NUMERICAL SIMULATIONS OF HEAT AND MASS TRANSFER IN CONDENSING HEAT EXCHANGERS FOR WATER RECOVERY IN POWER PLANTS

Kwangkook Jeong
Department of Mechanical Engineering, CESUR, Arkansas State University
2713 Pawnee Street, Jonesboro, AR, 72401, USA

Harun Bilirgen and Edward Levy
Department of Mechanical Engineering and Mechanics, Energy Research Center, Lehigh University
117 ATLSS Drive, Bethlehem, PA, 18015, USA

ABSTRACT

Power plants release a large amount of water vapor into the atmosphere through the stack. The flue gas can be a potential source for obtaining much needed cooling water for a power plant. If a power plant could recover and reuse a portion of this moisture, it could reduce its total cooling water intake requirement. One of the most practical way to recover water from flue gas is to use a condensing heat exchanger. The power plant could also recover latent heat due to condensation as well as sensible heat due to lowering the flue gas exit temperature. Additionally, harmful acids released from the stack can be reduced in a condensing heat exchanger by acid condensation.

Condensation of vapors in flue gas is a complicated phenomenon since heat and mass transfer of water vapor (H$_2$O) and various acids simultaneously occur in the presence of non-condensable gases such as nitrogen and oxygen. Design of a condenser depends on the knowledge and understanding of the heat and mass transfer processes. A computer program for numerical simulations of water (H$_2$O) and sulfuric acid (H$_2$SO$_4$) condensation in a flue gas condensing heat exchanger was developed using MATLAB. Governing equations based on mass and energy balances for the system were derived to predict variables such as flue gas exit temperature, cooling water outlet temperature, mole fraction and condensation rates of water and sulfuric acid vapors. The equations were solved using an iterative solution technique with calculations of heat and mass transfer coefficients and physical properties.

An experimental study was carried out in order to yield data for validation of modeling results. Parametric studies for both modeling and experiments were performed to investigate the effects of parameters such as flue gas flow rate, cooling water flow rate, inlet cooling water temperature and tube configurations (bare and finned tubes) on condensation efficiency. Predicted results of water and sulfuric acid vapor condensation were compared with experimental data for model validation, and this showed agreement between experimental data and predictions to within a few percent. The most important parameters affecting performance of the condensing heat exchangers was the ratio of cooling water to flue gas flow rates, since this determines how much heat the cooling water can absorb.

The computer program simultaneously calculates both water vapor condensation and sulfuric acid condensation in flue gas along downstream. Modeling results for prediction of sulfuric acid vapor concentration in the flue gas were compared with measured data obtained by the controlled condensation method. An analytical model of sulfuric acid condensation for oil-firing showed two trends - steep reduction within the high temperature heat exchanger and smooth reduction within lower temperature heat exchanger, which is in agreement with experimental data.

INTRODUCTION

Condensation in flue gas is a complicated phenomenon since heat and mass transfer of water vapor (H$_2$O) and various acid vapors simultaneously occur in the presence of non-condensable gases. As a result of the mass transfer of condensable vapor toward the surface in the presence of non-condensable gases, the non-condensable gases tend to be more concentrated near the cold surface. The condensable vapor has to diffuse through the non-condensable gas layers in order to reach the cooled condensing surface. To design a full scale heat exchanger for a power plant, the condensation process must be modeled correctly with consideration of the non-condensable...
gas effects. Numerical simulations would be also essential to predict the performance of a condensing heat exchanger, such as condensation rates of water vapor and sulfuric acid ($H_2SO_4$) within the power plant operating conditions.

Several investigators have proposed analytical solutions. In 1974, Verhoff and Banchero [1] developed correlations to predict the dew point temperatures of acid gases including hydrobromic acid (HBr), hydrochloric acid (HCl), nitric acid (HNO3), sulfurous acid ($H_2SO_3$) and sulfuric acid ($H_2SO_4$). For sulfuric acid, over 85% of the predicted dew point temperatures agreed within 7 K of actual values [2].

In 1980, Webb et al.[3] developed a one dimensional numerical model to predict the effect of non-condensable gases in a 10-row by 10-column finned tube heat exchanger by solving the Colburn-Hougen equation for refrigerant R-11 and air mixture. An iterative solution procedure was applied to solve the equation. The modeling results were not verified with measured data.

In 1987, D’Agostini [4] developed a theoretical model to analyze the condensation of sulfuric acid from power plant flue gas onto the surfaces of a regenerative air preheater (APH). The model was based on the Chilton-Colburn heat and mass transfer analogy and provided a means of predicting the mass transfer response of the APH to variations in the power plant operating conditions. The results of the model showed good qualitative agreement with the findings of two other theoretical analyses.

