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A comparative study on main lobe and side lobe of frequency response curve for FIR Filter using Window Techniques

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***Abstract*— Finite Impulse Response (FIR) filters are Digital filters which act as frequency selective systems. The design of FIR filter is a non-recursive structure because there is no feedback connection. The response of FIR filter depends on the present and past input samples. In this paper FIR filters are designed by window methods. The desired time domain response with infinite number of sequence is truncated at some point by multiplying by a window sequence. The length of the resultant sequence will be fixed and finite. Now the use of window function is reasonably straight forward to get filter impulse response with minimal computational effort. There are many window sequences like Rectangular Window, Hanning Window, Hamming Window, Blackman Window, Kaiser Window, Bohman Window, Taylor Window and Tukey Window etc. These windows are helping to approximate the desired characteristics. Basically the window function is a weighting function for an N-point sequence. The spectrum of any window can be obtained by taking Fourier Transform and the obtained frequency response curve can be low pass, high pass, band pass and band stop type. Now the width of the main lobe of the response curve is inversely proportional to the length of the window sequence. In this paper, the width of the main lobe is being varied by changing the value of the length (N) of any window function. The characteristic features for different types of filters are studied and the generated frequency responses are compared with respect to the length of the window sequence.**

***Index Terms*— Digital Filter, Finite Impulse Response, Window function, Frequency Response Curve, Central Lobe, Side Lobe**

I. INTRODUCTION

In advanced communication system, digital filter plays an important role. Depending upon the impulse response, the digital filter can be two types: Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) filters. In impulse response, the attenuation level is very low or ideally zero for desired signal components and the attenuation level is high for unwanted signal components. In this paper, the design of FIR filters is discussed using window method. The smoothness in pass band and stop band will be obtained when sharp transition in edges will occur. By taking proper value of number of taps the following windows are taken: Rectangular window, Blackman window, Hamming window, Bartlett window, Modified Bartlett-Hanning window, Bohman window, Chebyshev window, Flat Top window, Gaussian window, Nuttall defined minimum 4-term Blackman-Harris, Taylor window and Tukey window [1-3]. All the normalized magnitude response curves for low pass, high pass, band pass and band reject filters are compared for different number of taps. From magnitude response curves, the maximally flat value in main lobe and minimal side lobe value are observed and compared.

II. FIR FILTER DESIGN

The design of FIR filter can be done by using window functions in complex domain [4-6]. The impulse response h(n) of FIR filter for N-samples can be obtained by multiplying desired impulse response $h_d(n)$ with the window function w(n) and it is given in equation (1) where the desired impulse response $h_d(n)$ is obtained by taking Inverse Fourier transform of desired frequency response $H_d(e^{j\omega})$,shown in equation (2)

$$h(n) = h_d(n) \times w(n) \text{ ------------------ } (1)$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega \text{ ------------- } (2)$$

Now the selection of window function is important. The following window functions are used for testing and discussed briefly.

Sl No	DESCRIPTION OF WINDOW FUNCTIONS	Sl No	DESCRIPTION OF WINDOW FUNCTIONS
1.	**Rectangular Window:** This function is defined by $\omega(n) = \omega_0 \left(n - \frac{N-1}{2} \right), 0 \le n \le (N-1)$ **N** represents the width i.e. the number of samples in discrete-time.	2.	**Blackman Window:** The following equation defines the Blackman window of length n $\omega(n) = 0.42 - 0.5\cos(2n\pi/(N-1)) + 0.8\cos(4n\pi/(N-1)),$ $0 \le n \le M-1$
3.	**Hamming Window:** The window is given by the function $\omega(n) = \alpha - \beta \cos\left(\frac{2n\pi}{N-1} \right)$ $with, \alpha = 0.54, \beta = 1 - \alpha = 0.46$	4.	**Taylor Window:** This window is given by [7] $\omega(n) = (-1)^{n+1} \frac{1}{n} \exp(\frac{-n^2}{2r^2})$ $where, -\frac{(N-1)}{2} \le n \le \frac{(N-1)}{2}$
5.	**Bartlett-Hanning window:** The window function is $\omega(n) = a_0 - a_1 \left\| \frac{n}{N-1} - \frac{1}{2} \right\| a_2 \cos\left(\frac{2n\pi}{N-1} \right)$ $a_0 = 0.62; a_1 = 0.48; a_2 = 0.38$	6.	**Bohman Window:** The equation for computing coefficient of Bohman window function is $\omega(x) = (1 - \|x\|)\cos(\pi \|x\|) + \frac{1}{\pi}\sin(\pi \|x\|), -1 \le x \le 1$ Where x is a length – L vector of linearly spaced values generated using linspace the first and last elements of bohman window are forced to be identically zero.
7.	**Chebyshev Window:** The window is described in frequency domain by the expression $\omega(k) = \frac{cheb\left(L-1, \beta * \cos\left(\pi * \frac{k}{L}\right)\right)}{cheb(L-1, \beta)}; with, \beta = \cosh \frac{1}{L-1} * a\cosh\left(10^{\left(\frac{at}{20}\right)}\right)$ and Cheb(m,x) denoting the m-th order Chebyshev polynomial calculated at the point x.	8.	**Flattop Window:** The flattop window function is expressed by the equation , $\omega(n) = a_0 - a_1 \cos\left(\frac{2n\pi}{N-1}\right) + a_2 \cos\left(\frac{4n\pi}{N-1}\right) - a_3 \cos\left(\frac{6n\pi}{N-1}\right) + a_4 \left(\frac{8n\pi}{N-1}\right);$ $a_0 = 1, a_1 = 1.93, a_2 = 1.29, a_3 = 0.388, a_4 = 0.028$
9.	**Gaussian Window:** The function of Gaussian window given by $\omega(n) = \exp^{\left(\frac{1}{2}\right)\left(\frac{n-(N-1)/2}{\sigma(N-1)/2}\right)^2}$	10.	**Nuttal Window:** The Nuttal window function is given by $\omega(n) = a_0 - a_1 \cos\left(\frac{2n\pi}{N-1}\right) + a_2 \cos\left(\frac{4n\pi}{N-1}\right) - a_3 \cos\left(\frac{6n\pi}{N-1}\right)$ $a_0 = 0.3635819, a_1 = 0.4891775, a_2 = 0.1365995, a_3 = 0.0106411$

