Improved reservoir sizing utilizing observed and reconstructed streamflows within a Bayesian combination framework

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Abstract Reservoir sizing is a critical task as the storage in a reservoir must be sufficient to supply water during extended droughts. Typically, sequent peak algorithm (SQP) is used with observed streamflow to obtain reservoir storage estimates. To overcome the limited sample length of observed streamflow, synthetic streamflow traces estimated from observed streamflow characteristics are provided with SQP to estimate the distribution of storage. However, the parameters in the stochastic streamflow generation model are derived from the observed record and are still unrepresentative of the long-term drought records. Paleo-streamflow time series, usually reconstructed using tree-ring chronologies, span for a longer period than the observed streamflow and provide additional insight into the preinstrumental drought record. This study investigates the capability of reconstructed streamflow records in reducing the uncertainty in reservoir storage estimation. For this purpose, we propose a Bayesian framework that combines observed and reconstructed streamflow for estimating the parameters of the stochastic streamflow generation model. By utilizing reconstructed streamflow records from two potential stations over the Southeastern U.S., the distribution of storage estimated using the combined streamflows is compared with the distribution of storage estimated using observed streamflow alone based on split-sample validation. Results show that combining observed and reconstructed streamflow yield stochastic streamflow generation parameters more representative of the longer streamflow record resulting in improved reservoir storage estimates. We also generalize the findings through a synthetic experiment by generating reconstructed streamflow records of different sample length and skill. The analysis shows that uncertainty in storage estimates reduces by incorporating reconstruction records with higher skill and longer sample lengths. Potential applications of the proposed methodology are also discussed.

1. Introduction

The primary purpose of a reservoir is to supply water for the specified uses (including ecological needs) even under severe drought and also to protect the downstream from severe floods. Typically, reservoir sizing requires estimation of storage for the design yield/releases with specified reliability. The required storage is often estimated using the sequent peak algorithm (SQP) using the observed streamflow at a given location [Thomas and Burden, 1963; Vogel and Stedinger, 1987; Vogel, 1988]. SQP is a sequential process in which the reservoir storage is estimated based on the maximum cumulative deficit accounted for a given streamflow series under the desired annual yield. One limitation of this approach is that only one storage estimate can be obtained from an observed streamflow trace. To overcome this, many studies have employed synthetic streamflow traces so that the probability of meeting a given demand can be calculated [Stedinger and Taylor, 1982; Vogel and Stedinger, 1987]. Typically, multiple synthetic streamflow realizations having the length of observed annual streamflows are generated from a distribution and multiple realizations of storage estimates are obtained using SQP [Vogel, 1988]. Most commonly, a lognormal model is used to generate synthetic streamflow traces since it ensures generated streamflows to be positive and also accommodates for the lag-one correlation and skewness in the observed annual streamflow [Vogel, 1988].

Although synthetic streamflow generation provides distribution of reservoir storage, storage estimate for the desired reliability is derived only using the observed streamflow characteristics. Multiple traces generated by the stochastic streamflow generation models only address the sampling variability in storage estimates and they do not include information on extremes outside the envelope of instrumental record.
Further, observed streamflow is limited to around 80 years of data in the continental U.S., thereby resulting increased uncertainty on estimating reservoir yields corresponding to high reliabilities (i.e., 95% or more). For robust design and planning of water supply systems, information related to prehistoric events could be very useful in reducing the uncertainty on reliability estimates. Studies have used paleo-flood records with observed annual maximum time series for improving design flood estimates [Hosking and Wallis, 1986; Stedinger and Cohn, 1986]. Studies have also used paleo-proxies such as tree rings to extend the streamflow record beyond the observational period and such reconstructed series provides insight into the preobservational drought record [Cook and Jacoby, 1983; Gangopadhyay et al., 2009; Woodhouse, 2001; Woodhouse et al., 2006, 2010; Devineni et al., 2013]. However, the reconstructed streamflow using tree-ring chronologies underestimates high flow events indicating underestimation of annual streamflow characteristics, which obviously limits their use in the design of water resources systems [Prairie et al., 2008]. This limitation can be partially addressed by including Sea Surface Temperature (SST) data such as El Nino Southern Oscillation (ENSO) or Pacific Decadal Oscillation (PDO) as streamflow reconstruction predictors [Anderson et al., 2012a, 2012b; Patskoski, 2014].

Apart from understanding the past hydroclimatic extremes using paleo-information, another potential utility of preinstrumental records is in improving the water resources design [Hosking and Wallis, 1986; Stedinger and Cohn, 1986]. Even though construction of new reservoirs is very limited in the U.S., we believe combining paleo and observed information to obtain stochastic streamflow generation parameters with the SQP will be useful for reallocation of existing storages within the existing system. The utility of reconstruction records could also be of increased value in developing countries where still there is scope for designing new reservoir systems and also will provide a way to augment limited streamflow records. Despite several efforts in improving the reconstructed streamflow, the values in the reconstructed time series typically underestimate low flow values and also the observed variance. One way to reduce the uncertainty in streamflow reconstruction is to resample observed flows based on the state of the reconstructions. Tercile categories of reconstructed flow (above normal or below normal) have been incorporated within a Bayesian framework for better representing the frequency of prehistoric droughts in generating synthetic streamflow traces for the Colorado River [Prairie et al., 2008]. Still, this approach obtained generated streamflow using observed streamflow records conditional on the generated tercile categories from reconstructed flows. Henley et al. [2011] used a hierarchical Bayesian framework to generate seasonal rainfall using observed and reconstructed Interdecadal Pacific Oscillation and Pacific Decadal Oscillation indices. Like in Prairie et al. [2008], the reconstructed data were used to generate the state of the precipitation (wet/dry), and observed precipitation was used to generate the precipitation time series conditional on the generated state.

The goal of this study is to investigate the utility of reconstructed streamflow obtained from tree-ring chronologies in reducing the uncertainty in reservoir sizing. For this purpose, we consider reservoir storage estimation for two Hydro-Climatic Data Network (HCDN) basins that has long tree-ring chronologies over the southeast U.S. We estimate the parameters of the stochastic streamflow generation model by combining the reconstructed streamflow time series and the observed streamflow within a Bayesian framework. Bayesian frameworks have been used to generate streamflow on an annual scale due to the ability to incorporate model uncertainty [Vicens et al., 1975; Valdés et al., 1977]. Given the inherent ability of Bayesian approach in maximizing the joint likelihood of the observed and reconstructed streamflows, we consider them for the estimation of posterior distribution of lognormal parameters conditional on both observed and reconstructed flows. The synthetic streamflow traces generated from the parameters of the combined model will be used with the SQP to obtain the distribution of reservoir storage conditional on both observed and reconstructed streamflow. Based on split-sample validation, the estimated distribution of storage from the combined approach for the selected stations will be evaluated against a known true distribution of storage obtained from the entire observational record. The added value of reconstructed streamflow in reducing the uncertainty in storage estimates is also quantified by comparing the distribution of storage estimated using the observed streamflow alone under the split-sample experiment. To generalize the findings from the study, we consider additional experiments to understand how the uncertainties in storage estimates could be reduced as a function of the skill and length of reconstructed streamflow time series. Here, the skill is defined based on coefficient of determination, which indicates the ability of the streamflow reconstruction model in explaining the observed streamflow variance.
The manuscript is organized as follows: section 2 describes the Bayesian framework considered for demonstrating the utility of reconstructed streamflows in improving the reservoir design problem. Following that, we present the experimental design and results from a split-sample validation that quantify the added value of reconstructed flows in reducing the uncertainty in reservoir storage estimates for the selected two USGS streamflow stations. Section 4 discusses the results from a synthetic experiment that generalizes the findings on the role of sample length and skill of reconstructed flows in reducing the uncertainty in reservoir sizing. Finally, we summarize the salient findings and conclusions along with the potential utility of reconstructed streamflows in improving water supply system design.

