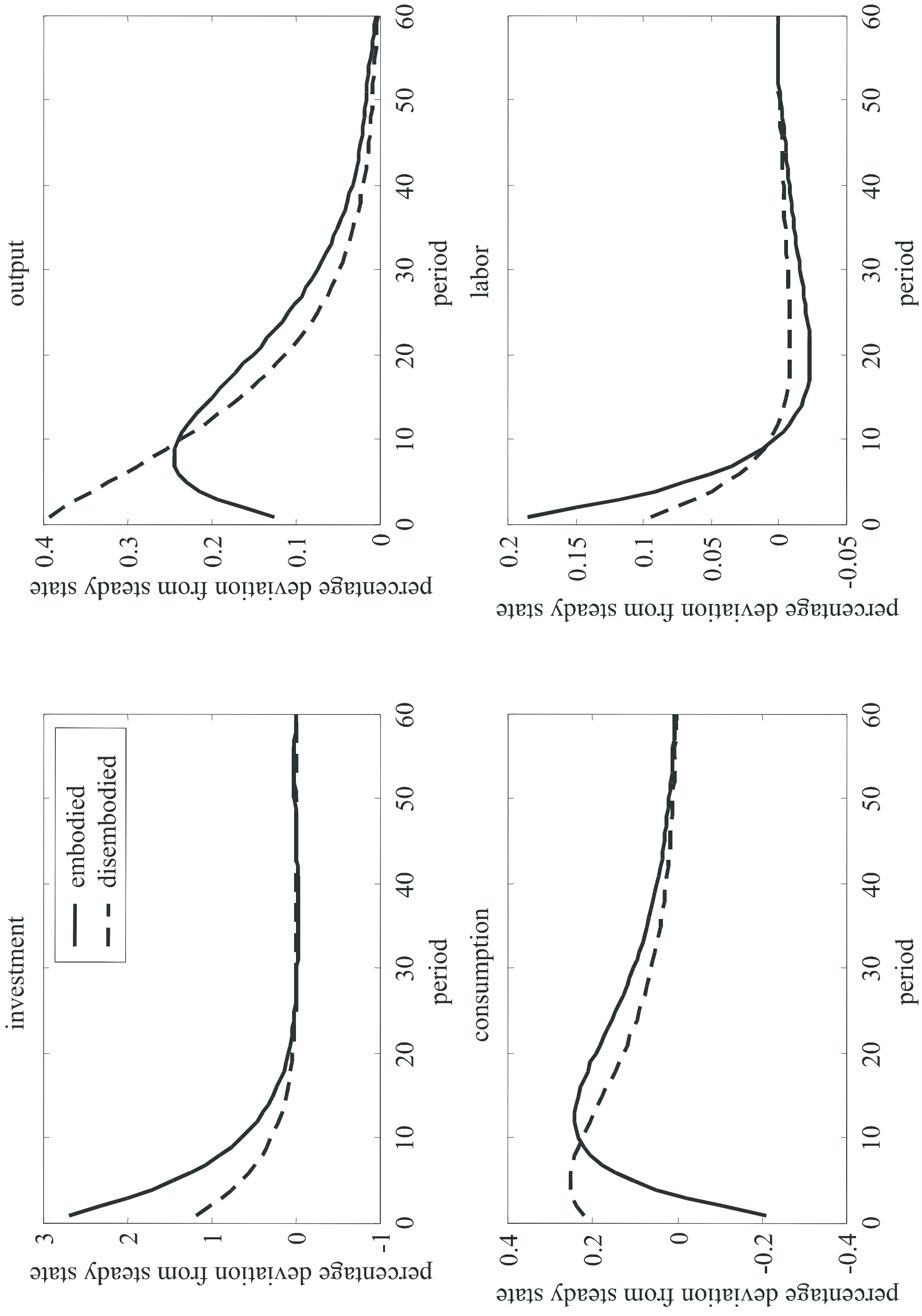


Figure 3: Impulse responses for embodied and disembodied shocks

in Q_t ($\varepsilon_{Q,0} = 0.01$). It exhibits a sharp spike in investment, a real investment boom. Consumption, after dropping on impulse, smoothly climbs and settles at a higher level. The initial drop in consumption is caused by the increased labor supply that is induced by the embodied shock. Since consumption and leisure are complements a decrease in leisure lowers the marginal utility of consumption and consumption drops on impulse. Employment tracks the investment dynamics. There is a sharp increase in labor supply in order to produce a lot of output that can be converted into new and cheap capital goods. The most interesting dynamic is exhibited by output. It rises with the initial increase in investment and continues to climb because the new vintages of capital come online while older less efficient vintages are scrapped. As the investment vintage profile evens out, a humped shape response in output emerges, where on the downside of the hump output and employment contracts as if in a recession. The persistence of the transient component of the innovation, ρ_Q, affects not only the size of the impact effect of the shock on investment and output, but also the shape of the hump in the response of output. The effects are small for $\rho_Q = 0$, because in that case the investment response is not sufficient to increase the capital stock and output after the initial period. The effects do become large as ρ_Q approaches unity. Thus a more persistent rate of innovation, typical in the early phases of the introduction of a new technology, is likely to generate a larger hump-shaped response of output. Note that the hump-shaped response of output, caused by the echo-effects, is in stark contrast to the monotonically declining impulse responses of output to embodied shocks reported in Greenwood, Hercowitz, and Krusell (1999) and DeJong, Ingram, and Whiteman (2000).

While we have modelled the economic lifetime of vintages, T, as exogenous, an acceleration in the rate of technological innovation is also likely to reduce T and increase the rate of adoption of new innovations, as is documented by Tevlin and Whelan (2000). Figure 4 depicts the impulse responses to a shock to Q_t for various lifetimes of capital. The earlier the equipment is scrapped, the more profound the echo effects. Lengthening the economic life of equipment, that is making T larger, increases the initial