Sl No	DESCRIPTION OF WINDOW FUNCTIONS	Sl No	DESCRIPTION OF WINDOW FUNCTIONS
11.	**Bartlett window:** The window function is given by For even value of n, $\omega(n) = \begin{cases} \frac{2n}{N-1}; 0 \le n \le \frac{N}{2} \\ 2 - \frac{2n}{N-1}; \frac{N}{2} \le n \le N \end{cases}$ For odd value of n, $\omega(n) = \begin{cases} \frac{2n}{N-1}; 0 \le n \le \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}; \frac{N-1}{2} + 1 \le n \le N-1. \end{cases}$	**12.**	**Tukey Window:** This window function is given by, $\omega(n) = \begin{cases} \frac{1}{2}\left[1+\cos\left(\pi\left(\frac{2n}{\alpha(N-1)}-1\right)\right)\right], 0 \le n \le \frac{\alpha(N-1)}{2} \\ 1, \frac{\alpha(N-1)}{2} \le n \le (N-1)\left(1-\frac{\alpha}{2}\right) \\ \frac{1}{2}\left[1+\cos\left(\pi\left(\frac{2n}{\alpha(N-1)}-\frac{2}{\alpha}+1\right)\right)\right], (N-1)\left(1-\frac{\alpha}{2}\right) \le n \le (N-1) \end{cases}$

III. IMPLEMENTATION OF THE DESIGN

The normalized magnitude response curves are obtained and observed for low pass filter (LPF), high pass filter (HPF), band pass filter (BPF) and band reject filter (BRF) using the above mentioned window functions and by varying the number of samples for designing the FIR filters. To implement and design of FIR filter MATLAB 7 has been used. The number of samples are taken as N=50 and N=100. The maximum magnitude in the main lobe and the maximum magnitude in the side lobe are observed for every case and are shown in Table IA and Table IB respectively.

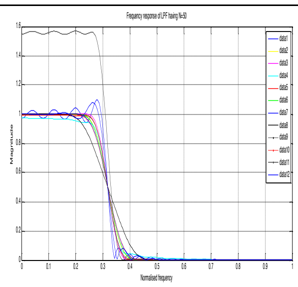
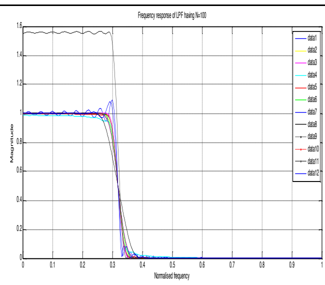
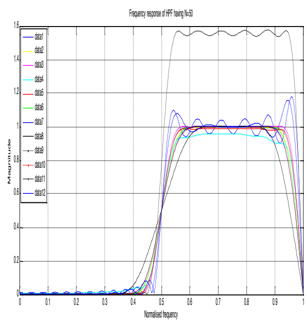
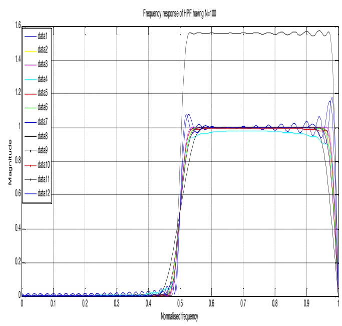
TABLE IA. TABLE FOR COMPARATIVE STUDY OF MAXIMUM MAGNITUDE OF MAIN LOBE