2. Reservoir Storage Estimation Using Observed and Reconstructed Streamflows

The intent of this study is to assess the utility of reconstructed streamflows in reducing the uncertainty on reservoir sizing using SQP. For this purpose, we consider observed and reconstructed streamflows from two undeveloped basins from the Hydro-Climatic Data Network (HCDN) [Slack et al., 1993; Vogel and Sankarasubramanian, 2003; Sankarasubramanian and Vogel, 2003; Vogel and Sankarasubramanian, 2005] and combine them within a Bayesian framework for estimating the reservoir storage. Streamflow stations in the HCDN are nearly void of upstream storage and groundwater pumping thereby modulating well with the variability exhibited in tree-ring chronologies. For this study, we selected reconstructed streamflow time series considered by Patskoski et al. [2015] who considered both tree-ring chronologies and Nino3.4, anomalous Sea Surface Temperature (SST) conditions in the tropical Pacific, which influence the hydroclimatology of the southeastern U.S. [Oh and Sankarasubramanian, 2012; Almanaseer and Sankarasubramanian, 2012]. The locations of the selected streamflow stations and the tree-ring chronologies used for reconstruction can be seen in Figure 1 and Table 1. The tree-ring chronologies used for reconstruction were obtained from the National Atmospheric and Oceanic Administration (NOAA) International Tree Ring Data Bank (ITRDB) [available online at http://www.ncdc.noaa.gov/paleo/treering.html]. Details of the streamflow reconstruction methodology using tree-ring chronologies and Nino3.4 for these two stations along with the comparison between the observed streamflow ($Q_t$) and reconstructed flows ($Q_r$) are provided in the next section. Following that, we discuss the SQP algorithm and the Bayesian framework for merging reconstructed flows and observed flows.

Table 1. List of USGS Streamgage Stations and the Corresponding Tree-Ring Chronologies Used for Annual Streamflow Reconstruction

<table>
<thead>
<tr>
<th>Station Index</th>
<th>USGS Station Number</th>
<th>Average Annual Streamflow (cfs)</th>
<th>Drainage Area (mi²)</th>
<th>Tree Chronologies Used (ITRDB Number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>02132000</td>
<td>1099</td>
<td>1030</td>
<td>SC004, SC006, NC008</td>
</tr>
<tr>
<td>2</td>
<td>02236000</td>
<td>3055</td>
<td>3066</td>
<td>FL007, FL008</td>
</tr>
</tbody>
</table>

Figure 1. Location of USGS streamflow stations and the tree-ring chronologies used for reconstruction in the study.
2.1. Description of Streamflow Reconstruction Model

Tree-ring chronologies serve as the primary indication of moisture availability in streamflow reconstruction. Studies have shown the strong association between tree-ring chronologies El-Nino Southern Oscillation (ENSO) and annual streamflow over the Southeast U.S. [Stahle et al., 2012; Patskoski et al., 2015]. Patkoski et al. [2015] considered eight virgin basins from the Southeast U.S. which are part of the Hydro-Climatic Data Network (HCDN) whose streamflow data are not influenced by groundwater pumping or upstream storage [Vogel and Sankarasubramanian, 2005]. Patkoski et al. [2015] proposed a hybrid reconstruction methodology that uses both Nino3.4 and tree-ring chronologies (SST-TR) for reconstructing the annual streamflow at eight HCDN sites and found that the SST-TR methodology performed better than the traditional principal component regression model. We briefly describe the methodology and the performance of SST-TR model.

2.1.1. Reconstructed Streamflows Using SST and Tree Rings (SST-TR)

The SST-TR model considers the addition of Sea Surface Temperatures (SSTs) to the predictors to reduce the underestimation of high flow values. Given that wet and dry periods in the basins from the Southeast U.S. are primarily influenced by ENSO, the inclusion of SSTs improve the high flow estimation based on the ENSO conditions [Patkoski et al., 2015]. The ENSO state can be described by the Nino3.4 Index which is the average SST anomaly over 120°W–170°W and 5°S and 5°N and has a frequency of 3–7 years. For this study, we consider the average Nino3.4 over the period October–September for reconstructing the annual streamflow during the corresponding water year. The role of ENSO in modulating seasonal precipitation and temperature [Devineni and Sankarasubramanian, 2010a,b], thereby the observed streamflow and groundwater have been well documented over the southeastern U.S. [Almanaseer et al., 2012; Oh and Sankarasubramanian, 2012]. Since ENSO, is quasiperiodic, its component in annual streamflow can be identified. Singular Spectrum Analysis (SSA), a time series approach that quantifies the lag dependence in streamflow through estimates of lagged cross-covariance [Ghil et al., 2002], is used in the SST-TR model to identify the ENSO component of annual streamflow. Nino3.4 is then used to estimate the identified component of ENSO, and tree-ring chronologies are used to estimate the remaining component of streamflow. However, the tree-ring chronologies also have an ENSO component, so SSA is used to remove the ENSO component from the tree-ring chronologies. The non-ENSO component of the tree-ring chronologies serves as the predictor for the non-ENSO component of streamflow. Finally, the ENSO and non-ENSO components are added together for each year to obtain the SST-TR estimate. Finally, a model combination was performed and the reconstructed flows from the combined SST-TR model had a correlation of 0.62 (0.64) for station 02132000 (station 02236000). For this study, we considered the reconstructed annual streamflow time series for the stations 02132000 and 02236000 from the combined SST-TR model (Figure 2). For additional details about the SST-TR algorithm-based reconstruction, see the full dissertation in the NCSU digital repository [Patkoski, 2014].

2.2. Reservoir Storage Estimation Using Sequent Peak Algorithm

Reservoir storage is typically estimated using SQP based on multiple traces of synthetic streamflows which are generated using the parameters of the lognormal distribution [Vogel and Stedinger, 1987; Lall and Miller, 1988]. Alternate optimal reservoir sizing techniques have also been proposed that include application of optimal control theory [Mousavi and Ramamurthy, 2000] as well as based on satellite imagery [Baban and Wan-Yusof, 2002] and remote sensing techniques [Schumann and Geyer, 1997]. In this study, we merge the time series of observed annual streamflows and reconstructed annual streamflows for estimating the parameters of the lognormal distribution, which are usually specified based on the mean (μ), standard deviation (σ), and lag-one correlation (ρ) of the log of the annual flows. Bayesian frameworks have been used to generate streamflow on an annual scale due to the ability to incorporate model uncertainty [Vicens et al., 1975; Valdés et al., 1977]. Bayes theorem obtains posterior probability density functions (PDFs) of parameters, $θ = (\mu, \sigma, \rho)$, of the lognormal distribution for a given data set, Y, which consists of both observed and reconstructed streamflow time series. Given $\mu$, $\sigma$, and $\rho$, flows in the log space having a sample length of n (t = 1, 2, ..., n) could be generated using equation (1) where $ε_t$ follows standard normal distribution.

