How effective is a rank-based filter with a frequency- and orientation-selective response?

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To my dear wife
Charmaine
Acknowledgements

I would like to express my gratitude to all those who gave me the possibility to complete this study.

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Abstract

The pioneering study about the functions of the primary visual cortex by Hubel and Wiesel [38], motivated scientists to design machine vision models which tentatively simulate the same functions.

In visual perception, there are two kinds of stimuli, usually referred to as first-order stimuli, characterized by a difference in luminance (the object tends to be lighter or darker than the background) and second-order stimuli that are characterized by difference in texture (the object and background share the same luminance but differ in texture).

The literature shows that the acceptable models for first- and second-order stimuli are 2-D Gabor linear filters and the linear-nonlinear-linear (LNL) approach respectively. However, this is still not entirely satisfactory because first- and second-order stimuli are processed through different channels, which require an a priori knowledge of the nature of the image; this is not plausible either from the perceptual or from the application point of view.

The existing orientation-selective, nonparametric features, called ranklets, which are based on the computation of Wilcoxon rank-sum test statistics [64] has motivated us to investigate the applicability of other rank statistics which would be suitable for both kinds of stimuli.

This study proposes an innovative rank-based filter, with orientation- and frequency-selective response. The filter is based on an approximation of a 2-D Gabor filter and on a combination of the Wilcoxon (location) and the Siegel-Tukey (dispersion) nonparametric statistics. The promising results obtained from experiments on perceptual stimuli show that the proposed filter is sensitive to both first- and second-order stimuli.
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<td><em>Discrete Fourier Transform</em></td>
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<td>FFT</td>
<td><em>Fast Fourier Transform</em></td>
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<tr>
<td>fMRI</td>
<td><em>Functional Magnetic Resonance Imaging</em></td>
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<tr>
<td>FRF</td>
<td><em>Filter-Rectify-Filter</em></td>
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<tr>
<td>IT</td>
<td><em>Inferior Temporal Cortex</em></td>
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<td>LGN</td>
<td><em>Lateral Geniculate Nuclei</em></td>
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<td>LNL</td>
<td><em>Linear-Nonlinear-Linear</em></td>
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<td>M-LGN</td>
<td><em>Magno-cells</em></td>
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<tr>
<td>MRI</td>
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<td>RMS</td>
<td><em>Root Mean Square</em></td>
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<td>SNoW</td>
<td><em>Sparse Network of Winnows</em></td>
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<td>SNR</td>
<td><em>Signal-To-Noise Ratio</em></td>
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<td>SVM</td>
<td><em>Support Vector Machine</em></td>
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Chapter 1  Introduction
1.1 Vision

“What does it mean, to see? The plain man's answer (and Aristotle's, too) would be, to know what is where by looking.” This famous quote by David Marr defines the ability to process visual information. Although, seeing is something that is considered trivial for animals and human beings, research is still far from a comprehensive explanation of how people process the visual information. This research area is of great interest and is very popular in various biological (Neurophysiology, Human Psychophysics, Neuroscience …) and computer vision domains.

Since the pioneering classic studies of David Hubel and Torsten Wiesel in the early 1960s, a lot of research has been done in the understanding of the functioning of higher visual areas. Numerous investigations have advanced the design and implementation of models for the analysis of digital images.

Texture segmentation, which is the area of interest for this study, is a biologically inspired research area within computer vision domain used to identify different textures representing objects and boundaries in a digital image. This section provides an overview of texture perception together with an understanding of basic visual mechanisms that respond to some visual stimuli. A brief overview of the visual pathway is also provided to put the reader in the right biological context before explaining the motivation, aims and deliverables of this project.

1.2 Visual Pathway

The processing of visual information, as illustrated in Figure 1.1, starts from the retina and performs object recognition through the ventral system. The retina contains densely packed receptors in the fovea region that are used to generate centre surround ON and OFF centre cells. A detailed description of this structure is provided in [37,56].

The retina receptors are connected to the bipolar cells which will subsequently feed into retinal ganglion cells through the optic nerve onto the lateral geniculate nuclei.
The LGN includes two types of cells namely Magno-cells (M-LGN) and Parvo-cells (P-LGN) which are primarily responsible for shape information. Both cells in the LGN feed to the primary visual cortex, which is usually referred to as striate cortex or V1.

![Visual Pathway](image)

**Figure 1.1: Visual Pathway**

### 1.2.1 Primary Visual Cortex (V1)

In their pioneer study, Hubel and Wiesel [38] studied the primary visual cortex of cats and categorized receptive fields of cells into *simple* and *complex* cells. The simple cells are defined as:

- Segregated spatially into excitatory and inhibitory regions
- There is summation in either types of regions
- Likely to cancel the effect when two opposing regions are illuminated at once
- The simulation of the receptive field is little effected by diffused light

On the contrary, the complex cells are those that do not meet at least one of the above criteria. It is stated that the above four criteria defined by [38] are partly subjective as none of them classifies neurons distinctly [63]. Further to this limitation, neurons
were later objectively classified by applying drifting bars response or sinusoidal gratings [22;63].

The following images show a comparison between the responses of simple and complex cells to a given image.

![Figure 1.2: (a) Simple Cells and (b) Complex Cells Response](adapted from [31])

Both images in Figure 1.2 contain the receptive field together with a white rectangle representing the stimulus and a blue line denoting a time trace that plot the onset and offset of the stimulus. Moreover, the black vertical lines indicate the respective cell responses.

As clearly shown above, simple cells are very sensitive to orientation, position and size [56]. A study by de Valois and de Valois [56] also shows that simple cells are sensitive to the spatial frequency of the visual stimuli. On the other hand, complex cells, which are in the majority of V1, are not very particular about the position, but they are still sensitive to orientation and size. Furthermore, complex cells are also direction-selective as they respond only when the stimulus moves in one direction.

There are also other cells referred to *end-stopped* cells which are found to be sensitive to specific curvatures, edge crossings, vertices and line endings [58].

### 1.2.2 Other Extra-Striate Regions

Further to numerous studies, region V1 is said to be well-understood. However, there is still lack of knowledge in the behaviour of extra-cortical circuits. The studies [51;52;60] have explored these neurons, including V2, V4 and Inferior Temporal (IT) and showed that they are sensitive to combinations of features.
The V2 region is known to be sensitive to low order combinations of the afferent cells from the V1 (primary visual cortex) region [56;60]. Several architectures including the VisNet [56] and the Standard Model [9;54;55;60;61] have shown the growing complexity and invariance of the architecture in Figure 1.1.

The V4 area, which was originally discovered by Semir Zeki [74], is found to respond to low order feature combinations [16;52]. It is understood that some V4 cells are sensitive to simple features like oriented line segments while other cells respond to more complex curvatures. In his study [16], Connor states that the V4 cells respond to angular position of a feature, the normal at the feature, the curvature at the feature and the context (curvatures of the segments adjacent to features). There are also other cells that are colour selective, which indicate a role in the colour analysis.

IT is the final higher visual area that is important to shape processing. In this region, the cells are sensitive to complex features and demonstrate significant translational invariance [56] together with scale invariance of up to two octaves [60]. In their study, Serre et al. [60] state that this region includes a number of view-tuned cells whose responses are united to produce view-independent cells that can identify specific objects. Also, in [56], Rolls claimed that the objects representation in the IT leads to a significantly larger number of shapes.

### 1.3 Visual Texture

This section aims to provide an overview of visual texture and visual perception.
Figure 1.3a shows a natural scene where the sky, grass and zebras can be easily distinguished by difference in luminance. However, the borders between the zebras do not involve changes in colour or in luminance. Also, Figure 1.3b shows a road in which the old and new sections of paving differ by the size and pattern, however the average luminance intensity is roughly constant across the regions.

### 1.3.1 First-Order versus Second-Order Stimuli

It is well-understood that the visual cortex is able to process two kinds of stimuli referred to as first- and second-order. A first-order stimulus is characterized by variations in local difference in luminance and can be perceptually depicted as a sine wave grating as shown in Figure 1.4a. It can be noted that it is spatially one-dimensional, and parameters of spatial frequency, orientation, contrast and spatial phase are used for static images. The attributes of the first-order stimulus including the spatial frequency and orientation are effected by the luminosity changes across space [2].

![Figure 1.4: (a) First- and (b) Second-Order Stimulus](image)

On the other hand, visual stimuli, which are characterized by difference in contrast or texture are referred to as second-order stimuli [10;15]. Figure 1.4b shows an example of a second-order stimulus where it consists of a sine wave grating similar to Figure 1.4a, but the crests contain random noise. In this case, there is roughly constant average luminance intensity across all regions of the image.
1.4 Motivation for the Investigation

Texture Segmentation is a popular research area and has attracted a lot of researchers to explore and implement acceptable models which simulate the operations of the visual pathway discussed in section 1.2. The primary visual cortex (V1) was the first area to be discovered and has since motivated a lot of studies in the computer vision domain to simulate its operations. Two acceptable models namely 2-D Gabor filter [68] and Linear-Nonlinear-Linear (LNL) model [42] have been successfully applied for first- and second-order stimuli respectively. Although these models provide successful results, the literature review (see Chapter 2) shows a research gap as there is still not a single filter which is suitable for both kinds of stimuli. This challenge is the motivation for this investigation which aims to design and implement a single filter which would be close this research gap.

1.5 Aims of the Project

The main aim of this project is to develop a single filter with orientation- and frequency-selectivity based on the approximation of a 2-D Gabor filter and combined with non-parametric statistics. This innovative filter is expected to be suitable for both first- and second-order stimuli and it would not require an *a priori* knowledge to perform texture segmentation. An array of experiments is conducted out to investigate the effectiveness of the new filter.

1.6 Changes to Original Proposal

Although certain sections required more thought than expected, the original project proposal was adhered to throughout.

1.7 Project Plan

The proposed and actual project plans for this study are provided in the form of Gantt charts in Appendix B.
1.8 Organisation of the Dissertation

This chapter aimed at providing the necessary background in order to put the reader in the right context. Below is a brief outline for each of the following chapters.

Chapter 2 - Perceptual Models
This chapter provides a thorough literature review of the successful techniques applied for first- and second-order stimuli including 2-D Gabor filters and the linear-nonlinear-linear model which is also referred to as the *back-pocket* model.

Chapter 3 - Analytical Tools
This chapter provides an overview of the linear and non-linear filtering techniques with particular focus on 2-D Gabor linear filters and rank-based filters together with Smeraldi’s work on the design of ranklets and their successful application in face recognition.

Chapter 4 - Hypothesis
Further to the literature above, this chapter identifies a research gap and provides a hypothesis for this project.

Chapter 5 - Methodology
This chapter includes a detailed explanation of three filter designs based on 2-D Gabor filters. Additionally, a further explanation is provided for non-parametric dispersion statistics commonly used in quantitative studies. Finally, it provides an innovative combination of Wilcoxon and Siegel-Tukey statistics which is later tested for both first- and second-order stimuli.

Chapter 6 - Implementation and Results
This chapter illustrates and compares various results obtained by a 2-D Gabor linear filter against rank-based filters. The response obtained from the rank-based filter, referred to as Wilcoxon-Siegel-Tukey, was indeed found to effective for both kinds of stimuli.

Chapter 7 - Discussion
The discussion chapter provides an analysis on the effectiveness of the innovative filter by comparing the obtained results to other established models. It also provides a discussion on the computational cost of the rank-based filtering algorithm together with any limitations of the study.

Chapter 8 - Conclusion
This final chapter summarises the findings, contributions of the work carried out and recommends any potential future related work.
Chapter 2  Perceptual Models
2.1 Introduction

The purpose of this review is to explore and understand the computer vision and image processing perceptual models which have similar sensitivity to the primary visual cortex (V1) for both first- and second-order stimuli.

2.2 First-Order Stimuli

In the last thirty years, extensive research has been conducted in spatial frequency (a measure of how often the structure repeats per unit distance) channels [23;35]. The following is a summary list of the applied methods:

1. Adaptation - As cited by [42], [6] states that sensitivity is minimized for gratings that are close to spatial frequency of the main stimulus being observed. However, sensitivity is not reduced for gratings that are far removed in spatial frequency, indicating that the adapter desensitized a narrowband mechanism.

2. Summation - The degree of summation for gratings that are close in the spatial frequency is greater than for those with more distant spatial frequency [57].

