3D Reconstruction From Non-Euclidian Distance Fields

Anders Sandholm* Ken Museth†

Graphics Group @ Linköpings University‡

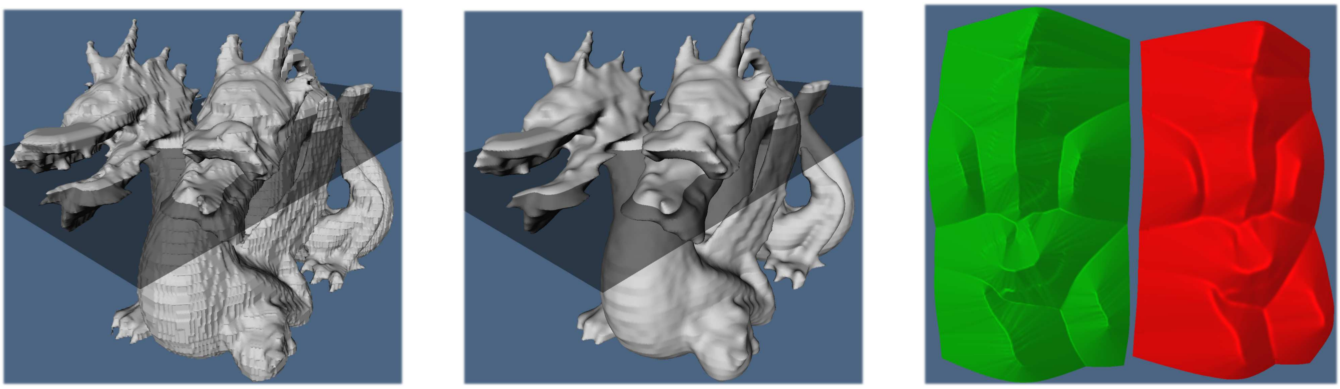

Figure 1: Left: Naive reconstruction by linear interpolation of 2D Euclidian distances to aliased (i.e. binary) input contours of a two-headed dragon. Note the semi-transparent image plane corresponding to one of the horizontal sampling planes. Middle: Our improved spline interpolation of Non-Euclidian distances. Right: Hight-field representations of the two distance fields shown as semi-transparent planes. Green is the Euclidian distance and red is the non-Euclidian distance. Note how the smoothing suppresses the medial-axis on the red surface.

Introduction

We present preliminary results on a robust method for 3D reconstruction from binary images of contours that sparsely sample 3D geometry. Our approach consists of the following steps: First Euclidian signed distance, $\Phi(x,y)$, is computed to each of the input contours. Next we smooth these distance fields by convolution with a filter kernel. The resulting non-Euclidian images are then interpolated to create a uniform volumetric implicit representation of the geometry which is finally rendered indirectly by different mesh extraction techniques.

Step 1: Deriving signed Euclidian distances

First we solving the Eikonal equation, $|\nabla\Phi| = 1$, subject to the boundary condition $\Phi(x,y) = 0$ for $(x,y) \in$ the corresponding contour. The Fast Marching Method[Sethian 1996] is a very efficient algorithm to solve such Hamilton-Jacobi equations. This produces implicit representations of the discrete contours in the whole image plane. In regions of the image plane where a pixel is equidistant from at least two other pixels on a contour $\nabla\Phi$ is undefined and one has a so-called medial-axis, see green figure. Consequently there typically only exist weak solutions to the Eikonal equation which leads to the fact that the distance transform of the contours is only Lipschitz continuous (i.e. not C1 everywhere). The presence of such medial-axis singularities in the derivative typically create noticeable kinks in 3D reconstructions directly from the Euclidian fields. To address this problem our next step is to smooth the distance fields.

Step 2: Filtering the Euclidian distance fields

As explained above we need to suppress artifacts from the presence of medial-axis in the signed Euclidian distance fields. Since the input contours are binary we also need to anti-alias the corresponding implicit representation. This all amounts to applying a smoothing filter on the Euclidian distance fields. We have experimented with the following different filter kernels:

- Uniform and anisotropic Gaussian filters.
- Adaptive Gaussian filter where the width is a functions of $|\phi(x,y)|$, i.e. the Euclidian distance to the contour.
- Bi-Laplacian filter kernel, i.e. $\Delta^2\Phi$ is minimized.

Step 3: Interpolation and mesh extraction

Once we have computed smoothed non-Euclidian distance fields we can produce an implicit 3D representation of the corresponding geometry by simple 1D interpolation between the 2D images to produce a uniform volume. The only constraint is that input contours embedded in the 2D image have to overlap to produce connected components in the 3D reconstruction. We have experimented with the following different interpolation techniques:

- Simple linear interpolation.
- Natural Cubic Spline interpolation.
- Monotonicity constrained interpolation.

As the final step in our reconstruction scheme we use mesh extraction techniques on the volume to produce a final polygonal model. We are experimenting with both Marching Cubes[Lorenson and Cline 1987] and an Extended Marching Cubes[Kobbelt et al. 2001].

Result

Figure 1 shows our reconstruction technique on horizontal slices of a two-headed dragon. Note that the contours in the slices are binary and deliberately under-sampled by a factor of five. As can be surmised from this figure our approach shows very promising results, and we plan to further develop and exploit these ideas in several exciting graphics applications.

References

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*e-mail: andsa344@student.itn.liu.se

†e-mail: museth@acm.org

‡http://www.gg.itn.liu.se