$$Y_{t+1} = Y_t + \mu_t + \epsilon_t \sigma Y_t \left(1 - \rho_t^2\right)^{1/2}$$  (1)
Multiple realizations of streamflows with each having length \( n \) could be generated based on equation (1) by transforming the flows back into the original space \( (Q_{0t} = \exp(Y_t)) \) [Stedinger and Taylor, 1982; Vogel and Stedinger, 1987; Vogel, 1988]. Each realization of generated streamflow can be used with SQP (equations (2) and (3)) to obtain one estimate of reservoir storage [Thomas and Burden, 1963].

\[
S = \text{Max}(S_t)
\]

\[
S_t = \begin{cases} 
S_{t-1} + D + E - Q_{0t} & \text{if positive} \\
0 & \text{otherwise}
\end{cases} \quad t = 1 \ldots n
\]

The reservoir storage, \( S_t \), required at time \( t \) is the sum of the storage required at the previous time step plus the specified water supply demand \( (D) \) and annual evaporation \( (E) \) minus the generated streamflow \( (Q_{0t}) \). If the generated inflow is greater than the previous storage, demand, and evaporation, the required storage at time \( t \) is zero. The reservoir storage \( (S) \) required for the entire trace \( (t = 1 \ldots n) \), where \( n \) is the length of the streamflow record) is found by taking the maximum of the storage over the entire trace. The initial storage \( (S_0) \) is typically set to zero, which could result in an underestimation of storage in the first few realizations that could be discarded. Thus, repeating this synthetic streamflow generation (equation (1)) and storage estimation (equations (2) and (3)) above for \( M \) realizations results in a distribution of storage from which an estimate of storage for the desired reliability could be estimated.

2.3. Bayesian Merging of Reconstructed Flows and Observed Flows

Recently, Henley et al. [2011] used a hierarchical Bayesian framework to obtain the parameters of gamma distribution representing the combined instrumental and paleo indices representing the Interdecadal Pacific Oscillation and Pacific Decadal Oscillation indices. Given the observed annual streamflow, \( Q_n \) for \( n \) years and reconstructed annual streamflow, \( Q_r \), for \( n_R \) years for a given station, we merge these two time
series and estimate the parameters, $\theta$, of the lognormal distribution using a Bayesian framework for obtaining the storage distribution. This could be written using the Bayes formulation as

$$P(\theta|Y_t, Y'_t) = \frac{P(Y_t, Y'_t|\theta)P(\theta)}{P(Y_t, Y'_t)}$$

where $Y_t$ and $Y'_t$ denote the log-transformed time series of $Q_t$ and $Q'_t$, respectively, $\theta$ denotes the parameters of interest, $P(\theta|Y_t, Y'_t)$ is the the posterior distribution of parameters $\theta$ given both data sets, $P(Y_t, Y'_t|\theta)$ denotes the joint likelihood of log-transformed observed flows and log-transformed reconstructed flows, and $P(\theta)$ indicates the prior distribution. The prior distribution is a PDF representing previous knowledge on the parameters. It can represent a strong belief, or it can be uninformative where each parameters has equal probability of occurrence. The marginal likelihood, $P(Y_t, Y'_t)$, is a constant, so equation (4) could be written as

$$P(\theta|Y_t, Y'_t) \propto P(Y_t, Y'_t|\theta)P(\theta)$$

Next, we discuss the steps involved in estimating the posterior distribution conditional on both reconstructed flow and observed flow.

### 2.3.1. Step A: Calculate the Likelihood

Given $n_R$ years of reconstructed flows and $n$ years of observed flows, we model the annual flows as lognormal (i.e., log-transformed flows follows normal). Consider a matrix, $Y$, of size $N \times 1$ with $N = n_R + n$ denoting the length of the merged time series of log-transformed reconstructed flows and observed flows. If the log-transformed flows, $Y$, are independent, then the joint likelihood function, $P(Y|\theta)$, for the merged data set can be found by multiplying the likelihood of each individual $Y$ for the sampled parameters, $\mu$ and $\sigma$, from the prior distribution. Since annual streamflow exhibits year-to-year dependence, the joint likelihood estimate, $P(Y|\theta)$, will be estimated based on the $N$ correlated normal variables with each following mean $\mu_Y$ and variance-covariance matrix ($\Sigma_Y$) of size $N \times N$ (equation (6)).

$$P(Y|\mu_Y, \Sigma_Y) = \frac{1}{(2\pi)^{n/2}\sqrt{\det(\Sigma)}} \exp\left[ -\frac{1}{2} \left( Y_1 - \mu_Y, \ldots, Y_N - \mu_Y \right) \Sigma_Y^{-1} \left( Y_1 - \mu_Y, \ldots, Y_N - \mu_Y \right)^\top \right]$$

where $\Sigma_Y =

\begin{bmatrix}
\sigma^2_Y & \rho_Y \sigma^2_Y & 0 & 0 & \cdots & 0 \\
\rho_Y \sigma^2_Y & \sigma^2_Y & \rho_Y \sigma^2_Y & 0 & \cdots & 0 \\
0 & \rho_Y \sigma^2_Y & \sigma^2_Y & \cdots & \cdots & 0 \\
0 & 0 & \cdots & \cdots & \cdots & \rho_Y \sigma^2_Y \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \cdots & \cdots & \sigma^2_Y \\
0 & 0 & \cdots & \cdots & \cdots & \rho_Y \sigma^2_Y \end{bmatrix}

(7)

Given that annual streamflow exhibit significant dependency only up to lag-one autocorrelation for the selected stations, thus the estimated covariance matrix results in a banded matrix as shown in equation (7).

### 2.3.2. Step B: Obtain the Prior Distributions

The prior distributions of parameters, $\theta = \{\mu_Y, \sigma_Y, \rho_Y\}$, will be uninformative indicating each parameter follows a uniform distribution. Each parameter will initially be set to the mean, standard deviation, and lag-one correlation of the observed streamflow data, and will be updated using a proposal distribution discussed in the following section. The prior distributions are used to define the range for each parameter. For the mean, since it represents the mean of the log of the flow values, the prior distribution of $\mu_Y$ will follow uniform distribution which makes the probability of all values of the mean a constant thereby canceling out in the proportionality expressed in equation (5). Given that standard deviation, $\sigma_Y$, must be greater than...
zero, we assume all values over zero are equally likely. Increased values of interannual variability in streamflow, $\sigma_r$, result in requirement of additional reservoir storage. For instance, in humid (arid) basins, the interannual variability in streamflow is smaller (larger), thereby within-year (over-year) reservoir systems with relatively smaller (larger) storages are common over the eastern (western) U.S. Thus, the prior distribution for the standard deviation could be written based on equation (8). The value of $c_r$ in equation (8) is a very small number since there are infinite possibilities for the standard deviation value. For the purpose of this study, we assumed the range of standard deviation was assumed to be 3 times the observed standard deviation of log-transformed flows.

$$P(\sigma) = \begin{cases} c_r & \text{if } \sigma > 0 \\ 0 & \text{otherwise} \end{cases}$$  

(8)

The lag-one autocorrelation, by definition, must have a value between $-1$ and 1. The short-term persistence, $\rho$, provides the characteristics of the inflows to change from one year to next and high values of lag-one correlation could be misinterpreted as a trend [Matalas and Sankarasubramanian, 2003]. For instance, high positive values of lag-one correlation indicates flood (or drought) in a given year to be followed by the same type of event in the next year, which usually requires increased storage to carry over the surplus/deficit from one year to the next [Vogel and Stedinger, 1987]. Thus, all values in this range are equally likely which results in the uniform density being equal to 0.5 (equation (9)).

$$P(\rho) = \begin{cases} 0.5 & \text{if } |\rho| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$  

(9)

