# The Supply-Driven Input-Output Model: A *Re*interpretation and Extension*

**JiYoung Park**

Von Kleinsmid Center 382
School of Policy, Planning, and Development
University of Southern California
Los Angeles, CA 90089-0001
Email: jiyoungp@usc.edu
Office: (213) 821-1351
Cell: (213) 550-9979

***This draft should be revised a little to submit

* The author wishes to acknowledge the support of the University of Southern California's Center for Risk and Economic Analysis of Terrorist Events (CREATE) and the intellectual support of Profs. Peter Gordon, Harry W. Richardson, and James E. Moore II.

Abstract

Most previous efforts have focused on constructing various demand-driven IO models because of their widely accepted usefulness in regional science. After Ghosh's suggestion of the supply-driven IO model, a debate over its plausibility ensued. Much of this was resolved with Dietzenbacher's (1997) suggestion of the interpretation of a price model, equivalent to Leontief's price model. However, in static market equilibrium, producers will not change the current technical relationships that are based on historical sales during the immediate period after an exogenous event. This reflects the fact that Ghosh's supply-driven model is in terms of monetarily expressed quantities and hence applicable when using the supply-side IO in the circumstance of static market equilibrium with abnormal economic cessations. To suggest a new interpretation for the supply-driven IO model, a four-quadrant space of economic situations is introduced, using 'price vs. price-quantity' and 'increase-decrease' axes. Furthermore, even in the case that normal market equilibrium is not maintained, instead of the direct use of supply-side quantity model, Ghosh's case can be switched to a price-type supply-driven model, and play a role in estimating economic impacts. To address this switching process, exogenous price elasticities of demand are combined with the supply-driven model, adjusting quantity responses to price impacts. This logic will underlie the theoretical background necessary to utilize the supply-side model, and hence the first essay highlights the power and the usefulness of linear models by clarifying the applicability of the supply model.

## I. Introduction

Regional scientists are often engaged in the study of economic impacts (Hewings, 1985). In recent years, the increased threat of terrorist attacks and the possibility of a higher frequency of natural disasters due to climate change have renewed policy makers' interest in impact studies. Yet, have our analytic tools really improved? The picture is mixed. While we have better data, better software and hardware than ever, still is it easy to apply currently existing models to those systems? In this paper, I argue that a fundamental reinterpretation of some old models does shed new light and does make it possible to develop more insightful and useful impact analyses.

In the Input-Output (IO) world, two standard models have been developed since Leontief's first contributions (1936, 1941). One is the 'Leontief' or demand-driven IO model, generalizing interdependences between industries in an economy. To address "the highly complex network of interrelationships which transmits the impulses of any local primary change into the remotest corners of the economic system", the general static equilibrium for an economy can be represented (Leontief, 1976: 34). In the classic IO system, therefore, the interrelations between industries take account of 'technical' relations throughout an economy via fixed-coefficient production functions.

Two key assumptions implicit in the Leontief model, a competitive market system and non-scarce resources, were noted by Ghosh (1958) who suggested another version of the model to identify the interrelationships among industries. The technical coefficients from the Leontief model are assumed fixed and yield new industrial total inputs necessary for an economy as in response to changed final demands. This requires conditions as "so long as there is no scarce factor and so long as suppliers are able to offer more of any commodity at the existing price" (*ibid*: p.59) on the production side, even in the short-term. However, a monopolistic or a centrally planned economy, where all resources are scarce except for one sector, considers the best feasible combination of the non-scarce sector based on the rest of the scarce resources with respect to a welfare function, not the optimized technical combination of production for the non-scarce sector (Ghosh, 1964). The economic situation can, therefore, denote a new approach to examining the ‘allocation’ of a non-scarce resource, so called ‘Ghosh’ model or supply-driven IO model. Here, two new key conditions are found from ‘Ghosh’ model, that depart from Leontief’s assumptions:

i. Fixed (by authority or stable equilibrium market during the short run not allowing any behavioral adjustments between industries) allocation coefficients not affected by final demand changes.

ii. Scarce capacity for all industrial sectors except the sectors targeted.

Although the two assumptions are basic when interpreting economic conditions in order to apply the supply-driven model, the theoretical criticisms (especially by Oosterhaven (1988; 1989)) on the implausibility of the supply-driven model still do not contain full considerations of them in their interpretation of specific economic conditions, and remain to be solved.

The rest of this paper, therefore, will deal with

The difference between the Leontief and Ghoshian IO model in basic terms and which economic conditions are taken into consideration in the empirical applications of supply-driven models.

What criticisms and defenses of supply-driven models have been offered.

Whether there are new approaches possible to reinterpret the supply-driven IO model or not.

What the appropriate conditions applied to impact studies might be.

Based on these discussions, a new price-type supply-driven IO model combining traditional supply-driven IO with price elasticities of demand is suggested, and then some conclusions are followed.

## II. Overview of the Supply-Driven IO Model

II-1. Demand-driven vs. supply-driven IO model

To discuss the demand-driven and supply-driven IO models, it is helpful to begin with definitions of the interindustry flows matrix expanded to include final demand and value added sectors. Table 1 shows the national expanded economic transaction flows for an economy, along with matrix (in parentheses) and notation.

