REVIEW OF ASME III CODE CASE FOR THE APPLICATION OF FINITE ELEMENT BASED LIMIT LOAD ANALYSIS

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ABSTRACT

The use of finite element based limit load analysis for the assessment of the primary load capacity of a pressure vessel is well established and numerous papers on the subject, including experimental results, have been published in the last decade. Finite element based limit load analysis is often used in the context of NB-3228.1 Limit Analysis to demonstrate a margin against ductile burst as an alternative to satisfaction of the NB-3200 limits on general, local and primary membrane plus bending stress intensity. However, although NB-3200 permits the use of ‘limit analysis’, no specific guidance on the use of finite element methods for this purpose is provided. Other pressure vessel codes, including ASME VIII Division 2 and EN13445 contain explicit guidance on the use of finite element methods for limit load analysis. To address this, a Code Case is currently under development to provide technical guidance on the use of finite element based limit load analysis within the context of NB-3200 assessments. The Code Case provides a step-by-step procedure which guides the analyst in the application of limit load analysis and ensures that a valid analysis has been undertaken. The topics of geometric weakening, yield surface selection, tentative wall thickness, element selection and selection of $S_m$ are accounted for in the Code Case and discussed. This paper provides a detailed review of the Code Case and shows how it can be used in practice.

INTRODUCTION

Although provision for limit analysis exists in ASME III NB-3228.1 [1], and the use of plastic analysis for shakedown and fatigue are considered in NB-3228.4, a detailed user procedure for the handling of plasticity with Finite Element (FE) methods is not present in Subsection NB. The Hierarchical Finite Element Framework (HFEF) [2] for the assessment of pressure vessels to the ASME III code primary and secondary stress limits uses a range of modern numerical techniques commensurate with the assessment of stresses in a complex three-dimensional structure. Within the HFEF approach, FE based Limit Load Analysis (LLA) is used to assess against plastic collapse, in conjunction with automated FE direct shakedown techniques and strain based fatigue post-processing to assess the structural response to complex thermo-mechanical cyclic loading. The HFEF approach removes the requirement to position stress classification lines (SCLs), or ‘cutlines’ in the structure, and the associated linearisation of stresses.

To support the HFEF methodology, a series of three draft ASME III Code Cases have been developed to provide explicit guidance on the application of finite element techniques for: (i) Limit Load Analysis, (ii) Direct Shakedown Prediction and (iii) Strain Based Fatigue Assessment. This paper considers the Limit Load Analysis Code Case in more detail. Rolls-Royce Power Engineering plc have been using finite element based limit load analysis in pressure vessel design for a number of years and have developed an internal procedure for its...
application; this procedure provides the foundation for the LLA Code Case described here. Further development of the three Code Cases is being pursued via the ASME Committee Working Group Design Methodology SG-D BPV III (WGDM). It is noted that this paper reflects the authors’ summary of LLA and the wording proposed may change throughout the review and approval process.

EXISTING ASME III NB-3228.1 GUIDANCE

ASME III NB-3228.1[1] states that the limits on General Membrane Stress Intensity (NB-3221.1), Local Membrane Stress Intensity (NB-3221.2) and Primary Membrane Plus Primary Membrane Stress Intensity (NB-3221.3) need not be satisfied at a specific location if it can be shown by limit analysis that the specified loadings do not exceed two-thirds of the lower bound collapse load. The yield strength to be used is $1.5S_m$ where $S_m$ is the design stress intensity.

ASME III NB-3228.1 does not refer explicitly to the use of finite elements, nor provide any guidance on how geometrical weakening should be handled. The proposed Code Case described in this paper addresses this.

REVIEW OF OTHER AVAILABLE GUIDANCE

Extent guidance for the application of FE based LLA is provided in the following pressure vessel standards and guides:

ASME Section VIII Division 2 [3] clause 5.2.3 describes the ‘Limit Load Analysis Method’ as an approach for providing protection against plastic collapse. ASME Section VIII Division 2 states that the limit load should be calculated based on an elastic-perfectly plastic material model, small displacement theory, von Mises theory and a yield strength defining the plastic limit of $1.5S$, where $S$ is the allowable stress. It is also recommended that full elastic-plastic analysis is used where geometric weakening is observed to occur.

