A Self-tuning Robust Track-following Control of Sampled-Data Hard Disk Drive Servo System

Qi Hao, Guoxiao Guo, Shinxin Chen, and Teck-Seng Low

Data Storage Institute
Singapore 117608, Singapore

Abstract

This paper presents a self-tuning scheme based on response surface method (RSM) to find the optimal finite impulse response (FIR) Youla parameter for an observer based state feedback track-following controller that minimizes the 3 times standard deviation of the PES in an HDD servo system. All the tested Youla parameters to construct the response surface were selected within a robust stable region which was defined by an artificial neural network (ANN) trained off-line. Such that the $H_\infty$ bound of some sampled-data system channels could be kept during the response data collection. The experimental data showed that such a self-tuning scheme could improve the position accuracy considerably in a short tuning time without any prior knowledge of the disturbance and noise, while robustly stabilizing the system.

1 Introduction

A myriad of technologies have been employed to achieve the increase in recording densities of hard disk drives (HDDs). The track density in HDDs is expected to be beyond 100,000 TPI in the next few years. One of the prerequisites of moving to higher TPI in HDDs is to improve the servo system performance for lower track mis-registration (TMR). A higher track density can be achieved through a reduction in the total TMR.

To cope with the challenge of the actuator pivot nonlinearity, high frequency uncertainty, the effects of various external disturbances and noises, improved servo control designs have been studied extensively as a cost effective way towards higher track density. Those control techniques includes PID, LQG/LTR ([1], [2]), $H_2/H_\infty$ ([3]), and multirate control ([4],[5],[6]).

In most control systems, performance and robustness are two vital factors limiting the use of a controller in the commercial mass-produced hard disk drives. Usually, the time domain performances require to minimize $H_2$ norm of some system channels, while robust stability of the closed loop system in the face of actuator uncertainty requires the bound of the $H_\infty$ norm of some other channels be constrained. Such a multiobjective $H_2/H_\infty$ problem has been proposed and solved by optimization of Youla parameter ([7]) or alternatively through solving Linear Matrix Inequality (LMI) ([8]).

With the development of modern optimization theory, more and more control problems are solved by applying optimization methods, in which certain control law structure and dynamic order are prescribed and the parameters of control law are optimized considering both performance and robustness of the system ([9],[10]). In practical sense, HDDs are mass-produced and as such, the exact parameters of their servo systems are unknown in advance. Due to the rapid development of digital micro-computer, to remain cost effective while performance competitive, various numerical optimization methods, such as simplex method, random neighborhood search (RNS), gradient methods and least square method (LSM), are studied recently to get a fine-tuned controller for each drive ([4],[11],[12],[13]). In contrast, a generic fixed controller is likely to be conservative.

Among these real-time tuning schemes, random optimization methods do not need convex performance surface and thus could be applied to any controller structure. However, they suffer from the disadvantage of slow convergence speed. For the PID and observer-based state feedback control system, in which the $H_2$ measurements of some system channels are the convex functions of the feedback/estimation gains, gradient methods could be employed for their fast convergence speed. But the estimation of step-size and gradient at each iteration during the gradient-based self-tuning also would spend much computation time of digital signal processor (DSP), especially for high-order or multirate observer-based state feedback controllers.

In this paper, the extra freedom degrees of suboptimal $H_\infty$ controller for sampled-data system are exploited to optimize the $H_2$ measurement of performance. After selecting suitable feedback gain and estimator gain, the square of system output $H_2$ norm is a nearly quadratic function of the feedback gain and estimator gain.
function of FIR Youla parameters. Therefore, when such a function is estimated from plant response data, we could obtain the optimal FIR Youla parameters in real time within a pre-known tuning time without requiring any prior knowledge on disturbances and measurement noise.

2 Control Design Method

In this section, a sampled-data HDD servo system controlled by a digital observer based state feedback controller parameterized in the terms of Youla parameter is presented. The performance index and robust stability for the track-following servo system are also described.

2.1 Sampled-Data System Model

The block diagram of sampled-data HDD servo system is shown in Fig. 1. The analog plant with sampled output for feedback is modeled as

\[
\begin{align*}
\dot{x}(t) &= \tilde{A}x(t) + \tilde{B}_1 w_1(t) + \tilde{B}_2 u(t), \\
y(t) &= Cx(t) + w_2(t), \\
y_m(k) &= y(k) + v_d(k),
\end{align*}
\]

where \(\tilde{A}\) means perturbed variables. \(A \in \mathbb{R}^{n \times n}, \ B_1, \ B_2, \ C^T \in \mathbb{R}^n\) is the nominal plant model. \(x(t) \in \mathbb{R}^n\) is plant state. \(y(t), \ y_m(k), \ u(t) \in \mathbb{R}^1\) are true, measured system output and control. \(w_1(t), \ w_2(t), \ v_d(k) \in \mathbb{R}^1\) are process and measurement noise respectively, which are modeled as continuous-time and discrete-time filtered unit white noise process.

