ABSTRACT

Emanating from a discussion of local ratcheting effects and appropriate limiting criteria this paper examines existing ratcheting rules based on local (or material) response that are currently in Section III of the ASME B&PV Code, German KTA rules, and other design codes. The objective is to offer clarification of the rules that may be useful to the designer, their bases and practical application. Among these rules, different ratcheting checks are required that use different ratcheting measures. The paper considers only the rules for which the ratcheting checks can be evaluated by a direct cycle-by-cycle elastic-plastic FEA. Examples are provided to illustrate the procedures.

1 INTRODUCTION

The ratcheting check in pressure vessel design is a topic that has received much attention for many years, especially during the last decade. Numerous papers were published on ratcheting in ASME Pressure Vessel & Piping Division Conferences and journals. An excellent summary of the basic issues involved in ratcheting can be found in a paper by Reinhardt1.

The simplified elastic-plastic fatigue analysis based on elastic FEA and plasticity correction is known to yield often overly conservative results. Furthermore, the $3S_m$ elastic shake down criterion is often exceeded. As a consequence, elastic-plastic analyses based on non-linear FEA have to be applied both for the fatigue and the ratcheting check.

Examination of the major design codes reveals that two different kinds of ratcheting are being employed in the ratcheting checks. One is based on the global structural response and the other on the local material response. Note that the definitions used for these terms are sometimes inconsistent in the related technical literature. For instance, the terms structural and material are referred to in a paper by Garud2, which reports experimental data on ratcheting of an elbow-pipe assembly. These responses are referred to as global and local, respectively in this paper. In reference3 the term “structural” would include both global and local ratcheting responses and the term "material ratcheting" would apply only to homogeneous stress distributions that are used in material tests. In contrast, ratcheting phenomena which do not appear in material tests but require an inhomogeneous distribution of stress are categorized as "structural ratcheting"3. The use of the global structural response in the ratcheting check means that failure is caused by conditions related to the structure as a whole. The mode of failure that is being prevented by this kind of ratcheting check is the same as that of global collapse according to limit analysis. It is thus called Global Ratcheting.

The use of the local material response in the ratcheting check means that failure is caused by conditions limited to a small volume (theoretically a point). The mode of failure that is being prevented by this kind of ratcheting check is governed by the material properties. Hence, exceeding the (local) strain limits or reaching ductility exhaustion are the potential failure modes3. This will be called Local Ratcheting. This entire paper is limited to Local Ratcheting, which will be the only one discussed from this point on. An associated but separate paper (PVP2011-57196) concentrates on Global Ratcheting.

Note that Section III of the ASME B&PV Code and the German KTA rule 3201.2 both address Local Ratcheting within the elastic-plastic ratcheting check. Furthermore, the code based ratcheting check is closely related to the fatigue check.

Further rules are given in EN 13445-34.
2 LOCAL RATCHETING EFFECTS, MATERIAL MODELS AND APPROPRIATE LIMITING CRITERIA

Local ratcheting effects are one of a couple of phenomena related to cyclic plasticity. The occurrence of cyclic elasto plastic stresses and strains induces fatigue as a (short) crack history. The fatigue check often presumes cyclically stabilized material behavior and thus relies on the cyclic stress strain curve (CSSC) in connection with the strain life fatigue curve. However, cyclic stabilization is often preceded by cyclic (isotropic) hardening or softening, non-Masing behavior, out-of-phase-hardening (additional hardening under non-proportional loading conditions) and last not least progressive plastic deformation (ratcheting)\(^5\). This last-mentioned ratcheting phenomenon may induce material failure if no (plastic) shakedown occurs after a certain number of cycles. This failure is reached by either exceeding the admissible (local) strains or by ductility exhaustion. As an analogy to failure criteria under monotonic (static) loading conditions there is a global (global plastic deformation, plastification of cross sections) and a local (strain limits at locations such as notches) aspect. Experimental evidence on the local ratcheting phenomenon is very scarce particularly concerning component like structures. Ratcheting effects are often experimentally induced by uniaxial combination of a constant mean load and a cyclically superposed loading of unnotched specimens\(^6\). Biaxial load combinations are often set up by combinations of tension/compression and torsion on hollow specimens. It can be stated that available and published experimental evidence is still far away from real world examples of components of complex 3D geometry under complex combined multi axial mechanical and thermal loading. Failure cases with local ratcheting as the generic root cause of components are not reported. Of course, the complex (local) interaction of fatigue and ratcheting may influence (accelerate) the fatigue process\(^7\). That’s why a detailed mechanistic cycle-by-cycle fatigue analysis\(^8\) should consider superposed ratcheting effects by modeling the material behavior.