In the general IO model, it is assumed that $X^d = X^{sT}$, where superscript $T$ means the transpose of the matrix, and the standard Leontief IO model is easily expressed in matrix form of which notations shown in Table 1.

$$X^d = Zu_N + Yu_k \qquad (1.)$$

Because the Leontief technical coefficients are interindustry coefficients to produce total inputs corresponding to demand requirements, the input coefficients matrix $A$ can be obtained from a flow matrix $Z$ but divided by the total inputs, that is, $A = Z(\hat{X}^d)^{-1} = Z(\hat{X}^s)^{-1}$, where the $\hat{X}$ means the diagonal matrix of $X$ and hence $\hat{X}^s = \hat{X}^{sT} = \hat{X}^d$. Note that the input coefficients matrix $A$ examines the backward effects of interindustry relationships, because the coefficient $a_{ij}\,(= z_{ij} / x_j^s)$ is based on total input.

**Table 1. General Expanded Flow Matrix of a National Economy**

| | | |
|---|---|---|
| $\begin{matrix} z_{11} & z_{12} & \cdots & z_{1N} \\ z_{21} & \ddots & & \\ \vdots & & z_{ij} & \\ & & & \ddots \\ z_{N1} & & & z_{NN} \end{matrix}$ $(Z)$ | $\begin{matrix} y_{11} & y_{12} & \cdots & y_{1K} \\ y_{21} & \ddots & & \\ \vdots & & y_{ik} & \\ & & & \ddots \\ y_{N1} & & & y_{NK} \end{matrix}$ $(Y)$ | $\begin{matrix} x_1^d (= \sum_j \sum_k (z_{1j} + y_{1k})) \\ x_2^d (= \sum_j \sum_k (z_{2j} + y_{2k})) \\ \vdots \\ x_N^d (= \sum_j \sum_k (z_{Nj} + y_{Nk})) \end{matrix}$ $(X^d)$ |
| $\begin{matrix} v_{11} & v_{12} & \cdots & v_{1N} \\ v_{21} & \ddots & & \\ \vdots & & v_{lj} & \\ & & & \ddots \\ v_{L1} & & & v_{LN} \end{matrix}$ $(V)$ | - | $\begin{matrix} \bar{v}_1 (= \sum_j v_{1j}) \\ \bar{v}_2 (= \sum_j v_{2j}) \\ \vdots \\ \bar{v}_L (= \sum_j v_{Lj}) \end{matrix}$ $(\bar{V})$ |
| $x_1^s \quad x_2^s \quad \cdots \quad x_N^s$ [1] $(X^s)$ | $\bar{y}_1 \quad \bar{y}_2 \quad \cdots \quad \bar{y}_K$ [2] $(\bar{Y})$ | - |

Note: 1. $x_j^s = \sum_i \sum_l (z_{ij} + v_{lj})$

2. $\bar{y}_k = \sum_i y_{ik}$

3. Description of notation

$Z$ is the NxN matrix of intermediate interindustry flows and its element $z_{ij}$ denotes the deliveries in dollar values from industry sector $i$ to $j$.

$Y$ is the NxK matrix showing various kinds of final demands and has its element $y_{ik}$ denoting the deliveries in dollar values from industry sector $i$ to final users $k$. Generally, $k$ contains private consumers, governments, investments, and exports.

$V$ denotes the LxN matrix showing various kinds of value added factors and has its element $v_{lj}$ meaning the dollar values going to product sector $j$ with factor inputs $l$. Generally, $l$ contains various kinds of labor, capital, taxes by governments, and imports.

$X^d$ denotes the monetary value column vector of total outputs for each sector and its elements are expressed as $x_i^d$, which is the column sum of intermediate flows and final demands of sector $i$.

$X^s$ denotes the monetary value row vector of total inputs for each sector and its elements are expressed as $x_j^i$, which is the row sum of intermediate flows and value added factors of sector $j$, as shown in Note 1.

$\overline{V}$ is a column vector, that is, column sum of value added factor $l$ and same as $Vu_N$, where $u_N^T$ is $N$ element unit row vector, *i.e.* (1, …, 1) and superscript $T$ means the transpose of $u$.

$\overline{Y}$ is a row vector, that is, row sum of final demands $k$ and same as $u_N^T Y$.

Then, the equation (1.) can be rewritten as equation (2.)

$$X^{sT} = A\hat{X}^{sT} u_N + Yu_k \qquad (2.1)$$

$$= AX^{sT} + Yu_k \qquad (2.2)$$

From equations (2.), it is simple to obtain the 'Leontief' inverse matrix which is fixed due to the assumption of constant input coefficients matrix $A$ as shown in equation (3).

$$X^{sT} = (I - A)^{-1} Y_k u_1 \qquad (3.)$$

If final demands change exogenously only for the $k^{\text{th}}$ final user ($Y_k$), new total inputs necessary to satisfy the required changes can be derived via equation (4.).

$$\Delta X^{s^T} = (I - A)^{-1} \Delta Y_k \tag{4.}$$

Then, to obtain the 'Ghoshian' supply-driven model requires construction of an allocation (output) coefficients matrix $B$, which allocates (sales) the current total inputs to each sector. Hence the allocation coefficients matrix $B$ should be measured as a fraction of total outputs ($b_{ij} = z_{ij} / x_i^d$) in order to examine allocation processes of input industry sectors. This examines 'bottleneck' effects according to the change in value added factors. Then, $B = (\hat{X}^s)^{-1} Z = (\hat{X}^d)^{-1} Z$, where interindustry relationships are examined in a forward direction. Note the relation between the allocation coefficients matrix and the Leontief technical coefficients, $B = (\hat{X}^d)^{-1} A \hat{X}^d = (\hat{X}^s)^{-1} A \hat{X}^s$ from $Z = \hat{X}^d B = A \hat{X}^d$.