![Figure 1: Block diagram of the robust sampled-data control system.](image)

It is known that the set of all internally stabilizing discrete time controllers can be parameterized in terms of a free parameter \(Q \in RH_{\infty}\) as (7), (15)

\[
K = F_l(J(z), Q(z)).
\]

Such a controller with update period \(T_s\) has the following realization

\[
\begin{align*}
\dot{x}(k+1) &= (\Phi - \Gamma_2 F - LC)x(k) + L(y_m(k) - y_r) \\
\end{align*}
\]

\[
\begin{align*}
u(k) &= -Fx(k) - u_d(k), \\
e_d(k) &= -Cx + (y_m(k) - y_r), \\
u_d(z) &= Q(z)e_d(z),
\end{align*}
\]

where \(\Phi = e^{AT_s} \in \mathbb{R}^{n \times n}, \ \Gamma_2 = \int_0^{T_s} e^{AT_2} B_2 d\tau \in \mathbb{R}^n\) and \(C\) are the step invariant discrete-time nominal plant model. \(x(k) \in \mathbb{R}^n\) is controller state and \(y_r \in \mathbb{R}^2\) is the reference. \(F\) and \(L\) are selected such that \(\Phi - \Gamma_2 F\) and \(\Phi - LC\) are stable. \(e_d(k)\) and \(u_d(k)\) are the input and output of Youla parameter \(Q(z)\) respectively.

2.2 Performance Index

The objective of the track following servo control is to maintain a minimum tracking error. This problem normally had been solved by using standard methods such as pole placement, LQG/LTR, \(H_2/H_{\infty}\), etc, based on a nominal plant model. Such controllers, when combined with proper seeking controllers and mode switching mechanism ([14]), can ensure that the disk drive read/write head reach a desired track quickly and remain on the track accurate enough for reliable read and write.

However, since the actuator parameter and disturbance/noise model variations are very common under different operation conditions for the disk drives, the control design based on nominal plant models may not be suitable to achieve the best performance. In our work, the measured TMR, defined statistically as the \(\pm 3\sigma\) of measured position error signal (PES) \(e_m = y_m - y_r\), was chosen as the performance index for online tuning the HDD servo controller ([1], [12]). It could be expressed as ([15])

\[
\sigma_{PES}^2 = \|T_{yy}^\infty\|_2^2 - \|T_{11}^\infty + T_{12}^\infty(I - T_{22}^\infty Q)^{-1}Q^T T_{12}^\infty\|_2^2 = \|T_{11}^\infty + T_{12}^\infty(I + O(Q))Q^T T_{21}^\infty\|_2^2 \\
= q^T H q + b^T q + c + O(q),
\]

where \(T_{11}^\infty, T_{12}^\infty, T_{21}^\infty\) are the models of disturbances and noise process.\(T_{11}^\infty, T_{12}^\infty, T_{21}^\infty\) and \(T_{22}^\infty\) are systems independent of \(Q\). Furthermore,

\[
\hat{T}_{22} : \begin{bmatrix} \Phi - \Gamma_2 F - LC & LC & -\Gamma_2 \\
\Gamma_2 F & \Phi & -\Gamma_2 \\
-C & 0 & 0 \end{bmatrix}.
\]

Obviously, when the plant is not perturbed, \(\hat{T}_{22} = 0\) and thus \(O(q) = 0\). At that time, the deviation square of PES \(\sigma_{PES}^2\) is just a 2nd-order function of \(q\) without any approximation. \(q = [q_0 ... q_{N-1}]^T\) is the impulse response of Youla parameter; \(H, H \in \mathbb{R}^{N \times N}; b, b \in \mathbb{R}^N; O(Q)\) is the sum of a series of system \(Q; O(q)\) is a larger-than-2nd-order function of \(Q\)'s impulse response \(q\). Equation (4) shows that the deviation square of PES \(\sigma_{PES}^2\) could be approximated as a quadratic function of FIR Youla parameters under some conditions that let the \(H_2\) norm of \(O(Q)\) less than a small positive real.
number. Such conditions usually require that the real plant is not perturbed too much.

2.3 Robust Stability

Beside the requirement on dynamic performance, the robust stability is usually required for control design. A realistic way to express plant uncertainty is to describe the plant transfer function as having a multiplicative uncertainty as

$$\tilde{P} = \{P(1 + \Delta W_d) : \|\Delta\|_\infty < 1\}, (5)$$

where the causal, stable and fixed weighting system $W_d = C_w(I - \Lambda_w)^{-1}B_w$, in which $\Lambda_w \in \mathbb{R}^{n_w \times n_w}$ and $B_w, C_w^T \in \mathbb{R}^{n_w}$, models uncertainty envelope in the magnitude of the plant, and $\Delta$ models uncertainty in the phase. As it follows, the bound for the stability margin we considered was

$$\gamma_{rs} = 1/\|W_\delta T_i\|_\infty = 1/\|T_{cw}\|_\infty, (6)$$

where $T_i$ is the complementary sensitivity function of plant input ([15], [16]).