A major presumption for the analytical description of local strain accumulation by ratcheting processes is the application of an appropriate material model. The applied material model has to be able to simulate the strain increment on a cycle-by-cycle basis for uniaxial and multiaxial loading conditions, for mechanical as well as thermal and combined loading and complex geometries. The available material models (see Figure 1) rely on the classical theory of plastic material behavior. Central notions are the yield function (limitation of elastic states of stress in the 3D stress space), the yield condition (Tresca or von Mises hypothesis), the elastic deformation law (Hooke’s law), the hardening rule (decisive module for local ratcheting simulation) and the flow rule (connection between stresses and plastic strains). Usually, so called “associated flow rules” are applied which allow for the derivation of plastic strain increments from the flow rule and thus their orthogonal direction.

Figure 1 shows a chronological listing of some relevant material models. The detailed mathematical description of the material models is published in numerous references such as\(^9,10,11\). Reference \(^10\) focuses on the reasons for the appropriateness or non-appropriateness of different material models for the description of (local) ratcheting effects and contains a detailed explanation of the anatomy of the constitutive material models.

The global monotonic and cyclic plastic deformation behavior of a structure, local strains under monotonic loading, simple plasticity effects such as the Bauschinger effect or stabilized cyclic hysteresis loops can be satisfactorily described by application of simple kinematic models (see upper box in Figure 1). The further development was decisively driven by insufficiencies with respect of realistic ratcheting description\(^1,11\). These development activities are ongoing\(^14,15\).

![Figure 1: Chronological listing of available material models according to\(^16\)](image)

The appropriate description of the kinematic hardening behavior – i.e. the translation of the yield surface in stress space – is crucial for ratcheting simulation\(^10\). The simple kinematic models (Prager, Besseling etc.) are insufficient in this respect. Depending on the load case and load history strong underestimations as well as overestimations are possible\(^16,16\).

Models relying on discrete yield surfaces (see box on right hand side of Figure 1) yield constant ratcheting rates at higher numbers of load cycles where experiments prove decreasing ratcheting rates. E.g. the Mróz model\(^17\) was discarded in \(^18\) because it is not able to describe the ratcheting behavior caused by non-proportional loading, particularly the fact that no ease of the ratcheting range can be detected. As a consequence, the crack opening and crack closure behavior cannot be described realistically in a mechanistic cycle-by-cycle fatigue analysis\(^18\).

The biggest development potential towards a realistic simulation of the (local) ratcheting behavior is accredited to the nonlinear kinematic (NLK) material models based on the nonlinear kinematic hardening rule by Armstrong and Frederick\(^19\) (see box on left hand side of Figure 1). Nowadays, the material models of Chaboche\(^20,21,22\) (different model variations) and Ohno&Wang\(^23,24\) are applied and implemented in finite element software as non standard applications. If temperature cycles are the principal cause of cyclic loading conditions (typical for power plant applications) particular care has to be taken with regard to the appropriate formulation of the material model including characteristic thermal relaxation terms. This constitutes the basis for the description of the thermal ratcheting phenomenon. The formulation and implementation of both the models of Chaboche and Ohno&Wang are comprehensively...
described in\textsuperscript{25}. Note that the appropriate identification of the associated material parameters remains a separate issue. Special optimization procedures are required for this purpose (see e.g.\textsuperscript{25}). See the given references for the detailed mathematical description of the constitutive material models and the approaches for parameter identification. Actually, AREVA relies on both the Chaboche and the Ohno\&Wang models. The Ohno\&Wang model allows for an optimization of the exponents of the material law by fitting available results of uniaxial and multiaxial ratcheting experiments. The limitations of the available material models are known as well, particularly for the simulation of complex loading situations and superposition of global and local ratcheting effects as it is shown for instance in\textsuperscript{11}. In other words, it cannot be expected that an optimized set of parameters for an advanced non-linear kinematic material model will deliver optimal results for all possible complex geometry and multiaxial loading situations.