Assuming that the mean, standard deviation, and lag-one autocorrelation are independent, their joint prior can be found by multiplying each individual prior distribution (equation (10)).

$$P(\mu, \sigma, \rho) = P(\mu)P(\sigma)P(\rho) = \begin{cases} c = 0.5 + c_r & \text{if } \sigma > 0 \& |\rho| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$  

(10)

2.3.3. Step C: Combining the Likelihood and Prior

Combining the likelihood and the prior distribution, the posterior distribution of the lognormal parameters could be expressed as follows (equations (11) and (12)):

$$P(\mu, \sigma, \rho | Y) \propto P(Y | \mu, \sigma, \rho) P(\mu, \sigma, \rho)$$  

(11)

$$P(\mu, \sigma, \rho | Y) \propto \begin{cases} P(Y | \mu, \sigma, \rho) & \text{if } \sigma > 0 \& |\rho| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$  

(12)

2.3.4. Setting Up the Markov Chain Monte Carlo (MCMC) Algorithm

The posterior distribution of lognormal parameters will be obtained using the Bayesian framework in equation (11) using MCMC based on Metropolis-Hastings algorithm [Metropolis et al., 1953; Hastings, 1970]. MCMC operates by changing the parameters in a Bayesian framework using a proposal distribution. With an initial set of streamflow parameters, the likelihood of the log-transformed streamflow time series conditional on those streamflow parameters (equation (12)) is calculated. New parameters are then generated using the selected proposal distributions, which are assumed to follow independent normal distribution. MCMC basically samples from the proposal distribution many times according to Metropolis-Hastings algorithm, so that the likelihood of the parameter conditioned on the log-transformed streamflow time series is increased at each iteration of the Markov chain. The proposal distribution updates mean, standard deviation, and lag-one autocorrelation one at a time with all distributions being normal with a mean of the previous value of a parameter and a standard deviation, which is specified so that the acceptance rate of a given parameter is between 0.2 and 0.4. This acceptance rate is desired because it is a good indication that the posterior distribution has converged. We also looked at trace plots and used a burning period of 1000 iterations for each MCMC run to ensure that each posterior distribution had converged. A posterior distribution should also be independent, so each posterior distribution was filtered so that the lag correlation between
the posterior distribution values was not significant. MCMC was run so that the posterior distribution after filtering had a sample size above 3000.

The Bayesian model can be used on any observed streamflow time series, but combining the observed streamflow and reconstructed streamflow poses serious challenges since the variance of the reconstructed streamflow time series, which is a model-based estimate, is always lesser than the variance of the observed streamflow. To address this issue, a Gaussian noise with mean equal to zero and variance equal to the residual variance from the reconstruction model will be added to the reconstructed streamflow time series at the beginning of each MCMC iteration. Given this stationarity assumption on the variance, the Bayesian methodology primarily updates the mean and lag-one correlation of lognormal flows by combining reconstructed streamflows with observed streamflow. However, the variance and the assumed probability distribution function could change under future climate change. We discuss the implications of these assumptions and how to model under such situations in section 5. Thus, the reconstructed streamflow with noise will subsequently be merged with the observed streamflow time series chronologically before estimating the likelihood of the merged sample conditional on the new set of parameters generated from the proposal distribution.

3. Utility of Reconstructed Flows in Reservoir Sizing—A Split-Sample Experiment

One of the limitations of using reconstructed streamflow in reservoir sizing is that the true values of streamflows are unknown. A split-sample validation was designed to evaluate the added utility of reconstructed streamflows in improving the reservoir sizing along with observed flows (Figure 3). Table 2 provides the details time periods considered for the split-sample validation experiment. We first estimated the storage distribution using the entire observed streamflow time series and we refer this as “All Instrumental (AI)” storage distribution (white in Figure 3). For a given station having both entire streamflow and reconstructed streamflow over the instrumentation period, we performed split-sample validation by estimating the storage distribution using the instrumental streamflows for a limited period of the record (light gray in Figure 3) and we refer this as the (“Limited Instrumental (LI)”) LI storage distribution (dark gray in Figure 3). Next, we combined the “LI” period streamflow with the reconstructed streamflow and estimated the storage distribution, which we refer to as “Instrumental + Reconstructed (IR)” storage distribution. To begin with, we first obtained the posterior distribution of the parameters of the streamflow generation model from the Bayesian model (equation (5)) by considering Y as “AI” streamflow alone without any reconstructed streamflows. To obtain a synthetic streamflow trace from the posterior distribution, a set of mean, standard deviation, and autocorrelation was selected and a streamflow trace corresponding to the length of “AI” streamflow was generated using equation (1). Then, SQP (equations (2) and (3)) algorithm was used on this realization to get a storage value. This process was repeated over all the samples from the posterior distribution for obtaining the “AI” storage distribution.

For the split samples (Table 2), we first obtained the posterior distribution of the parameters of the streamflow generation model from the Bayesian model (equation (5)) by considering Y as the “LI” streamflow alone without any reconstructed streamflows. Based on this posterior distribution of parameters, we basically generated synthetic streamflow traces corresponding to the length of the “AI” streamflow and then applied the SQP algorithm to get the storage for the generated values. The primary reason that we generated synthetic streamflows corresponding to the length of “AI” streamflow was to eliminate the bias in comparing the three different (“AI”, “LI”, and “IR”) storage distributions due to difference in sample lengths. By repeating this process over all the converged chain from the posterior distribution of parameters under “LI” streamflow, we obtained the “LI” distribution of storages (dark gray in Figure 3) which did not include any reconstructed streamflow information.

To obtain reconstructed streamflow records for the “IR” period, reconstructed streamflow from the SST-TR model was used. For additional details on the SST-TR methodology, see Patksosk et al. [2015]. Brief description of the methodology is also provided as supporting information S1. Thus, for the split-sample validation, reconstructed streamflow time series for the left-out period were obtained only using the tree-ring chronologies, SST, and streamflow available over the “LI” period. Figure 3 shows the reconstructed streamflows for both approaches for the “LI” period and for the left-out period. Similar to the procedure described earlier, the “LI” streamflow record and the reconstructed streamflow records were combined to develop “IR” streamflow record, which was substituted in Y in the Bayesian model to obtain posterior distribution of
parameters for the stochastic streamflow generation model. Thus, the length of the combined streamflow, $Y$, is equal to the length of the “AI” streamflow. Using the posterior distributions of lognormal parameters, we obtained the “IR” distribution of storages (light gray Figure 3) using SQP based on the synthetic streamflow data of “AI” record length.

Table 2. Time Periods and the Reconstruction Skill for Each Station Used for the Split-Sample Validation Experiment and for the Synthetic Experiments

<table>
<thead>
<tr>
<th>Station Index</th>
<th>Split-Sample Validation</th>
<th>Synthetic Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“Limited Instrumental”</td>
<td>All Instrumental</td>
</tr>
<tr>
<td></td>
<td>Reconstructed Flows Period</td>
<td>Flows Period</td>
</tr>
<tr>
<td>2</td>
<td>1964–2003</td>
<td>1934–1963</td>
</tr>
</tbody>
</table>

*The variable, $n_l$, denotes the number of years of lognormal flows generated for estimating the storage using SQP under different streamflow combination procedures. Reported $\rho$ under split-sample validation (synthetic experiments) indicate the correlation between SST-TR-based flows for the reconstruction period and the observed flows during the “Limited Instrumental” (All Instrumental) period.
To apply the SQP for each of the three streamflow scenarios in Figure 3, we assumed the annual demand and evaporation (terms D and E in equation (2)) to be 80% of the mean annual streamflow (Table 2). Thus, for each parameter set of the lognormal parameters from the posterior distribution, we obtained a storage value by generating synthetic streamflow trace having length corresponding to the “AI” streamflow scenario. Thus, by synthetic generating streamflow of length corresponding to “AI” streamflow scenario under all the three cases of split-sample validation, we ensured that the information content of the reconstructed streamflow was carried out only in the estimation of lognormal parameters without influencing the sample length associated with the storage estimation. Based on the 10,000 values of lognormal parameters from the posterior distribution, we thus obtained 10,000 values of storages under each streamflow scenario. Using these 10,000 values of storage, we basically estimated different quantiles of the storages for the three streamflow scenarios. The performance of the storage distribution under “LI” streamflow and “IR” streamflow will be compared with the distribution of storage obtained under “AI” streamflow.

