Simultaneous tracking and RSS model calibration by robust filtering

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Abstract—Received Signal Strength (RSS) localization is widely used due to its simplicity and availability in most mobile devices. The RSS channel model is defined by the propagation losses and the shadow fading. These parameters might vary over time because of changes in the environment. In this paper, the problem of tracking a mobile node by RSS measurements is addressed, while simultaneously estimating a two-slope RSS model. The methodology considers a Kalman filter with Interacting Multiple Model architecture, coupled to an on-line estimation of the observation’s variance. The performance of the method is shown through numerical simulations in realistic scenarios.

I. INTRODUCTION

The need for localization is not just confined to persons or vehicles of transportation in outdoor environments where Global Navigation Satellite Systems (GNSS) play an important role for this purpose. But accurately estimating location indoors, GNSS features remains a difficult problem because of signal blockage or severe attenuations.

Due to the present ubiquitous availability of powerful mobile computing devices, the bloom of personalized context- and localization-aware applications has become an active field of research. A way of localization in indoor environments is using signals of opportunity such as WLAN (IEEE 802.11x), Zigbee, UWB, etc. The advantage of working with signals of the IEEE 802.11 as the primary source of information to approach the localization problem is the inexpensive hardware and the already dense deployment of WLAN Access Points (APs) in urban areas.

The goal of this work is tracking a mobile path in an indoor environment using an existing WLAN infrastructure where several position-related measurements are available. Here we are interested in algorithms that use Received Signal Strength (RSS) observations for locating and tracking the mobile node, since most of mobile devices are equipped with wireless capability [1]. To achieve this aim, a Kalman filtering is considered to perform the sequential tracking in a two-slope RSS model, for which we use an Interacting Multiple Model (IMM) architecture.

There are several channel models in the literature to characterize the indoor propagation environment [2], [3]. In this paper, the IEEE 802.11x model is considered because does not require an accurate floor plan of the indoor scenario and can be implemented without using a third party software [4].

This work uses the path loss two-slope model [5], which is a mathematical model of RSS that relates the path loss attenuation with distance. This means that the distance data between the mobile target and the AP can be described by two models which depend on a breakpoint distance value. This channel model is used to design two kalman filters and an Interacting Multiple Model (IMM) is used to dynamically combine the outputs from these two filters [6]. IMM technique is used to estimate the mode (mixing) probabilities for each model based Kalman filters and mix the two filter results based on the mode probabilities. Several works used IMM techniques for location and control applications [7], [8].

The paper is organized as follows. In Section II the System model and state-space model are presented, which has two path loss regions depending on the distance to the Access Point. The EKF-IMM algorithm and mobile tracking model are explained in Section III. Chanel calibration is presented in Section IV. Section V shows computer simulations and Section VI concludes the paper.

II. SYSTEM MODEL

We are interested in tracking a mobile device using RSS measurements from a set of N APs. The estimation is performed in two steps: i) estimation of relative distances to the set of visible APs; ii) fusion of these distance measurements into a blended tracking solution. In this section, we present the peculiarities of the two-slope RSS model and the state-space formulation of the distance estimation problem.

A. Two-slope RSS model

The widely used model for RSS observations is the path loss model, which is a simple yet realistic model for such measurements. It is parameterized by the path loss exponent and the shadowing. However, it has been observed in experimental campaigns that these parameters fluctuate and are

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indeed distance dependent. As a conclusion, the parameters employed in the traditional path loss model are highly site-specific [9] [10].