European Standard EN 13445-3:2002, Unfired Pressure Vessels Part 3: Design [5] Annex B Direct Route B8.2 describes the use of an elastic-perfectly plastic material model, small displacement theory, Tresca theory and a yield strength defining the plastic limit of $RM_d\gamma_R\gamma_K$ where $RM_d$ and $\gamma_R$ and material dependent parameters. EN13445-3 states that von Mises theory may be used, if the design strength parameter $RM_d$ is multiplied by $\sqrt{3}/2$. If weakening is observed to occur, non-linear effects should be taken into account.

Kalnins has also provided guidance for the application of limit load analysis to pressure vessels in WRC Bulletin 464 – Guidelines for Sizing of Vessels by Limit Analysis [4].

OVERVIEW OF CODE CASE CONTENT

This section provides a review of the draft code case wording. The Code Case wording seeks to achieve a balance between the provision of high level procedural guidance and detailed finite element software instruction. For example the wording provides high level guidance on the selection of element types allowing the user to utilise an appropriate finite element package with suitable elements without identifying specific element types.

LLA-1 SCOPE

This Code Case provides rules for the application of finite element based limit load analysis to satisfy the requirements of NB-3228.1 Limit Analysis. As such, this Code Case provides rules for the use of finite element based limit load analysis as an alternative to satisfaction of the limits on General Membrane Stress Intensity (NB-3221.1), Local Membrane Stress Intensity (NB-3221.2) and Primary Membrane Plus Primary Bending Stress Intensity (NB-3221.3). This Code Case does not require the application of stress classification lines, or cutlines, and their associated linearisation procedures.

LLA-2 BACKGROUND AND TERMINOLOGY

Finite element based limit load analysis is a structural analysis method that can be used as described in this Code Case to predict the plastic collapse failure mode of a pressure vessel.

In the context of this Code Case, limit load analysis refers uniquely to a numerical solution based on the following assumptions:

(a) An elastic-perfectly plastic material model is used
(b) Small displacement linear geometry is assumed such that the collapse load is based on the original undeformed configuration of the structure. Changes in loading resulting from geometry changes or other forms of non-linearity such as changing contact are not accounted for.

Small displacement linear geometry is a reasonable assumption for thick-walled pressure vessels which are not expected to exhibit deflections sufficient to change the shape of the structure or direction of load application. However, some slender structures exhibit non-linear behaviour, potentially leading to a reduction in load capacity, or geometric weakening. This Code Case ensures that any propensity for geometric weakening is addressed by the analyst. Note that an increase in load carrying capacity, or geometric strengthening, may occur on the application of load.

The term limit load refers specifically to the plastic collapse load obtained from an elastic-perfectly plastic small displacement numerical solution. The term collapse load refers to the plastic collapse load obtained from a non-linear geometry or strain hardening numerical solution. Limit load analysis is a subset of non-linear analysis.

LLA-3 VALIDITY AND LIMITATIONS

The application of limit load analysis is subject to the following limitations:

(a) Limit load analysis should not be applied to structures that demonstrate significant geometric weakening on the application of load.

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In the case of a single load, this load should be ramped from zero using an appropriate load incrementation procedure. Where multiple loads are present, a series of analyses should be undertaken in which the ratio of loads is varied to enable the derivation of a conservative limit load set.

**LLA-9 FINITE ELEMENT MODEL**

The finite element model should include all structural features that contribute to the plastic collapse mode. Generally local stress raising features can be omitted.

As plasticity develops and incompressible regions are developed, hydrostatic locking may result in an over-stiff structure. Care should be given to the selection of element type to avoid this, typically by the use of reduced integration high order elements.

Boundary conditions should be selected to ensure that support is not derived from interfacing components. Sensitivity of the failure mode to boundary conditions should be assessed; it is noted that variation in boundary conditions can lead to completely different collapse modes, particularly in the case of non-linear geometry.

In order to establish the limit load or collapse load of a particular region of a structure, it is acceptable to test other regions of the structure to have purely elastic properties, provided that doing so does not influence the limit load or collapse load of the region of interest.