3 CODE BASED ELASTIC-PLASTIC FATIGUE AND RATCHETING ANALYSES

In the following the code rules for elastic-plastic fatigue and ratcheting analyses are considered in more detail.

The ASME code\textsuperscript{27}, paragraph NB-3228.4 (b) addresses elastic-plastic shakedown analysis without referring to what material model to use or not to use (thus allowing the use of any appropriate cyclic plasticity algorithm):

\begin{itemize}
  \item \textbf{NB-3228.4 Shakedown Analysis}
  \item \textbf{...}
  \item \textbf{(b) In lieu of satisfying the specific requirements of NB-3221.2, NB-3222.2, NB-3222.5, and NB-3227.3 at a specific location, the structural action shall be calculated on a plastic basis, and the design shall be considered to be acceptable if shakedown occurs (as opposed to continuing deformation). However, this shakedown requirement need not be satisfied for materials having a minimum specified yield strength to specified minimum ultimate strength ratio of less than 0.70 provided the maximum accumulated local strain at any point, as a result of cyclic operation to which plastic analysis is applied, does not exceed 5.0\%. In all cases, the deformations which occur shall not exceed specified limits.}
\end{itemize}

Concerning the German KTA rules, paragraph 7.8.1.2.2, subsection d) of KTA 3211.2\textsuperscript{28} regulates the application of elastic-plastic material models in the framework of the fatigue analysis in general and states the requirement of a ratcheting check for components outside the reactor coolant pressure boundary of light water reactors (secondary circuit):

\begin{itemize}
  \item \textbf{d) General elastic-plastic fatigue analysis}
  \item While the abovementioned methods are based on linear-elastic material behavior, a fatigue analysis based on the actual behavior of the material may be made in lieu of the abovementioned methods in which case it shall be demonstrated that no progressive distortion (ratcheting) occurs.
  \item Referring to the ratcheting check the application of incremental plastic analyses over several cycles is a practical consequence of this paragraph. As cyclic elastic-plastic analyses applied to complex geometric and loading configurations are extremely time and disc space consuming, appropriate extrapolation schemes on the basis of few analyzed cycles are indispensable. A further question is the application of an adequate material model for the simulation of the ratcheting behavior. As discussed above, kinematic hardening is a significant parameter for the consideration of Material (or Local) Ratcheting\textsuperscript{11}.
  \item Further and more detailed prerequisites to the elastic-plastic ratcheting analyses are formulated in the KTA rule KTA 3201.2\textsuperscript{29} for components of the reactor coolant pressure boundary of light water reactors (primary circuit). The corresponding paragraph 7.13.3 says:
  \item \textbf{7.13.3 General evaluation by elastic-plastic analysis}
  \item \textbf{(1) For the determination of plastic strains at cyclic loading an elastic-plastic analysis may be made. The material model used in this analysis shall be suited to realistically determine the cyclic strains.}
  \item \textbf{(2) Where in the case of strain hardening materials the decrease of the strain increment from cycle to cycle is to be taken for the determination of the total strain, the load histogram shall comprise several cycles. From the strain history determined from the respective load histogram the maximum accumulated strain may be calculated by conservative extrapolation.}
  \item \textbf{(3) At the end of service life, the locally accumulated principal plastic tensile strain shall not exceed, at any point of any cross section, the following maximum value: 5.0\% in the base metal, 2.5\% in welded joints.}
  \item Accordingly, several cycles of the load histogram have to be analyzed before the determination of the maximum accumulated first principal plastic strain can be determined and assessed by means of conservative extrapolation under consideration of the real number of load cycles. Assessment criteria are the above mentioned plastic strain limits (5.0\% in the base metal and 2.5\% in welded joints according to KTA 3201.2\textsuperscript{29}) or plastic shakedown below these limits. Plastic shakedown may be difficult to demonstrate in case of very small plastic strain increments\textsuperscript{30}.
  \item The conventional European Code EN 13445-3\textsuperscript{4} offers guidelines for elastic plastic analyses in Annex B (normative), Design by Analysis – Direct Route. Amongst other possible failure modes and limit states (see table B.4-1 in\textsuperscript{4}) progressive plastic deformation (ratcheting) is addressed. The Progressive Plastic Deformation Design Check (PD-DC) is regulated in paragraph B.8.3: Progressive Plastic Deformation (PD). The following principles are specified in B.8.3.1:
  \begin{itemize}
    \item first-order-theory;
    \item a linear-elastic ideal-plastic constitutive law;
    \item von Mises' yield condition (maximum distortion energy criterion) and associated flow rule\textsuperscript{6}.
  \end{itemize}
  \item That means, in the case of EN 13445-3 the limitation regarding the constitutive law for the ratcheting analysis is very strict. The only valid material law is a linearly elastic ideally...
plastic material model with a yield strength of $R_{p,0T}$ for austenitic stainless steels.