In this work we consider an extension of the classical path loss model accounting for two regions of propagation, referred to as the two-slope model [5]. Path loss refers to the average loss in signal strength over distance. For indoor environments, the path loss depends on the relative distance between the AP and the sensing device [11]. For far distances (e.g., 5 ≥ d ≥ 30 meters), path reflections from the environment (specially reflections from surrounding walls) generally result in a steeper overall drop in the signal strength at the receiver. Under this model, the RSS for the r-th AP (r = 1, 2, . . . , N total number of APs) to the mobile target is modeled as [5]:

\[
\text{RSS}^r(d) = \begin{cases} 
  h^{(1)}(d) + \chi \sigma^2_d & \text{if } d \leq d_{bp} \\
  h^{(2)}(d) + \chi \sigma^2_d & \text{if } d > d_{bp}
\end{cases}
\] (1)

where \(d\) is the relative distance between the AP and the moving node were the RSS was measured, and

\[
h^{(1)}(d) = 10 \alpha_1 \log_{10}(d) \\
h^{(2)}(d) = 10 \alpha_2 \log_{10}(d_{bp}) + 10 \alpha_2 \log_{10}(d/d_{bp})
\] (2)

The first equation gives the path loss (in decibels) for close distances (distances less than \(d_{bp}\), known as the breakpoint distance) and the second equation gives the path loss beyond \(d_{bp}\). The \(\alpha_1\) and \(\alpha_2\) values are the path loss exponents, defining the slopes before and after \(d_{bp}\), respectively. The functions \(h^{(1)}(d)\) and \(h^{(2)}(d)\) were obtained by measurement campaigns using radio signal ray tracing methods, premeasured RSS contours centered in the receiver or multiple measurements at several base stations [1], [12], [13].

Shadow fading is the variation of the nominal RSS measurements due to the transmitter/receiver geometrical configuration and can be modeled by an additive zero-mean Gaussian random variable. The notation, \(\chi \sigma^2_d \sim N(0, \sigma^2_d)\) is used. As happens for the path loss exponents, the variance values differ before and after the breakpoint distance. Typically the values depend on the scenario but in all cases it is observed that \(\sigma^2_1 < \sigma^2_2\) and \(\alpha_1 < \alpha_2\). Figure 1 of [14] represents a simulation of real measurements taken from the two-slope path loss model at different relative distances between one AP and the mobile target.

B. State-space model

As previously stated, the proposed strategy to solve the localization problem uses a two-step approach. In the first step (i.e., distance estimation) and for the v-th AP, the observations correspond to the RSS measurements and the unknown states to be sequentially inferred are \(\theta^r_v = [\hat{d}^r_k \hat{d}_k^T]\) where \(\hat{d}^r_k\) is the distance between the mobile and the r-th AP and \(\hat{d}^r_k\) is the rate of change of this distance. We assume a linear evolution of states in the form of \(\theta^r_v = A \theta^{r-1}_v + B \xi^r_v\), where \(B \xi^r_v\) is the process noise accounting for possible modeling mismatches, such as a possible acceleration of the mobile. In other words, this noise term gathers different forces that could affect target’s dynamics and which are not explicitly modeled. The process noise is normally distributed with zero mean and covariance matrix \(Q_k = \sigma^2 \sigma^2_T\ [15]\); where \(\sigma^2\) models the uncertainty on the mobile dynamics. The state equation includes these matrices:

\[
A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \Delta t^2/2 \\ \Delta t \end{bmatrix}
\] (4)

where \(\Delta t\) is the sampling period.

To complete the state-space representation, the observation vector is defined. In this case, the RSS measurements per AP are precisely the observations used to infer \(\theta^r_v\), and thus \(y^r_k \triangleq \text{RSS}^r(d_k) = h(d_k) + n_k\) where we recall that the model for \(\text{RSS}^r(d)\) depends on the breakpoint distance. Therefore, \(h(\cdot)\) has to be selected according to (1) and the statistics of the measurement noise as well, that is whether its variance is \(\sigma^2_1\) or \(\sigma^2_2\).