3. Masking - A narrowband masker increases threshold whenever spatial frequency is similar to the masker. However, this is not the case when spatial frequency is far removed from the masker [42].

As stated in [43], most of the processing in the primary visual cortex is effected by changes in luminance. The underlying neuron mechanism can be estimated in terms of linear filters applied on the image. For instance, simple type receptive fields consisting of spatially segregated alternating regions (either ‘on’ or ‘off’) were found to be modelled by linear filtering. In study [68], it is showed that a 2-D Gabor filter, a member of the linear filters family, is an acceptable model which approximates simple cells in the primary visual cortex. This filter is modelled as a sinusoidal wave plane of some frequency and orientation within a two dimensional Gaussian envelope. Filtering is achieved by summing up the luminance intensity over all points within the receptive field weighted by the Gabor function values followed by thresholding the result.
Gabor filters were originally introduced by Dennis Gabor in his famous monograph in 1946 [28]. A remarkable finding about the Gabor filter is that the Gaussian envelope of a 2-D Gabor filter type impulse response function $G(x,y)$ has an identical functional form to its 2-D Fourier transform $F(u,v)$.

As cited by [21], an important finding is that the great majority of the mammalian cortical simple cells have 2-D receptive field profiles (based on statistical chi-squared tests) [20;39] which can well be modelled by 2-D Gabor functions. Figure 2.1 shows the similarity between 2-D receptive profiles of the simple cells in the cat visual cortex and the best fitting 2-D Gabor functions for each neuron. This similarity proves how Gabor functions are a suitable model to simulate the response of the mammalian simple cells.

Further to the above and an evaluation given in [5;20], Gabor filtering is the most acceptable model for texture classification. The capability of modelling the response of cortical cells (simple cells) and the optimal joint resolution of space and frequency allow Gabor functions to perform texture discrimination. Numerous studies have been carried out to investigate the correlation between Gabor functions and the visual system of mammals. For instance, Daugman [20] showed that Gabor functions can be used to model the behaviour of the simple cells in cats. Moreover, the experiments performed by Hubel and Wiesel [38] showed that, the simple cells of the cat are characterized by a spatial-angular bandwidth of about 30°. As cited by [5], Pollen and Ronner [53] argued that the frequency bandwidth of cortical cells is one octave. Other studies concluded different frequency bandwidths in the range of 0.5 to 2.5 octaves, gathering around 1.2 and 1.4 [44].

Although Gabor filters are an acceptable model for first-order stimulus, they have certain parameter constraints that need to be well configured. This issue is generally referred to as the filter bank design and requires the selection of a number of proper filters at different orientations and frequencies [5]. In particular, smoothing parameters may affect the system performance and thus should be chosen very carefully. Since different groups of textures were used in the literature, it is difficult to compare the performance of filter banks. In 2004, an attempt was done by Chen et al [11] but the significance of parameters was not clarified where the effects of the
smoothing parameters and of the frequency sampling were not considered. Three years later, Bianconi and Fernandez [5] performed extensive experiments to understand the effect of each Gabor filter parameter on texture classification. An interesting outcome of this study is that actually there is little effect on texture classification when the number of frequencies and orientations is increased. On the other hand, the smoothing parameters are very significant and play an important role in the design of the filter bank.

![Figure 2.1: Cat’s visual cortex receptive fields versus 2-D Gabor functions adapted from [21]](image)

### 2.3 Second-Order Stimuli

Although a linear filter, such as Gabor filter, proved to be suitable to simulate simple cells, there are many types of image structures that cannot be detected by a linear filter [43]. For instance, Figure 1.3 shows two natural scenes where a linear filter is not suitable to segment the zebras from each other (Figure 1.3a) and to segment the old paving from the new paving (Figure 1.3b). This is due to the fact that although the textures may differ in size and/or pattern, the average luminance intensity is roughly constant across the regions. These types of images, as described in section 1.3.1, are referred to as second-order stimuli.

Figure 2.2a shows a first-order stimulus superimposed by a cartoon receptive field which is tuned to the spatial frequency and orientation of the sinusoidal grating. When the same cartoon receptive field is applied on a second order stimulus such as a
sinusoidal grating with a contrast carrier shown in Figure 2.2b, a net zero response would be obtained after performing linear filtering [48].

![Figure 2.2: Receptive field superimposition on (a) First- and (b) Second-Order stimulus adapted from [48]](image)

An accepted model for second-order visual processing which has been applied by several studies including [3;7], is based on a three-stage mechanism referred to as filter-rectify-filter (FRF) or linear-nonlinear-linear (LNL) [42]. This model is often referred to as the “back-pocket model” named by Chubb and colleagues [13;14] (see Figure 2.3). Basically, it starts by filtering the input image through a set of linear spatial filters similar to the simple cells of primary visual cortex. The outputs of the filters are then processed by a nonlinear filter followed by second-order linear spatial filters (e.g. enhancing the difference in responses to neighbouring texture regions). The outputs are finally pooled for performance predictions depending on various experiments [42].

![Figure 2.3: Filter-Rectify-Filter FRF model for texture segregation adapted from [2]](image)
The model described above is rather involved and there is no general agreement over each stage of the LNL architecture. The studies [18;34] show that the initial spatial filters in the above model are tuned for spatial frequency and orientation. The pointwise nonlinearity in the second step of the LNL model is usually a squaring operation where the filtered image is computed as quadratic (2\textsuperscript{nd} order) polynomials of the pixels and fed into the second-order linear filter. However, Chubb et al [12] designed a technique namely histogram contrast analysis used to measure aspects of static nonlinearity. They concluded that their method included components of higher order other than squaring the intensity values.

As explained in the previous section, first-order spatial frequency channels were initially explored by sine wave grating stimuli with various techniques such as adaptation, masking and summation operations. Similar experiments were used to test the 2\textsuperscript{nd}-order linear filters. Scientists try to feed the 2\textsuperscript{nd}-order linear filter something similar to the first-order sine wave grating stimuli. The 1\textsuperscript{st}-order filter and the subsequent rectification (nonlinearity) transform the stimulus in a first-order noisy version which can be filtered by a linear filter (such as Gabor filter) in the third step.

The FRF model discussed above has been used in different computer vision domains including texture borders detection [33;40], terminators [47], illusory contours [71], and non-Cartesian stimuli [29;30;71]. Varying the tuning of early and late filters to other attributes, this FRF model has the ability to respond to other kinds of stimuli. For instance, in his study [73], Zanker showed that if the linear filters at the first stage are tuned to direction or speed of motion, the model would be suitable for theta motion. Moreover, Hess and Wilcox [36] reported that second-order stereopsis can be achieved if the second linear filters are tuned for disparity. Additionally, appropriate tuning of first and last stage filters could be sensitive to optic flow invariants (expansion – contraction, rotation and others) or for contour integration [27].

In their study [2], Baker and Mareschal argued about the limited functional significance of second-order processing. Their argument was based on the fact that ‘pure’ second-order stimuli are very rare where most of the natural contours contain differences in luminance and textures, and hence second-order processing would only be marginally useful. However, further to this argument, Baker and Mareschal [2]
highlighted the importance of second-order processing in computationally motivated problems. Although first-order luminance data is abundant, the unpredictability of illumination affects (shadows, gradients of shading, etc …) the definition of objects [49]. This leads to the argument that a good computational strategy for image understanding might be to focus only on second-order processing while ignoring first-order information. They concluded that the motivation for the design of the model must not be to solve one class of stimulus, but rather to serve a general-purpose for higher-order structural features occurring in natural images. In this regard, it is suggested that second-order processing may provide a common ground for such a model [2].

As cited by [43], several single studies have showed that the visual cortex of cats and monkeys contain neurons with properties consistent with the above mentioned LNL model. The majority of these neurons were found in the extra-striate visual areas (area 18 in the cat, V2 and IT in macaque monkey) and other neurons selective for second-order patterns were found in the primary visual cortex. As cited by [43] similar studies [24;25;50;59;70] using functional MRI (fMRI) have been performed on the human visual cortex and the results are rather conflicting. Most of these studies found that first- and second-order stimuli affected neurons in the same areas of visual cortex. There were only few studies that reported response in one kind of stimulus in different area. For instance, [67] stated that second-order stimuli effected stronger activation in areas V3 and V3 A/B than did first-order stimuli.
Chapter 3  Analytical Tools
3.1 Introduction

This chapter describes two techniques, namely linear and non-linear, used to enhance an image. In particular, it focuses on the complex 2-D Gabor linear filters and the rank-based filters which are our area of interest.

3.2 Linear Filters

Linear filters are very popular in image processing where they enhance an image through some linear operations. This section provides an overview of how linear filtering can be performed in both the spatial and the frequency domains together with the design of a complex 2-D Gabor linear filter.

3.2.1 Fourier Transform

The French mathematician Jean Baptiste Joseph Fourier is remembered for his great contribution, that is the discovery that a function can be decomposed in terms of sinusoidal functions having different frequencies, referred to as basis functions, which can be recombined to get the original function (see Figure 3.1). The decomposition of the function consists of the amplitude and the phase assigned to each basis function. Hence, the function’s value can be represented as a complex number in either rectangular or polar coordinates [32].

![Figure 3.1: Fourier Basis Functions](image)

The function at the bottom is the sum of the three functions above it.
The Fourier transform is the technique used to transform a function into the frequency domain representation. The following are two variants of Fourier transforms which are extensively used in Signal and Image Processing:

### 3.2.2 Continuous Fourier Transform

The Fourier transform $F(u,v)$ of the continuous two-dimensional function $f(x,y)$ and its inverse are defined respectively as:

$$
F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-i2\pi(ux+vy)} \, dx \, dy \quad f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{i2\pi(ux+vy)} \, du \, dv \quad \text{Eq. 3.1}
$$

### 3.2.3 Discrete Fourier Transform (DFT)

In image processing, the interest lies in the discrete Fourier transform as pixel values are used. The transform and its inverse are defined respectively as:

$$
F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-i2\pi ux / M} \quad f(x) = \sum_{u=0}^{M-1} F(u) e^{i2\pi ux / M} \quad \text{Eq. 3.2}
$$

Substituting the below Euler’s formula into Eq. 3.2a

$$
e^{i\theta} = \cos \theta + i \sin \theta \quad \text{Eq. 3.3}
$$

We can rewrite Eq. 3.2a as:

$$
F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) \left[ \cos \frac{2\pi ux}{M} - i \sin \frac{2\pi ux}{M} \right] \quad \text{Eq. 3.4}
$$

Thus, the DFT of an image contains a real and imaginary part in the frequency domain [32].

![Figure 3.2: Example of Discrete Fourier Transform](image)

(a) A 100x100 black image with a small white rectangle at the centre; (b) The respective centred Fourier Spectrum after applying the log transformation
3.2.4 Filtering in the Spatial Domain

Filtering in the spatial domain involves scanning an $M \times N$ image with an $m \times n$ kernel as shown below.

![Kernel scanning the entire image](image)

**Figure 3.3:** Linear Filtering using convolution in the spatial domain

The convolution expression is given as:

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)f(x + s, y + t)$$ \hspace{1cm} Eq. 3.5

where $a = (m-1)/2$ and $b = (n-1)/2$. The complete filtered image can be obtained by applying the equation for $x=1,2,\ldots M-1$ and $y=1,2,\ldots N-1$ [32].

3.2.5 Filtering in the Frequency Domain

One application of Fourier transform, explained in section 3.2.1, is the ability of performing linear filtering in the frequency domain which is more efficient than convolution in the spatial domain. Linear filtering in the frequency domain is obtained by taking the inverse DFT of the scalar product between the image DFT and the kernel DFT (see Figure 3.4). The Fast Fourier Transform (FFT), also referred to as Cooley-Tukey algorithm, allows an efficient computation of the DFT and its inverse making linear filtering in the frequency domain much faster [17].