Amongst further shakedown rules, paragraph B.8.3.2 Application rule 1: Technical adaptation practically addresses a local elastic-plastic ratcheting check:

The principle is fulfilled, if it can be shown that the maximum absolute value of the principal structural strains is less than 5% after the application of the number of cycles specified for the considered load case. If the number is not specified, then a reasonable number, but at least 500 shall be assumed.

NOTE Total strains in any model which deviates only in the local stress/strain concentrations may be used instead of structural strains.

Remarkably, first-order-theory in connection with the evaluation of local structural strains may be used. The determination of structural strains is regulated in paragraph B 7.6. It is similar to a quadratic surface stress extrapolation used in the context of structural stress approaches to the fatigue assessment of welds (e.g. \cite{31}). Another option – not explicitly addressed in EN 13445-3 would be a path linearization with respect of strains across the wall thickness (volumetric FE model presumed). The application of elastic plastic structural strains in the context of the ratcheting check is not obvious. An additional note in \cite{4} says:

NOTE In case of doubt, or in case of obviously meaningless extrapolation values, the total stress/strain in any model which deviates solely in the local stress/strain concentrations may be used.

The quadratic extrapolation procedure (see Figure 2) requires the determination of all components of the strain tensor at distances $0.4\,\varepsilon$, $0.9\,\varepsilon$ and $1.4\,\varepsilon$ ($\varepsilon$ is the wall thickness) from the hot spot (location to be evaluated). The related strain values are denoted by $y_1$, $y_2$ and $y_3$. The strain values $y_0$ at the hot spot location are calculated as follows assuming the vertex of the parabola at $y_3$:

$$y_0 = 2.52\,y_1 - 2.24\,y_2 + 0.72\,y_3$$  \hspace{1cm} (1)

Figure 2: Hot spot extrapolation procedure \cite{4}

**4 DETERMINATION OF STRAIN INCREMENTS AND QUALIFIED EXTRAPOLATION**

The German KTA 3201.2 \cite{29} refers the locally accumulated plastic tensile strain. Hence, the principal plastic strain increment is not available as standard postprocessing output of finite element program systems. In contrast, it is to be determined from the difference between the plastic strain tensor at the end of one cycle $i$ and the previous cycle $i-1$ as described. In a first step, the strain increments for all components $C$ of the plastic strain tensor have to be calculated according to

$$\Delta\varepsilon_{C,pl,i} = \varepsilon_{C,pl,i} - \varepsilon_{C,pl,i-1}$$ with $C=x,y,z,xy,yz,xz$  \hspace{1cm} (2)

Hence, the calculation starts with the plastic strain tensor at the end of the first cycle and continues with the difference between the second and the first cycle. Possibly existing plastic prestrains can be considered in equation 1 as $\varepsilon_{C,pl,0}$. Note that the shear components (indexes $xy$, $yz$, $xz$) in equation 1 are shear strains and not the shear distortions $\gamma$ (e.g. $\gamma_{xy} = y_{xy} / 2$)! For instance, the standard ANSYS output refers to the shear distortions (e.g. ANSYS EPPL,XY = $\gamma_{xy}$)! The principal plastic strain increments ($\Delta\varepsilon_{C,pl,i}$ with $C=1,2,3$) are determined from the component plastic strain increments (see equation 2) e.g. according to reference \cite{33}. The accumulated first principal plastic strain at the end of the $N^{th}$ cycle results from summing up the increments:

$$\varepsilon_{i,pl,cum} = \sum_{i=1}^{N} \Delta\varepsilon_{i,pl,i}$$  \hspace{1cm} (3)