burst of investment, because the increase in the lifetime of capital implies an increase in the return to investment, but it dampens as well as delays the replacement echoes. The reason for the milder echoes, which would disappear in the limit, is the depreciation of vintages while in use. The longer lifetime decreases the quantity scrapped, and reduces investment needed for replacement. The shorter the T, the more profound are the investment booms and busts caused by echo effects.

More persistence in the responses of investment, consumption, output, and labor can be introduced by adding complementarities between capital vintages. Figure 5 depicts the impulse responses for three different values of γ. The smaller the γ, the more complimentary current investment levels are with future investment, hence the more incentive there is to drag out the investment response. This leads to increased persistence in all impulse responses, and also allows for more consumption smoothing.

So far we have focused on the impulse responses of our model to an embodied shock. However, we want to compare the relative importance of the sources of growth for economic fluctuations. There are two ways to look at this issue. The first is to ask how the different sources of shocks affect for the transitional dynamics of the economy. The second is to inquire whether the transitional dynamics of the economy depend on the composition of embodied and disembodied technological change.

In order to answer the question of whether the dynamic response of the economy is very different in response to an embodied shock than it is to a disembodied shock, we refer again to Figure 3. The dashed line in this figure depicts the response of the economy to an impulse in X_t $(\varepsilon_{X,0} = 0.01)$,the disembodied shock. In this case echo effects in investment are still present, but are hardly noticeable. They are much milder than the impulse resulting from a $Q-$shock. Responses of output, consumption and employment are close to monotonic in this case. The reason that the hump-shaped response of output is absent is that investment reacts much less to such a shock, leading to much smaller increase in the Jelly capital stock. The increase in the Jelly capital stock is in fact so much less that it fails to generate any subsequent increases in output after the initial shock. The bottom line is that embodied shocks are much more likely to lead to echo effects and a hump shaped output response than disembodied shocks.

