## SAS/STAT 13.1 User's Guide The FREQ Procedure

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## Chapter 40 The FREQ Procedure

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## Overview: FREQ Procedure

The FREQ procedure produces one-way to $n$-way frequency and contingency (crosstabulation) tables. For two-way tables, PROC FREQ computes tests and measures of association. For $n$-way tables, PROC FREQ provides stratified analysis by computing statistics across, as well as within, strata.

For one-way frequency tables, PROC FREQ computes goodness-of-fit tests for equal proportions or specified null proportions. For one-way tables, PROC FREQ also provides confidence limits and tests for binomial proportions, including tests for noninferiority and equivalence.

For contingency tables, PROC FREQ can compute various statistics to examine the relationships between two classification variables. For some pairs of variables, you might want to examine the existence or strength of any association between the variables. To determine if an association exists, chi-square tests are computed. To estimate the strength of an association, PROC FREQ computes measures of association that tend to be close to zero when there is no association and close to the maximum (or minimum) value when there is perfect association. The statistics for contingency tables include the following:

- chi-square tests and measures
- measures of association
- risks (binomial proportions) and risk differences for $2 \times 2$ tables
- odds ratios and relative risks for $2 \times 2$ tables
- tests for trend
- tests and measures of agreement
- Cochran-Mantel-Haenszel statistics

PROC FREQ computes asymptotic standard errors, confidence intervals, and tests for measures of association and measures of agreement. Exact p-values and confidence intervals are available for many test statistics and measures. PROC FREQ also performs analyses that adjust for any stratification variables by computing statistics across, as well as within, strata for $n$-way tables. These statistics include Cochran-Mantel-Haenszel statistics and measures of agreement.

In choosing measures of association to use in analyzing a two-way table, you should consider the study design (which indicates whether the row and column variables are dependent or independent), the measurement scale of the variables (nominal, ordinal, or interval), the type of association that each measure is designed to detect, and any assumptions required for valid interpretation of a measure. You should exercise care in selecting measures that are appropriate for your data.

Similar comments apply to the choice and interpretation of test statistics. For example, the Mantel-Haenszel chi-square statistic requires an ordinal scale for both variables and is designed to detect a linear association. The Pearson chi-square, on the other hand, is appropriate for all variables and can detect any kind of association, but it is less powerful for detecting a linear association because its power is dispersed over a greater number of degrees of freedom (except for $2 \times 2$ tables).

For more information about selecting the appropriate statistical analyses, see Agresti (2007) and Stokes, Davis, and Koch (2012).

Several SAS procedures produce frequency counts; only PROC FREQ computes chi-square tests for one-way to $n$-way tables and measures of association and agreement for contingency tables. Other procedures to consider for counting include the TABULATE and UNIVARIATE procedures. When you want to produce contingency tables and tests of association for sample survey data, use PROC SURVEYFREQ. See Chapter 14, "Introduction to Survey Procedures," for more information. When you want to fit models to categorical data, use a procedure such as CATMOD, GENMOD, GLIMMIX, LOGISTIC, PROBIT, or SURVEYLOGISTIC. See Chapter 8, "Introduction to Categorical Data Analysis Procedures," for more information.

PROC FREQ uses the Output Delivery System (ODS), a SAS subsystem that provides capabilities for displaying and controlling the output from SAS procedures. ODS enables you to convert any of the output from PROC FREQ into a SAS data set. See the section "ODS Table Names" on page 2773 for more information.

PROC FREQ uses ODS Graphics to create graphs as part of its output. For general information about ODS Graphics, see Chapter 21, "Statistical Graphics Using ODS." For specific information about the statistical graphics available with the FREQ procedure, see the PLOTS = option in the TABLES statement and the section "ODS Graphics" on page 2777.

## Getting Started: FREQ Procedure

## Frequency Tables and Statistics

The FREQ procedure provides easy access to statistics for testing for association in a crosstabulation table.
In this example, high school students applied for courses in a summer enrichment program; these courses included journalism, art history, statistics, graphic arts, and computer programming. The students accepted were randomly assigned to classes with and without internships in local companies. Table 40.1 contains counts of the students who enrolled in the summer program by gender and whether they were assigned an internship slot.