From 1999 to 2003, Osakabe et al.[5-8] carried out one-dimensional heat and mass balance calculations for the condensation of flue gas in bare and finned tube heat exchangers. Experimental studies using actual flue gas from propane, natural gas and oil combustion were conducted to investigate the effect of parameters such as flue gas flow rate, cooling water temperature and cooling water flow rate. The results of both modeling and experiments were compared for variables such as flue gas temperature, cooling water temperature and condensation rate. For tests using flue gas obtained from oil combustion, cooling water flow rate was varied from 60.0 to 3,700 kg/h while flue gas flow rate was varied from 1,000 to 176 kg/h. It was found that cooling water flow rate was an important factor affecting condensation rate.

In 2004, Valencia [9] performed a CFD simulation for the condensation of water vapor and acids on the plate using a commercial code, FLUENT and a user defined subroutine. A numerical simulation using the commercial code and a simulation based on empirical correlations using the Engineering Equation Solver (EES) were carried out for a two dimensional (2D) vertical water cooled plate. Experiments were conducted for the condensation of nitric acid, sulfuric acid and water vapor in the presence of air on a vertical water-cooled plate. The discrepancies between experiments and simulation are in a range of 7-25% depending on the combustion conditions and the average surface temperature of the plate. The numerical model was applied to real 3D geometries including an annular fin heat exchanger and a pin fin heat exchanger.

From 2008 to 2010, Levy et al. [10-13] investigated use of condensing heat exchangers to recover water vapor from flue gas at coal-fired power plants. Pilot scale heat transfer tests were performed to determine the relationship between flue gas moisture concentration, heat exchanger design and operating conditions, and water vapor condensation rate. A theoretical heat and mass transfer model was developed for predicting rates of heat transfer and water/acid vapor condensation and comparisons were made with pilot scale measurements. Analyses were also carried out to estimate how much flue gas moisture it would be practical to recover from boiler flue gas and the magnitude of the heat rate improvements which could be made by recovering sensible and latent heat from flue gas.

This paper describes numerical simulations based on analytical modeling of the heat and mass transfer process for condensation of water and sulfuric acid vapors in a countercurrent cross flow tubular heat exchanger with cooling water inside and flue gas outside the tubes. The simulation results are compared to experiments made using a pilot-scale heat exchanger and the performance of the heat exchanger system is shown as a function of various parameters.

**NUMERICAL SIMULATIONS**

**Control Volume and Main Variables**

The control volume used for deriving the governing equations is defined as a fixed region in space that encompasses the flue gas and the cooling water tubes. A countercurrent cross flow heat exchanger with smooth-wall tubes is presumed. Figure 1 shows the independent and dependent variables in the control volume which affect water and sulfuric acid vapor condensation. Heat and mass transfer for water and sulfuric acid condensation are considered with this control volume. The flue gas temperatures at the inlet and outlet of this control volume are expressed as $T_{g,in}$ and $T_{g,out}$ respectively. The parameters $T_{c,in}$ and $T_{c,out}$ Express cooling water temperatures at the inlet and outlet, respectively. Tube wall temperature is expressed by $T_c$. The parameters $T_g$ and $T_c$ are average values of flue gas and cooling water temperatures between the inlet and outlet, respectively. Heat transfer in this control volume is controlled by heat transfer coefficients on the flue gas side ($h_g$) and cooling water side ($h_c$).

Mass transfer of water and sulfuric acid vapor towards the wall occurs at locations where the tube wall temperature is below the local water and sulfuric acid dew point temperature. In this paper, the mole fraction at the inlet and outlet are expressed as $y_{H_2O,in}$ and $y_{H_2O,out}$ for water vapor and $y_{H_2SO_4,in}$ and $y_{H_2SO_4,out}$ for sulfuric acid vapor, respectively. The parameters $y_{H_2O}$ and $y_{H_2SO_4}$ are an average mole fraction of $y_{H_2O,in}$ and $y_{H_2O,out}$ for water vapor and $y_{H_2SO_4,in}$ and $y_{H_2SO_4,out}$ for sulfuric acid vapor. The dew point temperature of water vapor ($T_{dew}$) and sulfuric acid ($T_{dew,H_2SO_4}$) are the saturation temperatures corresponding to the partial pressure of water and sulfuric acid vapor in the flue gas. The parameters $T_i$ and $T_i,H_2SO_4$ are temperatures of water and sulfuric acid vapor at the interface between the gas and liquid film. Mass transfer coefficients for
water and sulfuric acid vapor are expressed as \( k_m \) and \( k_m\text{H}_2\text{SO}_4 \), respectively.

**Figure 1. Control volume and variables for analytical modeling [12]**

In the analytical model, both mass transfer of water and sulfuric acid vapor and heat transfer from flue gas to cooling water are concurrently considered. Entering the heat exchanger, the flue gas is first cooled by sensible heat transfer to cooling water. Acid vapor in the flue gas condenses on the cooling water tube surface if the wall temperature is lowered below the dew point of the acid, simultaneously releasing latent heat that transfers to the cooling water. With further cooling along downstream, water vapors participate at multi-component condensation with acids releasing latent heat to the tube surface. Two-phase flow, which consists of the gas phase (uncondensed flue gas) and liquid phase (water/acid condensate), is assumed on the flue gas side [12].