Therefore, the 'Ghoshian' inverse allocation matrix is easily obtained from (6.) via equation (5.), and changes of total outputs according to the changes of value added vector $l$ are estimated via (7.), similarly to the process executed in the 'Leontief' inverse solution.

$$X^s = u_N^T Z + u_l^T V \tag{5.1}$$

$$X^{d^T} = u_N^T Z + u_l^T V \tag{5.2}$$

$$= u_N^T \hat{X}^{d^T} B + u_l^T V \tag{5.3}$$

$$= X^{d^T} B + u_l^T V \tag{5.4}$$

and hence,

$$X^{d^T} = u_1^T V_l (I - B)^{-1} \quad (6.)$$

and

$$\Delta X^{d^T} = \Delta V_l (I - B)^{-1} \quad (7.)$$

where $V_l$ is a row vector only containing $l^{\text{th}}$ value added factor.

According to equations (4.) and (7.), significant assumptions for both IO models are implicit. The positive changes of total input ($\Delta X^{s^T}$) in equation (4.) assume that newly required value added factors should be enough to support the changes, $\Delta X^{s^T}$, in the production function. In other words, from equation (4.) obtain the newly required value added vector $\Delta V^T = \hat{R}_{X^s}[(I - A)^{-1}\Delta Y_k]$ where $X^s = V(\hat{R}_{X^s})^{-1}$ under the assumption that $R_{X^s} = V(\hat{X}^s)^{-1}$. This requires that the condition, $\Delta V \leq U_V$, should be satisfied at least for the economy, where $U_V$ is the upper-bound of the available value-added factors for all sectors in an economy (Ghosh, 1958: 58~59). Therefore, only a perfect market system which happens to contain enough resources for all sectors can plausibly afford to support the final demand changes.

Similarly but differently, the supply-driven model rests on the assumption that the forward interindustry allocation processes work providing that the new changes of final demands via allocation distributions are only higher than the lower-bound required by final users due to its welfare conditions.

That is, $\Delta Y \geq L_Y$, where $\Delta Y^T = [\Delta V_l (I - B)^{-1}]\hat{R}_{X^d}$ from the equation (7.). The $X^d = (\hat{R}_{X^d})^{-1} Y$, based on the definition $R_{X^d} = (\hat{X}^d)^{-1} Y$. The important implication of the condition that $\Delta Y \geq L_Y$ is that it is not necessary any more to let value-added factors increase in order to increase total outputs if the required final demands satisfy the minimum requirements. Key assumption of enough supplies to produce total outputs, as shown in the Leontief IO model, do not matter any more with respect to the welfare of users, irrespective of the inefficiency. The condition $\Delta Y \geq L_Y$, therefore, will be useful for discussing and interpreting the 'Ghoshian' supply model as shown below.

The interpretation of the changes of 'total outputs' according to changes of value added, however, was severely criticized by Oosterhaven (1988) and various debates followed until Dietzenbacher's (1997) novel interpretation. Before looking into these debates, it useful to review some empirical discussions in order to understand some criteria that can apply to the 'Ghoshian' models. Those are addressed in Section II-2.

### II-2. Empirical applications of Ghoshian supply-driven model

Empirical applications since Ghosh's suggestion have been few, although there have been many possibilities, e.g., oil shocks, cartels, earthquakes, and so on, to apply the supply-driven model. The demand-driven IO model, meanwhile, has been used widely for various impact analyses. Besides, ever since Oosterhaven's consecutive various criticisms (1988, 1989, 1996) on the implausibility of supply-side model theoretically, it is hard to find any dominant studies of impact analysis. Although Dietzenbacher (1997) showed that the supply-driven model can be interpreted as a 'absolute price' model and that the interpretation is easier to understand its price effects than the 'Leontief' (relative) price model, empirical applications still seem to be limited.[1] In this sense, selecting and comparing two dominant

[1] Recently, Cai et al. (2005) reported a case study of economic impacts about regulation of longline fishing using Goshian supply-side model, but it cannot still avoid Oosterhaven's criticism in the sense they depend on the

modes for impact analysis will guide which criteria can be applied to the supply-driven model.

First, Giarratani (1976) applied the supply-driven national IO model to trace national economic impacts by restricting each energy sector (coal mining, crude petroleum and natural gas, and energy activity sector) based on the 1967 U.S. interindustry table. The study stressed two key criteria, monopolistic markets and scarce resources, the same as suggested by Ghosh (1958) when choosing study definitions. It is reasonable to accept that energy sectors are in some sense 'monopolistic' because it is not easy to find easy alternatives in the short-run (Giarratani, 1976: 449~450). As Ghosh (1958) and Chen and Rose (1991) noted, the balanced equilibria of industries in the long-run period even in competitive markets remain stable by rationing without substitutions during the short-run, because producers will depend on their previous sales even in the case that sudden disruptions do occur. However, scarcities among other sectors except the three-targeted sectors were not clarified in his application. Further, when simulating other impact analyses in the study, two row vectors of coefficient sectors were extracted and multiplied by the total inputs, in order to add the amounts to the remaining value added sectors as exogenous values. To rerun the model with the increased value added, the supply-driven IO model was reconstructed without the two sectors. This approach would avoid well the problem of 'output changes without input changes' criticized by Oosterhaven (1988) later. Still, this effort disregarded the interconnections related to the two deleted sectors (*ibid*: 221).

Another important analysis was conducted by Davis and Salkin (1984), who applied the supply-driven model to the Kern County in California so as to estimate the economic impacts from a hypothetical limitation of water supply to the agricultural sectors of the County. They clarified two points;

i. Water supply in this area for agricultural industry is subject to the Agency's distribution policy, denoting monopoly.

ii. No alternate sources of the water would be supported from the small area and hence

---

hypothetical sector extraction approach neglecting the impacts on the targeted sector(s), remarking the result of the Goshian model should be understood as the potential impacts of the regulation, instead of being ignored.

economic behaviors of producers would be preserved in the local area, not adapted efficiently to minimize the costs by choosing alternatives.