Table 40.1 Summer Enrichment Data

| Gender | Internship | Enrollment |  |  |
| :--- | :--- | :---: | :---: | :---: |
|  | Yes | No | Total |  |
| boys | yes | 35 | 29 | 64 |
| boys | no | 14 | 27 | 41 |
| girls | yes | 32 | 10 | 42 |
| girls | no | 53 | 23 | 76 |

The SAS data set SummerSchool is created by inputting the summer enrichment data as cell count data, or providing the frequency count for each combination of variable values. The following DATA step statements create the SAS data set SummerSchool:

```
data SummerSchool;
    input Gender $ Internship $ Enrollment $ Count @@;
    datalines;
boys yes yes 35 boys yes no 29
boys no yes 14 boys no no 27
girls yes yes 32 girls yes no 10
girls no yes 53 girls no no 23
;
```

The variable Gender takes the values 'boys' or 'girls,' the variable Internship takes the values 'yes' and 'no,' and the variable Enrollment takes the values 'yes' and 'no.' The variable Count contains the number of students that correspond to each combination of data values. The double at sign (@ @) indicates that more than one observation is included on a single data line. In this DATA step, two observations are included on each line.

Researchers are interested in whether there is an association between internship status and summer program enrollment. The Pearson chi-square statistic is an appropriate statistic to assess the association in the corresponding $2 \times 2$ table. The following PROC FREQ statements specify this analysis.

You specify the table for which you want to compute statistics with the TABLES statement. You specify the statistics you want to compute with options after a slash (/) in the TABLES statement.

```
proc freq data=SummerSchool order=data;
    tables Internship*Enrollment / chisq;
    weight Count;
run;
```

The ORDER= option controls the order in which variable values are displayed in the rows and columns of the table. By default, the values are arranged according to the alphanumeric order of their unformatted values. If you specify ORDER=DATA, the data are displayed in the same order as they occur in the input data set. Here, because 'yes' appears before 'no' in the data, 'yes' appears first in any table. Other options for controlling order include ORDER=FORMATTED, which orders according to the formatted values, and ORDER=FREQUENCY, which orders by descending frequency count.

In the TABLES statement, Internship*Enrollment specifies a table where the rows are internship status and the columns are program enrollment. The CHISQ option requests chi-square statistics for assessing association between these two variables. Because the input data are in cell count form, the WEIGHT statement is required. The WEIGHT statement names the variable Count, which provides the frequency of each combination of data values.

Figure 40.1 presents the crosstabulation of Internship and Enrollment. In each cell, the values printed under the cell count are the table percentage, row percentage, and column percentage, respectively. For example, in the first cell, 63.21 percent of the students offered courses with internships accepted them and 36.79 percent did not.

Figure 40.1 Crosstabulation Table


Figure 40.2 displays the statistics produced by the CHISQ option. The Pearson chi-square statistic is labeled 'Chi-Square' and has a value of 0.8189 with 1 degree of freedom. The associated $p$-value is 0.3655 , which means that there is no significant evidence of an association between internship status and program enrollment. The other chi-square statistics have similar values and are asymptotically equivalent. The other statistics (phi coefficient, contingency coefficient, and Cramér's $V$ ) are measures of association derived from the Pearson chi-square. For Fisher's exact test, the two-sided $p$-value is 0.4122 , which also shows no association between internship status and program enrollment.