**Governing Equations**

Governing equations based on heat/mass transfer and energy balances for the control volume are introduced here [13]. Colburn and Hougen developed a fundamental transport equation for condensation in the presence of a non-condensable gas [14]. When the wall temperature is lower than the dew point temperature, condensation occurs on the tube surface as a result of diffusion of vapor through the flue gas to the liquid-vapor interface. Therefore, vapor exists in the flue gas as a superheated vapor at \( T_g \). Sensible heat transfers from the flue gases to the liquid-vapor interface in addition to the latent heat transferred from the condensing vapors. The heat transfer to the cooling water is the sum of sensible and latent heat. The Colburn-Hougen equation is defined as follows.

\[
h_s(T_g - T_i) + k_m \cdot l \cdot (y_{H_2SO_4} - y_i) = U_o(T_i - T_c)
\]

Equation (1) includes the interfacial temperature, \( T_i \), for the cases when vapor is condensing. The parameter \( l \) is the latent heat of vapor. As shown Figure 2, \( U_o \) is the overall heat transfer coefficient, taking into account resistances from the condensate liquid film to the cooling water. These resistances expressed in terms of the associated heat transfer coefficients as shown below:

\[
\frac{1}{U_o A_{eff}} = \frac{1}{U_r} + F_s \left( \frac{1}{A_i} + R_w + \frac{1}{h_f A_{eff}} \right)
\]

where \( A_{eff} \) and \( A_o \) are the heat transfer areas based on the tube outer diameter including film thickness and the inner diameter of tube, respectively. \( h_s \) and \( h_f \) are the heat transfer coefficients on the cooling water side and liquid film side, respectively. \( F_s \) is the fouling factor on the inside of the tubes. \( R_w \) is the conductive resistance of the tube wall and is expressed as \( \ln(d_o/d_i)/(2\pi \lambda_w L) \).

The fouling factor and the tube wall resistance are assumed to be negligible in this analysis. The thermal resistance due to the condensate film is negligible since it contributes only about 1-3 % percent of the total thermal resistance [15]. Thicknesses of water film and tube are neglected in this research, and subsequently, \( A_{eff} \) and \( A_o \) are replaced by \( A_s \) (the heat transfer area based on the tube outer diameter). Then Equation (2) is simplified to Equation (3) with the assumptions.

\[
\frac{1}{U_o A_s} \approx \frac{1}{h_i A_o}
\]

**Figure 2. Thermal resistances from condensate to cooling water side [13]**

\[
\frac{1}{U_o A_s} \approx \frac{1}{h_i A_o}
\]
Applying these approximations, the first governing equation is established as shown in Equation (4). The heat transfer coefficients on the flue gas side \( (h_g) \) and cooling water side \( (h_c) \) are calculated using empirical correlations of forced convective heat transfer for tube banks and inside a tube, respectively.

\[
h_g(T_g - T_i) + k_m \cdot l \cdot (y_{H,O} - y_i) = h_c(T_i - T_c)
\]  

(4)

The second governing equation, Equation (5) is the energy balance between the enthalpy change of the flue gas and the heat transfer rate from the flue gas to the liquid film. This ordinary differential equation is integrated over the control volume, which is presumably small enough to assume that all other variables are constant.

\[
m_g \cdot c_{p,g} \cdot dT = h_g \cdot (T_g - T_i) \cdot dA
\]  

(5)

For the case of no water vapor condensation, heat transfer from the flue gas is directly transferred to the tube wall, which has a temperature, \( T_w \), instead of the liquid-film interface. The third governing equation is for the case when no condensation is occurring and is shown in Equation (6). Equation (6) reflects energy conservation between the enthalpy change of the flue gas and the heat transfer rate from the flue gas to the tube wall surface.

\[
m_g \cdot c_{p,g} \cdot dT = h_g \cdot (T_g - T_w) \cdot dA
\]  

(6)

The fourth governing equation, the overall energy balance equation is derived by modifying the Colburn-Hougen Equation, (4). Since the total heat transferred to the cooling water, represented by the right hand side of Equation (4), is equal to the enthalpy change of the cooling water, Equation (4) can be rewritten as Equation (7).

\[
[h_g \cdot (T_g - T_i) + k_m l(y_{H,O} - y_i)] \cdot dA = m_c \cdot C_{p,c} \cdot dT_c
\]  

(7)

For the case of no condensation of water vapor, the interfacial temperature, \( T_i \) in Equation (7), is replaced by tube wall temperature \( T_w \), and the mass transfer term is eliminated on the left hand side of Equation (7). The fifth governing equation is derived from Equation (7) as shown hereunder.

\[
h_g(T_g - T_w) \cdot dA = m_c \cdot C_{p,c} \cdot dT_c
\]  

(8)

The sixth governing equation balances the enthalpy change of cooling water with the convective heat transferred from tube wall to the cooling water. This is written in Equation (9).

\[
h_c(T_w - T_c) \cdot dA = m_c \cdot C_{p,c} \cdot dT_c
\]  

(9)

The ordinary differential equations, from Equation (5) to Equation (9), are integrated over a discretized cell, which is presumably small enough to assume that all other variables are constant.