In this work, a solution involving an Extended Kalman filter (EKF) is considered to deal with the nonlinear filtering problem, for which we have to derive the Jacobian matrix of the measurements function because \(h^{(1)}(d)\) and \(h^{(2)}(d)\) are nonlinear. The \(2 \times 1\) Jacobian matrices \(H^{(1)}_k\) and \(H^{(2)}_k\) are

\[
H^{(1)}_k = \begin{bmatrix} \frac{1}{\log_{10} 10} & 0 \\ \frac{1}{\log_{10} 10} & 0 \end{bmatrix} \quad H^{(2)}_k = \begin{bmatrix} \frac{\alpha_2}{\log_{10} 10} & 0 \\ \frac{\alpha_2}{\log_{10} 10} & 0 \end{bmatrix}
\] (5)

III. Interactive Multiple Model Approach

The main goal in the RSS-based localization problem is to infer the distance to each AP, and the corresponding distance rate, from a set of \(N\) RSS measurements. The main concern of this section is to present and justify the reasoning behind the use of an IMM-based approach to solve such problem.

A. Parallel IMM-based solution

The first approach that comes to mind to solve this problem is the use of a traditional filtering solution, such as the EKF, where the observation accounts for the full set of RSS measurements \(y_k = [y^1_k, . . . , y^N_k]^T\) and the global state evolution takes into account the \(N\) independent states \(\theta_k = [\theta^1_k, . . . , \theta^N_k]^T\). But it is straightforward to see from (1) that this is not a valid approach, because the measurement model directly depends on the breakpoint distance. The two-slope model can be used to model the distance between the AP and the mobile node but both implicit models must be treated separately. The natural solution to overcome this model-switching problem is to use an IMM-based approach.

The key idea behind the IMM is to use a bank of \(M\) KFs, each one designed to cope with a specific model (or model set), and to obtain the state estimation as a clever combination of the individual estimates. If the full set of \(N\) independent observations \(z_k\) is considered, the question that arises is how many KFs should be considered into the IMM. As each independent observation may obey model 1 or model 2, the answer is \(2^N\) filters (i.e., all the possible combinations of model 1 and 2 for the \(N\) observations). It is clear that this is not a practical solution for an arbitrary number of APs,
where $\sigma$ is the position of the $I$ IMM, each one involving 2 KFs according to the two path loss models (shown in Figure 1).

At each discrete-time instant $k$, the IMM algorithm follows a clear three-step architecture: Reinitialization, KF and Model probability. The final estimates are obtained as a combination of the individual KF outputs using the corresponding model likelihoods. Mathematically, one cycle of the standard IMM associated to the $r$th RSS measurement is sketched in Algorithm 1, where $\pi_{ji}$ (for $i, j = 1, 2$) is a two-state Markov model transition probability matrix and is set in the proposed algorithm as, $\pi_{ji} = [0.9950, 0.0050; 0.0050, 0.995]$ for each AP.

Notice that in its standard form, both the two-slope model and the observation error $\nu$ is the variance related to the mobile acceleration, and $\sigma_0^2$ is the variance related to the mobile acceleration, and

$$\sigma_0^2 = \sigma_N^2 + \sigma_\nu^2.$$

Assuming that the mobile target has an average velocity of 1 m/s in the simulations presented in this work, so a small initial value for $\sigma_0^2$ of 0.7 m/s$^2$ is chosen.

The observation equation is $z^k_i = h_k(x_k) + \nu_k$ where the observation error $\nu_k$ is modeled as an uncorrelated white Gaussian noise with covariance $R_{pos,k} = diag(P_{pos,k}^0(1,1), ..., P_{pos,k}^N(1,1))$.

The nonlinear observation function $h_k(x_k)$ is defined as the distance of the mobile to every anchor point,

$$h_k = \begin{bmatrix} \sqrt{(x_k - x_{AP}^1)^2 + (y_k - y_{AP}^1)^2} \\ \vdots \\ \sqrt{(x_k - x_{AP}^N)^2 + (y_k - y_{AP}^N)^2} \end{bmatrix} \equiv \begin{bmatrix} d_{1,k} \\ \vdots \\ d_{N,k} \end{bmatrix},$$

where $\{x_{AP}^1, y_{AP}^1\}$ is the position of the $r$th AP, and the jacobian used to implement the EKF is given by

$$J_k = \begin{bmatrix} \frac{d_k}{d_{1,k}} & \frac{y_k - y_{AP}^1}{d_{1,k}} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{d_k}{d_{N,k}} & \frac{y_k - y_{AP}^N}{d_{N,k}} & 0 & 0 \end{bmatrix}.$$

Therefore, a divide-and-conquer strategy treating independent measurements separately is the best solution. In this contribution a parallel IMM-based approach is adopted, considering $N$ IMM$s$ each one involving 2 KFs according to the two path loss models (shown in Figure 1).