Figure 40.2 Statistics Produced with the CHISQ Option

| Statistic DF | Value | Prob |
| :---: | :---: | :---: |
| Chi-Square 1 | 0.8189 | 0.3655 |
| Likelihood Ratio Chi-Square 1 | 0.8202 | 0.3651 |
| Continuity Adj. Chi-Square 1 | 0.5899 | 0.4425 |
| Mantel-Haenszel Chi-Square 1 | 0.8153 | 0.3666 |
| Phi Coefficient | 0.0606 |  |
| Contingency Coefficient | 0.0605 |  |
| Cramer's V | 0.0606 |  |
| Fisher's Exact Test |  |  |
| Cell (1,1) Frequency ( $F$ ) | 67 |  |
| Left-sided Pr <= F | 0.8513 |  |
| Right-sided $\operatorname{Pr}>=\mathrm{F}$ | 0.2213 |  |
| Table Probability (P) | 0.0726 |  |
| Two-sided Pr <= P | 0.4122 |  |

The analysis, so far, has ignored gender. However, it might be of interest to ask whether program enrollment is associated with internship status after adjusting for gender. You can address this question by doing an analysis of a set of tables (in this case, by analyzing the set consisting of one for boys and one for girls). The Cochran-Mantel-Haenszel (CMH) statistic is appropriate for this situation: it addresses whether rows and columns are associated after controlling for the stratification variable. In this case, you would be stratifying by gender.

The PROC FREQ statements for this analysis are very similar to those for the first analysis, except that there is a third variable, Gender, in the TABLES statement. When you cross more than two variables, the two rightmost variables construct the rows and columns of the table, respectively, and the leftmost variables determine the stratification.

The following PROC FREQ statements also request frequency plots for the crosstabulation tables. PROC FREQ produces these plots by using ODS Graphics to create graphs as part of the procedure output. ODS Graphics must be enabled before producing plots. The PLOTS(ONLY)=FREQPLOT option requests frequency plots. The TWOWAY=CLUSTER plot-option specifies a cluster layout for the two-way frequency plots.

```
ods graphics on;
proc freq data=SummerSchool;
    tables Gender*Internship*Enrollment /
            chisq cmh plots(only)=freqplot(twoway=cluster);
    weight Count;
run;
ods graphics off;
```

This execution of PROC FREQ first produces two individual crosstabulation tables of Internship by Enrollment: one for boys and one for girls. Frequency plots and chi-square statistics are produced for each individual table. Figure 40.3, Figure 40.4, and Figure 40.5 show the results for boys. Note that the chi-square statistic for boys is significant at the $\alpha=0.05$ level of significance. Boys offered a course with an internship are more likely to enroll than boys who are not.

Figure 40.4 displays the frequency plot of Internship by Enrollment for boys. By default, frequency plots are displayed as bar charts. You can use PLOTS $=$ options to request dot plots instead of bar charts, to change the orientation of the bars from vertical to horizontal, and to change the scale from frequencies to percents. You can also use PLOTS= options to specify other two-way layouts (stacked, vertical groups, or horizontal groups) and to change the primary grouping from column levels to row levels.

Figure 40.6, Figure 40.7, and Figure 40.8 display the crosstabulation table, frequency plot, and chi-square statistics for girls. You can see that there is no evidence of association between internship offers and program enrollment for girls.

Figure 40.3 Crosstabulation Table for Boys


Figure 40.4 Frequency Plot for Boys


Figure 40.5 Chi-Square Statistics for Boys


Figure 40.6 Crosstabulation Table for Girls


Figure 40.7 Frequency Plot for Girls


Figure 40.8 Chi-Square Statistics for Girls


These individual table results demonstrate the occasional problems with combining information into one table and not accounting for information in other variables such as Gender. Figure 40.9 contains the CMH results. There are three summary (CMH) statistics; which one you use depends on whether your rows and/or columns have an order in $r \times c$ tables. However, in the case of $2 \times 2$ tables, ordering does not matter and all three statistics take the same value. The CMH statistic follows the chi-square distribution under the hypothesis of no association, and here, it takes the value 4.0186 with 1 degree of freedom. The associated $p$-value is 0.0450 , which indicates a significant association at the $\alpha=0.05$ level.

Thus, when you adjust for the effect of gender in these data, there is an association between internship and program enrollment. But, if you ignore gender, no association is found. Note that the CMH option also produces other statistics, including estimates and confidence limits for relative risk and odds ratios for $2 \times 2$ tables and the Breslow-Day Test. These results are not displayed here.