An exemplary plot of the accumulated first principal plastic strain (equation 3) and the first principal plastic strain increment as a function of the number of cycles is shown in Figure 3 (taking a thick walled pipe made of austenitic stainless steel 1.4550 and subjected to constant internal pressure and cyclic transient temperature loading as an example using a Chaboche type material model with five backstress terms).

In this example, ten cycles are analyzed and the typical trend of decreasing principal plastic strain increments is visible. Typically, appropriate extrapolation schemes emanating from few analyzed cycles are indispensable for the determination and evaluation of accumulated plastic strains within the ratcheting check. Complex 3D geometries typically allow for the elastic-plastic analysis of about ten cycles on a cycle by cycle basis. The ratcheting behavior beyond this number of cycles has to be assessed by appropriate extrapolation schemes.

The following extrapolation methods are proposed:

- linear method with constant strain increment,
- logarithmic method and
- approximation by a power function.

In the case of the linear method the strain increment of the last analyzed cycle is (conservatively) transferred to all subsequent cycles. Evidently, a possible plastic shake down tendency...
cannot be modeled by this approach. This approach is comparable to the application of a material law yielding constant ratcheting rates at higher numbers of load cycles\textsuperscript{15}.

\[ \Delta \varepsilon_{1,pl,cum}(N) = a + b \cdot N^c \] \hspace{1cm} (5)

and delivers the approximation parameters \(a\), \(b\) and \(c\). As a result, this equation can be used in a straightforward way to calculate the accumulated plastic principal tensile strain \(\varepsilon_{1,pl,cum}\) as a function of the desired number of load cycles.

The differentiation of equation 5 yields

\[ \frac{d\varepsilon_{1,pl,cum}(N)}{dN} = b \cdot c \cdot N^{c-1} \triangleq \Delta \varepsilon_{1,pl}(N) \] \hspace{1cm} (6)

and can be used to calculate the principal strain increment as a function of the number of cycles. A criterion \(c<0\) for the occurrence of plastic shake down can directly be derived from equation 6. In other words, regression coefficients \(c<0\) are an indicator of plastic shake down but do not automatically satisfy the additional strain limit criteria nor give the number of cycles when plastic shake down will occur.

\section{APPLICATION EXAMPLES}

\subsection{Description of examples}

In the examples, the models shown are subjected to a (constant) mechanical load (internal pressure) and temperature transients (mainly characterized by the temperature range with respect of time, the temperature change rate and the hold time). The internal pressure loading is derived from typical NPP operating conditions. The material models of Chaboche and Ohno&Wang as described in section 2 are applied for the Nibium stabilized austenitic stainless steel X6CrNiNb18-10 (1.4550, ANSI 347). Temperature dependent physical properties are taken from KTA rule 3201.1\textsuperscript{13}.

\subsection{Thick walled pipe}

In the first example, the ratcheting analyses are carried out for a two-dimensional axially symmetric model of a thick walled pipe section (disc model) as it is shown in Figure 4. The model consists of 13 elements including a surface layer of four elements (elements number 1 to 4 according to Figure 4) in order to capture high gradient effects on the inner surface of the pipe. The inside radius \(r_i\) amounts to 122mm and the wall thickness \(t\) to 40mm. The transient temperature load as well as a constant internal pressure is applied to the inner wall.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Finite element model of a thick walled pipe section (disc model)}
\end{figure}

The thermal analyses are based on the element type PLANE77 of the ANSYS\textsuperscript{\textregistered} elements library while type PLANE82 elements are used for the structural analyses. The axially symmetric option of both element types is activated.

The analyses described in this example are based on the Chaboche\textsuperscript{21} material model.