The state equation is $\dot{x} = \eta R$ for each AP, and the process noise variance must be specified according to the problem at hand. The error covariance matrix has a initial value assigned as $P_{pos,k}^0 = 4Q_k$ for each AP. The initial value state vector for the filter is $\hat{y}^{(1),r}_0 = \theta^r_0 + \omega$ where $\omega \sim \mathcal{N}(0, 0.8 I_2)$. The initial model probabilities are set as $\eta_{r}^{(1),k} = \eta_{r}^{(2),k} = 0.5$ for every AP also.

**B. Location Calculation Model**

The location calculation is solved with a KF using the $N$ distance estimates obtained from the bank of IMM$s$ as observations (see Figure 1). In the following, the location calculation model is detailed:

The state vector gathers the mobile position and velocity, $x^i_k = [x^i_k, y^i_k, \dot{x}^i_k, \dot{y}^i_k]$, and the observations vector is defined as $z^i_k = [d_{1,k} \ldots d_{N,k}]$, where $d_{r,k} \equiv \hat{y}^{r}_k(1)$ is the distance obtained from the $r$th IMM. The state equation is $x^i_k = A_{pos} x^i_{pos,k} + B_{pos} w^i_k$, where the resulting Gaussian process noise has a covariance matrix $Q_{pos,k} = \sigma_\nu^2 B_{pos} B_{pos}^T$, $\sigma_\nu^2$ is the variance related to the mobile acceleration, and

$$A_{pos} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_{pos} = \begin{bmatrix} \Delta t^2 & 0 \\ 0 & \Delta t^2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}.$$  

(18)

Assuming that the mobile target has an average velocity of 1 m/s in the simulations presented in this work, so a small initial value for $\sigma_\nu^2$ of 0.7 m/s$^2$ is chosen.

The observation equation is $z^i_k = h_k(x_k) + \nu_k$ where the observation error $\nu_k$ is modeled as an uncorrelated white Gaussian noise with covariance $R_{pos,k} = diag(P_{pos,k}^0(1,1), ..., P_{pos,k}^N(1,1))$.

The nonlinear observation function $h_k(x_k)$ is defined as the distance of the mobile to every anchor point,

$$h_k = \begin{bmatrix} \sqrt{(x_k - x_{AP}^1)^2 + (y_k - y_{AP}^1)^2} \\ \vdots \\ \sqrt{(x_k - x_{AP}^N)^2 + (y_k - y_{AP}^N)^2} \end{bmatrix} \equiv \begin{bmatrix} d_{1,k} \\ \vdots \\ d_{N,k} \end{bmatrix},$$

(19)

where $\{x_{AP}^1, y_{AP}^1\}$ is the position of the $r$th AP. The Jacobian used to implement the EKF is given by

$$H_k = \begin{bmatrix} \frac{\dot{x}_k - \dot{x}_{AP}^1}{d_{1,k}} & \frac{y_k - y_{AP}^1}{d_{1,k}} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\dot{x}_k - \dot{x}_{AP}^N}{d_{N,k}} & \frac{y_k - y_{AP}^N}{d_{N,k}} & 0 & 0 \end{bmatrix}.$$

$$x_k = x_{pos,k} = x_{pos,k-1} + \hat{y}^{(1),k}_k + \nu_k.$$  

(18)
Algorithm 1 Cycle \( k \) of the IMM for the \( r^{th} \) AP

1. For \( i = 1,2 \) and \( j = 1,2 \)
2. Reinitialization:

   Calculation of the predicted mode probability, mixing weights, mixing estimates and mixing covariances, respectively,