![Image processing](image)

**Figure 3.4:** Filtering in the Frequency Domain
3.2.6 Complex 2-D Gabor Linear Filter

As mentioned in Chapter 2, Gabor filter, which are members of the linear filters family, proved to be an acceptable model sensitive to first-order stimulus. Their main strength is the ability to detect different orientations and frequencies in both the spatial and frequency domains. A complex Gabor filter contains a real and an imaginary part where the impulse response is the product of a Gaussian function with a harmonic function (see Figure 3.5). It is defined as:

\[
\begin{align*}
\text{Re}(g(x,y)) &= \exp\left\{ -\frac{1}{2} \left[ \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right] \right\} \cos\left( 2\pi \frac{x'}{\lambda} + \psi \right) \\
\text{Im}(g(x,y)) &= \exp\left\{ -\frac{1}{2} \left[ \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right] \right\} \sin\left( 2\pi \frac{x'}{\lambda} + \psi \right)
\end{align*}
\]

Eq. 3.6

Real Part

Imaginary Part

where \( x' = x \cos \theta + y \sin \theta \) and \( y' = -x \sin \theta + y \cos \theta \). Moreover \( \lambda \) represents the wavelength of the harmonic function, \( \theta \) represents the orientation, \( \psi \) is the phase offset, and \( \gamma \) represents the spatial aspect ratio.

Figure 3.5: Complex Gabor Function in the Spatial Domain

(a, b) and (c, d) and (e, f) are 3D illustrations of the Real and Imaginary parts of the complex Gabor, where the Real part is the product of (a) and (b), while the Imaginary part is the product of (a) and (e).

The Fourier transform of a complex 2-D Gabor function is a real Gaussian function (with no imaginary part) as illustrated below:

Figure 3.6: Fourier Transform of a Complex 2-D Gabor Filter
The following figure demonstrates the Fourier transform of the real and imaginary parts of the complex Gabor function separately.

Hence the complex Fourier transform of the Gabor function shown in Figure 3.6, can be obtained by the linear operation:

\[ F(G) = F(\text{Re}(g)) + [i \times F(\text{Im}(g))] \]  

Eq. 3.7

This follows from the fact that in the spatial domain, the complex Gabor function can be expressed as:

\[ G = \text{Re}(g) + (i \times \text{Im}(g)) \]  

Eq. 3.8

### 3.3 Rank-Based Filters

Rank-based filters are non-linear filters based on non-parametric statistics and can only be applied in the spatial domain.

#### 3.3.1 Non-Parametric Statistics

The term *non-parametric* refers to statistical techniques that are distribution free as they are independent from the underlying data of a given probability distribution. The application of non-parametric methods is useful when the data is judged on the
ranking rather than the actual underlying numerical interpretation; e.g. assessing preferences or comparison. The lack of reliance on assumptions makes non-parametric statistics more robust. The simplicity and robustness provided by non-parametric methods attracted scientists to apply them in several learning theory techniques such as Support Vector Machines (SVM) [69] and statistical methods based on ranks [45].

Ranks statistics are usually the statistical technique used in medical research in order to decide whether a proposed innovation (such as drug, treatment, etc …) provides better results over the some standard procedure. For instance, in order to test the effectiveness of a new drug that is hypothesized to have a beneficial effect, the recruited participants suffering from the same disorder are categorized into two groups namely Treatment and Control. Based on random sampling, some of the participants (Treatment) receive the new drug and the others (Control) are given a placebo without letting them know who is having what in order to reduce the psychological effects. After some time the participants are ranked according to their severity of their condition; most severe is ranked first. The new treatment will be considered justified if the Treatment group of participants rank significantly high [45].

### 3.3.2 Mann-Whitney-Wilcoxon (MWW) Test Statistics

The MWW is a combination of Wilcoxon rank-sum test and Mann-Whitney U-Test. It is a non-parametric alternative to the *t*-test used to test the hypothesis for the comparison of two independent distributions. One advantage of this test is that the data does not have to be normally distributed. It assesses whether two samples of observations come from the same distribution; i.e. the location [45].

Consider a population of *N* observations divided into two independent distributions namely *Treatment* \( T_i (1 \leq i \leq m) \) and *Control* \( C_j (1 \leq j \leq n) \). We test for the alternative that the Treatment observations are significantly higher or lower than the Controls, where the null hypothesis \( H_0 \) is \( T \equiv C \).
Let $W_s$ be the Wilcoxon statistic defined by:

$$
W_s = \sum_{i=1}^{N} \pi_i V_i \\
\pi_i = \text{Rank of element } i \\
V_i = \begin{cases} 
0 & \text{for } \pi_i \in C \\
1 & \text{for } \pi_i \in T.
\end{cases}
$$  

Eq. 3.9

Subsequently, let $U$ be the Mann-Whitney statistics defined by:

$$
MW = W_s - \left[ \frac{m(m+1)}{2} \right]
$$  

Eq. 3.10

Suppose $T =\{5,9,11,10,15\}$ and $C =\{20,4,7,13,19,11\}$, then

<table>
<thead>
<tr>
<th>Sample</th>
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<th>C</th>
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<td>5</td>
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</table>

Hence, $W_s = 1 + 3 + 5 + 6 + 9 = 24$ and $MW = 24 - \left[ \frac{5(5+1)}{2} \right] = 9$

### 3.3.3 Rank-Based Features

The application of the above non-parametric statistics was found to be applicable in different domains, including computer vision and image processing (e.g. using pixel intensity values rather than the disorder severity of human participants). Akin to rank statistics are rank-based features which have been extensively applied in stereopsis [4,72]. They are very popular for their robustness in detecting outliers and invariance under monotonic transformations such as brightness and contrast changes along with gamma correction.

Fabrizio Smeraldi [64] had introduced a family of multi-scale, orientation-selective, non-parametric features, called ranklets, based on the computation of MWW rank-sum test statistics. A ranklet is the normalization of the Wilcoxon sum of ranks in the range $[-1, +1]$ defined as [65]:

$$
R = \frac{MW}{mn/2} - 1
$$  

Eq. 3.11

where $MW$ is the Mann-Whitney test statistics defined in Eq. 3.10, and $m$ and $n$ are the size of the Treatment (T) and Control (C) groups respectively. Well, the choice of the T and C sets is a crucial element.

Ranklets achieve similar response to Haar wavelets [19] as they share the same pattern of orientation selectivity, multi-scale nature and a suitable notion of
completeness. The technique involves dividing an image window $W$ into Treatment and Control regions (see Figure 3.8). Subsequently, Wilcoxon rank-sum test statistics [45] are computed in order to determine the intensity variations among conveniently chosen regions of the samples in $W$. The intensity values of both regions are then replaced by the respective ranking scores defined in Eq. 3.11. These ranking scores determine a pairwise comparison between the T and C regions. This means that a ranklet essentially counts the number of $T \times C$ pairs which are brighter in the $T$ set.

Figure 3.8 shows three ranklets used to detect vertical (a), horizontal (b) and diagonal (c) dark-to-bright edges. Smeraldi [65] showed that ranklets are complete and thus all pixels in the image can be unambiguously represented by their rank values. Furthermore, in case of ties (more than one pixel have the same intensity value) the standard approach is to use mid-ranks by assigning the average of the ranks to each group of tied values.

![Figure 3.8: Ranklets sensitive to vertical, horizontal and diagonal edges. Adapted from [64]](image)

Multi-scale ranklets have been successfully applied in Face Recognition and outperformed other rank features including Haar wavelets, Sparse Network of Winnows (SNoW) and linear SVMs [64].

A further generalization of ranklets has been proposed in [66] on hexagonal pixel lattices. Figure 3.9 shows the three preferential directions in an image using a six-fold rotational symmetry of the lattice where the shaded pixels represent the C sets and the central pixel is ignored.

![Figure 3.9: Ranklets on Hexagonal Pixels. Adapted from [66]](image)
Chapter 4  Hypothesis
4.1 Research Gap

The above literature review described two successful acceptable models namely 2-D Gabor Filters and FRF models used for first- and second-order stimuli respectively. Figure 2.3 illustrates a complete model applied in [2] having separate paths for processing first- and second-order stimuli. However, this model requires an a priori knowledge of the image in order to select the appropriate path. For instance, a simple luminance grating can be detected only through the first-order pathway. Likewise, a contrast envelope stimulus can provide a response on the final output only through the FRF stream [2].

Further to the above and to the best of our knowledge, there is no single model suitable for both kinds of stimuli without a priori knowledge of the nature of the image.

4.2 Hypothesis

Given the successful application of rank-based features in image processing and the popular 2-D Gabor filters, we are postulating that a single filter with frequency- and orientation-selectivity (i.e. based on the approximation of a 2-D Gabor filter) combined with non-parametric statistics may be effective for both first- and second-order stimuli.
Chapter 5   Methodology
5.1 Introduction

This chapter provides the techniques used to design the frequency- and orientation-selective rank-based filter combined with location and dispersion non-parametric statistics.

5.2 Filter Design

As mentioned in section Chapter 3, Wilcoxon-Mann-Whitney test statistics were successfully applied in face recognition using square ranklets sensitive for horizontal, vertical and diagonal orientations. Although rectangle features are sensitive to the presence of edges and bars, they are not suitable for complex image structures (e.g. detailed analysis of borders and texture analysis) when compared to alternative filters, such as Gabor filters which can be tuned to any frequency and orientation.

So, the first challenge is to design a filter with frequency- and orientation-selective response as a 2-D Gabor filter, which can be categorized into Treatment and Control regions. This section provides three alternative filter designs which are then tested in Appendix D in order to establish the best design achieving the highest performance. The results of these experiments are then analysed in Chapter 7. These filter designs are referred to as approximations of the 2-D Gabor linear filter.

5.2.1 Approx 1: Naïve Approximation of Complex 2-D Gabor Filter

The ranklets proposed by Smeraldi (see Figure 3.8) consist of a square filter where half of the pixels are assigned a +1 value and the other half is assigned a -1 value. In order to have a similar +1/-1 filter, the 2-D Gabor function is approximated by setting all coefficients $\geq c_T$ equal to +1 and all coefficients $\leq c_T - T_c$ equal to -1. The remaining coefficients are set to 0; they do not participate in filtering.

The following diagrams illustrate different Gabor filter approximations by varying the threshold $T_c$. If the $T_c$ is 0, the undesirable rectangular filter will be obtained. Ideally $T_c$ has to be just above 0; e.g. 0.005 or 0.01 but not too high such as 0.1.
Furthermore, the filters can be tuned to different orientations and frequencies as demonstrated below.

Given the symmetrical shape of the imaginary filter, the Treatment and Control regions always contain identical number of pixels. However, this approximation does not guarantee identical sized regions for the real filter; i.e. naïve approximation.

### 5.2.2 Approx 2: Adaptive Approximation of Complex 2-D Gabor Filter

As it had been proposed in [64], ideally, the positive (+1) and negative (-1) areas must be equal. This alternative approach aims at achieving the same (or very close) pixel count for both positive and negative regions by the following iterative technique:

- Let $T_d$ be a threshold representing the maximum accepted difference between the pixel count of the positive and negative regions.
- Let $T_c$ be the threshold where the Gabor coefficients $\geq T_c$ are set to +1 and Gabor coefficients $\leq -T_c$ are set to -1.
- **Repeat**
  - **filter** = approximated Gabor Filter using $T_c$
  - **pCount** = number of +1 pixels in filter
  - **nCount** = number of -1 pixels in filter
  - $T_c = T_c + 0.001$
- **Until** $\text{abs}(\text{pCount} - \text{nCount}) \leq T_d$
The above algorithm presumably obtains a better approximation as the count of the negative and positive regions are the same (or very close; at most differ by $T_d$ pixels).

Considering the complex Gabor function in Figure 5.1, when $T_c = 0.01$ and $T_d = 3$ the following approximation is achieved where the positive and negative regions in the real filter differ only by one pixel.

![Real Filter: 213 + 213 = 426 black pixels](image1)

![Imaginary Filter: Balanced regions](image2)

$91 + 243 + 91 = 425$ white pixels

Figure 5.3: Adaptive Filter Approximation

### 5.2.3 Approx 3: Weighted Gabor Coefficients

This filter, which is referred Approx 3 to in this study, is designed exactly as discussed in section 3.2.6 but is used in a non-linear (non-parametric) fashion rather than in its original linear mode. The difference between this filter and the 2-D Gabor linear filter is in the filtering process. In this case, the positive coefficients form the Treatment group, while the Control group consists of the negative coefficients. The spatial filtering process performs a scalar product for each sub-image with the modulus coefficients of this filter before proceeding with the non-parametric statistics explained below. Figure 5.4 shows an illustration of a snapshot during the filtering process.