The second point attached to using the supply-driven model has significant implications especially when an economy depends on imports severely, because it is hard for producers to find alternatives in the economy itself, and hence the scarcity condition might be achieved at least during short run.

In the circumstances that our current economic system is not a centralized planned economy, it is important to specify the relevant conditions applying to supply-driven model. From Ghosh's idea and previous studies, therefore, four major conditions applying to supply-driven model can be summarized:

i. Monopoly.
ii. Scarcity (of other inputs except targeted sectors).
iii. Short period.
iv. Small region (depending targeted resources much upon other regions).

Those four criteria should be applied to a case study in order to examine the applicable possibilities of the supply-driven model. The next section will highlight the debates on the plausibility of the supply-driven model and the state of the art.

## III. Debate on the Implausibility of Supply-Driven IO Models

While some comments on the supply-driven model since Ghosh had been addressed, serious criticisms of the implausibility of the supply-driven model were made by Oosterhaven (1988). He convincingly questioned that based on given final demands, if "local consumption or investment reacts perfectly to any changes in supply" for example, "purchases are made, e.g., of cars without gas and factories without machines", it does not require any production function because the final demands and input factors might be combined without any technological relationship. Using Taylor's expansion, he concluded that "both as a general description of the working of any economy and as a way to estimate the effects of loosening or tightening the supply of one scarce resource, the supply-driven model may not be used" and suggested an alternative model instead of using the supply-driven model directly.

In the following year, two studies by Gruver (1989) and Rose and Allison (1989) added comments on the Oosterhaven's critique. Gruver (1989) suggested an alternative interpretation that the implicit production function, in spite of its perfect substitutability, and agreeing with Oosterhaven that the supply-driven IO model could be interpreted as a cost-minimizing choice to produce constant-returns-to-scale outputs, under the assumption of constant relative prices. Similarly, Rose and Allison (1989) argued that the supply model could still be useful to approximate the impacts of supply-driven disasters, if the allocation coefficients are tolerably fixed. However, both studies did not touch the core debate on the implausibility of the model and Oosterhaven had succeeded with his argument (1989, 1996) until Dietzenbacher's interesting interpretation appeared.

The contribution of Dietzenbacher's (1997) interpretation is that the supply-driven IO model is equivalent to the Leontief price IO model and hence that the supply model is to be interpreted as (absolute) price model instead of a quantity model. Then, the implausibility problem raised by Oosterhaven would vanish. This interpretation contains one condition that the changes in value added should be followed by the price changes for the value added inputs, not quantity changes. Given that quantities are fixed in the value-added vectors, here is a simple proof that follows from equation (7.) and the fact that $B = (\hat{X}^s)^{-1} A \hat{X}^s$.

$$\Delta X^{d^T} = \Delta V_l [I - (\hat{X}^s)^{-1} A \hat{X}^s]^{-1} \quad (8.1)$$

$$= \Delta V_l [(\hat{X}^s)^{-1} \hat{X}^s - (\hat{X}^s)^{-1} A \hat{X}^s]^{-1} \quad (8.2)$$

$$= \Delta V_l (\hat{X}^s)^{-1} (I - A)^{-1} \hat{X}^s \tag{8.3}$$

From equation (8.3), let price changes in the $l^{\text{th}}$ labor factor be $\delta v_i^p$, let the row vector without price changes in other value added sectors be $\Delta V_l{}^p (= \Delta V_l (\hat{X}^s)^{-1})$, and price changes of total outputs be $\Delta P$. Then,

$$\Delta X^{d\,T} (\hat{X}^s)^{-1} = \Delta V_l{}^p (I - A)^{-1} \qquad (9.)$$

and hence,

$$\Delta P = \Delta V_l{}^p (I - A)^{-1} \qquad (10.)$$

Equation (10.) is exactly the same as the Leontief price model, which suggests the supply-driven model is the 'Ghoshian' price model. Although this interpretation provides a theoretical defense against the criticism of the supply-driven model, the interpretation places limits on the empirical applications of impact analyses. This might be due to difficulties of interpretation that direct and indirect impacts caused by a disruption on the supply-side are presumably dollar quantity losses rather than price decreases. This is the reason why recent studies focus on forward linkages (Dietzenbacher, 2002; Cai et al., 2006) or structural changes within an economy (Wang, 1997; Bon and Yashiro, 1996; Bon, 2001) using the supply-driven model. Therefore, some further explanations are necessary to apply the supply-driven model to impact studies, beyond the interpretation of the 'Goshian' price model.

## IV. *Re*interpretation of Supply-Driven IO Model

The key assumption in Dietzenbacher's (1997) is the suggestion that there are only price changes among the value added changes. Corresponding changes of total outputs, consequently, result from price changes, not quantity changes. This interpretation successfully met the attacks on the implausibility of supply-driven IO models. However, accepting this interpretation limits our understanding of the results of impact analyses, because quantity losses, e.g. labor losses or capacity losses of a facility from unexpected disasters are general and cannot be applied to the 'Ghoshian' price model.

Unfortunately, it has not been suggested how to find applicable conditions of supply-driven models in the current market environment. Even Oosterhaven (1988) took some cautious steps when dealing with the two basic conditions, monopoly and scarcity, in his first criticism. Contrary to Ghosh's explanation of the welfare function, however, one defender (Gruver, 1989) concluded that producers behave to minimize their costs, but unfortunately his conclusion was undermined by Oosterhaven's next response (1989: 460~461), which missed Ghosh's point of the welfare function. Since Dietzenbacher (1997), any attempts to deal with these conditions have not been made.