\[
\hat{\eta}_{k|k-1}^{r,i} = \sum_{j} \pi_{j} \hat{\eta}_{k|k-1}^{(j),r} ; \quad \eta_{k}^{r,i} = \frac{\pi_{r(\hat{\eta}_{k|k-1}^{r,i})}}{\eta_{k|k-1}} \quad (6)
\]

\[
\hat{\theta}_{k|k-1}^{(j),r} = \sum_{j} \hat{\theta}_{k|k-1}^{(j),r} \quad (7)
\]

\[
\hat{P}_{k|k-1}^{(j),r} = \sum_{j} \hat{P}_{k|k-1}^{(j),r} + (\hat{\theta}_{k|k-1}^{(j),r} - \hat{\theta}_{k|k-1}^{(j),r})^{r} \quad (8)
\]

3. Model-conditioned std.KF:

   Prediction, innovations’ covariance matrix, Kalman gain, state estimate and the corresponding error covariance matrix, are given by

\[
\hat{\theta}_{k|k}^{(i),r} = A\hat{\theta}_{k-1|k-1}^{(i),r} + \hat{\theta}_{k|k-1}^{(i),r} \quad A\hat{\theta}_{k|k-1}^{(i),r} \quad (9)
\]

\[
S_{k}^{(i),r} = H_{k}^{(i),r}P_{k|k-1}^{(i),r}H_{k}^{(i),r} + Q_{k}^{r} \quad (10)
\]

\[
K_{k}^{(i),r} = P_{k|k-1}^{(i),r}H_{k}^{(i),r} \quad (11)
\]

\[
\hat{\theta}_{k|k}^{(i),r} = \hat{\theta}_{k|k-1}^{(i),r} - K_{k}^{(i),r}(y_{k}^{r} - H_{k}^{(i),r}\hat{\theta}_{k|k-1}^{r}) \quad (12)
\]

\[
\hat{P}_{k|k}^{(i),r} = P_{k|k-1}^{(i),r} - K_{k}^{(i),r}S_{k}^{(i),r}K_{k}^{(i),r} \quad (13)
\]

4. Model probability update:

   The model likelihood function and model probability are respectively

\[
L_{k}^{(i),r} = \mathcal{N}(\hat{\theta}_{k|k}^{(i),r}; 0, S_{k}^{(i),r}) \quad (14)
\]

\[
\eta_{k}^{r,i} = \hat{\eta}_{k|k-1}^{r,i} \quad (15)
\]

5. Estimate fusion:

\[
\hat{\theta}_{k|k}^{r} = \sum_{i} \hat{\theta}_{k|k}^{(i),r} \quad (16)
\]

\[
\hat{P}_{k|k}^{r} = \sum_{i} \hat{P}_{k|k}^{(i),r} + (\hat{\theta}_{k|k}^{r} - \hat{\theta}_{k|k}^{(i),r})(\hat{\theta}_{k|k}^{r} - \hat{\theta}_{k|k}^{(i),r})^{r} \quad (17)
\]

The initial value state vector for the filter is \( \hat{x}_{0|0} = x_{0} + \omega \), with \( \omega \sim \mathcal{N}(0, 0.8I_{k}) \).