The second way to pose the question in a manner similar to Phelps (1962), is to inquire how the dynamic response of the economy changes if we change the composition of growth, but leave the growth rate, G, constant. Output in this model is measured in terms of consumption goods and the average annual postwar growth rate of PCE deflated real GDP per capita is 2.1%[5]. Figure 6

[5] There is a recent discussion in the literature on how to properly measure the growth rate of real GDP in a world with embodied technological change. see Ho and Stiroh (2001), and Licandro et. al. (2001), for suggestions on how to measure real GDP growth in a way consistent with the National Income and Product Accounts.

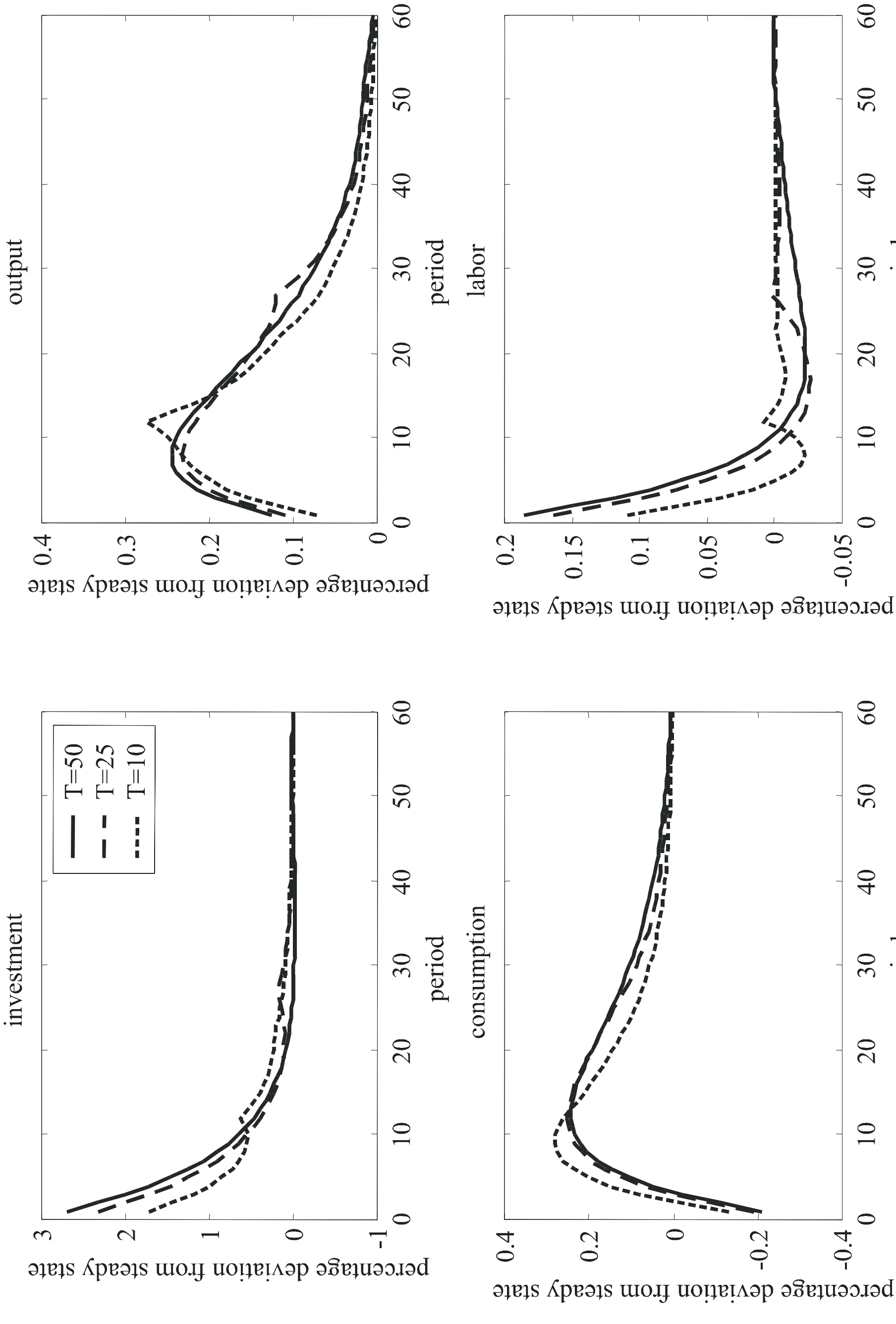


Figure 4: Impulse responses for various scrappage times

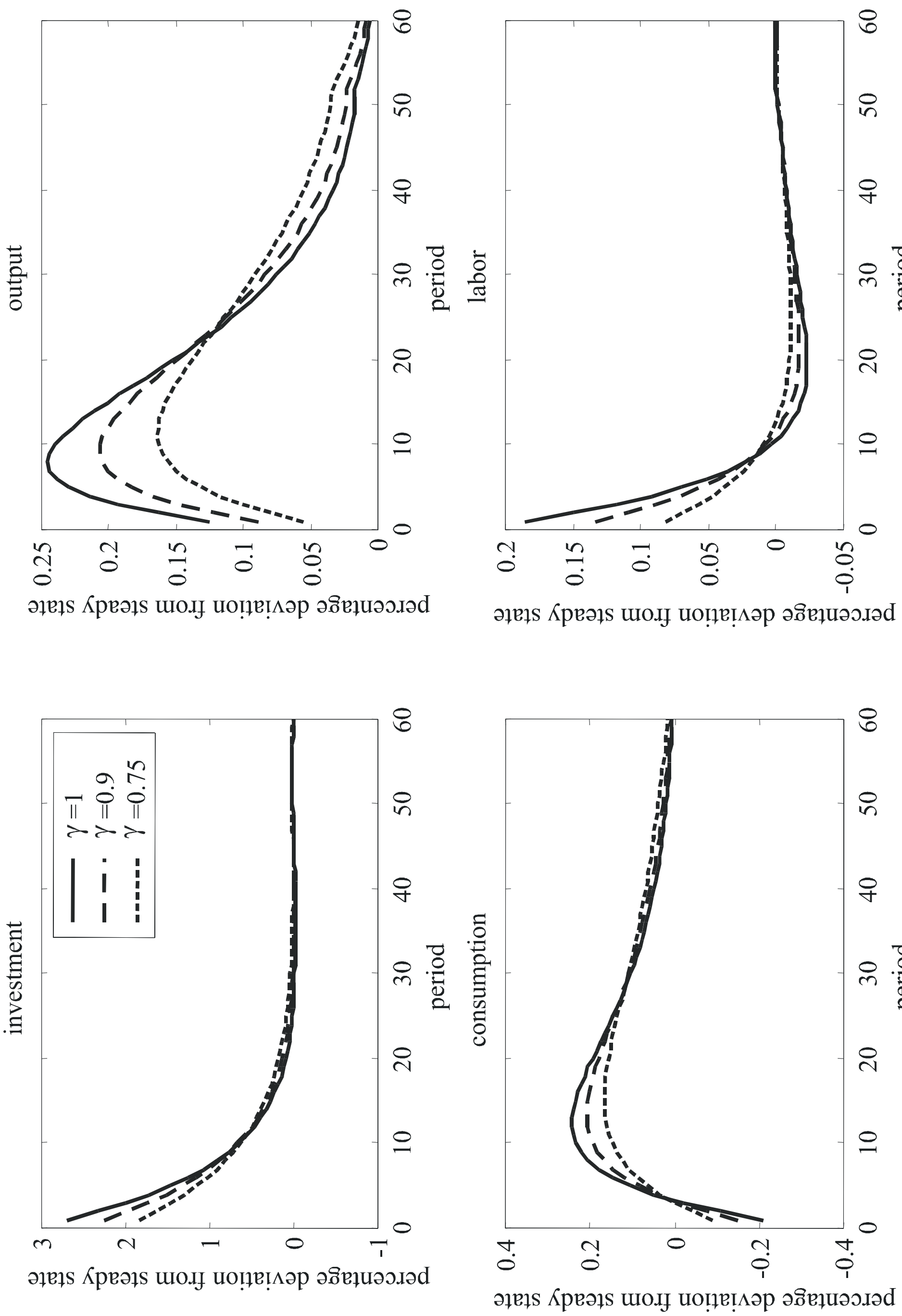


Figure 5: Impulse responses for different degrees of vintage complimentarities

depicts the impulse responses of the model for three different scenarios. Each scenario implies that $G = 1.021$, but the first is the one in which all growth is disembodied, the second is the case considered by Greenwood, Hercowitz, and Krusell (1997) in which $q = 0.035$, and the latter is the case in which all growth is embodied. As the figure clearly illustrates, the impulse responses are relatively insensitive to the composition of the sources of growth.