Figure 40.9 Test for the Hypothesis of No Association

| Cochran-Mantel-Haenszel Statistics (Based on Table Scores) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Statistic | Alternative Hypothesis | DF | Value | Prob |
| 1 | Nonzero Correlation | 1 | 4.0186 | 0.0450 |
| 2 | Row Mean Scores Differ | 1 | 4.0186 | 0.0450 |
| 3 | General Association | 1 | 4.0186 | 0.0450 |

## Agreement Study

Medical researchers are interested in evaluating the efficacy of a new treatment for a skin condition. Dermatologists from participating clinics were trained to conduct the study and to evaluate the condition. After the training, two dermatologists examined patients with the skin condition from a pilot study and rated the same patients. The possible evaluations are terrible, poor, marginal, and clear. Table 40.2 contains the data.

Table 40.2 Skin Condition Data

|  | Dermatologist 2 |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Dermatologist 1 | Terrible | Poor | Marginal | Clear |
| Terrible | 10 | 4 | 1 | 0 |
| Poor | 5 | 10 | 12 | 2 |
| Marginal | 2 | 4 | 12 | 5 |
| Clear | 0 | 2 | 6 | 13 |

The following DATA step statements create the SAS dataset SkinCondition. The dermatologists' evaluations of the patients are contained in the variables Derm1 and Derm2; the variable Count is the number of patients given a particular pair of ratings.

```
data SkinCondition;
    input Derm1 $ Derm2 $ Count;
    datalines;
terrible terrible 10
terrible poor 4
terrible marginal 1
terrible clear 0
poor terrible 5
poor poor }1
poor marginal }1
poor clear 2
marginal terrible 2
marginal poor 4
marginal marginal }1
marginal clear 5
clear terrible 0
clear poor 2
clear marginal 6
clear clear 13
;
```

The following PROC FREQ statements request an agreement analysis of the skin condition data. In order to evaluate the agreement of the diagnoses (a possible contribution to measurement error in the study), the kappa coefficient is computed.

The TABLES statement requests a crosstabulation of the variables Derm1 and Derm2. The AGREE option in the TABLES statement requests the kappa coefficient, together with its standard error and confidence limits. The KAPPA option in the TEST statement requests a test for the null hypothesis that kappa equals zero, or that the agreement is purely by chance. The NOPRINT option in the TABLES statement suppresses the display of the two-way table. The PLOTS= option requests an agreement plot for the two dermatologists. ODS Graphics must be enabled before producing plots.

```
ods graphics on;
proc freq data=SkinCondition order=data;
    tables Derm1*Derm2 /
            agree noprint plots=agreeplot;
    test kappa;
    weight Count;
run;
ods graphics off;
```

Figure 40.10 and Figure 40.11 show the results. The kappa coefficient has the value 0.3449 , which indicates some agreement between the dermatologists, and the hypothesis test confirms that you can reject the null hypothesis of no agreement. This conclusion is further supported by the confidence interval of ( 0.2030 , 0.4868 ), which suggests that the true kappa is greater than zero. The AGREE option also produces Bowker's test for symmetry and the weighted kappa coefficient, but that output is not shown here. Figure 40.11 displays the agreement plot for the ratings of the two dermatologists.

Figure 40.10 Agreement Study


Figure 40.11 Agreement Plot


## Syntax: FREQ Procedure

The following statements are available in the FREQ procedure:

```
PROC FREQ < options > ;
    BY variables;
    EXACT statistic-options < / computation-options > ;
    OUTPUT < OUT=SAS-data-set > output-options;
    TABLES requests < / options > ;
    TEST options ;
    WEIGHT variable < / option > ;
```

The PROC FREQ statement is the only required statement for the FREQ procedure. If you specify the following statements, PROC FREQ produces a one-way frequency table for each variable in the most recently created data set.

```
proc freq;
```

run;

Table 40.3 summarizes the basic functions of the procedure statements. The following sections provide detailed syntax information for the BY, EXACT, OUTPUT, TABLES, TEST, and WEIGHT statements in alphabetical order after the description of the PROC FREQ statement.