The condensation rate of water vapor is proportional to both the mass transfer coefficient, \( k_m \) [kg/s·m²·mol] and the concentration differences in the vapor phase between the interface and bulk flow. This can be calculated by integration of the following ordinary differential equation.

\[
dm_{aw} = k_m \cdot (y_{H,O} - y_i) \cdot dA
\]  

(10)

To calculate \( k_m \), the relationship between mass diffusion and heat transfer obtained from the Lewis relation is used. The Colburn \( j \) factors for heat and mass transfer with their applicable ranges are defined as, [16-17]:

\[
j_H = St \cdot Pr^{2/3} = \frac{h}{\rho \cdot c_p \cdot V} \cdot Pr^{2/3}
\]  

(11)

\[0.6 < Pr < 60\]

\[
j_m = St_m \cdot Sc^{2/3} = \frac{k_D}{V} \cdot (Sc)^{2/3}
\]  

(12)

\[0.6 < Sc < 3000\]

where \( h \) is convective heat transfer coefficient [W/m²·K] and \( k_D \) is convective mass transfer coefficient [m/s]. The Lewis analogy requires equating Equation (11) and (12), that is, \( j_H = j_m \). Then an expression is derived for the mass transfer coefficient as :

\[
k_m = \frac{h_j \cdot M_{H,O}}{c_{p,g} \cdot M_g \cdot y_{lm} \cdot Le_{H,O,gas}^{1/3}}
\]  

(13)

The mass transfer coefficient is shown to be a function of the gas side heat transfer coefficient, the logarithmic mole fraction difference of the non-condensable gas and the properties of flue gas side including Lewis number of water vapor in flue gas. To predict the convective heat transfer coefficient on the gas side of a bare tube bank, an empirical correlation, Equation (14), proposed by Zhukauskas [18] was used.

\[
\frac{Nu_g = 0.27 \cdot Re_g^{0.63} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_f}\right)^{1/4}}{[0.7 \leq Pr \leq 500] \quad [1000 \leq Re_{\rho,max} \leq 2 \times 10^7]}
\]  

(14)

The Reynolds number appearing in Equation (14) is based on the maximum velocity \( V_{g,max} \) which is in the minimum flow area between the tubes. The maximum velocity \( V_{g,max} \) ranged from 0.8 to 4.0 m/s, while the upstream flue gas velocity at the duct inlet ranged from 0.3 to 1.5 m/s in this study. All properties are calculated based as mixtures, and \( y_{lm} \), the logarithmic mole fraction difference of the non-condensable
gas between the free stream and the wall, is calculated as [3,19]:

\[ y_{lm} = \frac{y_{ai} - y_{ab}}{\ln(y_{ai}/y_{ab})} \tag{15} \]

where \( y_{ai} \) and \( y_{ab} \) are the mole fractions of non-condensable gases at the gas/film interface and in the bulk flow, respectively.

The parameter \( L e_{H_2O-gas} \), the Lewis number of water vapor in flue gas, is defined as Equation (16), in which \( D_{H_2O-gas} \) is the binary mass diffusion coefficient of water vapor in flue gas.

\[ L e_{H_2O-gas} = \frac{Sc}{Pr} = \frac{\alpha_s}{D_{H_2O-gas}} \tag{16} \]

The Lewis number is typically of order unity for gases. This implies that changes in the thermal and species distributions progress at approximately the same rates in gases that undergo simultaneous heat transfer and mass diffusion processes [20-21]. Therefore, unknown mass diffusivity \( (D*_{AB,unknown}) \) can be estimated with known other properties from Equation (17).

\[ L e_{gas} \approx 1 = \frac{\alpha_{known}}{D_{AB,known}} = \frac{\alpha_{unknown}}{D_{AB,unknown}} \tag{17} \]

Subsequently, the mass diffusivity of water vapor in flue gas is estimated from the known properties of water vapor in air and the calculated thermal diffusivity of flue gas,

\[ D_{H_2O-gas} = \left( \frac{\alpha_s}{\alpha_{air}} \right) \cdot D_{H_2O-air} \tag{18} \]

where \( \alpha_s \) and \( \alpha_{air} \) are the thermal diffusivities of flue gas and air, respectively.