**LLA-10 PROPENSITY FOR GEOMETRICAL WEAKENING**

Some structures may exhibit non-linear geometry behaviour on the application of load, leading to geometric weakening. Such structural non-linearity may arise from one, or a combination of, the following factors:

(i) Change in load bearing cross section
(ii) Change in second moment of area
(iii) Change in direction of load paths

To quantify the influence of geometric non-linearity, both linear (limit load) and non-linear (collapse load) solutions should be obtained. In the case of multiple loads, a series of analyses should be undertaken to ascertain the propensity for geometric non-linearity throughout the load space.

Geometric weakening is deemed to occur if the linear (limit load) solution exceeds the non-linear (collapse load) solution. If the non-linear solution is equal to or exceeds the linear solution, use of the limit load solution is conservative.

If strengthening occurs, it is acceptable to use the elastically perfectly plastic non-linear collapse load in order to meet the required design loads only if supporting validation data are available, possibly from testing.

In the case that the elastically perfectly plastic limit load or collapse load solutions do not meet the required design loads, further analysis using a strain hardening material model and appropriate acceptance criterion may be required. This is beyond the scope of this Code Case.
LLA-11 LIMIT LOAD ANALYSIS STEP-BY-STEP PROCEDURE

The following procedure should be followed in the application of limit load analysis, as illustrated in Annex A:

1) Build finite element model, in alignment with the guidance provided in LLA-9.
2) Define all relevant loads as defined in LLA-7.
3) Set the onset of plasticity to \( S_m \) at the design temperature of interest, as discussed in LLA-5 and LLA-6.

4) Determine the significance of geometrical non-linearity:
   a) Obtain an elastic-perfectly plastic small displacement limit load solution.
   b) Obtain an elastic-perfectly plastic non-linear (large displacement) collapse load solution.
   If the structure demonstrates significant weakening, proceed to step 5) else proceed to step 6).

5) If the structure demonstrates weakening on the application of load:
   a) Proceed to step 7) and use the non-linear collapse load in comparison to the required design load OR
   b) Undertake a full elastic-plastic analysis with strain hardening and non-linear geometry. This is beyond the scope of this Case.

6) If the structure demonstrates strengthening or the effects of non-linearity are insignificant:
   a) Proceed to step 7) and compare the small displacement limit load to the required design load OR
   b) If supporting validation data are available, proceed to step 7) and compare the non-linear collapse load to the required design load.

7) Compare the limit or collapse load with the required design load. If the limit load, or collapse load, exceeds the design load, the geometry is acceptable from a load capacity perspective, proceed to step 8) or proceed to step 9).

8) Check that all relevant functional requirements and design constraints are satisfied. If compressive stresses are generated, particularly in shell structures, buckling may occur and should be investigated. If all relevant functional requirements are satisfied, proceed to step 10) else proceed to step 9).

9) If the required design load, functional response or design constraints are not achieved, a modification to the geometry may be required and the procedure repeated from step 4). Revisit the functional and design constraints in the event of conflict.

10) Procedure complete, an acceptable limit load or collapse load has been obtained.

LLA-12 ALTERNATIVE NUMERICAL APPROACHES

The Code Case does not prescribe the particular numerical method to be used. Any suitable method including but not limited to static finite element analysis, arc-length methods or elastic modulus adjustment schemes such as those described in [6] may be used to obtain a limit load subject to the limitations stated above.

- End of proposed Code Case Content -

ADDITIONAL DISCUSSION

It is noted that the current guidance in NB-3228.1 Limit Analysis states that a value of \( 1.5 \times S_m \) should be used to represent the onset of plasticity and that the limit load should not exceed two-thirds of the lower bound collapse load. In the case of linear geometry assumptions, an analysis in which the onset of plasticity is represented by \( S_m \) is identical to an analysis in which the onset of plasticity is represented by \( 1.5 \times S_m \) and the resulting limit load is scaled by two-thirds. Differences in the finite element model and time stepping procedure may result in slightly different results.