![Spatial Filtering using Approx 3](image3)
5.3 Non-Parametric Statistics: Location and Dispersion Tests

As described in section 1.3.1, first-order stimuli are characterized by differences in luminance where one region may be brighter or darker than the other region. Similar to ranklets, this study applies the Wilcoxon statistics (see section 3.3.2) in conjunction with the filter designs discussed above to test their effectiveness for first-order stimuli. However, the Wilcoxon statistics may not be adequate to second-order stimuli (see Figure 1.4b) due to a roughly constant luminance across regions (but with different variance). A more suitable test statistics in this case would be any non-parametric statics sensitive to dispersion rather than location. Ansari-Bradley [1] and Siegel-Tukey [62] are two dispersion test statistics, developed in the 1960s, that are tested in this study for second-order stimuli.

Moreover, keeping the main objective of this study in mind, a combination of both location and dispersion test statistics is hypothetically suitable for kinds of stimuli.

5.3.1 Ansari-Bradley Dispersion Test

This is a non-parametric statistics alternative to the parametric $F$-test for differences in variance. It is used to test the hypothesis that two independent samples are members of populations which differ in variance or dispersion. Since the interest now is on dispersion, a different ranking scheme is applied differing from the ranking scheme applied for Wilcoxon test statistics where the interest was on location [1].

The proposed rank order starts ranking from the centre towards the two ends beginning with unities in case of an even sample and with a zero in case of an odd sample [1]. The following shows an example of the Ansari-Bradley ranking scheme for odd and even samples.

<table>
<thead>
<tr>
<th>Sample Value</th>
<th>T</th>
<th>C</th>
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<tbody>
<tr>
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<td>0</td>
<td>1</td>
<td>2</td>
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<td>4</td>
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Odd Population $P_o$

<table>
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<tr>
<th>Sample Value</th>
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<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Even Population $P_e$

Similar to Wilcoxon statistics, the sum of the ranks is obtained using Eq. 3.9.

\[
\text{Rank - Sum for } P_o = 4 + 2 + 0 + 1 + 4 = 11 \\
\text{Rank - Sum for } P_e = 4 + 2 + 1 + 2 = 9
\]
If $m$ is the size of the Treatment group, $n$ is the size of the Control group and $N = m + n$, the mean and variance for Ansari-Bradley ranking distribution are defined as [46]:

\[
\mu = \begin{cases} 
\frac{m(N+2)}{4} & \text{N is even} \\
\frac{m(N+1)^2}{4} & \text{N is odd}
\end{cases} \\
\sigma^2 = \begin{cases} 
\frac{mn(N^2 - 4)}{48(N-1)} & \text{N is even} \\
\frac{mn(N+1)(N^2 + 3)}{48N^2} & \text{N is odd}
\end{cases}
\]

Eq. 5.1

### 5.3.2 Siegel-Tukey Dispersion Test

Similar to Ansari-Bradley, Siegel-Tukey is another dispersion non-parametric test statistic but with a different ranking order having different mean and variance. The ranking scheme involves assigning low ranks to extreme observations and high ranks to central observations. If there were an odd number of observations, the middlemost score is not ranked in order to have an even maximum rank. The following shows an example of this scheme with odd and even populations consisting of two groups; Treatment (T) and Control (C) [62].

<table>
<thead>
<tr>
<th>Sample</th>
<th>T</th>
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<td>8</td>
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<td>6</td>
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</table>

Odd Population $P_o$

<table>
<thead>
<tr>
<th>Sample</th>
<th>T</th>
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<td>4</td>
<td>5</td>
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</table>

Even Population $P_e$

Similarly to the Wilcoxon statistics and Ansari-Bradley, the sum of the ranks is calculated using Eq. 3.9.

\[
\text{Rank-Sum for } P_o = 1 + 5 + 0 + 7 + 2 = 15 \quad \text{Rank-Sum for } P_e = 1 + 5 + 7 + 6 = 19
\]

Moreover, the mean and variance of the Siegel-Tukey ranking distribution are defined as [62]:

\[
\mu = \frac{1}{2}(N+1) \quad \sigma^2 = \frac{1}{12}(N^2 - 1)
\]

Eq. 5.2

### 5.3.3 Combined Location and Dispersion Test Statistics

Given that Wilcoxon test statistics may be applicable to first order stimulus and dispersion tests statistics (Ansari-Bradley and Siegel-Tukey) may be applicable to second-order stimulus, it is expected that a combination of both statistics would presumably be suitable for both stimuli.
Looking at the rank order of the three statistical tests, the Ansari-Bradley contains repetitive ranks while Wilcoxon and Siegel-Tukey contain unique ranks. This leads to the fact that both Wilcoxon and Siegel-Tukey test statistics have the same mean defined by Eq. 5.2. This latter was the main reason that motivated us to implement a combination of Wilcoxon and Siegel-Tukey rank statistics as explained in the following sub-section.

5.3.3.1 Innovative Combination of Wilcoxon-Siegel-Tukey (WST) Test

This section proposes a combination of Wilcoxon and Siegel-Tukey statistics, aimed to be suitable for both first- and second-order stimuli. The sum of ranks for this combination is defined to be equal to the average of both rank-sums.

Considering the following example

<table>
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<tr>
<th>Sample</th>
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</tr>
<tr>
<td>Wilcoxon Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Siegel-Tukey Rank</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The Wilcoxon and Siegel-Tukey sum of ranks are respectively:

\[ W_s = 1 + 3 + 5 + 6 + 10 = 25 \]
\[ ST_s = 1 + 5 + 9 + 10 + 2 = 27 \]

Hence, the average sum of ranks is equal to \( \frac{1}{2} (W_s + ST_s) = \frac{1}{2} (25 + 27) = 26 \).

The average sum of ranks can be obtained directly by averaging the Wilcoxon and Siegel-Tukey ranking as follows:

<table>
<thead>
<tr>
<th>Sample</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>T</th>
<th>C</th>
<th>C</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>11</td>
<td>13</td>
<td>20</td>
<td>21</td>
<td>24</td>
<td>29</td>
</tr>
<tr>
<td>Wilcoxon Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Siegel-Tukey Rank</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Wilcoxon-Siegel-Tukey Rank</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Thus, the average sum of ranks \( WST_s = 1 + 4 + 7 + 8 + 6 = 26 \)

5.3.4 Treatment of Ties

If more than one observation within the same sample has an equivalent score, then the rank does not affect the sum of ranks. However, in case there is a tie between observations coming from different samples, usually the mean rank is applied to the observations [62].
Suppose \( T = \{5, 9, 9, 8, 15\} \) and \( C = \{20, 4, 9, 13, 19, 5\} \), then

\[
\begin{array}{cccccccccccc}
\text{Sample Value} & C & T & C & T & C & T & C & T & C & C \\
\hline
\text{Wilcoxon Rank} & 4 & 5 & 5 & 8 & 9 & 9 & 13 & 15 & 19 & 20 \\
\text{Wilcoxon Mid-Ranks} & 1 & 2.5 & 2.5 & 4 & 6 & 6 & 6 & 8 & 9 & 10 & 11 \\
\text{Ansari-Bradley Rank} & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\text{Ansari-Bradley Mid-Ranks} & 4 & 3.5 & 3.5 & 2 & 0.67 & 0.67 & 0.67 & 2 & 3 & 4 & 5 \\
\text{Siegel-Tukey Rank} & 1 & 4.5 & 4.5 & 8 & 6.33 & 6.33 & 6.33 & 7 & 6 & 3 & 2 \\
\text{Siegel-Tukey Mid-Ranks} & 1 & 3.5 & 3.5 & 8 & 6.17 & 6.17 & 6.17 & 7.5 & 7.5 & 7.5 & 7.5 \\
\end{array}
\]

Table 5.1: Mid Ranks

Hence, the sum of ranks for Wilcoxon \( W_s \), Ansari-Bradley \( AB_s \), Siegel-Tukey \( ST_s \), and Wilcoxon-Siegel-Tukey \( WST_s \) are:

- \( W_s = 2.5 + 4 + 6 + 6 + 9 = 27.5 \)
- \( AB_s = 3.5 + 2 + 0.67 + 0.67 + 3 = 9.84 \)
- \( ST_s = 4.5 + 8 + 6.33 + 6.33 + 6 = 31.16 \)
- \( WST_s = 3.5 + 8 + 6.17 + 6.17 + 7.5 = 31.34 \)

Michael and Edwardes [26] claimed that when data is tied in the comparison of dispersion, the statistics should be tied-centered. They claimed that for statistics with score sums, the highest score must be assigned to the median both when there are ties and when there are no ties. They proposed a new tie-centered rank test statistic with a Wilcoxon distribution for ordered categorical data together with Fligner-Killeen test for data consisting meaningful point values with many ties.

### 5.3.5 Ranklet Calculation

In his study [64], Smeraldi defined the ranklet value for Wilcoxon statistics as:

\[
R = \frac{MW}{\sqrt{mn} - 1}
\]  
Eq. 5.3

The ranklet value is rescaling the Wilcoxon sum of ranks \( W_s \) between -1 and 1 while assuming a symmetrical distribution. This is achieved by first subtracting the minimum rank-sum in order to set the minimum to 0; i.e. Mann-Whitney statistics \( MW \). Subsequently subtract the mean so that it is centered at 0 and finally divide by the mean. The following illustrations show a step-by-step of this technique used to rescale the sum of ranks.
In general, the rescaled rank value is defined as:

$$R = \frac{W_s - \text{Min}_m - \text{Mean}}{\text{Mean}} = \frac{W_s - \text{Min}_m}{\text{Mean}} - 1$$  \hspace{1cm} \text{Eq. 5.4}$$

The following are the equations for the non-parametric statistics used in this study. The derivations of the equations are given in Appendix C.

<table>
<thead>
<tr>
<th>Method</th>
<th>Minm</th>
<th>Maxm</th>
<th>Mean</th>
<th>R formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilcoxon</td>
<td>$m(m+1)/2$</td>
<td>$m(m+2n+1)/2$</td>
<td>$(m+n)/2$</td>
<td>$R = \frac{W_s - m(m+1)/2}{mn/2} - 1$ or $R = \frac{MW}{mn/2} - 1$</td>
</tr>
<tr>
<td>Ansari-Bradley</td>
<td>$\begin{cases} \frac{m(m+2)}{4} &amp; \text{m is Even} \ \frac{m(m+2)+1}{4} &amp; \text{m is Odd} \end{cases}$</td>
<td>$\begin{cases} \frac{mn}{4} &amp; \text{m is Even} \ \frac{(m-1)n}{4} &amp; \text{m is Odd} \end{cases}$</td>
<td>$\text{Mean} = (\text{Max} - \text{Min})/2 = mn/2$</td>
<td></td>
</tr>
<tr>
<td>Siegel-Tukey</td>
<td>$\begin{cases} \frac{m^2}{4} &amp; \text{m is Even} \ \frac{(m-1)(m+1)}{4} &amp; \text{m is Odd} \end{cases}$</td>
<td>$\text{Mean} = mn/4$</td>
<td>$\text{Min}_m$</td>
<td></td>
</tr>
<tr>
<td>Wilcoxon-Siegel-Tukey</td>
<td>$\begin{cases} \frac{m(m+1)}{2} &amp; \text{m + n is Even} \ \frac{m(m-1)}{2} &amp; \text{m + n is Odd} \end{cases}$</td>
<td>$\text{Mean} = mn/2$</td>
<td>$\text{Mean} = (\text{Total} - \text{Min}_m - \text{Min}_n)/2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Ranklet Calculation for Non-Parametric Statistics
5.4 Rank-Based Filtering in the Spatial Domain

Filtering is performed in the spatial domain with both the real and imaginary filters separately. Following Eq. 3.8, a constant response $H$ is obtained by taking the modulus of the complex filtered image as follows:

$$H = \text{abs}[\text{Re}(I) + (i \times \text{Im}(I))]$$

Eq. 5.5

The convolution applies a zero-padding scheme for off-the-edge image pixels. The image is scanned with the kernel and for each window the middle pixel is set to be the ranklet score (see Eq. 5.4) explained above. Figure 5.6 shows the pseudo code for this algorithm:

1. Set $W = 0$. This represents the sum of ranks
2. Set $L = \text{rank-order according to the non-parametric statistics type}$
3. Initialise $I$ to zeros as the result filtered image
4. Start scanning the image by the filter
5. Repeat for all sub-images
   5.1 Let $y$, $x$ be the row and column of the sub-image middle pixel relative to the entire image
   5.2 Let $S$ be the current sub-image
   5.3 If filter type equals Approx 3
       5.3.1 $S$ is the scalar product of $S$ and filter absolute coefficients
       End If
   5.4 Select the Treatment and Control pixels from $S$ corresponding to the positive and negative filter coefficients respectively
   5.5 Sort the Treatment group in ascending order
   5.6 Let $U$ be the union of the Treatment and Control sets and sort it in ascending order keeping track of the group where each value belongs
   5.7 For each value $t$ in the Treatment set
       5.7.1 If current value $t_i$ is the first in the set or not equal to $t_{i-1}$
           5.7.1.1 Let $F$ be the set of all instances of $t_i$ in $U$
           5.7.1.2 Let $S$ be the set of the corresponding rankings of $F$ from $L$
           5.7.1.3 Let $R$ be the mean of set $S$
           End If
       5.7.2 $W = W + R$
   End For
5.8 Set $I(y,x) = [(W - \text{Min}_m)/\text{RankingMean}] - 1$ (i.e. Ranklet: see Eq. 5.4)
End Loop

- Perform steps 3 to 5 for both the real and imaginary filter in order to create two filtered images referred to $\text{Re}(I)$ and $\text{Im}(I)$ respectively.
- The final output $H$ is constructed using Eq. 5.5

Figure 5.6: Pseudo code for Rank-Based Spatial Filtering
5.5 Conclusion

This chapter provided a detailed methodology of the three filter designs which are based on the approximation of a 2-D Gabor filter. It has also introduced the innovative combination of Wilcoxon (location) and Siegel-Tukey (dispersion) non-parametric test statistics; referred as Wilcoxon-Siegel-Tukey WST. The chapter concludes by giving the ranklet calculation for each of the non-parametric statistics.

The following chapter shows the experiments that were carried out in order to test the effectiveness of the above filter designs and filtering techniques.
Chapter 6  Implementation and Results
6.1 Software Tools

Matlab 7.4.0.287 (R2007a) is the single programming software tool used in this project. Matlab is very popular in the academic world and is very suitable for image processing applications where it allows reasonably easy interface through matrix manipulation, plotting of data, and other useful functions.

6.1.1 Software Tools Summary

Table 6.1 shows a summary of software components used in this project.

<table>
<thead>
<tr>
<th>Software Component</th>
<th>Name</th>
<th>Version</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating System</td>
<td>Windows</td>
<td>XP</td>
</tr>
<tr>
<td>Programming Language</td>
<td>Matlab</td>
<td>7.4.0.287 (R2007a)</td>
</tr>
</tbody>
</table>

Table 6.1: Software Tools Used

6.2 Execution of Experiments

The filtering technique described in the methodology requires a lot of processing mainly due to the pixels sorting of each window required before calculating the ranklet value. This affected a slow filtering process for each image where an image of 128x128 pixels usually takes around 10 minutes to be filtered. Further to this limitation, the execution of numerous experiments was a bit restricted on one machine.

This issue was overcome by running concurrent experiments on multiple machines at the computer laboratory of Queen Mary University. The same Matlab script with different parameters was executed on different machines, and subsequently each filtered image was stored in a common repository as shown in Figure 6.1.

Figure 6.1: Concurrent execution of experiments
6.3 2-D Gabor Filter - Biologically Motivated Linear Filter

Further to the literature review given in Chapter 2, 2-D Gabor filters are found to be suitable for first-order stimulus but unsuitable for second-order stimulus. The following experiments illustrate the response achieved for both stimuli where a constant response is successfully obtained for first-order, while a noisy response is obtained for the second-order stimulus. The experiments include two kinds of first-order stimuli (round and square object) and a second-order stimulus (round object). For each of the stimuli, two experiments were carried out in order to get a constant response for both patterns; i.e. the object at the centre and its background. This is achieved by tuning the filters to \( \theta = 45^\circ \) and \( \theta = 0^\circ \) respectively as shown below.

![Real Filter Imag. Filter Real Filter Imag. Filter](image)

**1st-Order Stimulus: Round object**

(a) SNR = 19.0574

(b) SNR = 13.5986

**1st-Order Stimulus: Square object**

(c) SNR = 17.9743

(d) SNR = 12.7827

**2nd-Order Stimulus: Round object**

(e) SNR = 1.303

(f) SNR = -1.0236

Figure 6.2: 2-D Gabor: Response to (a-d) 1st- & (e-f) 2nd-Order Stimulus
6.4 Wilcoxon Rank-Based Filter: Approximation of Linear Filter

As already mentioned, Smeraldi [64] showed how rank-based features, which he called ranklets, were found to perform better in face recognition than alternative wavelet linear filters. In this study, the same concept of ranklets, based on Wilcoxon statistics, is applied with the approximation of a Gabor filter. The same experiments as above are carried out, using the Approx 2 filter type, to test the effectiveness of the Wilcoxon rank-based filter.

![Real filter Imag. Filter](image1)
![Real filter Imag. Filter](image2)

1st-Order Stimulus: Round object
(a) SNR = 17.1209
(b) SNR = 10.3635

1st-Order Stimulus: Square object
(c) SNR = 16.1179
(d) SNR = 10.172

2nd-Order Stimulus: Round object
(e) SNR = 1.9212
(f) SNR = -0.6279

Figure 6.3: Wilcoxon - Response to (a-d) 1st & (e-f) 2nd-Order Stimulus
6.5 Rank-Based Filters sensitive to Second-Order Stimulus

This section shows how dispersion test statistics, such as Ansari-Bradley and Siegel-Tukey, can be applied in a rank-based filter to detect second-order stimulus which is characterized by variations in texture. Furthermore, the experiments show that the non-parametric statistics are not suitable for first-order stimulus which is characterized by variations in luminance.

6.5.1 Ansari-Bradley Rank-Based Filter

The following figures show the successful responses of Ansari-Bradley rank-based filter to first-order stimulus but poor response to second-order stimulus.

Figure 6.4: Ansari-Bradley -Response to (a-d) 1st- & (e-f) 2nd-Order Stimulus
6.5.2 Siegel-Tukey Rank-Based Filter

Siegel-Tukey is another dispersion test statistic but with a different ranking order than Ansari-Bradley (see section 5.3.2). These experiments were carried out to investigate the effect of a different ranking scheme against the response to second-order stimulus.

<table>
<thead>
<tr>
<th>2nd-Order Stimulus: Round object</th>
<th>1st-Order Stimulus: Square object</th>
<th>1st-Order Stimulus: Round object</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Real filter" /></td>
<td><img src="image2" alt="Imag. Filter" /></td>
<td><img src="image3" alt="Real filter" /></td>
</tr>
<tr>
<td><img src="image5" alt="Real filter" /></td>
<td><img src="image6" alt="Imag. Filter" /></td>
<td><img src="image7" alt="Real filter" /></td>
</tr>
<tr>
<td>(b) SNR = 15.7228</td>
<td>(b) SNR = 9.9051</td>
<td>(a) SNR = 15.1592</td>
</tr>
<tr>
<td><img src="image9" alt="Real filter" /></td>
<td><img src="image10" alt="Imag. Filter" /></td>
<td><img src="image11" alt="Real filter" /></td>
</tr>
<tr>
<td><img src="image13" alt="Real filter" /></td>
<td><img src="image14" alt="Imag. Filter" /></td>
<td><img src="image15" alt="Real filter" /></td>
</tr>
<tr>
<td>(a) SNR = 1.9776</td>
<td>(b) SNR = 11.5220</td>
<td>(a) SNR = 1.9776</td>
</tr>
</tbody>
</table>

Figure 6.5: Siegel-Tukey - Response for (a-d) 1st- & (e-f) 2nd-Order Stimulus

The above figures show that the responses obtained by Siegel-Tukey rank-based filter are very similar to the ones obtained by Ansari-Bradley. This confirms that any dispersion test statistic is applicable to second-order stimulus.
6.6 Combined Filter sensitive to First- and Second-Order Stimulus

The above results show effective responses by the application of location and dispersion non-parametric statistics to first- and second-order stimuli respectively. Consequently, a combination of both location and dispersion non-parametric statistics is motivated to have a single rank-based filter suitable for both kinds of stimuli. Section 5.3.3 provides an explanation of how Wilcoxon (location) and Siegel-Tukey (dispersion) non-parametric statistics are combined within a single rank-based filter, which we refer to it as Wilcoxon-Siegel-Tukey WST.

The responses illustrated below show that the innovative WST rank-based filter is indeed effective for both stimuli. For the first-order stimulus, a slightly better response is obtained due to a more uniform sinusoidal grating.

![Figure 6.6: WST - Response to 1st- & 2nd-Order Stimulus with round object](image)

(a) SNR = 16.6145
(b) SNR = 10.3743
(c) SNR = 13.9971
(d) SNR = 9.7064

Figure 6.6: WST - Response to 1st- & 2nd-Order Stimulus with round object
The following figures are further responses showing that the innovative filter is also sensitive when sharp-edged objects are present in the stimuli.

![Figure 6.7: WST - Response to 1st- & 2nd-Order Stimulus with sharp-edged object](image)

**6.6.1 Detuned WST Rank-Based Filter Response**

Further experiments are carried out to investigate the response of the combined WST rank-based filter when it has different wavelength or different orientation (i.e. detuned) to that of the stimuli.

The experiments are performed on both first- and second-order stimuli. The experiments on first-order stimulus are also repeated with 2-D Gabor filter for comparison purposes. The stimuli (both first and second-order) used in these experiments have wavelength $\lambda = 8$, the background has an orientation of $\theta = 0^\circ$ and the orientation of the middle object is $\theta = 45^\circ$. On the other hand, the filters are configured with $\lambda = 12, 16$ and $\theta = 90^\circ, 135^\circ$.

Figure 6.8 and Figure 6.9 show the detuned responses to first-order stimulus using 2-D Gabor filter and WST rank-based filter respectively, while Figure 6.10 shows the detuned response to second-order stimulus using WST rank-based filter. The responses clearly show that when the filters are detuned with the object, a poor
response is achieved for both filters and both stimuli. This shows that the innovative WST rank-based filter is congruent with the function of a 2-D Gabor filter, where the best response is achieved only when the filter is exactly tuned to the region of interest.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>λ = 8; θ = 90°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 8; θ = 135°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 12; θ = 45°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 16; θ = 45°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.8: 2-D Gabor - detuned response to first-order stimulus

<table>
<thead>
<tr>
<th>1st-Order Stimulus</th>
<th>SNR = -2.13</th>
<th>SNR = 0.7701</th>
<th>SNR = 10.6756</th>
<th>SNR = -1.2696</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ = 8; θ = 90°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 8; θ = 135°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 12; θ = 45°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 16; θ = 45°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.9: WST - detuned response to first-order stimulus

<table>
<thead>
<tr>
<th>2nd-Order Stimulus</th>
<th>SNR = -1.4563</th>
<th>SNR = -0.084</th>
<th>SNR = 6.9536</th>
<th>SNR = -1.6122</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ = 8; θ = 90°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 8; θ = 135°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 12; θ = 45°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ = 16; θ = 45°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.10: WST - detuned response to second-order stimulus
6.6.2 Mixed Visual Stimulus

This section shows a direct comparison between a 2-D Gabor filter and the WST rank-based filter. A mixed stimulus (see Figure 6.11), containing the two kinds of stimuli with different orientations and different object types (round and sharp-edged object) is used in this scenario.

![Figure 6.11: Mixed Stimulus](image)

The responses below show that although a Gabor filter manages to obtain a better response for first-order stimulus, it fails to respond to second-order stimulus. For instance, Figure 6.12a shows only a response to the circle object but no response to the square object even though it has the same frequency and orientation. On the other hand, the proposed filter showed that it can provide frequency- and orientation-selective response to both stimuli.

![Figure 6.12: WST rank-based filter effective for both stimuli](image)
6.7 Conclusion

Through the illustrated experimental results, this chapter showed that the innovative frequency- and orientation-selective WST rank-based filter, which is designed on the approximation of a Gabor filter, is indeed effective for both first- and second-order stimuli.