Next we will discuss the control design for the above constraint optimization problem.

3 Self-tuning Robust Control

In this section, firstly the transform of the original hybrid system to a norm-equivalent pure discrete-time system to bound the robust stability parameter space is outlined. Next by training an artificial neural network (ANN) off-line to bound the robust stable tuning parameter space is proposed to reduce the computation in implementation. After that, the response surface method (RSM) is introduced to estimate the quadratic function of performance with respect to FIR Youla parameters using plant response data and to find optimal solution. Finally, the control stratagem is summarized.

3.1 The Equivalence of Sampled-Data System

Although most control designs are in continuous-time domain or discrete-time domain, the HDD servo systems are in fact sampled-data systems. By using the continuous lift technique ([15]), which can reduce the hybrid continuous/discrete-time problem to a norm-equivalent discrete-time problem, the effect of continuous-time plant parameter variations on the hybrid system stability could be taken into estimation.

The hybrid HDD servo system with multiplicative uncertainty is shown in Fig. 2. The $H_\infty$ norm of transfer function $T_{cw}$ to calculate the robust stability bound is ([15])

$$\|T_{cw}\|_\infty = \|W_\delta H_{T_2}(1-KS_T P_H T_2)^{-1}S_T P\|_\infty = \|N_{R_d} M\|_\infty, (7)$$

where $H_{T_2}$ and $S_T$ are the zero-order hold and sampler respectively, $R_d = (1-KP_d)^{-1}K$, $P_d = C(Iz - \Phi)^{-1}\Gamma_2$, $N = H_w(Iz - \Phi_w)^{-1}T_w + J_w$, $M = C(Iz - \Phi)^{-1}\Gamma_2^*$,

$$\Gamma_w = \int_0^{T_f} e^{A_w t} B_w d\tau \in \mathbb{R}^{n_w},$$

$$[H_w^* J_w^*]T[H_w^* J_w^*] = \int_0^{T_f} e^{A_w^T T} [C_w 0]^T [C_w 0] e^{A_w} B_w d\tau \in \mathbb{R}^{(n_w+1) \times (n_w+1)},$$

$$\Lambda_w = \begin{bmatrix} A_w & B_w \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n_w+1) \times (n_w+1)},$$

$$\Gamma_2^T \Gamma_2 = \int_0^{T_f} e^{A T} B_2 B_2^T e^{A^T} d\tau \in \mathbb{R}^{n \times n}.$$
points (controller parameters) must be at least \( p = (N + 1)(N + 2)/2 \).

This problem also could be solved in real time via recursive least square algorithm (RLSA). After \( p \) step iterations, we would obtain the parameters of quadratic function \( T^2 \) consisted of the elements of \( H, b \) and \( c \) defined in (4),

\[
\Theta = (X^T X)^{-1} X^T Y. \tag{9}
\]

where regressor \( X = [x^T(1) \ldots x^T(p)]^T \), \( x(i) = [q_0^T q_i^{q_k} \ldots q_{V-1}(i) q_0^T] \) (\( j, k = 0, \ldots, N - 1; i = 1, \ldots, p \)) and regressand \( Y = [y(1) \ldots y(p)]^T = [\sigma_{PES1}^2 \ldots \sigma_{PESp}^2]^T \). In implementation, \( \Theta \) could be obtained via the following recursive algorithm:

\[
\hat{\Theta}(i) = \hat{\Theta}(i - 1) + K(i)x^T(i)\epsilon(i),
\]

\[
K(i) = \frac{K(i - 1)x^T(i - 1)x(i - 1)K(i - 1)}{1 + x(i - 1)K(i - 1)x^T(i - 1)},
\]

\[
\epsilon(i) = y(i) - x(i - 1)\hat{\Theta}(i). \tag{10}
\]

Once the parameters of the quadratic function are known, the optimal solution of FIR Youla parameter could be obtained as

\[
q_{\text{opt}} = -\frac{1}{2}H^{-1}b. \tag{11}
\]

Nevertheless, the optimal Youla parameter for minimal PES deviation may not necessarily satisfy the robust stability requirement. In that case, the Youla parameter could be updated along the gradient direction of quadratic performance surface using deepest descent method (DDM) until it touches the boundary of robust stability region.

### 3.4 Summary

**Figure 2:** Block diagram of the adaptive robust control system.

Fig. 2 shows the block diagram of the whole control system. For each step of controller tuning, the disk drive was doing short track seeks \( m \) times from different directions in a particular zone of the disk surface and hence \( m \times (I + 1) \) PES samples were collected for optimization. The following steps were adopted to find the optimal controller.