In the general case of geometrically non-linear analysis, the collapse load will not scale in proportion to the value chosen to represent the onset of plasticity and is dependent on both elastic properties and yield strength. As a result, using \( 1.5 \times S_m \) to represent the onset of plasticity and dividing the subsequent collapse load by 1.5 is not identical to an analysis in which \( S_m \) is used from the outset. The sensitivity of the non-linear geometry structural collapse load to changes in the onset of plasticity stress should be assessed, but is expected to be small for thick-walled structures.

It is recognised that the NB-3222.1 \( S_m \) limit on primary membrane stress intensity, in conjunction with the NB-3222.2 3\( S_m \) limit on primary plus secondary stress intensity range, provides assurance against ratcheting based on the Bree criteria repeated in NB-3222.5 Thermal Stress Ratchet. The user of the LLA Code Case should be aware of the dependency between the strength and ratchet limits, particularly when the LLA Code Case is used to satisfy the strength limits and the existing ASME III NB-3200 clauses are used to satisfy the remaining limits. If direct shakedown methods are used to derive the ratchet boundary directly using the HFED approach, LLA becomes independent from the evaluation of shakedown.

A TYPICAL EXAMPLE

The potential benefits of LLA are well known, including the reduction of non-standard section thickness, improved thermal fatigue performance, weight reduction and a reduction in machining operations. Reference [7] provides a good demonstration of the application of LLA to a perforated plate structure. In a structure of this type a reduction in plate thickness can result in a significant reduction in drilling operations. The example provided here serves to demonstrate a simple application of the Code Case procedure, not the benefit of using LLA in the context of structural optimisation.
This example considers a simple intersection between cylinders, typical of a nozzle in a pressure vessel as shown in Figure 1. Internal pressure is applied to the shell and an external moment is applied to the end of the nozzle; as such a multiple load scenario is encountered. The shell radius to thickness ratio is six, whilst the corresponding ratio for the nozzle is 3. The ratio of the shell to nozzle outside radii is eight.

In accordance with the multiple load guidance of LLA-8, the failure locus in pressure / moment space has been determined by undertaking a number of limit load analyses, resulting in Figure 2. Both linear and non-linear geometry analyses have been undertaken in accordance with the guidance of LLA-11 Step 4) using a commercially available FE package. The pressure and moment axes have been normalised to the linear geometry zero moment and zero pressure cases respectively. In this example, the primary failure mode of the structure occurs in the closure end to shell intersection. To enable investigation of the pressure versus moment interaction in the vicinity of the nozzle, elastic material properties have been set in the elastic closure end region.

This structure demonstrates two further failure modes, the first in the shell, dominated by internal pressure, and the second in the nozzle, dominated by the external moment.

It can be seen that the influence of non-linearity is small, and providing functional requirements are satisfied, the linear geometry locus defines the design space.

CONCLUSIONS

The use of limit load analysis is well established as an alternative means of addressing the ASME III NB-3200 primary general membrane, local membrane and primary membrane plus primary bending stress intensity limits, although explicit guidance on the use of finite element methods to achieve this is not provided in NB-3228.1. This proposed Code Case described in this paper describes a procedure for the application of modern finite element methods to be used to support NB-3228.1.

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REFERENCES

ANNEX A – LIMIT LOAD ANALYSIS FLOW CHART

Step 1
Build finite element model

Step 2
Define loads of interest

Step 3
Set $S_m$

Step 4
Determine the extent of geometric non-linearity

Step 4a
Obtain limit load from elastic-perfectly plastic small displacement analysis

Step 4b
Obtain collapse load from elastic-perfectly plastic non-linear geometry analysis

Step 5b
Analysis with strain hardening material model
Beyond the scope of this Case

Step 6
Test data to support geometrical strengthening?

Step 6a
Use elastic-perfectly plastic limit load from Step 4a

Step 7
Collapse load OK?

Step 8

Functional requirements, eg deflections, sealing, other components

Design constraints and drivers, eg weight, cost

Step 8
Functions satisfied?

Step 12
Satisfactory design

Step 9

Conflict?

Step 9
Adjust design

Step 9
Re-visit functional and design constraints

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