3.1. Split-Sample Validation—Results

Under split-sample validation, posterior distributions of lognormal parameters for the given streamflow scenario were first obtained and then followed by the storage distribution using SQP. We estimated the likelihood of the “AI” streamflow ($Y$, $p(Y|\theta)$, equation (6)) occurring using the posterior distribution of the parameters, $\theta$, obtained from the “LI” streamflow and the “IR” streamflow. Under this case, we simply considered the log-transformed “AI” streamflow ($Y_{AI}^*$) i.e., without generating the flows using the posterior distribution of parameters, $\theta$, under “AI” scheme) for each station to estimate the likelihood. Based on these two likelihood estimates, one can estimate the likelihood ratio (equation (13)) that denotes the ratio between the likelihood of “AI” streamflow occurring under the posterior distribution of lognormal parameters ($\theta_{AI}$) obtained from the “IR” streamflow to the likelihood of “AI” streamflow occurring under the posterior distribution of lognormal parameters ($\theta_{LI}$) obtained from the “LI” streamflow.

$$LR = \frac{p(Y_{AI}^*|\theta_{IR})}{p(Y_{AI}^*|\theta_{LI})}$$

(13)

This results in a distribution of likelihood ratio (Figure 4) whose value above 1 indicates that the lognormal parameters estimated using “IR” streamflow have higher likelihood of estimating the “AI” streamflow.

From the distribution of likelihood ratio (Figure 4), there is more mass above 1.0 meaning the streamflow generation parameters from the “IR” streamflow are more representative of the “AI” streamflow characteristics than the parameters from the “LI” streamflow alone. Furthermore, the mass above 1.0 increases as we increase the length of the reconstructed time series for each station. It is important to note that the addition of reconstructed streamflow with the “LI” streamflow only results in improved estimation of lognormal parameters as the sample length of $Y_{AI}^*$ being just the observed log-transformed flows available at each station. This validates the argument that combining observed and reconstructed streamflow will reduce uncertainty in estimating the lognormal parameters for generating synthetic streamflow traces. We also infer that the posterior distribution of the lognormal parameters estimated using “IR” streamflow improves the estimation of the likelihood of the “AI” streamflow as we keep adding more reconstructed streamflows with the “IR” streamflow (Figure 4). This confirms that including reconstructed streamflows with the instrumental flows could provide additional information toward the better estimation of annual streamflow parameters for the preinstrumental period.

Posterior distributions of lognormal parameters from “IR” streamflow, “LI” streamflow, and “AI” streamflow were subsequently used to generate 10,000 sets of synthetic time series of lognormal streamflow corresponding to the length ($n_i$ shown in Table 2 for each station) of “AI” streamflow for estimating the posterior distribution of storage using SQP. We ensure that under all the three schemes, the generated lognormal streamflow length always corresponds to the length “AI” streamflow to eliminate the bias due to difference in sample length in the estimation of storage distribution. Figure 5 shows the difference between the quantiles of the storage distribution obtained using the “LI” streamflow and using “IR” streamflow with the quantiles of the storage distribution using “AI” streamflow. From Figure 5, we infer that the storage estimates obtained using the “IR” streamflow are closer to the storage estimates from the “AI” streamflow in comparison to the storage estimates obtained using “LI” streamflow alone (Figure 5). More importantly, uncertainty reduction resulting from “IR” streamflow on the upper quantiles is quite significant. This is very critical for
reservoir sizing since reservoirs are often sized based on the 95th quantile (i.e., 95% reliability for the assumed demand). The magnitude of this difference can be particularly attributed to the posterior distributions of the mean parameter, \( \mu \), standard deviation, and lag-one autocorrelation from the Bayesian combination. In reservoir storage estimation, the upper quantiles of storage depends on the streamflow traces that correspond to lower mean parameters. Therefore, the streamflow combination which best estimates the lower quantiles of the \( \mu \) parameter would yield the best reservoir storage estimates. In both stations 1 and 2, the “IR” streamflow better estimates the 10th and 25th quantiles than the “LI” streamflow (Table 3), yielding better reservoir storage estimates in the upper quantiles.

Figure 5 also shows that addition of longer reconstruction records produces better storage estimates. More precisely, in station 2, the deviation from the true storage distribution decreases as the length of the used reconstructed streamflow increases for each quantile. This confirms the argument made in section 1 that including reconstructed streamflow in reservoir design could potentially improve storage estimation for the desired reliability. Furthermore, it demonstrates that longer reconstructed streamflow records will reduce the uncertainty in storage estimation. It is important to note that the uncertainty reduction on storage estimates in station 1 is lesser as we add more reconstruction data for reservoir sizing. One reason for this is due to the length of the “AI” streamflow considered for split-sample validation for station 1 and also due to the limited skill of reconstructed flows in explaining the observed variability. Station 1 has 56 years of streamflow data, which means only 16 years of reconstructed flows were added since the remaining 40 years were used for model regression. On the other hand, in station 2, we can add up to 30 years of reconstructed streamflows under split-sample validation, thereby resulting in smaller difference in storage estimates between the “IR” streamflow and the “AI” streamflow. It is important to note however, that including reconstructed streamflow in station 1 still improved the storage estimates in the upper quantiles. Thus, the split-sample validation experiment shows that adding reconstructed streamflows reduces the uncertainty in reservoir sizing. As we add more reconstruction record, we further reduce the uncertainty in storage estimation for reservoir sizing.

To understand why addition of reconstruction streamflow records reduces the uncertainty in storage estimation, we compare the posterior distribution of lognormal mean (Table 3) parameter from the “IR” streamflow and from the “LI” streamflow with the “AI” streamflow. The posterior distributions in Table 3 for the “IR” streamflow are given as a function of the length of reconstructed streamflow years (\( n_R \)) added. From Table 3, we can see that the posterior distribution of lognormal mean under the “IR” streamflow approaches the “AI” streamflow as we add more number of years of reconstructed streamflow under “IR” streamflow. This is particularly clear for station 2 under which the posterior distribution of lognormal mean from the “IR” streamflow approaches...
the quantiles of the posterior distribution of the lognormal mean from the "AI" streamflow. It is important to note that under all the three schemes ("AI", IR," and "LI"), the generated lognormal streamflow length always corresponds to the length "AI" streamflow to eliminate the data redundancy issues in the estimation of storage distribution. Reduced uncertainty on the lognormal mean will result in reduced uncertainty estimates of the storage estimated for sizing the reservoir. Thus, the split-sample experiment clearly demonstrates that adding reconstructed streamflows with the observed streamflows within the Bayesian framework to obtain the lognormal distribution parameters for synthetic streamflow generation results in improved storage estimates for sizing the reservoir.