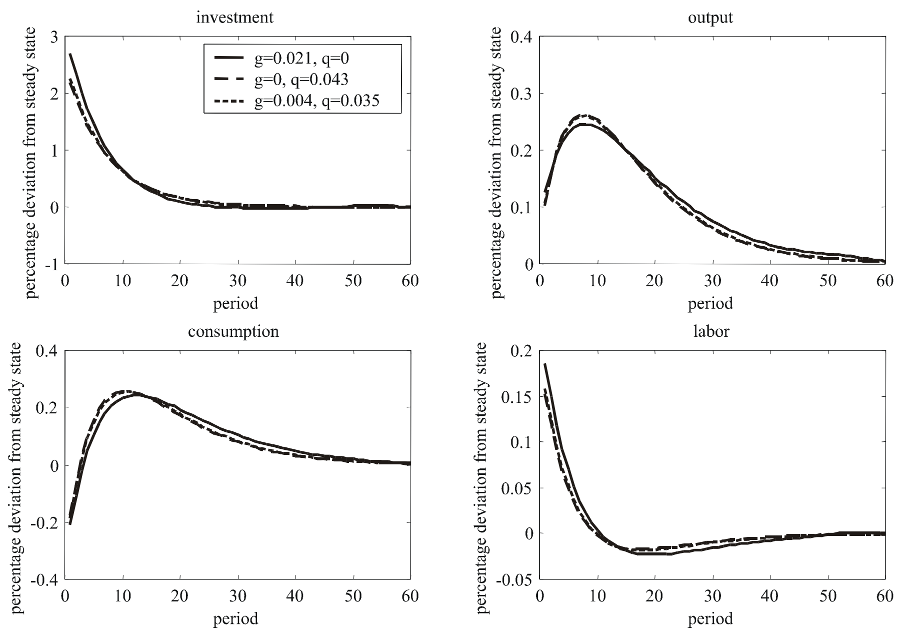
6 Conclusion

We started this paper with the question "Does it matter if the growth of productivity is driven by disembodied technological change that raises the productivity of all factors of production, including all capital in place, or it is driven by continuous improvements the quality of new capital goods?" Our conclusion is a resounding 'Yes! ', both for the long-run, as well as for the short-run.

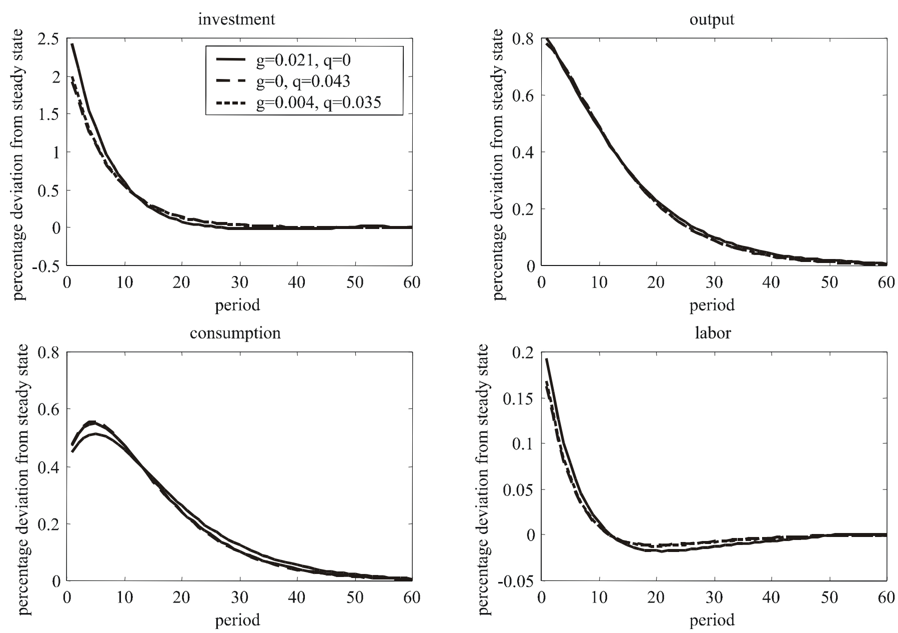
By endogenizing the savings rate and the labor supply, we showed that, contrary to Phelps (1962), the long run effects of policies affecting the savings rate depend on the relative importance of embodied and disembodied technological change.