However, the market mechanism, although often modeled as "perfect", includes some market power at a given prices and quantities. Due to limited accesses to market information between sellers and buyers or even between industries, that is, due to asymmetry of information, various shades of market power are common. This is the reason why a long-run solution maintains that the equilibrium is a result of best negotiations among numerous efforts by the actors in normal economic environments and induces resource scarcity without efficient distributions especially during the short-run due to capacity constraints (Pindyck and Rubinfeld, 1998: 21). This leads to, as Ghosh (1958) mentioned, the result that producers will not decrease their previous outputs or factors during the short run, even if there are huge shocks to the economy. The effort for finding substitute products requires many other unexpected costs, e.g. costs of searching for substitutes, additional transportation costs, and so on, and hence unless those are expected for the long-run experience, these reactions will not happen. Even in the market system, therefore, two conditions of monopoly and scarcity of resources can be verified during the short-run. Of course, because smaller regions are more dependent on other regions, they will loose some market power and be subject to exogenous market power which is not easily changed. So, it can be said ‘the more common the market power, shorter the period and smaller the region’.

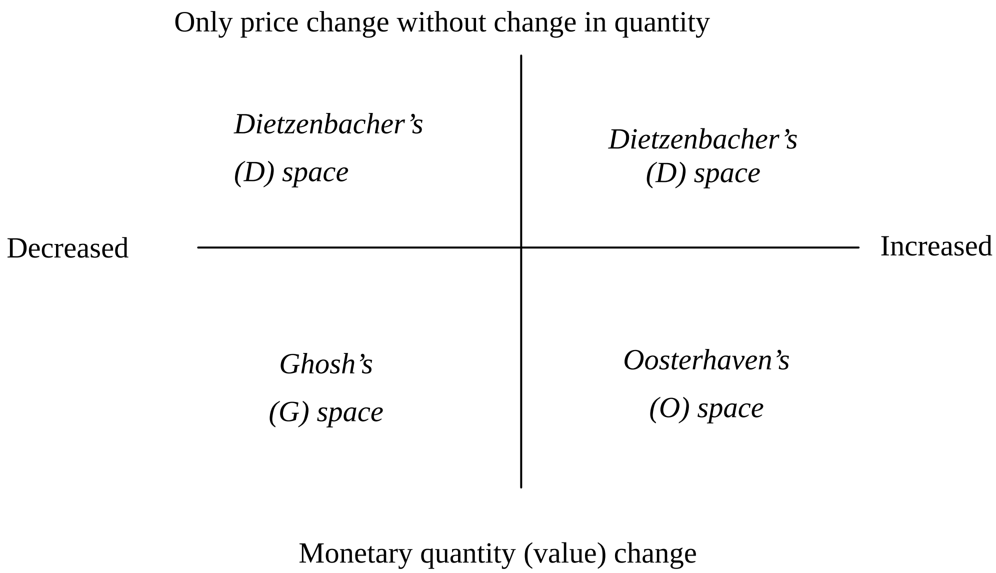
Therefore, under normal equilibrium economic status and characteristics, four possible quadrants according to the change of value added factors could be identified as shown in Figure 1. Because economic factors are expressed as money, I assumed two changes:

- Only price change without changes in quantity.
- Monetary quantity (=quantity x price) changes.

These changes could be increased or decreased. Among the quadrants, the upper quadrants are only affected by the price changes under the assumption that quantities are fixed, while the lower quadrants refer to monetary quantity changes. The right sides of the quadrants indicate the increases of price or monetary quantity, while the left sides show the decreases. Although Dietzenbacher’s (1997) suggestions with respect to the ‘Goshian’ price model still plays an important role for all kinds of spaces, because under normal economic equilibrium conditions, quantities are more or less fixed, but prices change, it is not enough to understand some of the cases and hence requires some additional explanation in order to be extended to the study of impact analyses. The detailed discussions on each quadrant are followed

The upper-left side, the first quadrant, of decreased value added factors in price might be experienced in a deflated economic state. The U.S. experienced two sustained deflations associated with depressions during the 1890s and 1930s and a temporary deflation experience was experienced for one year 1954~1955 (Samuelson and Nordhaus, 1995: 575~576). This space is rarely observed in reality and therefore will be hard to find an appropriate case to use the supply-driven IO model, although Dietzenbacher’s suggestion can be applicable.

Figure 1. Four possible cases according to the change of value added factors




The upper right quadrant shows normal economic equilibrium conditions, where only price increases are generally observed for the value added factors, given the quantity fixed. Most economic systems experience this sort of inflation and it might be investigated whether or not there are different structures in an economic system during time intervals based on such a price-deflator or decomposition method. Also, sudden increases of labor price in one industry sector due to e.g. a labor strike without increasing the number of labors will lead increases of output prices in other industry sectors even if there are no changes in quantities in all other value added factor. Therefore, this quadrant wholly matches Dietzenbacher's explanation and is labeled as Dietzenbacher's (D) space.

However, the assumption of only price changes without quantity changes is idealized compared to an actual economic world, and monetary quantity changes including price changes in value added factors are common. Although Dietzenbacher noted the mathematical relation between the Leontief price model and the supply-driven model, it is more generally true that the only price increase in value added is not reflected wholly in total output, because market equilibrium still works during the short term period and hence would not sustain the price increase. Rather, the D-space is more useful to examine these cases e.g.

change of economic structures or linkages of multipliers for long-run analysis than short-run impact analysis.