One exemplary model temperature transient (specified bulk temperature of the fluid) is applied to the inner surface of the piping section. The details are summarized in Figure 5. The load is applied by the specification of convective boundary conditions. The heat transfer coefficient is assumed to be 10000W/(m\textsuperscript{2}K). The outer surface in the analysis model is assumed to be ideally isolated.

The complete transient temperature fields resulting from the thermal analysis are applied as mechanical loadings to the structural model (unidirectional coupling of thermal and structural analysis). In addition, a constant pressure load (\(164\text{bar} = 16.4\text{N/mm}\text{\textsuperscript{2}}\)) is applied to the inner surface of the piping section.
Symmetry boundary conditions (no displacements in the vertical direction) are assumed for the lower surface of the model. Coupled degrees of freedom (COF) are assumed for the upper surface of the model representing a plane cross section (equal vertical displacement of all nodes of the upper surface). In addition, the axial component of the stress resulting from the internal pressure loading is applied to the upper surface as a constant external load (see Figure 4). The axial stress component $\sigma_{ax}$ amounts to 21.5N/mm².

The transient shown in Figure 5 may have a real number of occurrences of 110. The elasto-plastic analysis on a cycle by cycle basis may be realistically limited to 10 cycles in case of complex geometric and loading situations. In the case of the simple disc model shown in Figure 4 the complete number of occurrences of 110 can be analyzed and compared to the results of different extrapolation methods.

The accuracy of the different extrapolation procedures with respect to a conservative determination of the maximum accumulated strain and the compliance with the given strain limits as well as the plastic shake down check was investigated as follows. Three different elasto-plastic analyses on a cycle by cycle basis were carried out for 10 cycles, 55 cycles and 110 cycles.

As expected, the accuracy of results increases with an increasing number of analyzed cycles. Hence, all three methods meet the strain limit criteria if the extrapolation procedure starts after the 55th analyzed cycle (see Figure 6). There is still a tendency of overestimation of the real accumulated strain in the case of linear extrapolation with constant strain increment. The logarithmic extrapolation method yields nearly exact results compared to the cycle by cycle finite elements analysis up to 110 cycles.

There is no tendency of plastic shake down after 110 calculated cycles according to Figure 6. This may be due to the applied Chaboche material law with a known tendency of overestimating the real accumulated plastic strains.

The calculated accumulated principal plastic strains are the criterion for the evaluation of the extrapolation accuracy. The finite elements program ANSYS® provides an evaluation per element (extrapolation of results at integration points to the nodes within the element by way of the ESOL command) as well as an averaging of results with neighboring elements by way of the ANSOL command. Figure 6 shows that the differences in the results are fairly small with respect to the accumulated principal plastic strains.

The linear extrapolation with constant strain increment after the 10th analyzed cycle yields an unrealistic slope of the accumulated plastic principal strain. The strain limit criterion cannot be satisfied by application of this method. The logarithmic approach and the power function method are comparable with regard to the resulting strain values after 110 cycles. The comparison with the finite elements results from the cycle by cycle analysis including 110 cycles shows that the code requirement of conservative extrapolation is fulfilled by both methods without significant overestimation. The given strain limits can be met by application of these two methods. In other words, the ratcheting check — by means of local strain limits — would pass.

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5.2 Heat Exchanger tube sheet

The component to be considered as a second example is a heat exchanger classified as a class 2 component, i.e. part of a system outside the primary circuit of a NPP. Internal pressure and thermal transients are the significant loads. The general elasto-plastic analyses both for fatigue and ratcheting are carried out for a 2D axially symmetric model for economic reasons. The perforated part of the plate is replaced by a solid plate with modified values of the elastic constants. This approach is in accordance with the ASME Code A-8000 and requires the determination of effective elastic constants. The main effect of the thermal transients is a radial expansion and contraction of the tube sheet relative to the cylindrical shell of the heat exchanger. Therefore a uniform temperature was chosen as a representative load for the determination of the effective elastic constants. The tubes were included in the model so that the stiffening effect of the tubes is taken into account and causes later in the main calculation using the axially symmetric model conservatively higher stresses/strains in the junction area between tube sheet and cylindrical shell. The thermal expansion in radial direction was locked at the boundary of both models, as result the reaction forces were compared. The value of the ligament efficiency in figure A-8131-1 of article A-8000 was chosen in that way, that the resulting effective Poisson’s ratio $\nu^*$ and effective Young’s modulus $E^*$ effectuate the same reaction forces at the boundary of the solid plate as they occur at the boundary of the perforated plate. The determined values of the elastic constants are $E^*/E = 0.5$ and $\nu^* = 0.3009$. Both models used are shown in Figure 7.