The interfacial mole fraction of water vapor used in Equation (10) can be calculated by Equation (19), the Antoine equation [22], which is a vapor pressure equation and describes the relation of saturated vapor pressure and temperature for pure components:

\[ y_i \exp\left( \frac{a - \frac{b}{T_{i,H_2O}}} {P_{sat}} \right) \tag{19} \]

\[ a = 16.262 \]
\[ b = 3799.89 \]
\[ c = 226.35 \]

where \( T_i \) is interfacial temperature in °C and \( P_{sat} \) is in kPa.

Verhoff and Banchero [1] performed a detailed investigation to obtain a correlation for predicting the acid dew point of flue gas. The equation which reflects a least squares fit of their experimental data is presented below:

\[ \frac{1}{T_{dew,H2SO4}} = 0.002276 - 0.00002943 \cdot \ln(P_{H_2O}) - 0.0000858 \cdot \ln(P_{H_2SO4}) \tag{20} \]

\[ + 6.2 \times 10^{-4} \cdot \ln(P_{H_2O}) \cdot \ln(P_{H_2SO4}) \]

where \( T_{dew,H2SO4} \) is the dew point temperature of sulfuric acid in degree Kelvin, \( P_{H_2O} \) and \( P_{H2SO4} \) are partial pressures of water vapor and sulfuric acid, respectively, in mmHg.

Equation (20) can be inverted to yield an expression for mole fraction of sulfuric acid at the interface, \( y_{i,H2SO4} \), corresponding to a given interfacial temperature of water vapor, \( T_{i,H2O} \), and a partial pressure of water vapor at the interface, \( P_{i,H2O} \). The equation for interfacial mole fraction of sulfuric acid is expressed by Equation (21):

\[ y_{i,H2SO4} = \exp\left( \frac{1}{T_{i,H2O}} - 0.002276 + 0.00002943 \cdot \ln(P_{i,H2O}) \right) / P_{sat} \tag{21} \]

\[ + 6.2 \times 10^{-4} \cdot \ln(P_{i,H2O}) \cdot \ln(P_{i,H2SO4}) \]

where \( y_{i,H2SO4} \) is in vol\% wet, \( T_{i,H2O} \) is in degree Kelvin, \( P_{i,H2O} \) and \( P_{sat} \) are in mmHg.

Mass transfer coefficient for sulfuric acid condensation is derived as Equation (22):

\[ k_{m,H2SO4} = \frac{h_s \cdot M_{H2SO4}} {C_{p,g} \cdot M_{g} \cdot y_{lm} \cdot L e_{H2SO4-gas}^{2/3}} \tag{22} \]

The parameters \( h_s \) and \( C_{p,g} \) are the heat transfer coefficient and the specific heat of wet flue gas side, respectively. The parameters \( M_{H2SO4} \) and \( M_{g} \) are the molecular weight of sulfuric acid and wet flue gas mixture, respectively. The parameter \( y_{lm} \) is the logarithmic mean mole fraction of the non-condensable gas between the free stream and the wall, as shown in Equation (15).

The parameter \( L e_{H2SO4-gas} \) is the Lewis number of sulfuric acid, in which \( D_{H2SO4-gas} \) is the mass diffusion coefficient of sulfuric acid in flue gas.

\[ L e_{H2SO4-gas} = \frac{Sc}{Pr} = \frac{\alpha_s}{D_{H2SO4-gas}} \tag{23} \]

The mass diffusivity of sulfuric acid in flue gas was estimated with the well-known mass diffusivity of water vapor in air as,

\[ D_{H2SO4-gas} = D_{H2SO4-air} \left( \frac{\alpha_s}{\alpha_{air}} \right) \tag{24} \]
where $\alpha_g$ and $\alpha_{air}$ are the thermal diffusivities of flue gas and air, respectively. The diffusivity of sulfuric acid in air can be expressed as,

$$D_{H_2SO_4-air} = 5.0032\times10^{-4} + 1.04\times10^{-8}\cdot T + 1.64\times10^{-11}\cdot T^2 - 1.566\times10^{-14}\cdot T^3$$  \hspace{1cm} (25)

where $T$ is in Kelvin and $D_{H_2SO_4-air}$ is in $m^2/s$. Calculated values of thermal diffusivity and mass diffusivity for water vapor in flue gas are listed in Table 1.