The next chapter provides a complete and detailed discussion of the obtained results, cost of the algorithm and any limitations of the study.
Chapter 7  Discussion
7.1 Introduction

The objective of this chapter is to analyse the results obtained in Appendix D and the one illustrated in the previous chapter. The analysis is focused on the effectiveness of the proposed rank-based filtering technique for both kinds of stimuli. Additionally, any limitations of our approach are also discussed below.

7.2 Analysis of Results: Comparison of Proposed Filter Types

Initially, a set of experiments, given in Appendix D, were carried out to test which filter design (see section 5.2) contributes to the best response for each of the filtering techniques. Other comparisons include the filter size characterized by the standard deviation $\sigma$ of the Gaussian envelope, together with its wavelength $\lambda$ and orientation $\theta$. The following sub-sections explain the outcome of each of these comparisons.

7.2.1 Linear Filtering

The purpose of the filter types designed in section 5.2, especially Approx 1 and Approx 2, is not to replace the 2-D Gabor linear filter but to be applied in a rank-based fashion. However, these experiments investigated the response when the proposed filter types are used in a linear mode. As expected, the best response in these experiments is obtained by the 2-D Gabor linear filter but a reasonable good quality response is also recorded for proposed filter types.

7.2.2 Rank-Based Filtering

The effectiveness of the proposed filter types is actually shown in rank-based filtering. When the filters were compared against first-order stimulus, very close responses were obtained for all the filters. Although the difference is not significant, Approx 2 provided the best overall performance for both Wilcoxon and Wilcoxon-Siegel-Tukey filtering techniques followed by Approx 1 and Approx 3. For second-order stimulus, Approx 1 and Approx 2 again outperformed Approx 3. In particular, Approx 2 produced the best responses when combined separately with Ansari-Bradley and Siegel-Tukey rank statistics.
These experiments clarified that both Gabor filter approximations, especially Approx 2, are more suitable for rank-based filtering than the actual Gabor filter referred to as Approx 3.

### 7.3 Effects of Filter Wavelength (λ) and Gaussian Envelope Size (σ)

The abovementioned experiments were performed with various filter and stimulus configurations. Five different filter sizes (σ = 4, 5, 6, 7, 8) were used to test their response effect. The results clearly show that when λ = 4 and 8 of both stimulus and filter, there is a correlation trend between σ and SNR when linear filtering, Wilcoxon and WST rank-based filtering are applied on first-order stimulus, and also when Siegel-Tukey and Ansari-Bradley rank-based filtering are applied on second-order stimulus. In general, the smaller the σ the better the response. However, when Wilcoxon-Siegel-Tukey was applied on second-order stimulus, the trend of the σ effect was less significant. Probably, with more testing scenarios a trend could also be established in this case.

Figure 7.1 and Figure 7.2 illustrate the above finding when a WST rank-based filter was applied for both first- and second-order stimuli where the filter was tuned to detect the circle object with θ = 45° and λ = 8.

![Figure 7.1: Filter Size Response Effect to first-order stimulus](image1)

![Figure 7.2: Filter size response effect to second-order stimulus](image2)
On the other hand, Figure 7.2 shows that the best SNR, in relation to second-order stimulus, is obtained when $\sigma = 6$.

However, when $\lambda = 16$ for both the filter and the stimulus, the correlation trend seems to be reverted between $\sigma$ and the response. This suggests that there may be a correlation between $\lambda$ and $\sigma$, where their values may be proportional to each other. These relationships must all be considered in the design of a filter bank.

### 7.4 Analysis of Results: Comparison of Filtering Techniques

The experimental results given in Chapter 6 show the effectiveness of the proposed rank-based filters. The responses obtained by the Wilcoxon rank-based filter are relatively comparable with the responses of the popular 2-D Gabor linear filter where it has shown high sensitivity to first-order stimulus but poor response to second-order stimuli.

Conversely, dispersion rank-based filters were found to be suitable for second-order stimulus but not sensitive to first-order stimulus. The Ansari-Bradley and Siegel-Tukey rank-based filters obtained very similar responses and thus one can conclude that any dispersion non-parametric test statistics is adequate to detect second-order stimulus.

Finally, the responses obtained in section 6.6 show that the innovative combination of Wilcoxon and Siegel-Tukey statistics within a rank-based filter is sensitive to both first- and second-order stimuli. Subsequently, further experiments were carried out to investigate the response of this combined rank-based filter when it is detuned with the region of interest. The poor responses obtained in this scenario show the congruence of the proposed filter with that of 2-D Gabor linear filter.

### 7.5 Rose Criterion

Rose criterion, which was defined by Albert Rose, states an SNR of 5db is enough to guarantee that the image features are distinguishable. In almost all experiments performed, as shown in Appendix D, the SNR was always above 5db when linear
filter together with Wilcoxon and Wilcoxon-Siegel-Tukey rank-based filters were applied on first-order stimulus, and also when Wilcoxon-Siegel-Tukey, Siegel-Tukey and Ansari-Bradley rank-based filters where applied on second-order stimulus.

This shows that the responses obtained by the proposed rank-based filters, especially the combined Wilcoxon-Siegel-Tukey, are effective for texture segmentation.

### 7.6 Computational Cost of Filtering Process

The pseudo code given in section 5.4 is the main filtering process used in this project. This algorithm scans the image in the spatial domain with the rank-based filter and for each window the pixels inside it are first sorted before proceeding to the calculation of the ranklet value. This involves a lot of computations for each window. A stimulus of 128x128 pixels, the size used for our experiments, takes approximately 10 minutes to complete. However, first-order stimuli took less to complete due to more repetitive pixel values.

#### 7.6.1 Hardware Specifications

The processing time, depends on the machine specifications. For the purpose of this study, multiple computers located at the laboratory of Queen Mary College, each equipped with the below hardware specifications, were used to run experiments concurrently:

<table>
<thead>
<tr>
<th>Hardware</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>Intel Core 2 CPU 6420 2.13GHz</td>
</tr>
<tr>
<td>Physical Memory</td>
<td>2GB of RAM</td>
</tr>
</tbody>
</table>

| Table 7.1: Hardware Specifications |

### 7.7 Limitations

Due to time constraints, this project contains a number of limitations which are discussed below.

The experiments were performed on synthetic stimuli which were particularly chosen and implemented by the author of this study. These stimuli were considered to
provide a good basis for the purpose of this study. However, it may be the case, that
the obtained results are influenced by the chosen stimuli and thus may not generalise
their effectiveness to other several stimuli.

Additionally, the proposed rank-based filters were tested only on synthetic stimuli.
The short period of time restricted the author from proceeding to the design of a filter
bank in order to be able to perform texture segmentation on real images.

With regards to Wilcoxon-Siegel-Tukey rank-based filter no closed form expression
has been derived for the calculation of the ranklet value (see Table 5.2). Although it
did not significantly affect the efficiency of the filtering process, a closed form
expression would still be desirable.

7.8 Conclusion

The above discussion emphasized on the effectiveness of the Gabor filter
approximation combined with non-parametric statistics in detecting first and second-
order stimuli.

The final chapter provides a summary of the entire study and outlines the
contributions along with future work recommendations.
Chapter 8   Conclusion
8.1 Summary

A background overview was initially given in the first chapter where it was explained how scientists were originally motivated to design models which tentatively simulate the functions of the primary visual cortex discovered by several fMRI studies. Research within the biological and neuroscience domains show that the primary visual cortex (part of the brain; also known as striate cortex or V1) processes two kinds of natural visual stimuli. These allow object delineation based respectively on differences in luminance (referred to as first-order stimuli) and on differences in contrast (referred to as second-order stimuli).

Subsequently a thorough literature review was provided which discussed and compared related studies in the field of texture segmentation based on the accepted models namely 2-D Gabor linear filter, which is mainly suitable for first-order stimulus, and the LNL model which is mainly suitable for second-order stimulus. Furthermore, the literature survey covers the work of Smeraldi [65] in which he developed a new type of rank features (namely ranklets) characterized by orientation selectivity. Ranklets are based on the Wilcoxon statistics where one set of measurements is generally characterized by higher values than the other.

The promising results obtained by ranklets in face recognition [64] were the main motivation to design and implement an innovative rank-based filter characterized by orientation- and frequency-selectivity (an approximation of a Gabor filter) which would be sensitive for both visual stimuli mentioned above. Given the fact that dispersion statistics are useful to test for difference in variance between the two groups, they were hypothetically considered to be suitable for second-order stimulus. In practice, experiments shown that the dispersion rank statistics namely Siegel-Tukey and Ansari-Bradley were effectively sensitive to second-order stimuli.

The above positive finding furthered our motivation to implement a single filter which would be sensitive for both stimuli. An innovative single rank-based filter based on the combination of Wilcoxon and Siegel-Tukey statistics was successfully designed and found to be very effective for both first- and second-order stimuli without a priori knowledge of the image.
8.2 Contributions

The main contributions of this study include:

2. Application of Siegel-Tukey or Ansari-Bradley non-parametric statistics which respond to second-order stimulus only.
3. The innovative combination of Wilcoxon-Siegel-Tukey rank statistics sensitive to both stimuli.

The above innovative filters, especially the Wilcoxon-Siegel-Tukey rank-based filter may contribute in the field of texture segmentation where second-order is important.

8.3 Future Work and Recommendations

The promising results obtained by this study encourage us to develop this finding further by testing the above rank-based filters in real-world applications more than just perceptual experiments.

However, before proceeding to the above, it is recommended that a more efficient filtering algorithm would be investigated. As explained in section 7.6, the current algorithm involves numerous computations which are mainly characterized by sorting the pixel intensities in ascending order for each sub-image. Since spatial filtering is performed by moving the kernel one pixel at a time on the image, it is evident that there is large overlap between successive windows. Having already sorted the previous window, it would be smarter to adopt a different sorting technique in order to insert just the new row or column within the overlapped sorted pixels rather than re-sorting each window. This would theoretically lead to significantly less number of computations, thus a more efficient algorithm.

Furthermore, as pointed out in section 7.7, it would be desirable to derive a closed form expression for the ranklet calculation of Wilcoxon-Siegel-Tukey combined statistics. Although the ranking order is the average between the ranking of Wilcoxon
and Siegel-Tukey, the ranklet calculation for their combination is not simply the average of their ranklet values.

Another important recommendation would be to investigate how robust the proposed rank-based filters are. For instance, it would be interesting to compare the Wilcoxon rank-based filter with 2-D Gabor linear filter when applied to noisy (e.g. Gaussian noise) first-order stimuli. The same investigation would follow for the other filters especially for the WST rank-based filter.

Moreover, it would be beneficial in future works to include also an investigation of the comparison between the implemented single rank-based filter with the commonly accepted LNL model for second-order stimuli.

Having established an efficient algorithm would allow the exploration of applications to practical computer vision problems, such as texture segmentation, tracking, and facial image processing amongst others.
References


Appendices
A. System and User Manuals

This appendix provides a user manual on how to use the implemented Matlab scripts to filter a given image with the proposed innovative rank-based filter. Other comments and help can be found within each script.

A.1 Generate a perceptual stimulus

Two scripts are available to create first- and second-order stimuli respectively. The stimuli used for our experiments were generated by the following two scripts.

A.1.1 Generating First-Order stimulus:

- **msc_StimulusFirstOrder.m**

  ```
  function stimulus = msc_StimulusFirstOrder(wavelength,angle,rotation,side,type,width)
  ``

  To generate a first-order stimulus with wavelength $\lambda = 8$, angle (background) $\theta = 0^\circ$, rotation (middle object) = $45^\circ$, side = 128, type = 1 (circle) and width (in this case width is the radius of the circle object) = 16, then call the above function as follows:

  ```
  f = msc_StimulusFirstOrder(8,0,45,128,1,32);
  ```

A.1.2 Generating Second-Order stimulus:

- **msc_StimulusSecondOrder.m**

  ```
  function stimulus = msc_StimulusSecondOrder(wavelength,angle,rotation,side,type,width)
  ``

  Using the same parameters as above, the second-order stimulus can be obtained as:

  ```
  s = msc_StimulusSecondOrder(8,0,45,128,1,32);
  ```

The above function calls generate the below first- and second-order stimuli:

![Figure A.1: Generated (a) First- and (b) Second-Order Stimuli](image)
A.2 Generate the complex approximation of a 2-D Gabor filter

- msc_2DGaborFilterApproximator.m

```
function gbApproxComplex = 
    msc_2DGaborFilterApproximator(sigma_x,sigma_y,theta,lambda,psi, 
        gamma, posT,negT,T)
```

The following parameters are used to generate a sample approximated complex Gabor filter tuned to the object of the above stimuli.