More importantly, we showed that if one considers the interaction of vintage effects with scrappage of capital goods, shocks to embodied technological change can generate echo effects that take the form of investment booms and busts and hump-shaped impulse responses in output, but that with disembodied shocks these effects are much more subdued. This is an important observation if one realizes that the 1990's expansion was mainly investment driven. Hence, if the source of the 1990's expansion was indeed embodied technological change rather than the disembodied technological change associated with other postwar expansions, then our results provide a possible interpretation for the different character of the transitional dynamics of the current slowdown.

The emphasis in this paper is on the interaction between scrappage and embodied technological change in economic fluctuations. However, just like Phelps (1962) took the savings rate as exogenous, we took the scrappage time as exogenous. A worthwhile extension of the paper here would be to endogenize the scrappage time, as in Campbell (1999), and get a better insight for the response of T to the composition of the sources of growth.



(a) response to embodied shock



(b) response to disembodied shock

Figure 6: Effect of sources of growth on impulse responses

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A Mathematical details

This appendix contains some of the mathematical details needed to obtain the results presented in the paper. It contains 3 subsections. In the first subsection we solve for the steady state equations (9) through (13). In the second subsection, we present the detailed derivation of the long-run effects of the composition of the sources of growth. Finally, in the third subsection we briefly describe our log-linearization. procedure.

A.1 Derivation of steady state

The system of equations (9) through (13) can be solved sequentially. We can solve for the steady state Jelly capital to labor ratio by defining $f_J(x) = F_J(x, 1; 1)$ and using (13) to find

$$\frac{J^*}{L^*} = f_J^{-1}\left(\frac{(1-\tau_I)\left[\sum_{\tau=0}^{T}\left((1-\delta)^{\tau}\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^{\gamma}\right]^{\frac{\gamma-1}{\gamma}}}{(1+q)\sum_{s=1}^{T+1}\left(\beta(1+g)^{\phi(1-\sigma)-\gamma}(1+q)^{\frac{\alpha}{1-\alpha}\phi(1-\sigma)-\frac{\gamma}{1-\alpha}}\right)^{s}(1-\delta)^{\gamma(s-1)}}\right)$$

Once we have the 'Jelly capital-labor' ratio, we can easily solve for the investment ratio k^*/L^*. That is

$$\frac{k^*}{L^*} = \left[\sum_{\tau=0}^{T}\left((1-\delta)^{\tau}\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^{\gamma}\right]^{-\frac{1}{\gamma}}\frac{J^*}{L^*}$$

and output per hour equals

$$\frac{Y^*}{L^*} = F\left(\frac{J^*}{L^*}, 1\right)$$

while consumption per hour equals

$$\frac{C^*}{L^*} = \frac{Y^*}{L^*} - \frac{1}{1+q}\frac{k^*}{L^*}$$

The intratemporal optimality condition can now be solved to obtain the steady state labor supply such that

$$1/L^* = 1 + \frac{1-\phi}{\phi}\frac{C^*/L^*}{F_L(J^*/L^*, 1; 1)}$$

and

$$L^* = \frac{\phi F_L(J^*/L^*, 1; 1)}{\phi F_L(J^*/L^*, 1; 1) + (1-\phi)C^*/L^*}$$