Therefore, it is more useful to deal with the monetary changes among value added factors because they are more realistic. The lower quadrants are the cases including both price and quantity changes simultaneously. Under normal economic equilibrium conditions, the third (lower right) quadrant indicates that monetary value added factors have increased. I labeled this space as O-space, because this is the space criticized by Oosterhaven due to its implausibility as discussed in section III. However, if monetary increases in value added factors due to, e.g., an increase of number of laborers for one sector temporarily induces prices in all other sectors increased in the forward direction without increases of value added factors in all other sectors, because all economic factors only recognize the 'dollar' values and hence Dietzenbacher's interpretation might be still useful to defend Oosterhaven's concerns.

However, the monetary increases in factors would be relevant with the demand-driven model because in many cases increases of value added factors might result from the market signals required by final demands. That is, although the supply-driven model might be still useful in O-space to verify the price increases in total outputs of all sectors due to the increase of only one value added factor during the short-run, the fundamental changes from normal market equilibrium should be noticed at both supply and demand sides. For example, an increased demand for cars will induce an increase of the number of labors and thus value added and output increases of other linked sectors. These changes require movement of market equilibrium for each period. Therefore a new approach reflecting both sides at the same time might be helpful to investigate the impacts. In the sense, an alternative model by Oosterhaven (1988) combining supply- and demand-driven models is understandable, but his implausibility suggestion on the O-space was met with Dietzenbacher's suggestion.

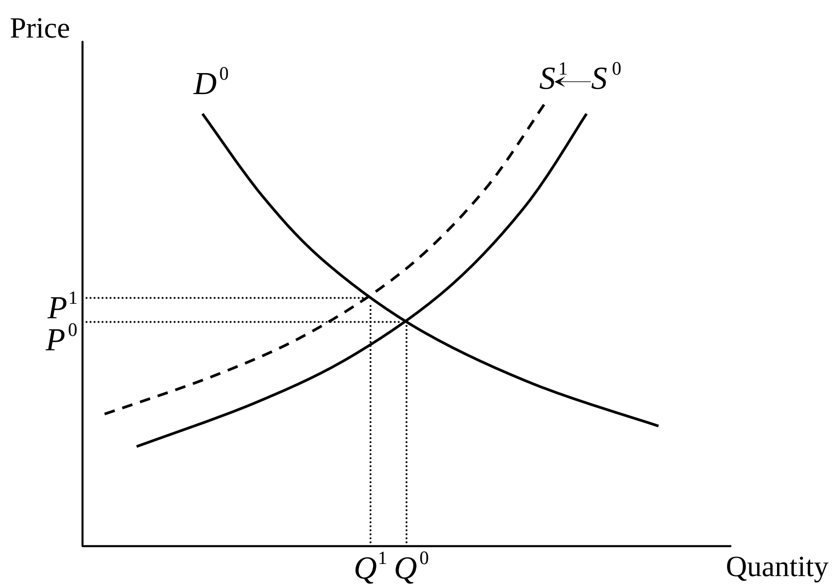
For the above three quadrants, the supply-driven model is surely applicable using the Ghoshian price-model. However, for the final remaining case, the Ghoshian price-model is not as easily applied as the other quadrants. According to the Ghoshian price-model, a sudden decrease in the monetarily expressed value added factors will decrease the absolute price of total output losses. This result is wholly opposite to actual experience, because a sudden decease in valued added sectors induces decreases in total output for the sector via the allocation interrelations, and hence the absolute price for the sector generally would increase.

Therefore, the fourth lower left-side quadrant indicates the exceptional economic situation of sudden monetary value added losses such caused by terrorist attacks or unexpected natural disasters, requiring the four conditions for an impact analysis. While the basic interpretation of the supply-driven model might be focused on the price interrelations, under static market equilibrium, producers will not change their current technical rationing during the short term, after a man-made or natural disaster, as discussed for the four conditions. In other words, only if they verify changes of final demands by the factor losses ( $\Delta Y$ ) are higher than the lower-bounds ( $L_Y$ ) required by final users at least, or $\Delta Y \geq L_Y$, not upper-bound (or maximum) requests, will suppliers continue their sales until the market loses its power from other pressures, even in the case that they lose their benefits during short-term. This examination of Ghosh's supply-driven model shows a relation with monetary quantity losses and is theoretically applicable to the man-made or natural impact analyses under the four conditions. Therefore, this quadrant might be labeled as Ghosh's (G) space. The outstanding defense of Dietzenbacher, seeing the supply-driven model as a Goshian (absolute) price model is very useful for other spaces except the G-space. But taking into consideration that many impact analyses are conducted in this space, the G-space should be differentiated.

## V. Extension of Supply-Driven IO Model

The common limitation in IO models is their linear characteristics which can be expected to over-estimate total impacts in a relatively long-term duration, because of the fixed coefficients assumption. Or market system might react relatively fast, loosening one or two conditions among the four conditions. It is common to note that market power is relatively weak because sizable regions or a relatively long-run period enabling production substitutions and violating static market equilibrium will require more or less adjusted equilibrium via markets. Empirically, however, we observe consumer behaviors as price changes according to quantity changes in the market as price elasticity. Using this price elasticity, even in the case of loosening the four conditions in the G-space, the supply-driven IO model still might be useful. However, the supply-driven model using a price elasticity is not the quantity-type supply-driven model, but price-type supply-driven model, which is possibly converted from the quantity-type demand-driven model.