IV. Maximum Likelihood Covariance estimator for model calibration

In the proposed methodology, each IMM inherently treats this model uncertainty by computing the model likelihood from the innovations of each KF. For each AP \( r \) and model \( i \), the model probability is given by \( \eta_{k}^{r,i} \). These probabilities are used into the filter to weight the outputs of the individual KFs and for the model calibration. At each time step and using these model probabilities, two subsets of RSS measurements are constructed: \( \eta_{k}^{r,1} > \eta_{k}^{r,2} \). The RSS measurement \( y_{k}^{r} \) is associated to \( y_{k}^{r,1} \) (i.e., which represents the RSS measurements subset obeying \( \mathcal{Y}_{1,k}^{r} \), otherwise, it is associated to \( \mathcal{Y}_{2,k}^{r} \)). The cardinality of these sets is upper bounded by the present time instant, \( L_{i} = |\mathcal{Y}_{i,k}^{r}| < k ; \ i = 1,2 \), and that their sum is precisely \( |\mathcal{Y}_{1,k}^{r}| + |\mathcal{Y}_{2,k}^{r}| = k \). For the sake of clarity in the forthcoming derivations, the elements in the sets are defined as \( \mathcal{Y}_{1,k}^{r} = \{y_{1,k}^{r}, \ldots, y_{L_{1}}^{r}\} \). For the \( r^{th} \) AP, the \( k^{th} \) sample of the \( i \) model subset \( Y_{L_{1}}^{r} \) at time \( k \) is Gaussian distributed, \( y_{1,k}^{r} \sim \mathcal{N}(\mu_{1}^{(r)}(d_{e})^{r}, \sigma_{1}^{r}) \) and \( y_{2,k}^{r} \sim \mathcal{N}(\mu_{2}^{(r)}(d_{e})^{r}, \sigma_{2}^{r}) \).

Using this subset and assuming a known distance to the \( r^{th} \) AP, \( d_{e}^{r} \), at instant \( k \) the ML \( \sigma_{1}^{r} \) and \( \sigma_{2}^{r} \) estimators is given by

\[
\sigma_{1,k}^{2} = \frac{1}{L_{1}} \sum_{\ell=1}^{L_{1}} (y_{1,k}^{r} - \hat{y}_{1,k}^{r})^{2} \quad (20)
\]

\[
\sigma_{2,k}^{2} = \frac{1}{L_{2}} \sum_{\ell=1}^{L_{2}} (y_{2,k}^{r} - \hat{y}_{2,k}^{r})^{2} \quad (21)
\]

V. Results

The method proposed in this work was validated by computer simulations in an scenario depicted in Fig. 2, where the \( N = 6 \) APs were deployed in a \( 30 \times 30 \) m\(^{2} \) area at known locations. The duration of the trajectory consisted in 18 seconds with a sample period \( \Delta t \) of 100 ms. The first approach in this work was estimating the distance to every AP at every instant \( k \). A single realization was performed and the Figure 3 shows \( \theta_{k}^{r}[1,0] \), illustrating the case when the mobile node is close to the breakpoint distance. When this happens, the model probabilities \( \eta_{k}^{(1),r} \) and \( \eta_{k}^{(2),r} \) exhibit nervous behaviors. The estimated distance shown in Figure 3 corresponds to AP number 5. The top plot presents the estimated distance and the bottom plot shows the performance of the decision process in \( \mathcal{Y}_{1} - \mathcal{Y}_{2} \) switching. This indicates that a switch model has occurred.

The IMM performance was evaluated with the RMSE values. Focusing in AP number 5 as an illustrative example, the RMSE for the estimating of \( \sigma_{1} \) and \( \sigma_{2} \) is shown in Figure 4 where is observed the convergence of our algorithm after short period of time where we observe the convergence of the ML-based method after some instants. Figure 5 shows the average RMSE of the distance estimation over all 6 APs and position estimation. From this Figure, it is notable that the IMM algorithm implemented under covariance estimation has good accuracy in terms of mobile tracking.

![Fig. 2. Real mobile trajectory versus estimated trajectory.](image-url)
VI. CONCLUSIONS

The mobile location via RSS measurements and the covariance calibration in a realistically wireless scenario has been formulated as a switching non-linear state problem. This work proposes an EKF-IMM algorithm to face the problem when the RSS values measured by the mobile are switching from a model to another. The proposed method determines the probabilities of the two models improvising the distance estimation between the mobile and the Anchor Point. Simulation results shows that the EKF-IMM algorithm gives a good mobile tracking estimation together with a covariance calibration of the channel. However, it is possible to estimate the path-loss model parameters and the $d_{bp}$ using the same EKF-IMM algorithm. Using other smoothing algorithms are recommended as future work as well as implementing the proposed EKF-IMM algorithm under realistic WLAN scenarios.

REFERENCES