- sigma; σ = 5
- orientation; θ = π/4
- spatial aspect ratio; γ = 1
- wavelength; λ = 8
- phase offset; ψ = 0
- posT and negT; Tc = 0.005
- T; Td = 3

```
rankFilter = 
    msc_2DGaborFilterApproximator(5,5,pi/4,8,0,1,0.005,0.005,3);
```

This creates a complex Gabor filter approximation containing the following real and imaginary parts:

![Real and Imaginary Parts](image-url)

**Figure A.2:** Generated complex filter: approximation of a 2-D Gabor filter

A.3 Execute the rank-based filtering algorithm

- msc_RankBasedFiltering.m

```
function filteredImage = 
    msc_RankBasedFiltering(image,filter,type,tie,weighted,border)
```

Let

- image = f, s (generated above) or else any other image
- type = 3 (i.e. Wilcoxon-Siegel-Tukey)
- weighted = 1 (without scalar product between the sub-image and the filter absolute coefficients - see section 5.2.3)
- filter = rankFilter (generated above)
- tie = 1 (i.e. Mean scheme for ties)
- border = 1 (i.e. zero-padded border)

Then

```
filteredImage = msc_RankBasedFiltering(f,rankFilter,3,1,1,1)
```
B. Project Plan

Project tasks were carried out according to the project plan depicted in the Gantt chart below. Figure B.1a and Figure B.1b show two Gantt charts of the proposed and the actual project plans respectively. The implementation for the Rank-Based filter took longer than predicted due to complex implementation. However, the slack time planned for contingency was enough to cater for delaying this major task.

Figure B.1: Project Plan (a) Planned and (b) Actual
C. Derivation of Equations used for Ranklet Calculation

Let $m$ and $n$ represent the size of the Treatment (T) and Control (C) groups.

### C.1 Wilcoxon Statistics

The Wilcoxon statistics ranking scheme is an ordered sequence of $m+n$ observations:

<table>
<thead>
<tr>
<th>Sample</th>
<th>C</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Minimum sum of ranks for T is the arithmetic series of $m$.

$$Min_m = m(m+1)/2$$

Likewise, minimum rank score for C is:

$$Min_n = n(n+1)/2$$

Total sum of ranks is:

$$Total = (m+n)(m+n+1)/2$$

Maximum rank score for T is:

$$\begin{align*}
\text{Max}_m &= \text{Total} - \text{Min}_m \\
\text{Max}_m &= (m+n)(m+n+1)/2 - n(n+1)/2 \\
\text{Max}_m &= m(m+2n+1)/2 \\
\text{Mean score is:} & \quad \text{Mean} = (\text{Max}_m - \text{Min}_m)/2 \\
\text{Mean} &= (m(m+2n+1)/2 - m(m+1)/2)/2 \\
\text{Mean} &= mn/2
\end{align*}$$

### C.2 Siegel-Tukey Statistics

The Siegel-Tukey ranking scheme for dispersion statistics is given as:

<table>
<thead>
<tr>
<th>Sample</th>
<th>C</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>T</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Minimum sum of ranks for T is equal to $Min_m$ of Wilcoxon statistics.

$$Min_m = m(m+1)/2$$

Similarly, minimum rank score for C is:

$$Min_n = n(n+1)/2$$

Total sum of ranks is:

$$Total = (m+n)(m+n+1)/2$$

Maximum rank score for T is:

$$\begin{align*}
\text{Max}_m &= \text{Total} - \text{Min}_m \\
\text{Max}_m &= (m+n)(m+n+1)/2 - n(n+1)/2 \\
\text{Max}_m &= m(m+2n+1)/2 \\
\text{Mean score is:} & \quad \text{Mean} = (\text{Max}_m - \text{Min}_m)/2 \\
\text{Mean} &= (m(m+2n+1)/2 - m(m+1)/2)/2 \\
\text{Mean} &= mn/2
\end{align*}$$

Minimum sum of ranks for T is equal to $Min_{m-1}$ for an even sample.

$$\begin{align*}
\text{Min}_{m-1} &= (m-1)(m-1+1)/2 \\
\text{Min}_{m-1} &= m(m-1)/2 \\
\text{Max}_m &= \text{Total} - \text{Min}_{m-1} \\
\text{Max}_m &= (m+n)(m+n-1)/2 - (m-1)(m-1+1)/2 \\
\text{Max}_m &= m(m+2n-1)/2 \\
\text{Mean score is:} & \quad \text{Mean} = (\text{Max}_m - \text{Min}_{m-1})/2 \\
\text{Mean} &= (m(m+2n-1)/2 - m(m-1)/2)/2 \\
\text{Mean} &= mn/2
\end{align*}$$

Minimum sum of ranks for T is equal to $Min_n$.

$$Min_n = n(n+1)/2$$

Similarly, minimum rank score for C is:

$$Min_n = n(n-1)/2$$

It follows that total sum of ranks is:

$$Total = (m+n)(m+n-1)/2$$

Maximum rank score for T is:

$$\begin{align*}
\text{Max}_m &= \text{Total} - \text{Min}_n \\
\text{Max}_m &= (m+n)(m+n-1)/2 - n(n-1)/2 \\
\text{Max}_m &= m(m+2n-1)/2 \\
\text{Mean score is:} & \quad \text{Mean} = (\text{Max}_m - \text{Min}_n)/2 \\
\text{Mean} &= (m(m+2n-1)/2 - m(m-1)/2)/2 \\
\text{Mean} &= mn/2
\end{align*}$$
C.3 Wilcoxon-Siegel-Tukey Statistics

The Wilcoxon-Siegel-Tukey ranking scheme is the average ranking between the Wilcoxon ordered sequence and the Siegel-Tukey sequence shown below:

<table>
<thead>
<tr>
<th>Sample</th>
<th>C</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>T</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilcoxon Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Siegel-Tukey Rank</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Combined</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Since the combined order does not match an arithmetic progression, no generic solutions are defined for \( \text{Min}_m \) and \( \text{Mean} \). However, these values can be obtained by first sorting the combined order in ascending order and subsequently set:

- \( \text{Min}_m = \text{Sum of the first } m \text{ sorted ranks} \)
- \( \text{Total} = \text{Sum of the entire ranking list} \)
- \( \text{Mean} = (\text{Total} - \text{Min}_m - \text{Min}_n)/2 \)

C.4 Ansari-Bradley Statistics

The Ansari-Bradley ranking scheme for dispersion statistics is given as:

<table>
<thead>
<tr>
<th>Sample</th>
<th>C</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample</th>
<th>C</th>
<th>C</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>T</th>
<th>C</th>
<th>T</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Even Sample

- Minimum sum of ranks for T is the arithmetic progression for half \( m \).
  - \( \text{Min}_m = m/2(m/2 + 1)/2 \)
  - \( \text{Min}_m = m(m + 2)/4 \)

- The minimum rank score for C is:
  - \( \text{Min}_n = n(n + 2)/4 \)

- Total sum of ranks is:
  - \( \text{Total} = (m + n)(m + n + 2)/4 \)

- Maximum rank score for T is:
  - \( \text{Max}_m = \text{Total} - \text{Min}_n \)
  - \( \text{Max}_m = (m + n)(m + n + 2)/4 - n(n + 2)/4 \)
  - \( \text{Max}_m = m(m + 2n + 2)/4 \)

- Mean score is:
  - \( \text{Mean} = (\text{Max}_m - \text{Min}_n)/2 \)
  - \( \text{Mean} = (m(m + 2n + 2)/4 - m(m + 2)/4)/2 \)
  - \( \text{Mean} = mn/4 \)

Odd Sample

- Minimum sum of ranks for T is:
  - \( \text{Min}_m = \text{Min}_{m-1} + (m + 1)/2 \)
  - \( \text{Min}_m = (m - 1)(m - 1 + 2)/4 + (m + 1)/2 \)
  - \( \text{Min}_m = (m(m + 2)/4 \)

- The minimum rank score for C is:
  - \( \text{Min}_n = (n(n + 2) + 1)/4 \)

- Total sum of ranks is:
  - \( \text{Total} = (m + n)(m + n + 2)/4 \)

- Maximum rank score for T is:
  - \( \text{Max}_m = \text{Total} - \text{Min}_n \)
  - \( \text{Max}_m = (m + n)(m + n + 2)/4 - (n(n + 2) + 1)/4 \)
  - \( \text{Max}_m = (m(m + 2n + 2) - 1)/4 \)

- Mean score is:
  - \( \text{Mean} = (\text{Max}_m - \text{Min}_m)/2 \)
  - \( \text{Mean} = ((m(m + 2n + 2) - 1)/4 - m(m + 2)/4)/2 \)
  - \( \text{Mean} = (m - 1)/4 \)
### Odd Sample

<table>
<thead>
<tr>
<th></th>
<th>$m$ is even</th>
<th>$m$ is odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum sum of ranks for $T$ is equivalent to $\text{Min}_{m-1}$ when $m$ is odd in an even sample plus 0.</td>
<td>Minimum sum of ranks for $T$ is equivalent to $\text{Min}_{n}$ when $m$ is even in an odd sample:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{Min}_m = ((m-1)(m-1+2)+1)/4$</td>
<td>$\text{Min}_m = (m-1)(m+1)/2$</td>
</tr>
<tr>
<td></td>
<td>$\text{Min}_m = m^2 / 4$</td>
<td>The minimum rank score for $C$ is equivalent to $\text{Min}_{n-1}$ when $m$ is even in an odd sample:</td>
</tr>
<tr>
<td></td>
<td>The minimum rank score for $C$ is equivalent to $\text{Min}_{n-1}$ when $m$ is even in an odd sample plus 0.</td>
<td>$\text{Min}_n = n^2 / 4$</td>
</tr>
<tr>
<td></td>
<td>$\text{Min}_n = (n-1)(n-1+2)/4$</td>
<td>Total sum of ranks is independent of $m$ and thus is equal the other:</td>
</tr>
<tr>
<td></td>
<td>$\text{Min}_n = (n-1)(n+1)/4$</td>
<td>$\text{Total} = (m+n+1)(m+n-1)/4$</td>
</tr>
<tr>
<td>Total sum of ranks is equal to the sum of N-1 ranks of an even sample plus 0</td>
<td>Maximum rank score for $T$ is</td>
<td>$\text{Max}_m = \text{Total} - \text{Min}_n$</td>
</tr>
<tr>
<td></td>
<td>$\text{Total} = (m+n-1)(m+n-1+2)/4$</td>
<td>$\text{Max}_m = (m+n+1)(m+n+1)/4 - (n-1)(n+1)/4$</td>
</tr>
<tr>
<td></td>
<td>$\text{Total} = (m+n+1)(m+n-1)/4$</td>
<td>$\text{Max}_m = (m+n+1)(m+n+1)/4 - n^2 / 4$</td>
</tr>
<tr>
<td>Maximum rank score for $T$ is:</td>
<td>$\text{Max}_m = \text{Total} - \text{Min}_n$</td>
<td>$\text{Max}_m = (m+n+1)(m+n+1)/4 - n^2 / 4$</td>
</tr>
<tr>
<td></td>
<td>$\text{Max}_m = (m+n-1)(m+n+1)/4 - (n-1)(n+1)/4$</td>
<td>$\text{Max}_m = (m+n+1)(m+n+1)/4 - n^2 / 4$</td>
</tr>
<tr>
<td>Mean score is:</td>
<td>$\text{Mean} = (\text{Max}_m - \text{Min}_m)/2$</td>
<td>Mean score is:</td>
</tr>
<tr>
<td></td>
<td>$\text{Mean} = (m(m+2n)/4 - m^2/4)/2$</td>
<td>$\text{Mean} = (\text{Max}_m - \text{Min}_m)/2$</td>
</tr>
<tr>
<td></td>
<td>$\text{Mean} = mn/4$</td>
<td>$\text{Mean} = (m(m+2n) - 1)/4 - (m-1)(m+1)/4)/2$</td>
</tr>
</tbody>
</table>
D. Filter Type Comparison

Further to the three filter types proposed in section 5.2, namely Approx 1, Approx 2 and Approx 3, an array of experiments were carried out in order to test which filter type provides the best performance. Each filter was tuned to the frequency of the respective stimuli and oriented to detect the background (θ = 0°) and the object (θ = 45°) in two separate experiments. Moreover, multiple of the mentioned filters were designed with various standard deviations including σ_1 = 4, σ_2 = 5, σ_3 = 6, σ_4 = 7, and σ_5 = 8 in order to test the effect of the filter size.