Table 1. Calculated and reference properties of water vapor and sulfuric acid

<table>
<thead>
<tr>
<th>Properties</th>
<th>Water Vapor</th>
<th>Sulfuric Acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated thermal diffusivity</td>
<td>1.96$\times10^{-5}$</td>
<td>1.96$\times10^{-5}$</td>
</tr>
<tr>
<td>of flue gas at 15 °C in this study</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reference of thermal diffusivity</td>
<td>2.22$\times10^{-5}$</td>
<td>2.22$\times10^{-5}$</td>
</tr>
<tr>
<td>of air at 15 °C [20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated mass diffusivity</td>
<td>2.56$\times10^{-5}$</td>
<td>0.91$\times10^{-5}$</td>
</tr>
<tr>
<td>of water vapor in flue gas at 15 °C in this study</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reference of mass diffusivity</td>
<td>2.61$\times10^{-5}$</td>
<td>0.94$\times10^{-5}$</td>
</tr>
<tr>
<td>of water vapor in air at 15 °C [18,23]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated $Le$ at 15 °C in this study</td>
<td>0.77</td>
<td>2.15</td>
</tr>
<tr>
<td>Reference of $Le$ [24,25]</td>
<td>0.845</td>
<td>2.33</td>
</tr>
</tbody>
</table>

Condensation rate of sulfuric acid is proportional to the concentration driving forces in the vapor phase and can be calculated by integration of the following ordinary differential equation, Equation (26):

$$dn_{id,H_2SO_4} = k_{m,H_2SO_4}\cdot(y_{H_2SO_4} - y_{i,H_2SO_4})\cdot dA$$  \hspace{1cm} (26)

where the parameter $dn_{id}$ is the condensation rate of sulfuric acid and the parameter $k_{m,H_2SO_4}$ is the mass transfer coefficient of sulfuric acid. The parameters $y_{H_2SO_4}$ and $y_{i,H_2SO_4}$ are the mole fractions of sulfuric acid in flue gas and interface, respectively.

**Numerical Scheme**

The governing equations for the boundary value problem were solved using an iterative numerical scheme and a one dimensional finite difference method with forward differencing. The known variables were: inlet flue gas temperature, inlet flue gas flow rate, inlet cooling water temperature, cooling water flow rate, and inlet mole fraction of water vapor and sulfuric acid vapor.

The inlet cooling water temperature was used as a target value and served as the criterion for convergence. In order to calculate inlet cooling water temperature, a value for the exit cooling water temperature was initially assumed, and the calculations were carried out. The scheme iterated until the correct inlet cooling water temperature was calculated. The entire system was discretized with control volumes in each cell. For this study, the total heat transfer area, $6.74 \text{ m}^2$, was discretized into 1,000 cells.

**EXPERIMENTAL**

An experimental study was carried out to validate the analytical model. Figure 3 shows the overall experimental setup. The experimental setup consisted of six stages of heat exchangers, an inlet and outlet duct, an induced draft fan, cooling water lines, and an electric hot water heater. Flue gas was channeled from the exit duct of the boiler to the inlet of the heat exchanger system. All heat exchangers were countercurrent cross flow, with flue gas flowing around the cooling water tube bundles. Cooling water was distributed to an inline tube bundle arrangement with 8 parallel rows of multiple U-bend tubes (0.0127 m O.D.). Manifolds were at the inlet and exit of each counter-cross-flow heat exchanger to distribute the water. Heat exchangers one through six had 3, 5, 9, 13, 13, and 13 U-bends. The tube banks had a transverse pitch $S_T$ of 0.7 and longitudinal pitch $S_L$ of 2.0. The cooling water was taken from the local municipal water line. The duct housing the heat exchangers was thermally insulated using flexible fiber glass and rigid foam insulation. The first heat exchanger (HX1) was not needed in the experiments described in this paper because of a relatively low inlet flue gas temperature.
of the heat exchangers, an eight foot straight section of PVC pipe was used to create a fully developed flow region for an accurate flow rate measurement. In this section, a pitot tube and manometer were used to measure flue gas flow velocity.

Rigid plastic containers were used to collect condensate from the bottom of each heat exchanger. The mass of condensate and elapsed time for the test were measured to determine condensation rates. Moisture content of the flue gas at the inlet of the heat exchanger was calculated by adding the measured condensation rate in each heat exchanger to the vapor flow rate in the flue gas, on a molar basis. Wet bulb and dry bulb temperature measurements made in the first few tests showed the flue gas at the exit of the apparatus was saturated. As a result, the exit vapor flow rate in the flue gas was calculated by assuming the flue gas to be fully saturated at the cooling water tube-wall temperature at the exit of the heat exchanger system.

Tests were carried out with flue gas from three different fuels: #6 oil, natural gas and coal. Tests with the flue gas from #6 oil and natural gas were performed in the Boiler House at Lehigh University [9,10]. Tests using coal combustion gas were conducted at a coal-fired power plant. In this paper, test results related to flue gas derived from coal and oil firing are presented. All of the data were obtained by using bare tube bank heat exchangers. The parameters which could have an impact on the performance of condensing heat exchangers were selected for experimental study: 1) inlet cooling temperature, 2) cooling water flow rate, and 3) flue gas flow rate.

A term used in this study, condensation efficiency, is the weight % ratio of total condensation rate to inlet water vapor flow rate. This is used to evaluate the performance of the condensing heat exchangers. Condensation efficiency [12] is expressed as follows:

$$\eta_{cd}[\text{wt}\%] = \sum_{i=1}^{n} \frac{m_{cd,i}}{m_{i,o,in}} \times 100$$

(27)

The numerator represents total condensation rate, which is calculated by summing individual condensation rates from the various heat exchanger stages. The denominator represents the mass flow rate of water vapor at the system inlet.