This is derived under the assumption that the steady state exists for appropriate restrictions on parameters. The assumption is warranted because the first level Cobb-Douglas production function satisfies Inada conditions, in the sense that

$$\infty = \lim_{x \downarrow 0} f_J(x) > \frac{(1-\tau_I)\left[\sum_{\tau=0}^{T}\left((1-\delta)^{\tau}\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^{\gamma}\right]^{\frac{\gamma-1}{\gamma}}}{(1+q)\sum_{s=1}^{T+1}\left(\beta(1+g)^{\phi(1-\sigma)-\gamma}(1+q)^{\frac{\alpha}{1-\alpha}\phi(1-\sigma)-\frac{\gamma}{1-\alpha}}\right)^{s}(1-\delta)^{\gamma(s-1)}}$$

Hence, the steady state exists and is unique.

A.2 Effect of investment tax credit on steady state values

We will decompose the effect of an investment tax credit on the steady state values in two parts. In the first part, we will consider the effect of the investment tax credit on the savings rate and show how this effect depends on the composition of the sources of growth. In the second part, we then derive how the savings rate affects the steady state level of output. Throughout, we will substitute G following (**??**) for all relevant instances. The steady state savings rate in this economy, which we will denote by s^*, is given by

$$s^* = 1 - \frac{C^*}{Y^*} = \frac{1}{1+q}\frac{k^*}{Y^*}$$

for the investment to output ratio k^*/Y^* we can derive

$$\begin{aligned}
\frac{k^*}{Y^*} &= \frac{J^*/L^*}{\left[\sum_{\tau=0}^{T}\left((1-\delta)^{\tau}\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^{\gamma}\right]^{\frac{1}{\gamma}} F(J^*/L^*, 1; 1)} \\
&= \frac{\alpha}{\left[\sum_{\tau=0}^{T}\left((1-\delta)^{\tau}\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^{\gamma}\right]^{\frac{1}{\gamma}} f_J(J^*/L^*)}
\end{aligned}$$

where we have used the fact that the production function is Cobb-Douglas. In the steady state

$$f_J(J^*/L^*) = \frac{(1-\tau_I)\left[\sum_{\tau=0}^{T}\left((1-\delta)^{\tau}\left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^{\gamma}\right]^{\frac{\gamma-1}{\gamma}}}{(1+q)\sum_{s=1}^{T+1}\left(\beta(1+g)^{\phi(1-\sigma)-\gamma}(1+q)^{\frac{\alpha}{1-\alpha}\phi(1-\sigma)-\frac{\gamma}{1-\alpha}}\right)^{s}(1-\delta)^{\gamma(s-1)}}$$

such that the savings rate equals

$$\begin{aligned}
s^* &= \frac{\alpha}{(1-\tau_I)} \frac{\sum_{s=1}^{T+1} \left(\beta (1+g)^{\phi(1-\sigma)-\gamma} (1+q)^{\frac{\alpha}{1-\alpha}\phi(1-\sigma)-\frac{\gamma}{1-\alpha}}\right)^s (1-\delta)^{\gamma(s-1)}}{\left[\sum_{\tau=0}^{T} \left((1-\delta)^\tau \left[\left(\frac{1}{1+g}\right)\left(\frac{1}{1+q}\right)^{\frac{1}{1-\alpha}}\right]^{\tau+1}\right)^\gamma\right]} \\
&= \frac{1}{(1-\tau_I)} \alpha \frac{\sum_{s=1}^{T+1} \left((1-\delta)^{s-1} \left(\frac{1}{G(1+q)}\right)^s\right)^\gamma \left(\beta G^{\phi(1-\sigma)}\right)^s}{\left[\sum_{\tau=0}^{T} \left((1-\delta)^\tau \left(\frac{1}{G(1+q)}\right)^{\tau+1}\right)^\gamma\right]} \\
&= \frac{1}{(1-\tau_I)} \widetilde{s}^*
\end{aligned}$$

where $\beta G^{\phi(1-\sigma)} < 1$ is a necessary condition for boundedness of the solution to the representative agents problem.