D.1 Visual Stimuli

The following synthetic images are the first- and second-order stimuli used for testing.

![Figure D.1: (a) First- and b) Second-Order stimuli used for testing](image)

D.2 Filtering Technique

The filtering process applies a zero-padded border for off-the-edge image pixels. Furthermore, the following five techniques, including linear and non-parametric based approaches were applied in the spatial domain.

- Typical linear spatial filtering (see section 3.2.4)
- Filtering based on non-parametric statistics:
  - Wilcoxon (see section 3.3.2)
  - Wilcoxon-Siegel-Tukey (see section 5.3.3.1)
  - Siegel-Tukey (see section 5.3.2)
  - Ansari-Bradley (see section 5.3.1)
D.3 Quality of Filtered Image: Signal-to-Noise Ratio (SNR)

The signal-to-noise ratio is usually applied to measure the power ratio between the signal and the background noise of an image defined as:

\[
SNR(\text{db}) = 20 \log_{10} \left( \frac{A_{\text{signal}}}{A_{\text{noise}}} \right)
\]

where \(A\) is the root mean square (RMS) amplitude defined as:

\[
RMS = \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}
\]

In our case, the signal is the region where the tuned filter should have responded, while the noise is the remaining region where the filter should have not responded. The rose criterion states that an SNR of at least 5 db is required to guarantee a 100% detection rate for image features [8].

D.4 Experiments

This section shows six line charts, showing the comparison of the three filter types based on the SNR values for various filter and stimulus configurations. The following test scenarios where used to test the performance of the filter:

- **First-Order stimulus**
  - Linear filtering
  - Wilcoxon rank-based filtering
  - Wilcoxon-Siegel-Tukey rank-based filtering

- **Second-Order stimulus**
  - Wilcoxon-Siegel-Tukey filtering
  - Siegel-Tukey filtering
  - Ansari-Bradley filtering

Each experiment included a variation of filter and stimuli wavelength (\(\lambda = 4, 8, 16\)) together with different filter orientation (\(\theta = 0^\circ, 45^\circ\)) and standard deviation (\(\sigma = 4, 5, 6, 7, 8\)) and also two different stimuli square dimensions (width = 32, 64).

The green, red and blue lines represent the response of Approx 1, Approx 2, and Approx 3 filter types respectively.
based filtering was used the filter approximations, referred to Approx 1 and Approx 2, in Chapter 7.

Further discussion on these results is provided in Chapter 7.
E. Weekly Log

This appendix lists all the meetings performed with Dr. Fabrizio Smeraldi, who is the supervisor of this project.

<table>
<thead>
<tr>
<th>No</th>
<th>Meeting Place</th>
<th>Date</th>
<th>Duration</th>
<th>Minutes</th>
<th>Action</th>
</tr>
</thead>
</table>
| 1  | Room CS322 at Queen Mary | Thursday, 5th February 2008 | 2 hours | • Explanation of Gabor Filters  
• Agreed a plan and deliverables for the entire project  
• Agreed on a suitable conference: ICPR  
• Discussing the relationship of Rank Features, Ranklets, and Gabor Filters | • Read about Gabor Filters and Non-Parametric Ran Statistics |
| 2  | Room CS322 at Queen Mary | Wednesday, 27th February 2008 | 1.5 hours | • Change of Plans - Simplification of the study. Focus on Orientation-Selective filter rather than Gabor filter.  
• Aim and objectives remain the same | • Read about orientation-selective filters and Ranklets |
| 3  | Room CS322 at Queen Mary | Wednesday, 5th March 2008 | 1 hour | • Discussing the implementation of the shape of the orientation-selection filter, which is an approximation of a Gabor filter  
• Discussing how this new rank-based filter will adopt the ranklets approach to rank pixels within the filter. | • Start implementing the Gabor approximation in the shape of ellipses  
• Continue reading relevant studies |
| 4  | Room CS322 at Queen Mary | Thursday, 13th March 2008 | 1.5 hours | • Going through the planned methodology  
• Discussing how the rank-based filter may respond to both first-order and second-order images  
• Discuss any bugs of the implementation so far | • Fix bugs and continue reading relevant studies |
<table>
<thead>
<tr>
<th>No</th>
<th>Meeting Place</th>
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<tbody>
<tr>
<td>5</td>
<td>Room CS322 at Queen Mary</td>
<td>Thursday, 19\textsuperscript{th} March 2008</td>
</tr>
</tbody>
</table>

**Minutes**
- Discussing the implementation of Ranklets
- Discussing the similarity of the new filter with Ranklets

**Action**
- Implement Ranklets to have an understanding and compare the results with linear filter
- Continue reading relevant studies

<table>
<thead>
<tr>
<th>No</th>
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<tbody>
<tr>
<td>6</td>
<td>Room CS322 at Queen Mary</td>
<td>Monday, 31\textsuperscript{st} March 2008</td>
</tr>
</tbody>
</table>

**Minutes**
- Discuss the results of the ranklets implementation
- Discuss how the implementation of ranklets can be adapted for the Gabor approximation and applied for first-order stimulus.

**Action**
- Read again about Siegel-Tukey statistics

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<tr>
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<th>Meeting Place</th>
<th>Date</th>
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<tbody>
<tr>
<td>7</td>
<td>Room CS322 at Queen Mary</td>
<td>Friday, 4\textsuperscript{th} June 2008</td>
</tr>
</tbody>
</table>

**Minutes**
- Have a recap after the examination period
- Discussing again the Gabor Filter approximation
  - Positive area must be equal to the Negative area
  - Ellipses have an odd length
  - Discussing the Wilcoxon Statistics again

**Action**
- Make sure that the shape of the Gabor approximation is well implemented
- Read more on orientation-selective approaches and non-parametric rank statistics

<table>
<thead>
<tr>
<th>No</th>
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<tbody>
<tr>
<td>8</td>
<td>Room CS322 at Queen Mary</td>
<td>Friday, 9\textsuperscript{th} June 2008</td>
</tr>
</tbody>
</table>

**Minutes**
- Discuss the implementation of a Gabor filter again
- Discuss the implementation of the Gabor approximation using ellipses
- Normalization of the Gabor approximation to have the sum of positive area equal to the sum of negative area

**Action**
- Implement a Gabor filter and perform some experiments
No 9  
Meeting Place Room CS322 at Queen Mary  
Date Monday, 20th June 2008  

Duration 1 hour

Minutes
- Discussing the new Gabor filter approximation technique identified by the author of this project
  - Instead of implementing ellipses, generate the Gabor filter and take a cut for the positive (+1) and negative (-1) values. Then normalize to end up with a sum of 0.
- Supervisor agreed on this technique

Action
- Implement the new Gabor approximation technique and include Mann-Whitney statistics

No 10  
Meeting Place Room CS322 at Queen Mary  
Date Monday, 30th June 2008  

Duration 1 hour

Minutes
- Publicly available texture database at www.oulu.fi
- Discuss how to implement images for first and second order stimuli
- Discussing again Ranklets, Wilcoxon, Mann-Whitney and Siegel-Tukey
- Discussing the new ranking order used for this new filter which is the average between the Siegel-Tukey and an ordered sequence
- Discussing the report structure

Action
- Implement Siegel-Tukey and the new ranking order within the filter
- Start writing up the report; Introduction and Literature Review chapters

No 11  
Meeting Place Room CS322 at Queen Mary  
Date Monday, 7th July 2008  

Duration 1 hour

Minutes
- Going through the introduction and literature review chapters
- Discuss the methodology used for the experiments

Action
- Start doing experiments

No 12  
Meeting Place Room CS322 at Queen Mary  
Date Monday, 11th July 2008  

Duration 2 hours

Minutes
- Discussing various dispersion test statistics including Ansari-Bradley, Mood and Siegel-Tukey
- Analysis of experiments

Action
- Start writing up the methodology and experiments chapter
No 13  |  Meeting Place  | Room CS322 at Queen Mary
Duration 1.5 hours  |  Date  | Monday, 18th July 2008

Minutes
- Further Analysis of experiments
- Discussing second draft of Methodology chapter

Action
- Continue writing up and conducting more experiments

---

No 14  |  Meeting Place  | Room CS322 at Queen Mary
Duration 2 hours  |  Date  | Monday, 23rd July 2008

Minutes
- Identified a problem for the ranklet calculation of the combined Wilcoxon-Siegel-Tukey ranking
- Agreed to take an estimation and a generic equation that will be studied in the future

Action
- Continue with writing the report and conducting more experiments (again) for the Wilcoxon-Siegel-Tukey tests.

---

No 15  |  Meeting Place  | Room CS322 at Queen Mary
Duration 1 hour  |  Date  | Wednesday, 20th August 2008

Minutes
- Discussing the project report
- Including feedback and suggestions on re-structuring a couple of sections

Action
- Carried out the recommendations
- Print the dissertation
F. CD Contents
Attached with this report is a Compact Disk (CD) containing two copies of this report in different formats together with other data including the Matlab scripts and results.

1. Dissertation Document:
This dissertation is available in two formats as per following
- CD Root:\AMCM025\Dissertation\LA892_FINAL.pdf
- CD Root:\AMCM025\Dissertation\LA892_FINAL.doc

2. Implementation
All Matlab scripts used in this project are available on
- CD Root:\AMCM025\Implementation\Matlab\*.m

3. Results
The results are all compressed within the below single zip file
- CD Root:\AMCM025\Results\LA892_Results.zip

The above compressed file contains a further 120 compressed files where each folder contain experiments with different configurations. Each of these folders consists of:

<table>
<thead>
<tr>
<th>Stimulus1.txt</th>
<th>ImagFilter1.txt</th>
<th>RealFilter3.txt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulus2.txt</td>
<td>RealFilter2.txt</td>
<td>ImagFilter3.txt</td>
</tr>
<tr>
<td>RealFilter1.txt</td>
<td>ImagFilter2.txt</td>
<td>File*.txt</td>
</tr>
</tbody>
</table>

Stimulus1.txt and Stimulus2.txt are the first- and second-order stimuli used for that experiments, while the three filter pairs namely RealFilter?.txt and ImagFilter?.txt are the three filter types used in the experiments. The files named File*.txt are the actual responses after filtering Stimulus1.txt and Stimulus2.txt with the three filter types. The experiments can be loaded in Matlab using the following function:

```matlab
function F = msc_LoadExperiment(path, folder, max, show)
```

where the path and folder are the absolute path and respective folder name, while max can be set to 48 (load all files within the same folder) and show can be set to 1 (display images on screen).

The SNR results for each experiment together with the comparison of the three filter designs can be found:
- CD Root:\AMCM025\Results\LA892_Graphs.xls
Evaluation

I am very satisfied with the study carried out during the last months of my Masters degree. I managed to implement an innovative single rank-based filter with frequency- and orientation-selective response which showed to be effective for both first- and second-order stimuli. The literature survey was an essential research process to build strong grounding of the research area.

This study helped me to broaden my knowledge in several interesting areas. Although a lot of material was covered in Computer Vision and Image Processing modules, this project gave me a deeper understanding of certain techniques including Gabor filters and non-parametric statistics. Moreover, I believe that I have gained a good experience of Matlab which will definitely be useful for further research.

The systematic approach proposed by the project guidelines helped me to complete the dissertation in a methodical way. My self-discipline was another ingredient which helped to focus and stick to my project plan even though there were some delays in specific tasks.

The dissertation equipped me with stronger technical and transferable skills where I have broadened my knowledge in the field of image processing and improved significantly in academic writing and presentation skills.

This Masters degree was essential to equip me with the necessary skills to further my research studies. I am very looking forward to start my PhD studies in October 2008 as part of a long journey in the field of Pattern Recognition.