**Step 1:** A suitable feedback gain \( K \) and estimator gain \( L \) of the observer based state feedback controller (3) were selected such that the transfer function \( T \) is as insensitive as possible, given the nominal plant model (1) and parameter uncertainty range.

**Step 2:** An ANN defined in (8) was trained to bound the robust stability space of FIR Youla parameter, which was in a prescribed length, by using the training data obtained from (6) and (7).

**Step 3:** One available controller \( F(K, Q_0) \) was used to control the HDD. One-track seeks in a particular zone of the disk surface was done and the PES samples were selected. Then the deviation of measured PES \( \sigma_{PES_m} \) could be calculated.

**Step 4:** The quadratic function of PES deviation with respect to FIR Youla parameter (4) was estimated with sampled parameters selected from the candidate space (8) using RSM implemented with RLSA (10).

**Step 5:** The optimal solution of FIR Youla parameter could be gotten using (11) after \( p \) iterations of Steps 3 and 4.

**Step 6:** The robust stability of optimal controller parameters was checked with (8). If such a requirement could not be satisfied, controller parameters would be updated with DDM, that was \( q_{k+1} = q_k - \alpha (2Hq_k + b) \), where \( \alpha \) was the pre-set step size, until it arrived the boundary of robustly stable space.

It should be noted that during real time implementation only the standard deviation of measured PES \( \sigma_{PES_m} \) other than that of true PES \( \sigma_{PES} \) was minimized, but [12] showed that if the measurement noise was white or its bandwidth was larger than the system closed-loop bandwidth, the optimal controllers for them were same or almost same.

Next, the whole control stratagem was applied to an HDD to test its effectiveness.

### 4 An Application Example

In this section, an application of design method described in the previous section to a disk drive servomechanism is introduced.

#### 4.1 Plant, Uncertainty and Noise Models

A measured Bode diagram of an HDD actuator driven by a transconductance amplifier is shown as the solid line in Fig. 3. The dash-dot line was the identified plant model with a transfer function

\[
\frac{2}{k} = \frac{\omega_n^2}{s^2 + 2\zeta_n\omega_n s + \omega_n^2}, \tag{12}
\]

where \( k = 2.41 \times 10^7 \mu m/V \pm 10\% \) is the plant gain, \( \omega_n = 5400 \text{ Hz} \pm 10\%, \zeta_n = 0.03 \pm 10\% \).

From Fig. 4 we could find that the maximum magnitude of multiplicative uncertainty had two peaks, one at 4.9 kHz and another at 5.9 kHz. Since the reso-
Figure 3: Bode plot of an HDD actuator.

Figure 4: Uncertainty of the plant and uncertainty weight.

Figure 5: Disturbance and noise models.

Figure 6: Robustly stable region learned by an ANN.

Figure 7: The ANN training process.

Figure 8: Self-tuning process of a 2\textsuperscript{nd}-order Youla parameter (Dashed line: robustly stable region; Solid line: estimated performance surface).

Fig. 8 shows the self-tuning process. We can see that Youla parameter was tuned to the optimal solution within the robustly stable region after sampling 7 pairs Youla parameters through response surface method.

To determine the order of \( Q \), Fig. 9 illustrates the PES deviation with respect to the order of FIR Youla parameter, compared with that achieved by an \( H_2 \) optimal controller. It could be found that the PES deviation achieved by a 4\textsuperscript{th}-order FIR Youla parameter was almost the same as what the \( H_2 \) optimal controller did.

The sensitivity and complimentary sensitivity Bode plots of the control system before and after tuning are shown in Fig. 10. The PES deviation \( \sigma_{PESm} \) was improved by 34.2% on an average from 0.114 \( \mu \)m to 0.075 \( \mu \)m.
5 Conclusion

In this paper, a self-tuning robust control scheme based on the response surface method for HDD servo system is proposed and studied. To a plant with uncertainty, after selecting the suitable feedback gain and estimator gain, the deviation of PES is approximately a quadratic function of FIR Youla parameters. Such a quadratic function could be identified after testing several candidate FIR Youla parameters in a robustly stable space to obtain the optimal solution. The experimental data showed that for the given plant a 4th-order Youla parameter could achieve 97% performance of $H_2$ optimal control when without robustness restrictions. Another advantage of such a method is its pre-known tuning time that is longer than a 2nd-order function $(N+1)(N+2)/2$ of Youla parameter’s order $(N)$. Thus the tuning process will be shorter when $N$ is small. The proposed self-tuning method provides us a new scheme to optimize the track-following controller in real time without any prior knowledge on disturbances and measurement noise. The method also could be extended to the dual-stage and multirate HDD servo control systems.

References