Table 3. Quantiles of the Posterior Distribution of the Lognormal Mean, $l_n$, Under "Limited Instrumental" (LI) Streamflow, "Instrumental + Reconstructed" (IR) Streamflow, and "All Instrumental" (AI) Streamflow or Both Stations* 

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Station Index 1</th>
<th>Station Index 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AI</td>
<td>LI</td>
</tr>
<tr>
<td>10</td>
<td>24.07</td>
<td>24.07</td>
</tr>
</tbody>
</table>

*The reconstructed streamflow length is indicated as (LI + $n_R$) where $n_R$ indicates the number of reconstructed years added from the SST-TR approach.
3.2. Reservoir Sizing Using Entire Observed and Reconstructed Streamflows

Findings from the split-sample validation results show that including reconstructed streamflow in reservoir sizing estimation reduces the uncertainty in storage estimates with the improvement being significant as the length of the reconstructed streamflow used increases. Reconstructed streamflows using both SST and tree rings are available for both selected stations from 1857 onward (see Table 2 for the period of records). Therefore, the best storage estimate obtainable would be to combine the entire observed streamflow record with the entire reconstructed streamflow record. By repeating the Bayesian combination procedure, we obtained posterior distribution of lognormal parameters under combined streamflow and observed streamflow. Using the samples from the posterior distribution of parameters, we obtained the posterior distribution of storages by generating lognormal flows having the length \( n_R \) given in Table 2 under synthetic experiments for both cases of observed streamflow and combined streamflow. The storage estimates using only the observed period and the combination of the entire reconstructed and observed streamflow records are shown in Figure 6 for two different demand scenarios namely 80% and 95% of the mean annual flow. For both stations, the required storage found by using only the observed streamflow is higher than the required storage obtained using both the observed and reconstructed streamflow records. If reservoirs were to be sized according to the 95th quantile using the combination of observed and reconstructed streamflow rather than only the observed flows, the required reservoir size for stations 1 and 2 would decrease 0.82 (9.2) and 1.06 (12.4) million ac ft respectively to supply 80% (95%) of mean annual flow. We also infer as the annual demand increases from 80% to 95%, the required storage increases. This means that the size decrease seen in Figure 6 will result in a much smaller area being affected by a reservoir. Thus, combining observed streamflow with reconstructed streamflow for reservoir sizing provides useful information about prehistoric flow conditions for estimating the reservoir size.

4. Role of Sample Length and Skill of Reconstructed Flows in Uncertainty Reduction on Reservoir Sizing—A Synthetic Experiment

Split-sample validation for the two selected stations shows that Bayesian combination of observed streamflow and reconstructed streamflow for reservoir sizing resulted in reduced uncertainty on the storage quantiles for the desired reliability. Both selected stations had reasonable skill in reconstructing the streamflow record using tree rings and SST. On the other hand, it is important to understand how the uncertainty reduction on reservoir sizing would change if the reconstruction skill were not as high as the current skill. Similarly, one would be interested in understanding how many years of reconstructed streamflows would be required to provide effective reduction on uncertainty in reservoir sizing. In this section, we address these two important questions on the role of (a) sample length \( n_R \) and (b) skill of reconstructed streamflows \( \rho_R \) in reducing the uncertainty on reservoir sizing through a synthetic experiment. For addressing both these questions, we considered only the 95th quantile of the storage distribution for evaluation. Appendix A details the generation of synthetic reconstructed flows with a specified skill, \( \rho_R \), and sample length \( n_R \).

The synthetic study considers two experiments with the first experiment focusing on the storage distribution obtained when synthetic reconstructed flows with a specified skill and sample length were combined with the observed flows. Analysis from the first experiment shows that utility of synthetic reconstructed flows having a skill poorer than the current level of skill could be beneficial for the estimation of reservoir storage. In this case, the upper bound on the synthetic reconstructed flows is the current skill. The number of years of synthetic reconstructed flows \( (t = 1, 2, \ldots, n_R) \) added always begins at the beginning of the instrument period and goes backward in time into the preinstrumental period. Using equation (A1), we generated corrupted reconstructed streamflow \((Q'_{C})\) of length \( n_R \) and skill \( \rho_R \) for both stations. These flows were log transformed and merged with the log-transformed observed streamflow (see time periods for the synthetic experiments in Table 2) available with the respective station. For instance, \( n_R = 10 \) for station 1 (station 2), then we generated synthetic reconstructed flows with the specified skill, \( \rho_R \), for the period 1920–1929 (1924–1933). Both the synthetic reconstructed flows and the observed flows were combined using the Bayesian merging procedure described in section 2.3. Thus, if \( n_R \) represents the number of years of reconstructed flows added, then the merged matrix, \( Y \), of size \( N \times 1 (N = n + n_R) \) could be used to estimate the posterior distribution of lognormal parameters for the combined flow using equation (12). Similarly, we also obtained the posterior distribution of lognormal parameters for the observed flows (i.e., matrix
Y having observed flows alone with size \( n \times 1 \) using equation (12). Using the samples from the respective posterior distribution, we generated lognormal flows (equation (1)) of sample length \( n \) and estimated maximum storage using the SQP (equations (2) and (3)) for the case of observed streamflow as well as combined streamflow. From 1000 sets of storage obtained under both cases, we estimated the difference in the 95th quantile of the storage (Figure 7) and it is plotted as a function of \( n_R \) and \( q_j \). It is important to note that we generated same sample length, \( n_l \), for both cases so that no bias is introduced due to difference in sample length in the estimation of 95th storage quantile.

Results from the first experiment (Figure 7) show the difference in the 95th quantile of storage distribution obtained using the entire observed and reconstructed streamflow time series (95th quantile of the storage for 80% of mean annual flow demand shown in Figure 6) and the 95th quantile of the storage distribution obtained using the entire observed streamflow (Figure 3) and synthetic reconstructed streamflow records with different sample lengths (\( n_R \)) and skill \((\rho_j)\) generated using equation (A1). Figure 7 shows the contour of the difference in storage for the 95th between the two scenarios of streamflows. For stations 1 and 2, the storage estimate approaches the true storage estimate (i.e., 95th

![Figure 6](image.png)

**Figure 6.** Storage distribution obtained by the Bayesian combination of observed and reconstructed (OR-80% and OR-95%) flows from SST-TR (a) station 1 and (b) station 2 model with demand being 80% and 95% of mean annual flow of both stations. Storage estimates from observed streamflow (O-80% and O-95%) alone are also given.
quantile for 80% of mean annual flow demand in Figure 6) as the reconstructed years added and the skill increases (Figure 7). This is expected since the true storage estimate is obtained using all of the uncorrupted reconstructed years. However, the number of reconstructed years added has a larger effect on reducing the differences in the 95th quantile storage estimate as opposed to the improvements that could be obtained with increased skill. It is important to note that the number of years of reconstructed flows, $n_R$, added only influences the estimation of posterior distribution of lognormal parameters not on the sample length of lognormal flows ($n_o$) generated for estimating the storage using SQP. This could be inferred with the contour lines being parallel to the $x$ axis which represents the skill of the synthetic reconstructed flows. Basically, combining the reconstructed streamflow with the observed flow results in stochastic streamflow generation parameters that better preserve the long-term characteristics of annual streamflow. We can also see that as expected the difference in storage approaches zero (right-hand

Figure 7. Difference between the 95th storage quantile obtained using observed flows and reconstructed flows with current level of skill (Figure 6) and 95th storage quantile obtained using the combined flows having synthetic reconstructed flows with different correlation ($\rho$) and sample lengths ($n_R$) and the observed flows. Contours denote the difference in million acre-feet.
corners of Figures 7a and 7b) as the number of years of reconstructed flows added goes back to 1857 (Table 2) and the skill of reconstructed flows approaches the current skill. Further, the skill of the reconstruction model has to be high enough to capture the annual streamflow characteristics, but longer reconstruction data could also potentially improve the estimation of annual streamflow characteristics incorporating preinstrumental flow conditions.