The 2D axially symmetrical finite element model used for the fatigue analyses is shown in Figure 8. The 8-node thermal solid element type PLANE77 of the ANSYS® element library was used with axially symmetric option for the thermal transient analyses. The structural mechanical analyses were based on PLANE183 elements with axially symmetric option.

The effective elastic constants determined according to Figure 7 were used for the perforated part of the plate. The finite element model consists of 8563 nodes and 2712 elements.

The transient temperatures were applied on the inside of the model as convective boundary conditions with the corresponding heat transfer coefficients divided into different regions. For the perforated plate a uniform time-dependant temperature is determined and applied directly on every node (region 4). These boundary conditions for the thermal transient analyses are shown in Figure 9.

For mechanical calculations the axial degrees of freedom of the nodes at the shell sided end of the model were suppressed ($u_1 = 0$). The axial degrees of freedom of the nodes at the water chamber sided end of the model were coupled (CP) to ensure a planar cross section. The time dependent internal pressure was applied to the inside nodes of the region representing the water chamber and shell. The transient temperature fields resulting from the thermal calculations were applied in the mechanical analyses as transient thermal loads (unidirectionally coupled analyses).
The pressure loads were applied simultaneously to the thermal loads as a function of time. All mechanical boundary conditions are shown in Figure 10.

The fatigue analysis of this component is comprehensively described in [17]. In this context, results of the 2D representation were verified by application of a 3D model of the tube sheet (see Figure 11). This model represents a 30°-segment of the tube sheet including the tubes and the drainage hole. Representative fatigue analysis was carried out for the most penalizing event.

Effectively, the transient loading conditions considered for fatigue analysis are identical to those considered for the (local) ratcheting check. For this study, the thermal transients shown in Figure 12 and Figure 13 are chosen as covering ratcheting relevant loading conditions. The first evaluations (see Figure 15 and Figure 18) refer to transient no. 2 (see Figure 13).

Figure 11: 3D model of the tube sheet

On this model the thermal and mechanical boundary conditions were adopted from the axially symmetric model with a convective boundary condition inside the tubes instead of the uniform temperature condition of the 2D model. Results based on the axially symmetric model were confirmed this way. The results of the evaluated positions on the axially symmetric model are representative for the entire tube sheet.

Figure 12: Thermal transient no. 1

Figure 13: Thermal transient no. 2

Figure 14: Qualitative contour plot of von-Mises equivalent total strains

Figure 15: Accumulated first principal plastic strain

A qualitative contour plot of von-Mises equivalent total strains for t=5400s of transient no. 2 is shown in Figure 14. The transition radii at the junction between the tube sheet and the cylindrical shell are both the fatigue and (local) ratcheting critical locations. The cycle-by-cycle accumulation of plastic strains will be demonstrated for the transition radius at the location of maximum accumulated strain which is identical to the position “MX” in Figure 14 (Shell chamber side) and the radius on the shell side.

Ratcheting simulation results both for the material models of Chaboche and Ohno&Wang are shown in Figure 15. In fact,
100 cycles of transient no. 2 were analyzed on a cycle-by-cycle basis. Ratcheting evaluation is based on the KTA rule 3201.2 (paragraph 7.13.3)\textsuperscript{29}. Transient no. 2 delivers nearly identical accumulated strain results for Shell and Water chamber sides in the case of the application of the Ohno&Wang material model. As expected, the application of the Chaboche material model yields very conservative simulation results, which may exceed the local strain limit criteria (5\%) at very low numbers of cycles (45 cycles) while the Ohno&Wang material model predicts a tendency of (local) plastic shakedown. In other words, the component passes the ratcheting check with respect of the covering transient no. 2 by application of the Ohno&Wang material model. Controversely, it does not pass the ratcheting check by application of the Chaboche material model. As pointed out above, the Ohno&Wang material model may not be available in finite element codes and necessitate a user implementation.