It was found that the mass flow rate of cooling water is an important factor affecting condensation efficiency since it is related to the heat absorption capacity of the cooling water. However, the mass flow rate of cooling water ($m_i$) is more meaningful when known relative to the mass flow rate of wet flue gas at the inlet ($m_{e,in}$). Moreover, better condensation efficiencies were expected with higher ratios of cooling water to flue gas flow rates.

The ratio of the mass flow rate of wet flue gas to the total heat transfer surface area ($m_{e,in}/A_s$) is another important parameter. It is expected that higher ratios of flue gas flow rate to heat transfer surface area result in poor performance since it could mean relatively insufficient heat transfer surface area.

**RESULTS AND DISCUSSION**

Analytical modeling for water vapor condensation in pilot scale condensing heat exchangers was performed using the experimental test conditions. Five input variables are needed to simulate the pilot scale test: inlet flue gas flow rate and temperature; inlet cooling water flow rate and temperature; and inlet moisture/acid fraction. Experimental data selected to verify the analytical model were conducted with inlet cooling water temperatures ranging from 23.9 °C to 32.3 °C.

![Figure 4. Predicted and measured temperatures of flue gas and cooling water (inlet wet flue gas flowrate: 185.7 kg/h, inlet flue gas temperature: 149.5 °C, inlet moisture fraction: 14.4 vol%, cooling water flowrate: 542.9 kg/h and inlet cooling water temperature: 31.0 °C)](image)

Figure 4 shows axial variations of predicted and measured flue gas and cooling water temperatures. The x-axis represents the normalized cumulative surface area of the heat exchangers starting from the HX2 (shown in Fig. 3) in the flue gas flow direction. The operating conditions for the test are listed under the graph. For this particular test condition in Figure 4, the ratio of $m_{e,in}/m_{i,o}$ was 2.92. Given this high ratio, it was expected to have high condensation efficiency even though the inlet cooling water temperature was relatively high. The ratio of $m_{e,in}/A_s$ was 27.6 [kg/h·m²] for this test point.

Predicted and measured data are expressed with lines and symbols, respectively. It is shown with the measured data that the flue gas was cooled from 149.5 °C to 32 °C while the cooling water temperature was increased from 31 °C to 51.8 °C. As can be seen from this figure, there is good agreement between the measured and the predicted values of both flue gas and cooling water temperatures. These results show that the theoretical model used in this study can predict the heat transfer phenomenon in the condensing heat exchangers with good accuracy.
Figure 5. Individual and total condensation rates from each heat exchanger (inlet wet flue gas flowrate: 185.7 kg/h, inlet flue gas temperature: 149.5 °C, inlet moisture fraction: 14.4 vol%, cooling water flowrate: 542.9 kg/h and inlet cooling water temperature: 31.0 °C)

Figure 6. Effect of inlet cooling water temperature on condensation efficiency for coal flue gas condensation modeling (Inlet wet flue gas flowrate: 152 - 192 kg/h, inlet flue gas temperature: 140.9 – 150.9 °C, inlet moisture fraction: 11.9 - 14.5 vol%, cooling water flowrate: 281 - 404 kg/h and inlet cooling water temperature: 24.4 – 38.1 °C)

Figure 7. Effect of \( \frac{m_c}{m_{g,in}} \) on condensation efficiency for coal flue gas condensation modeling (Inlet wet flue gas flowrate: 177 - 190 kg/h, inlet flue gas temperature: 122.9 – 152.9 °C, inlet moisture fraction: 12.4 – 14.5 vol%, cooling water flowrate: 281 - 662 kg/h and inlet cooling water temperature: 30.9 – 32.9 °C)

Figure 8. Condensation efficiency as a function of the ratio of inlet wet flue gas flow rate to total heat transfer surface area, \( \frac{m_{g,in}}{A_s} \). It is seen that condensation efficiency

and the minimum occurring in HX2 (shown in Fig. 3). The condensation efficiency predicted by the analytical model was the same as what was measured in experiment, being 67.6 %. The results showed that the analytical model can accurately predict the mass transfer phenomenon for water condensation among the heat exchangers.

Figure 6 shows the effect of inlet cooling water temperature on the condensation efficiency. The results show strong dependence of condensation efficiency on inlet cooling water temperature since condensation efficiency linearly decreased from 74 to 46 wt% as inlet cooling water temperature increased from 24.4 °C to 38.1 °C. When inlet cooling water temperature was increased, both tube wall temperature and interfacial temperature increased. Then, interfacial water vapor mole fraction increased due to its dependence on interfacial temperature. So the smaller difference in moisture fraction (see Equation (10)) reduced the condensation rate and efficiency.