Now note that a change in g and q such that G is constant will affect $\widetilde{s}^*$. This is because in case of embodied technological change the capital, measured in efficiency units, depreciates faster (due to economic depreciation) than in case of disembodied technological change. In fact, an increase in the importance of embodied technological change, i.e. in q, will decrease the returns to capital, decrease $1/(G(1+q))$, and this will increase $\widetilde{s}^*$. The intuition behind this result is that if productivity growth can only be obtained through investing in new vintages then there is an increased incentive to invest and the savings rate will rise. Consequently, an increase in the investment tax credit τ_I will raise the steady state savings rate more than in the case of disembodied technological change. However, the elasticity of the savings rate with respect to the investment tax credit equals $\tau_I/(1-\tau_I)$ and is independent of the composition of growth into embodied and disembodied technological change.

Now that we have considered the effect of a change in the investment tax credit on the savings rate, we will rederive Phelps' (1962) result in the context of our model by considering the effect of the savings rate on steady state output. For the steady state labor supply we obtain

$$L^* = \frac{\phi(1-\alpha)}{\phi(1-\alpha) + (1-\phi)(1-s^*)}$$

The steady state level of the Jelly capital to labor ratio has to satisfy

$$\frac{J^*}{L^*} = \left(\left[\sum_{\tau=0}^{T} \left((1-\delta)^\tau \left(\frac{1}{G(1+q)}\right)^{\tau+1}\right)^\gamma\right]^{\frac{1}{\gamma}} s^*\right)^{\frac{1}{1-\alpha}}$$

such that

$$\begin{aligned}
Y^* &= (s^*)^{\frac{\alpha}{1-\alpha}} \left(\left[\sum_{\tau=0}^{T} \left((1-\delta)^\tau \left(\frac{1}{G(1+q)}\right)^{\tau+1}\right)^\gamma\right]^{\frac{1}{\gamma}}\right)^{\frac{\alpha}{1-\alpha}} \times \\
&\quad \left(\frac{\phi(1-\alpha)}{\phi(1-\alpha) + (1-\phi)(1-s^*)}\right)
\end{aligned}$$

where we have again used the fact that the production function is Cobb-Douglas. In this case the elasticity of the steady state level of output with respect to the savings rate equals

$$\left[\frac{\alpha}{1-\alpha}+\frac{(1-\phi)\,s^*}{\phi\,(1-\alpha)+(1-\phi)\,(1-s^*)}\right]$$

A.3 Log-linearization details

The problem with log-linearizing the model is that it contains a high-dimensional state space, because the relevant state variables are the shocks $X_t^{(t)}$, and $Q_{t+1}^{(t)}$, as well as the sequence of past T investment levels $\{k_{t-\tau}\}_{\tau=0}^{T}$. In order to deal with this efficiently, we apply a two step procedure. In the first step we log-linearize the transformed resource constraint, (4), production function, (5), Jelly capital identity, (6), and intratemporal optimality condition, (7). These four equations then allow us to solve for $\hat{C}_t$, $\hat{Y}_t$, $\hat{J}_t$, and $\hat{L}_t$ as a function of $\left\{\hat{k}_{t-\tau}\right\}_{\tau=-1}^{T}$ and $\hat{X}_t^{(t)}$, and $\hat{Q}_{t+1}^{(t)}$. Here $\hat{}$ denotes percentage deviations from the steady state. We then log-linearize the Euler equation, (8), and substitute in for $\hat{C}_t$, $\hat{Y}_t$, $\hat{J}_t$, and $\hat{L}_t$ and their leads. This yields a $(2T+2)$-order linear difference equation in $\hat{k}_t$ from which we eliminate the $T+1$ unstable roots to obtain the resulting linearized investment policy function that gives $\hat{k}_{t+1}$ as a function $\left\{\hat{k}_{t-\tau}\right\}_{\tau=-1}^{T}$ and $\hat{X}_t^{(t)}$, and $\hat{Q}_{t+1}^{(t)}$.