Table 40.3 Summary of PROC FREQ Statements

| Statement | Description |
| :--- | :--- |
| BY | Provides separate analyses for each BY group |
| EXACT | Requests exact tests |
| OUTPUT | Requests an output data set |
| TABLES | Specifies tables and requests analyses |
| TEST | Requests tests for measures of association and agreement |
| WEIGHT | Identifies a weight variable |

## PROC FREQ Statement

PROC FREQ < options > ;
The PROC FREQ statement invokes the FREQ procedure. Optionally, it also identifies the input data set. By default, the procedure uses the most recently created SAS data set.

Table 40.4 lists the options available in the PROC FREQ statement. Descriptions of the options follow in alphabetical order.

Table 40.4 PROC FREQ Statement Options

| Option | Description |
| :--- | :--- |
| COMPRESS | Begins the next one-way table on the current page |
| DATA $=$ | Names the input data set |
| FORMCHAR $=$ | Specifies the outline and cell divider characters for crosstabulation tables |
| NLEVELS | Displays the number of levels for all TABLES variables |
| NOPRINT | Suppresses all displayed output |
| ORDER $=$ | Specifies the order for reporting variable values |
| PAGE | Displays one table per page |

You can specify the following options in the PROC FREQ statement.

## COMPRESS

begins display of the next one-way frequency table on the same page as the preceding one-way table if there is enough space to begin the table. By default, the next one-way table begins on the current page only if the entire table fits on that page. The COMPRESS option is not valid with the PAGE option.

## DATA=SAS-data-set

names the $S A S$-data-set to be analyzed by PROC FREQ. If you omit the DATA= option, the procedure uses the most recently created SAS data set.

## FORMCHAR(1,2,7)='formchar-string'

defines the characters to be used for constructing the outlines and dividers for the cells of crosstabulation table displays. The formchar-string should be three characters long. The characters are used to draw the vertical separators (1), the horizontal separators (2), and the vertical-horizontal intersections (7). If you do not specify the FORMCHAR = option, PROC FREQ uses FORMCHAR( $1,2,7$ )=‘--+’ by default. Table 40.5 summarizes the formatting characters used by PROC FREQ.

Table 40.5 Formatting Characters Used by PROC FREQ

| Position | Default | Used to Draw |
| :--- | :--- | :--- |
| 1 | I | Vertical separators |
| 2 | - | Horizontal separators |
| 7 | + | Intersections of vertical and horizontal separators |

The FORMCHAR = option can specify 20 different SAS formatting characters used to display output; however, PROC FREQ uses only the first, second, and seventh formatting characters. Therefore, the proper specification for PROC FREQ is $\operatorname{FORMCHAR}(1,2,7)=$ 'formchar-string'.

Specifying all blanks for formchar-string produces crosstabulation tables with no outlines or dividersfor example, FORMCHAR $(1,2,7)=$ ' '. You can use any character in formchar-string, including hexadecimal characters. If you use hexadecimal characters, you must put an $x$ after the closing quote. For information about which hexadecimal codes to use for which characters, see the documentation for your hardware.

See the CALENDAR, PLOT, and TABULATE procedures in the Base SAS Procedures Guide for more information about form characters.

## NLEVELS

displays the "Number of Variable Levels" table, which provides the number of levels for each variable named in the TABLES statements. See the section "Number of Variable Levels Table" on page 2765 for details. PROC FREQ determines the variable levels from the formatted variable values, as described in the section "Grouping with Formats" on page 2698.

## NOPRINT

suppresses the display of all output. You can use the NOPRINT option when you only want to create an output data set. See the section "Output Data Sets" on page 2762 for information about the output data sets produced by PROC FREQ. Note that the NOPRINT option temporarily disables the Output Delivery System (ODS). For more information, see Chapter 20, "Using the Output Delivery System."

NOTE: A NOPRINT option is also available in the TABLES statement. It suppresses display of the crosstabulation tables but allows display of the requested statistics.