The second experiment under the synthetic study relaxes the upper bound on the skill of reconstructed flows to evaluate a scenario of having perfect skill (i.e., $\rho_j = 1$) with reconstructed flows. Analysis under this experiment will help us to understand if the improvement in skill is beyond the current level of predictability, then what would be the reduced sample length required to obtain the level of uncertainty reduction on reservoir sizing. Like that of the first experiment, the number of years of synthetic reconstructed flows ($t = 1, 2, \ldots, n_0$) added always begins at the beginning of the instrumentation period and goes backward in time into the preinstrumental period. An allied goal of this experiment is to investigate the utility of reconstructed streamflows for other regions (e.g., Colorado) with high skill in reconstructed flows. Under this experiment, the entire reconstructed streamflow time series available prior to the instrumental period was corrupted to the desired skill ($\rho_j$) and sample length $n_R$ using equation (A2). Given that the interest is to generate reconstructed streamflows with very high skill, we considered the reconstructed streamflow over the instrumental period as the primary series with which the corrupted streamflow would exhibit such a high skill. Thus, the corrupted reconstructed would exhibit the specified correlation ($\rho_j$) with the reconstructed streamflow time series available over the instrumental period (see Appendix A with regard to the discussion about equation (A2)). Thus, we have three sets of time series: (a) reconstructed streamflow for the entire period (i.e., station 1: 1857–1985; station 2: 1857–2003), (b) corrupted reconstructed streamflow corresponding to length $n_R$ with the specified skill ($\rho_j$) before the instrumental period (i.e., station 1: 1857–1929; station 2: 1857–1933), and (c) reconstructed streamflow during the instrumental period (i.e., station 1: 1930–1985; station 2: 1934–2003).

To begin with, we first obtained the true/baseline 95th storage for each station by obtaining the posterior distribution of lognormal parameters using the time series (a)—the entire reconstructed flow from 1857 till the end of the instrumental period—based on equation (12). Using the samples from the posterior distribution of parameters, we generated lognormal flows of length, $n_l$ (in Table 2 for synthetic experiments) and estimated the storage for each parameter set. Based on the 10,000 values of storages obtained, we obtained the 95th quantile of the storage, which we consider it as the true storage for the second experiment. Following this, we combined the time series (c)—the reconstructed flows during the instrumental period—and the time series (b)—corrupted reconstructed flows with specified correlation ($\rho_j$) and length ($n_R$)—using equation (12) to obtain the posterior distribution of lognormal parameters for the combined time series. Similar to the true storage distribution estimation, we generated lognormal flows of length, $n_l$ from the realizations of the lognormal parameters for each corrupted reconstructed flow having skill ($\rho_j$) and reconstruction years added ($n_0$) and obtained 95th quantile of storage from the posterior distribution of storage for the combined time series of (b) and (c). We obtained the difference between the true storage—95th quantile storage from the entire reconstructed flows (time series (a))—and the 95th quantile storage obtained from the combined time series corresponding to each $\rho_j$ and $n_R$ (Figure 8). Under this experiment also, the sample length, $n_0$, of lognormal flows for estimating the storage was kept as the same for each station, thereby the information added from the reconstruction flows were incorporated in the estimation of lognormal parameters.

Similar to the first experiment, this experiment shows how uncertainty in storage estimates improves as both the skill ($\rho_j$) increases and the number of years of reconstructed flows ($n_0$) added increases (Figure 8) especially in station 1. Although in station 2, the number of years of reconstructed flows added have a greater effect on the reservoir estimate performance than the reconstruction skill, it is important to note that the best estimate is obtained by using all of the reconstructed years at the highest skill. Thus, the difference in storage quantile obtained using the entire reconstructed flows (i.e., true storage) and the combined time series using the corrupted reconstructed flow with specified correlation ($\rho_j$) and sample lengths ($n_0$) are smallest at the right-hand upper most corner of the figure where the number of reconstruction records and the skill reaches the maximum. From Figure 8, we also understand if the skill of reconstructed flows is high, then the number of years of reconstructed records to be added is lesser for obtaining the same reduction in error in estimating the true storage. To summarize the findings from the second experiment, for
regions exhibiting higher skill in developing reconstructed flows, the number of years of reconstructed flows to be added ($n_R$) could be lesser to obtain the same level of uncertainty reduction in estimating the storage for the desired reliability.

5. Discussion

This study demonstrated that the combination of observed and reconstructed streamflow can improve stochastic streamflow generation and reservoir storage estimation based on a split-sample experiment and a synthetic experiment. We also show that estimated streamflow generation parameters and reservoir storage estimates improve as the skill in reconstructed flows and sample length included for Bayesian combination...
increase. This shows clearly as consideration of paleo-streamflow records beyond the instrumentation period should provide useful information for reservoir sizing. However, it is important to look at the feasibility of this Bayesian combination as it heavily depends on the skill of the reconstructed flows.

In practice, the reconstruction skill depends on the quality of the paleo-data available. Although Figures 7 and 8 show error in estimating reservoir storage estimation reduces if the correlation between reconstructed and observed streamflow is significant. Reconstructed streamflow data are estimated values, so careful consideration is necessary when using these values to design a project as large as a reservoir. In our experiments, the reconstructed streamflow explained almost half of the total variance in annual streamflows, which was shown to be enough to improve the mean and lag-one correlation of the lognormal parameters. We also emphasize that rigorous validation tests have to be performed on the streamflow reconstruction model before it can be considered for combining with the observed streamflow.

If the reconstruction skill is able to explain observed variability in streamflow, then the length of reconstructed streamflow used can be chosen. It was shown in section 4 that the reservoir sizing estimates improved as more reconstructed years were added (Figures 4 and 5), but it is important to note that in each experiment, the length of the observed period was longer than the reconstructed period. Even though a longer reconstructed record should better capture the lognormal parameters, reconstructed values are estimates and have uncertainty. By keeping the length of the reconstructed period shorter than the observed period, the observed values have a greater effect in the MCMC likelihood than the reconstructed values. However, a longer reconstructed streamflow record can be utilized without being given a greater effect than the observed streamflow by having two weighted likelihoods inside the MCMC. A formal way to address this in a Bayesian setting is to consider power priors that weigh the likelihoods independent of the sample length [Ibrahim and Chen, 2000]. We intend to consider this as part of our future work.

The difference in storage between using only the observed streamflow and the combined flow—reconstructed and observed streamflow—should also be considered when using this methodology in practice. If the difference is positive, as it happened in stations 1 and 2, then observed flow overestimates the storage requiring additional cost. On the other hand, if the difference is negative, then potential underestimation could result in severe shortfalls. If the significant differences primarily due to number of years of added, then a careful validation experiment could be necessary in understanding the role of reconstructed flows. Thus, a detailed alternate analysis may be necessary to go with a particular choice.