Different Window Functions	Maximum Magnitude of Main Lobes							
	LPF		HPF		BPF		BRF	
	Value of N		Value of N		Value of N		Value of N	
	50	100	50	100	50	100	50	100
Rectangular window	1.0963	1.0920	1.1797	1.1783	1.0963	1.0920	1.1833	1.1772
Blackman window	1.0002	1.0002	1.0004	1.0003	1.0002	1.0002	1.0002	1.0004
Hamming window	1.0021	1.0017	1.0038	1.0039	1.0024	1.0029	1.0036	1.0039
Bartlett window	0.9736	0.9868	0.9579	0.9793	0.9664	0.9836	0.9673	0.9840
Modified Bartlett-Hanning window	0.9938	0.9968	0.9899	0.9950	0.9921	0.9960	0.9923	0.9961
Bohman window	0.9997	1.0000	0.9992	0.9999	0.9994	0.9999	0.9994	0.9999
Chebyshev window	1.0022	1.0005	1.0022	1.0005	1.0022	1.0005	1.0022	1.0006
Flat Top window	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Gaussian window	1.0011	1.0011	1.0024	1.0020	1.0014	1.0011	1.0023	1.0027
Blackman-Harris window	0.9978	0.9994	0.9978	0.9994	0.9978	0.9994	0.9978	0.9994
Taylor window	1.5700	1.5681	1.5775	1.5752	1.5696	1.5674	1.5802	1.5782
Tukey window	1.0790	1.0777	1.1581	1.1571	1.0787	1.0791	1.1592	1.1570

TABLE IB. TABLE FOR COMPARATIVE STUDY OF MAXIMUM MAGNITUDE OF SIDE LOBE

Different Window Functions	Maximum Magnitude of side lobes							
	LPF		HPF		BPF		BRF	
	Value of N		Value of N		Value of N		Value of N	
	50	100	50	100	50	100	50	100
Rectangular window	0.0818	0.0856	0.0843	0.1008	0.0973	0.1016	0.0929	0.0927
Blackman window	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Hamming window	0.0009	0.0022	0.0024	0.0018	0.0023	0.0022	0.0017	0.0017
Bartlett window	0.0417	0.0135	0.0387	0.0432	0.0446	0.0462	0.0488	0.0371
Modified Bartlett-Hanning window	0.0089	0.0096	0.0083	0.0093	0.0076	0.0053	0.0092	0.0097
Bohman window	NA	NA	NA	NA	NA	NA	NA	NA
Chebyshev window	NA	NA	NA	NA	NA	NA	NA	NA
Flat Top window	NA	NA	NA	NA	NA	NA	NA	NA
Gaussian window	0.0007	0.0009	0.0012	0.0014	0.0017	0.0014	0.0013	0.0013
Blackman-Harris window	NA	NA	NA	NA	NA	NA	NA	NA
Taylor window	0.0053	0.0075	0.0108	0.0120	0.0137	0.0130	0.0109	0.0099
Tukey window	0.0791	0.0777	0.0793	0.0782	0.0793	0.0791	0.0788	0.0791

The graphs show the nature of normalized magnitude curves for different windows with different value of N.

Characteristics of filter	For N=50	For N=100
LPF		
HPF		

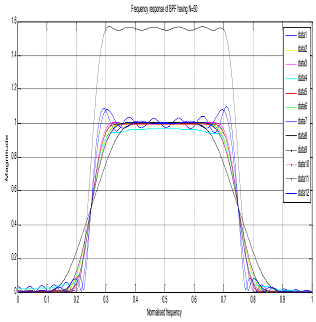
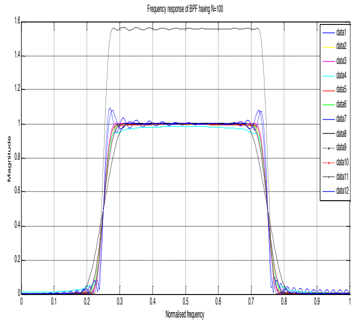
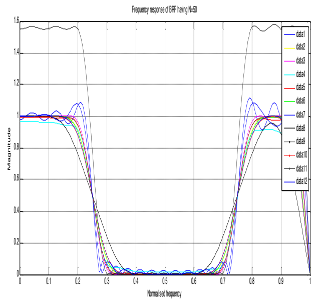
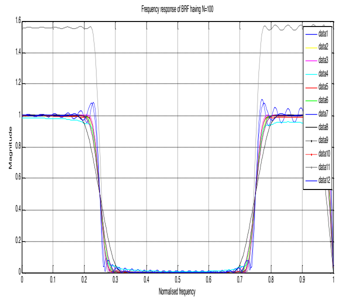
Characteristics of filter	For N=50	For N=100
BPF		
BRF		

Figure : Normalized Magnitude Response Curves of FIR Filters using different windows

CONCLUSIONS

The computational effort is minimal if we use window method to obtain the filter impulse response. From the comparison tables we observe that the Flat-top window and Blackman window are giving better response because the maximum desired value of main lobe is near to the normalized value which should be one and the maximum magnitude for side lobe is nearly zero. The response curve corresponds to Bohman window and Chebyshev window are also reaching towards the desired value as well. So the response of Fir filter is best when the Flat-top window and Blackman window are used.

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