## ORDER=DATA | FORMATTED | FREQ | INTERNAL

specifies the order of the variable levels in the frequency and crosstabulation tables, which you request in the TABLES statement.

The ORDER= option can take the following values:

| Value of ORDER $=$ | Levels Ordered By |
| :--- | :--- |
| DATA | Order of appearance in the input data set <br> FORMATTED <br> External formatted value, except for numeric variables with <br> no explicit format, which are sorted by their unformatted <br> (internal) value |
| INTER | Descending frequency count; levels with the most observa- <br> tions come first in the order |
| Unformatted value |  |

By default, ORDER=INTERNAL. The FORMATTED and INTERNAL orders are machine-dependent. The ORDER= option does not apply to missing values, which are always ordered first.

For more information about sort order, see the chapter on the SORT procedure in the Base SAS Procedures Guide and the discussion of BY-group processing in SAS Language Reference: Concepts.

## PAGE

displays only one table per page. Otherwise, PROC FREQ displays multiple tables per page as space permits. The PAGE option is not valid with the COMPRESS option.

## BY Statement

```
BY variables;
```

You can specify a BY statement with PROC FREQ to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the FREQ procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in SAS Language Reference: Concepts. For more information about the DATASETS procedure, see the discussion in the Base SAS Procedures Guide.

## EXACT Statement

EXACT statistic-options < / computation-options > ;
The EXACT statement requests exact tests and confidence limits for selected statistics. The statistic-options identify which statistics to compute, and the computation-options specify options for computing exact statistics. See the section "Exact Statistics" on page 2757 for details.

NOTE: PROC FREQ computes exact tests by using fast and efficient algorithms that are superior to direct enumeration. Exact tests are appropriate when a data set is small, sparse, skewed, or heavily tied. For some large problems, computation of exact tests might require a considerable amount of time and memory. Consider using asymptotic tests for such problems. Alternatively, when asymptotic methods might not be sufficient for such large problems, consider using Monte Carlo estimation of exact $p$-values. You can request Monte Carlo estimation by specifying the MC computation-option in the EXACT statement. See the section "Computational Resources" on page 2759 for more information.

## Statistic Options

The statistic-options specify which exact tests and confidence limits to compute. Table 40.6 lists the available statistic-options and the exact statistics that are computed. Descriptions of the statistic-options follow the table in alphabetical order.

For one-way tables, exact $p$-values are available for binomial proportion tests, the chi-square goodness-of-fit test, and the likelihood ratio chi-square test. Exact (Clopper-Pearson) confidence limits are available for the binomial proportion.

For two-way tables, exact $p$-values are available for the following tests: Pearson chi-square test, likelihood ratio chi-square test, Mantel-Haenszel chi-square test, Fisher's exact test, Jonckheere-Terpstra test, and Cochran-Armitage test for trend. Exact $p$-values are also available for tests of the following statistics: Pearson correlation coefficient, Spearman correlation coefficient, Kendall's tau- $b$, Stuart's tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, simple kappa coefficient, and weighted kappa coefficient.

For $2 \times 2$ tables, PROC FREQ provides the exact McNemar's test, exact confidence limits for the odds ratio, and Barnard's unconditional exact test for the risk (proportion) difference. PROC FREQ also provides exact unconditional confidence limits for the risk difference and for the relative risk (ratio of proportions). For stratified $2 \times 2$ tables, PROC FREQ provides Zelen's exact test for equal odds ratios, exact confidence limits for the common odds ratio, and an exact test for the common odds ratio.

Most of the statistic-option names listed in Table 40.6 are identical to the corresponding option names in the TABLES and OUTPUT statements. You can request exact computations for groups of statistics by using statistic-options that are identical to the TABLES statement options CHISQ, MEASURES, and AGREE. For example, when you specify the CHISQ statistic-option in the EXACT statement, PROC FREQ computes exact $p$-values for the Pearson chi-square, likelihood ratio chi-square, and Mantel-Haenszel chi-square tests for two-way tables. You can request an exact test for an individual statistic by specifying the corresponding statistic-option from the list in Table 40.6.