Figure 2. New Equilibrium via Market due to Economic Disturbance



As shown in Figure 2, total input losses $\Delta Q(= Q^1 - Q^0)$ due to a disaster will increase price by shifting the original supply curve $S^0$ to the left $S^1$ supply curve, following the $D^0$ demand curve. Then, new market equilibrium is decided at the new price $P^1$, where consumers' demands are reflected. From the changes of monetary quantity and price on demand curve and exogenous price elasticity of demand ($\bar{\varepsilon}_p$) vector, new total input losses are estimated as follows, reflecting consumers' demands.[2]

To use the exogenous price elasticity of demand, $\bar{\varepsilon}_p$, first, the quantity-type demand-driven model shown in the equation (4.) should be converted to a price-type supply-driven model as shown in equations (11.) and (12.).

$$\Delta X^{s^T} = [I - \hat{X}^{s^T} B (\hat{X}^{s^T})^{-1}]^{-1} \Delta Y_k \qquad (11.1)$$

$$= \hat{X}^{s^T} (I - B)^{-1} (\hat{X}^{s^T})^{-1} \Delta Y_k \qquad (11.2)$$

and hence,

$$(\hat{X}^{s^T})^{-1} \Delta X^{s^T} = (I - B)^{-1} (\hat{X}^{s^T})^{-1} \Delta Y_k \qquad (12.1)$$

$$\Delta P^{s^T} = (I - B)^{-1} \Delta P^{Y_k} \qquad (12.2)$$

Because the price elasticity of demand for sector $i$, $\varepsilon_{p,i}$, is defined as $\frac{\delta x_i^s / x_i^s}{\delta p_i / p_i}$, based on the

2 Price elasticities of demand for energy, for example, are available from http://www.eia.doe.gov/smg/asa_meeting_2004/fall/files/exe/Elasticity%20Estimates.htm

$Q = X^s$, the price change $\delta p_i$ is obtained based on the exogenous price elasticity of demand $\bar{\varepsilon}_{p,i}$ as,

$$\delta p_i = \frac{\delta x_i^s / x_i^s}{\bar{\varepsilon}_{p,i} / p_i} \tag{13.1}$$

$$= \frac{\delta x_i^s p_i}{\bar{\varepsilon}_{p,i} x_i^s} \tag{13.2}$$

$$= \delta x_i^s \pi_i \tag{13.3}$$

where $\pi_i = \frac{p_i}{\bar{\varepsilon}_{p,i} x_i^s}$ is exogenous for sector $i$, because it is relatively easier to find the fixed $p_i$ and $x_i^s$ right before the event than to find the exogenous price elasticities of demand.

Here, the column vector of price changes due to the disaster, $\Delta_{t=1} P^d$ (= $\Delta \hat{X}^s \Pi$, where $\Pi$ is a column vector of $\pi_i$), is changed to $\Delta_{t=1} P^{Y_k}$ as,

$$\Delta_{t=1} P^{Y_k} = \hat{R}_{X^d} \Delta_{t=1} P^d \quad \text{where} \quad R_{X^d} = (\hat{X}^d)^{-1} Y \tag{14.}$$

Therefore, based on the $\Delta_{t=1} P^{Y_k}$ and the equation (12.2), the vector of derived total (relative) price changes $\Delta_{t=1} \tilde{P}^{sT}$ are obtained as,

$$\Delta_{t=1}\widetilde{P}^{sT} = (I - B)^{-1}\Delta_{t=1}P^{Y_k} \qquad (15.)$$

Because the total input vector in the next period, $_{t=1}X^{sT}$, is the sum of total input vector of pre-disaster and the monetary quantity changes vector after-disaster, that is, $_{t=0}X^{sT} + \Delta X^{sT}$, the total input changes in post-disaster $\Delta_{t=1}X^{sT}$ can be obtained by multiplying of total inputs and price changes in the next period as $_{t=1}\hat{X}^{sT}\Delta_{t=1}\widetilde{P}^{sT}$.

Therefore, even in the case that markets are out of equilibrium due to quantity losses caused by a disaster, the price-type supply-driven model can still be applied to G-space to estimate the total input losses for consumers due to the increase of prices if there are exogenous price elasticities of demands.

## VI. Conclusions

In the Leontief comparative static analysis, we pass from one equilibrium to another. It is clear, however, that in reality, the economy traverses a period of disequilibrium in between. In fact, it is possible to comment on the temporary disequilibria in terms of the various models that have been discussed along with exogenously provided information on selected price elasticities of demand. Under normal economic equilibrium conditions, quantities are more or less fixed, but prices change, and hence Dietzenbacher's (1997) suggestion is useful.

However, abnormal economic cessations such as caused by natural disasters will temporarily produce quantity losses and lead to further economic losses via interindustrial and interregional relations. As Dietzenbacher has noted, the basic interpretation of the supply-driven model is via price interrelations. However, in static market equilibrium, producers will not change the current technical relationships that are based on historical sales during short run immediately after an exogenous event. This reflects the fact that Ghosh's supply-driven model is in terms of monetarily expressed quantities and hence is applicable.

Although Ghosh's allocation IO model is suggested based on limiting conditions, the implausibility debates have not focused on these aspects. According to his first conditions and the two dominant empirical applications of economic impacts by hypothetic inoperability of facilities, it is reasonable to assume four conditions when running the supply IO model: Monopoly characteristics, scarcity of inputs, short period, and small region depending much upon other regions. Based on these conditions, I have supplemented Dietzenbacher's interpretation that showed weaknesses in understanding economic impact analyses by monetary quantity losses.