Figure 7 shows the effect of the ratio of the mass flow rate of cooling water to the mass flow rate of inlet flue gas (\( \frac{m_c}{m_{g,in}} \)) on condensation efficiency. It is also shown that predicted condensation efficiencies are in good agreement with the corresponding measured efficiencies. As shown with the measured data, condensation efficiencies improved from 56 to 67 wt% as \( \frac{m_c}{m_{g,in}} \) increased from 1.5 to 3.5. Condensation efficiency is strongly affected by changing \( \frac{m_c}{m_{g,in}} \). The higher values of \( \frac{m_c}{m_{g,in}} \) result in higher condensation efficiencies owing to higher thermal mass (\( m_c \cdot c_p \cdot c_m \)) of cooling water side.

Figure 8 shows the condensation efficiency as a function of the ratio of inlet wet flue gas flow rate to total heat transfer surface area, \( \frac{m_{g,in}}{A_s} \). It is seen that condensation efficiency

Figure 5 shows a comparison between calculated and measured condensation rates for the same test point as Fig. 4, in terms of individual and total condensation rates. Predicted individual condensation rates agree with measured rates to within 16% and total condensation rates agree with measured values to within 0.1% according to the errors obtained by the least squares method. Individual condensation rates have relatively greater differences than the total condensation rate does. The average uncertainties of the measured individual and total condensation rates for all tests were 3.4% and 1.7%, respectively. The analytical model correctly predicted the distribution of condensate among the five heat exchangers; the maximum condensation occurring in HX4 (shown in Fig. 3),
linearly decreased from 77 to 70 wt% as \( \dot{m}_{w,in} / A_0 \) decreased from 22.4 to 28.3 kg/h\( \cdot \)m\(^2\), which implies a weak dependence on \( \dot{m}_{w,in} / A_0 \).

Figure 8. Effect of \( \dot{m}_{w,in} / A_0 \) on condensation efficiency for coal flue gas condensation modeling (Inlet wet flue gas flowrate : 150 - 192 kg/h, inlet flue gas temperature : 145.9 - 150.9 ºC, inlet moisture fraction : 11.9 - 13.5 vol%, cooling water flowrate : 363 - 401 kg/h and inlet cooling water temperature : 23.9 - 25.9 ºC)

Figure 9 illustrates predicted condensation efficiency versus measured condensation efficiency for all the coal fired tests. The dotted line in Figure 9 shows the ± 10 percent standard deviation band. All predicted data are within ± 10 % with an average discrepancy of 2.5 % between experimental data and predicted results. Therefore, the analytical model was able to predict heat and mass transfer in the condensing heat exchangers with good accuracy.

Figure 10. Predicted and measured mole fractions of sulfuric acid in flue gas for oil combustion tests (Inlet wet flue gas flowrate : 217 - 242 kg/h, inlet flue gas temperature : 156 – 167 ºC, inlet moisture fraction : 8.6 – 9.3 vol%, cooling water flowrate : 309 - 327 kg/h and inlet cooling water temperature : 8.4 – 9.4 ºC)

Figure 10 illustrates average data from computed concentrations and experimental sulfuric acid concentrations in flue gas based on oil-firing which were obtained by the controlled condensation method (EPA Method 8B). Modeling results of sulfuric acid condensation for oil-firing showed two trends - steep reduction within the high temperature heat exchanger and smooth reduction within lower temperature heat exchanger, which is in agreement with experimental data. Discrepancy between predicted and measured data is expected to come either from the measurement uncertainty of 2.0 ppmv due to controlled condensation method or from assumptions due to non-application of chemical reaction of sulfuric acid generation.

SUMMARY AND CONCLUSIONS

Numerical simulations based on an analytical model of heat and mass transfer processes for water and sulfuric acid vapor condensation in a flue gas condensing heat exchanger system was conducted and compared with experimental data. The modeling approach is based on conservation of energy and heat/mass transfer for the flue gas and cooling water. All governing equations were solved using an iterative solution technique with appropriate assumptions and simplifications.

Experiments were carried out to validate the analytical model. Flue gas exhausted from a boiler was channeled to the pilot scale heat exchanger. Measurements were made and experimental results were compared to the results from the simulation.
The term ‘condensation efficiency’ was defined to make a quantitative performance evaluation of the condensing heat exchangers. The ratio of the mass flow rate of cooling water to the mass flow rate of inlet flue gas was defined as $\frac{\dot{m}_c}{\dot{m}_{g,in}}$, which was more critical operating parameter used to evaluate the condensation efficiency than assessing with respect to only coolant flow rate ($\dot{m}_c$).

The results from the simulation, using the same operational conditions as the experiments, agreed well with experimental results. The condensation rates of water and sulfuric acid vapor in each heat exchanger and temperature distributions for the flue gas and cooling water were compared with the tests. The average discrepancy between the results of the analytical model and experiments were within a few percent.

REFERENCES