In a further step, transients no. 1 and no. 2 were considered separately and in the combinations 10 times transient no. 1 + 10 times transient no. 2. The evaluation is limited to the Shell chamber side. Again, a whole of 100 cycles were considered. The results are plotted in Figure 16.

The separate consideration of transients no. 1 and no. 2 reveals that transient no. 2 yields the higher accumulated principal plastic strain after 100 cycles. The alternate application of transient no. 1 and transient no. 2 yields a considerably higher accumulated principal plastic strain after 100 cycles. In other words, it is not conservative to analyze the transient of highest accumulated principal plastic strains in the sense of a covering transient. The effect of combined events is less pronounced in the case of applying a sequence of ten times transient no. 2 alone. Again, transient no. 2 alone is not covering for the combination of the two transients. These effects need to be investigated in more detail. Results for realistic load-time histories will be of particular interest in this context.

Finally, the ratcheting check for this example (covering transient no. 2, see Figure 13) was based on the procedure of EN 13445-3\textsuperscript{4} (see section 3). The associated quadratic extrapolation procedures for the water chamber side (left, index “E”) and the shell side (right, index “T”) of the tube sheet junction (three extrapolation points on both sides) are shown schematically in Figure 17.

A cycle by cycle analysis is carried out for the first 30 cycles while a linear extrapolation of accumulated strains is performed after the 30\textsuperscript{th} cycle. The results are shown in Figure 18.

The resulting accumulated strains are compared to the total principal strains and to the strains resulting from a linearization across the wall thickness (of the shell) along the paths shown additionally in Figure 17. The lowest values are evaluated by means of the quadratic hot spot extrapolation. The path linearization delivers considerably higher accumulated strain values than the hot spot extrapolation. The peak strain values die out rapidly and thus the surface strains at the hot spot locations deliver lower resulting strain values. This is a general problem of surface extrapolation methods\textsuperscript{31}. The courses of stresses/strains between the extrapolation points and the hot spot have to be verified with respect of plausibility and applicability of the extrapolation procedure\textsuperscript{31}. The locations of the positions \(y_1\) to \(y_3\) do not only depend on the wall thickness but also on the discretization. This would demand for additional discretization studies for tube sheet structures and thermal cyclic loading conditions as well as the elaboration of guidelines similar to\textsuperscript{38}. Based on these findings, the quadratic extrapolation procedure appears to be questionable for this example and is not recommendable. The option of using total strains (see additional note in\textsuperscript{4} and reference in section 3) should be applied.
and Ohno&Wang (see Figure 15) predict an initially degressive strain accumulation (decreasing strain increments).

Note that the component passes the ratcheting check with respect of the covering transient no. 2 by formal application of the quadratic extrapolation procedure but does not pass in case of using linearized or total strains.

6 CONCLUSIONS

General elastic-plastic structural analyses are an option of the fatigue and ratcheting check of components according to the ASME code and the German KTA rules. In this sense, the ratcheting check becomes practically part of the code forming fatigue analysis. However, the code does not deliver more than rough instructions for the evaluation of this kind of analyses. The ratcheting check requires fully elastic-plastic finite elements analyses over several cycles based on an appropriate (nonlinear kinematic) material model. As an extended post processing procedure the calculation of the principal tensile strain increment per cycle (in case of the German post processing procedure the calculation of the principal plastic strain increment per cycle given by the German KTA rule 3201.2(26)), appropriate extrapolation methods and formulae for the determination of the number of load cycles corresponding to given strain limits as well as the plastic shake down check are required. As cyclic elastic-plastic analyses applied to complex geometric and loading configurations are extremely time and disc space consuming the number of practically analyzable cycles may be rather small (typically about ten cycles). Consolidated extrapolation methods yielding reliable and manageable results without being overly conservative are required. The elastic-plastic procedure of EN 13445-3 based on linearly-elastic ideally-plastic constitutive law was applied for comparison. The material model and the proposed quadratic strain extrapolation procedure remain open issues in that context.

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