On the Rate-Distortion Region for Separate Encoding of Correlated Sources

João Barros  
Inst. for Comm. Engr. (LNT)  
Munich Univ. of Tech. (TUM)

Sergio D. Servetto  
School of ECE  
Cornell University

Let $U$ and $V$ be two correlated sources drawn i.i.d. according to the joint probability distribution $p(u,v)$. Furthermore, let the two sources $U$ and $V$ be encoded by two separate encoders, each of which observes only the one source it has been assigned to. If $U$ is encoded at rate $R_1$ with average distortion $D_1$ and $V$ is encoded at rate $R_2$ with average distortion $D_2$, what are the achievable rate-distortion tuples $(R_1, R_2, D_1, D_2)$?

This apparently simple question represents one of the longest standing open problems in network information theory. The first contribution to this problem was given in 1973 by Slepian and Wolf [1], who solved the case of perfect reconstruction at the receiver, i.e. $D_1 = D_2 = 0$. Wyner and Ziv [2] then provided a solution for the rate-distortion problem with perfect side information at the receiver, corresponding to the case in which the receiver is provided with a perfect copy of $V$, i.e. $R_2 \geq H(V)$ and $D_2 = 0$. Later Berger and Tung derived an inner and an outer bound for the rate region $(R_1, R_2)$ with respect to a fixed pair of arbitrary distortions $(D_1, D_2)$ ([3],[5]). Both bounds are given in terms of mutual informations involving two auxiliary random variables denoted as $W$ and $Z$. However, while the outer bound assumes two Markov chain conditions $W - U - V$ and $U - V - Z$ the inner bound requires $W$ and $Z$ to obey an additional long chain condition taking the form $W - U - V - Z$. The latter poses a very significant restriction on the set of auxiliary random variables over which the minimization of the mutual information terms giving the boundaries of the rate distortion region can be performed. Consequently, the thus obtained inner bound contains only a subset of the rate-distortion tuples suggested by the outer bound.

Our main result consists of the following theorem, which provides an inner bound for the rate-distortion region $R(D_1, D_2)$.

**Theorem** Let $(U, V)$ be drawn i.i.d. $\sim p(u,v)$. For a given distortion pair $(D_1, D_2)$ the rate pair $(R_1, R_2)$ is achievable if

$$
R_1 \geq I(U; V, W | Z) \\
R_2 \geq I(V; U, Z | W) \\
R_1 + R_2 \geq H(U, V) - H(U|V, W) - H(W|U, V)
$$

where $W$ and $Z$ are two auxiliary random variables, which obey the following conditions: (i) $W - U - V$ and $U - V - Z$ form two Markov chains, (ii) there exist $\hat{U}(V, W)$ and $\hat{V}(U, Z)$ such that $D_1 \geq E_d(U, \hat{U})$ and $D_2 \geq E_d(V, \hat{V})$.

This result goes beyond those obtained by Berger and Tung [3], because the characterization of our inner bound does not require the auxiliary random variables $W$ and $Z$ which help describe the achievable rate-distortion tuples to obey the aforementioned long chain condition $W - U - V - Z$.

The coding strategy used in our proof is based on time-sharing of rate-distortion codes, which can reach all points of the Berger-Yeung region [4], i.e., all rate-distortion tuples of the form $(R_1, R_2, D_1, 0)$ and, similarly, $(R_1, R_2, 0, D_2)$. Although this coding strategy does not achieve all the tuples $(R_1, R_2, D_1, D_2)$ promised by the outer bound of Berger and Tung, the region thus obtained does give the fundamental performance limits of a family of rate-distortion codes, which is likely to be simpler to implement than multi-terminal rate-distortion codes and therefore relevant from a practical point of view. Current work is focusing on the development of an alternative coding strategy, with the aim of closing the existing gap (cf. Fig.1) to the Berger-Tung outer bound.

A slightly revised version of the paper submitted for review can be downloaded from our web pages, at http://www.lnt.e-technik.tu-muenchen.de/mitarbeiter/barros/, http://people.ece.cornell.edu/servetto/.

**REFERENCES**