To suggest a new interpretation for the supply-driven IO model, I introduced four quadrant spaces of economic situations based on 'price vs. price-quantity' and 'increase-decrease' axes. The analysis of the model space shows that Dietzenbacher's suggestion is useful for three quadrants, although his focus is only on the first and second 'price increase' quadrant, or D-space. Also, Oosterhaven's focus on the supply-driven model is in the third 'price- (monetary) quantity increase' quadrant, or O-space. Finally, Ghosh's suggestion is most useful for (equivalent to) the 'price- (monetary) quantity decrease' quadrant, or G-space, in market systems, where supply-driven model can be used if normal static equilibrium continues. Furthermore, even in the case that normal equilibrium could not be maintained, the G-space can still be useful because the price-type supply-driven model plays an important role in estimating the economic impacts in the abnormal equilibrium status. To address the switching processes, I introduced an exogenous price elasticity of demand and combined it with the classic supply-driven model, adjusting monetary quantity impacts to the price impacts.

Input-output models are attractive because they can be made operational and accessible at low cost. I have tried to unscramble the various positions taken and have contrasted them in Figure 1. When the general price level moves up or down, the supply model highlights the actual absolute price transmission linkages. This is the Dietzenbacher view. Oosterhaven's implausibility criticism is most applicable when a positive value added change is presumed to increase all forward transactions. This leaves us with the lower-left quadrant. Ghosh's original position is most plausible in the downward direction: a *downward* shift in value added inputs *can* put limits on the forward transactions.

The power and the usefulness of linear models are enhanced once the applicability of the supply model is clarified.

## References


Bon, R. and T. Yashiro, 1995, Comparative Stability Analysis of Demand-side and Supply-side Input-Output Models: The Case of Japan, 1960-90, *Applied Economics Letters*, Vol. 3: 349-354.

Bon, R., 2001, Comparative Stability Analysis of Demand-side and Supply-side Input-Output Models: Toward Index of Economic "Maturity", in Lahr, M.L. and E. Dietzenbacher eds., *Input-Output Analysis: Frontiers and Extensions*, NY, Palgrave.

Cai, J., P. Leung, and J. Mak, 2006, Tourism's Forward and Backward Linkages, *Journal of Travel Research*, Vol. 45, No. 1: 36-52.

Cai, J., P. Leung, M. Pan, and S. Pooley, 2005, *Linkage of Fisheries Sectors to Hawaii's Economy and Economic Impacts of Longline Fishing Regulations*, University of Hawaii - NOAA, Joint Institute for Marine and Atmospheric Research, Honolulu, Hawaii.

Chen, C.Y. and A. Rose, 1991, The Absolute and Relative Joint Stability of Input-Output Production and Allocation Coefficients, in Peterson, W., ed., *Advances in Input-Output Analysis*. NY, Oxford University Press.

Davis, H.C. and E.L. Salkin, 1984, Alternative Approaches to the Estimation of Economic Impacts Resulting from Supply Constraints, *The Annals of Regional Science*, Vol.18: 25-34.

Dietzenbacher, E., 1997, In Vindication of the Ghosh Model: A Reinterpretation as a Price Model, *Journal of Regional Science*, Vol.37: 629-651.

Dietzenbacher, E., 2002, Interregional Multipliers: Looking Backward, Looking Forward, *Regional Studies*, Vol. 36, No.2:125-136.

Ghosh, A., 1958, Input-Output Approach to an Allocative System, *Economica*, Vol. XXV: 58-64.

Ghosh, A., 1964, *Experiments with Input-Output Models: An Application to the Economy of the United*

*Kingdom, 1948-55*, London, Cambridge University Press.

Giarratani, F., 1976, Application of an Interindustry Supply Model to Energy Issues, *Environmental Planning A*, Vol. 8: 447-454.

Gruver, G.W., 1989, On the Plausibility of the Supply-driven Input-Output Model: A Theoretical Basis for Input-coefficient Change, *Journal of Regional Science*, Vol.29, 441-450.

Hewings, G..J.D., 1985, *Regional Input-Output Analysis*. Beverly Hills, CA: Sage Publications, Inc.

Leontief, W., 1936, Quantitative Input and Output Relations in the Economic System of the United States, *Review of Economic Statistics*, Vol. XVIII, No. 3: p.105-125.

Leontief, W., 1941, *The Structure of American Economy, 1919-1929: An Empirical of Equilibrium Analysis*, Cambridge, MA, Harvard University Press.

Leontief, W., 1976, *The Structure of American Economy, 1919-1939: An Empirical of Equilibrium Analysis*, White Plains, NY, International Arts and Science Press, Inc.

Oosterhaven, J., 1988, On the Plausibility of the Supply-driven Input-Output Model, *Journal of Regional Science*, Vol.28: 203-217.

Oosterhaven, J., 1989, The Supply-driven Input-Output Model: A New Interpretation but Still Implausible, *Journal of Regional Science*, Vol.29: 459-465.

Oosterhaven, J., 1996, Leontief versus Ghoshian Price and Quantity Models, *Southern Economic Journal*, Vol. 62, No.3: 750-759.

Pindyck, R.S. and D.L. Rubinfeld, 1998, *Microeconomics*, NJ. Prentice-Hall, Inc.

Rose, A. and T. Allison, 1989, On the Plausibility of the Supply-driven Input-Output Model: Empirical Evidence on Joint Stability, *Journal of Regional Science*, Vol.29: 451-458.

Samuelson, P.A. and W.D. Nordhaus, 1995, *Economics*, McGraw-Hill, Inc.

Wang, E.C., 1997, Structural Change and Industrial Policy in Taiwan, 1966-91: An Extended Input-Output Analysis, *Asian Economic Journal*, Vol. 11, No. 2: 187-206.