Even though construction of new reservoirs has been very limited in the U.S., we expect combining paleo and observed information to obtain stochastic streamflow generation parameters with the SQP will be useful for reallocation of existing storages within the existing system. For instance, increasing demand at urban centers necessitates capacity expansion projects requiring reallocation of existing storages for multiple uses [Singh et al., 2014]. Perhaps the utility of reconstruction records is more in developing countries where still there is scope for designing new reservoir systems. In such situations, utilizing paleo-information with observed information is an effective way to improve reservoir system design since the preinstrumental drought information captured from the tree-ring chronologies reduce the uncertainty on estimating annual streamflows corresponding to very small nonexceedance probabilities (i.e., flow with high reliability).

The assumptions on the Bayesian combination methodology, stationarity on the variance and the form of the distribution function (i.e., lognormal distribution), pose challenges in estimating reservoir storages considering future streamflow conditions under climate change. Recently, Singh et al. [2014] show how near-term climate change projections could be utilized for reallocating existing storages. To address the stationarity assumption on the variance, reconstruction models that explicitly estimate the variance of streamflow as a function of predictors (i.e., tree-ring chronologies and SST) could be developed [Sankarasubramanian and Lall, 2003]. To apply this methodology, it is important that the variance of the annual streamflow over a particular period should exhibit dependency with the predictor. However, in our analyses, we did not find any association between the variance and the predictors [Patskoski et al., 2015] for the selected two stations. The next assumption on the form of annual streamflow distribution could also be modeled by using local density estimation methods [Lall, 1995; Sankarasubramanian and Lall, 2003].
2003]. Such procedures adaptively estimate the density of the annual streamflow using local semiparametric and nonparametric density estimation procedures without globally assuming a functional form for the entire data. Thus, combining paleo, observed and projected (i.e., future) annual streamflow information would be the most prudent approach for designing new reservoir systems as well as for the capacity expansion of existing systems. We intend to address these critical areas as part of our future research effort.

6. Summary and Conclusions

Given streamflow records in the U.S. are limited to about 80 years, the use of observed streamflow alone in reservoir sizing does not incorporate annual drought events with rare frequency. To overcome this limitation, we proposed a Bayesian framework for combining information from observed streamflow and reconstructed streamflow from paleo-data and SST data. For this purpose, we considered two virgin basins in the southeastern United States having a long record of observed streamflow and tree-ring chronologies. Reconstructed streamflows for the two sites were obtained using a hybrid approach based on Singular Spectrum Analysis (SSA) using SST and tree rings (SST-TR).

To evaluate the utility of reconstructed streamflow in improving the reservoir sizing, we considered two experiments—split-sample validation and a synthetic experiment—that combines the observed and reconstructed streamflow using the Bayesian combination framework. Combining the observed and reconstructed streamflow using the Bayesian framework provided stochastic streamflow generation parameters that were more representative of the long-term streamflow characteristics at the site. Based on split-sample validation, we showed that incorporating reconstructing streamflow with observed streamflow in the stochastic generation model resulted in more accurate estimates of storage distribution. Further, the accuracy in storage estimation improved as we added more length of reconstructed streamflow in the Bayesian combination.

Under the synthetic study, the study performed two experiments. The first one evaluated the role of reconstruction record length and skill of reconstruction in improving the accuracy of the storage estimates by comparing with the 95% quantile estimate of the storage obtained using current streamflow record and the reconstruction record with the current level of skill. The second experiment considered the entire reconstruction record as the "AI" streamflow, thereby providing a scenario of reconstructed streamflow with perfect skill. Differences in 95th quantile storage estimates between the reconstructed streamflows with perfect skill and the corrupted combined flows with prespecified skill and length was computed. The analysis showed that uncertainty in storage estimates reduces by incorporating reconstruction records with higher skill and longer sample lengths. Even though construction of new reservoirs are very limited, we expect the application of the proposed combination methodology for obtaining stochastic streamflow generation parameters with the SQP will be useful for reallocation of existing storages within the existing system. These are some of the potential areas of research on utilizing paleo-information for planning and management of water resources systems.

Appendix A: Generation of Synthetic Reconstructed Flows With a Specified Skill

To understand the role of skill of reconstructed flows on reservoir sizing, the time series of reconstructed flows was corrupted to develop a synthetic reconstructed flows having specified skill with observed/reconstructed streamflow. For the synthetic experiments, given two time series, observed streamflow \( Q_t \) and reconstructed streamflow \( Q_{R,t} \), the reconstructed streamflow needs to be corrupted to obtain the synthetic time series of reconstructed streamflow \( Q'_{R,t} \) which has a desired correlation, \( \rho_j \), with the reference time series which could be either observed streamflow or reconstructed streamflow. The number of years of synthetic reconstructed flows \( t = 1, 2, \ldots, n_R \) added always begins at the beginning of the instrumentation period and goes backward into the prehistoric period. In the first experiment (Figure 7), the skill of synthetic reconstructed streamflow had an upper bound, which was based on the actual correlation \( \rho_{AI} \) between the reconstructed streamflow and the observed streamflow. Given the desired skill, \( \rho_j \), synthetic reconstructed streamflow could be generated using equation (A1).
\[
Q^*_t = \mu_Q + \rho_j (Q_t - \mu_Q) + \left(1 - \left(\frac{\rho_j}{\rho_t}\right)^2\right) * \epsilon_t + \left(1 - \left(\frac{\rho_t}{\rho_t}\right)^2\right) * \nu_t
\]

where \(\mu_Q\) is the mean of the observed streamflow, \(\rho_j\) is the desired reconstruction skill, and \(\epsilon_t\) follows \(N(0, \Sigma_{obs})\) having zero mean and a covariance matrix, \(\Sigma_{obs}\), whose variance and lag-one correlation are derived from the observed streamflow. It is important to note that synthetic reconstructed flow values have an expected mean and variance equal to the mean and variance of the observed streamflow.

The second experiment (Figure 8) under the synthetic study focused on extending the desired skill to perfect correlation (i.e., \(\rho_j = 1\)) for assessing the utility of reconstructed flows. Given that the reconstructed streamflow is derived from a regression, the mean of the reconstructed streamflow will be equal to the mean of the observed streamflow over the period of model fitting. Using equation (A2), the synthetic reconstructed streamflow with desired correlation could be obtained as follows

\[
Q^*_t = \mu_Q + \rho_j (Q_t - \mu_Q) + \left(1 - \left(\frac{\rho_j}{\rho_t}\right)^2\right) * \nu_t
\]

where \(\mu_Q\) is the mean of the observed streamflow, \(\rho_j\) is the desired reconstruction skill, and \(\nu_t\) follows \(N(0, \Sigma_R)\) having zero mean and a covariance matrix, \(\Sigma_R\), whose variance and lag-one correlation are derived from the entire reconstructed streamflow available from 1857 till the entire period considered for analysis (Table 2).

Acknowledgments
All the data used from the study could be obtained from USGS and Tree rings database. Data from validation experiments could be obtained by contacting the authors. The first author’s PhD dissertation research was partially supported by the United States National Science Foundation CAREER grant CBET-0954405. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not reflect the views of the NSF. The authors also thank the Associate Editor and the anonymous reviewer whose valuable comments led to significant improvements in the manuscript. All data related to this manuscript could be obtained from Jason Patskoski (jpatskoski@gmail.com).

References
Patskoski, J. (2014), Utility of tree rings and future climate change projections in reservoir sizing, PhD dissertation, NCSU Libr. Digital Repos-
itory, Raleigh, N. C. [Available at http://www.lib.ncsu.edu/resolve/1840.16/96868.]
Patskoski, J., A. Sankarasubramanian, and H. Wang (2015), Reconstructed streamflow using SST and tree-ring chronologies over the south-
eastern United States, J. Hydrol., 527, 761–775.