## Using the EXACT Statement with the TABLES Statement

You must use a TABLES statement with the EXACT statement. If you use only one TABLES statement, you do not need to specify the same options in both the TABLES and EXACT statements; when you specify a statistic-option in the EXACT statement, PROC FREQ automatically invokes the corresponding TABLES statement option. However, when you use an EXACT statement with multiple TABLES statements, you must specify options in the TABLES statements to request statistics. PROC FREQ then provides exact tests or confidence limits for those statistics that you also specify in the EXACT statement.

Table 40.6 EXACT Statement Statistic Options

| Statistic Option | Exact Statistics |
| :---: | :---: |
| AGREE | McNemar's test (for $2 \times 2$ tables), simple kappa test, weighted kappa test |
| BARNARD | Barnard's test (for $2 \times 2$ tables) |
| BINOMIAL \\| BIN | Binomial proportion tests for one-way tables |
| CHISQ | Chi-square goodness-of-fit test for one-way tables; Pearson chi-square, likelihood ratio chi-square, and Mantel-Haenszel chi-square tests for two-way tables |
| COMOR | Confidence limits for the common odds ratio, common odds ratio test (for $h \times 2 \times 2$ tables) |
| EQOR I ZELEN | Zelen's test for equal odds ratios (for $h \times 2 \times 2$ tables) |
| FISHER | Fisher's exact test |
| JT | Jonckheere-Terpstra test |
| KAPPA | Test for the simple kappa coefficient |
| KENTB I TAUB | Test for Kendall's tau-b |
| LRCHI | Likelihood ratio chi-square test (one-way and two-way tables) |
| MCNEM | McNemar's test (for $2 \times 2$ tables) |
| MEASURES | Tests for the Pearson correlation and Spearman correlation, confidence limits for the odds ratio (for $2 \times 2$ tables) |
| MHCHI | Mantel-Haenszel chi-square test |
| OR I ODDSRATIO | Confidence limits for the odds ratio (for $2 \times 2$ tables) |
| PCHI | Pearson chi-square test (one-way and two-way tables) |
| PCORR | Test for the Pearson correlation coefficient |
| RELRISK | Confidence limits for the relative risk (for $2 \times 2$ tables) |
| RISKDIFF | Confidence limits for the proportion difference (for $2 \times 2$ tables) |
| SCORR | Test for the Spearman correlation coefficient |
| SMDCR | Test for Somers' $D(C \mid R)$ |
| SMDRC | Test for Somers' $D(R \mid C)$ |
| STUTC I TAUC | Test for Stuart's tau-c |
| TREND | Cochran-Armitage test for trend |
| WTKAP \\| WTKAPPA | Test for the weighted kappa coefficient |

You can specify the following statistic-options in the EXACT statement.

## AGREE

requests exact tests for the simple and weighted kappa coefficients and McNemar's exact test. "Tests and Measures of Agreement" on page 2743 and "Exact Statistics" on page 2757 for details.

The AGREE option in the TABLES statement provides the simple and weighted kappa coefficients (with their standard errors and confidence limits) and the asymptotic McNemar's test. The AGREE option in the TEST statement provides asymptotic tests for the kappa coefficients.

Kappa coefficients can be computed for square two-way tables, where the number of rows equals the number of columns. For $2 \times 2$ tables, the weighted kappa coefficient equals the simple kappa coefficient, and PROC FREQ displays only the simple kappa coefficient. McNemar's test is available for $2 \times 2$ tables.

## BARNARD

requests Barnard's exact unconditional test for the risk (proportion) difference for $2 \times 2$ tables. See the section "Barnard's Unconditional Exact Test" on page 2735 for details. The RISKDIFF option in the TABLES statement provides risk difference estimates, confidence limits, and asymptotic tests. See the section "Risks and Risk Differences" on page 2725 for more information.

## BINOMIAL

## BIN

requests exact tests for the binomial proportion (for one-way tables). See the section "Binomial Tests" on page 2721 for details. The BINOMIAL option in the TABLES statement provides confidence limits and asymptotic tests for the binomial proportion. See the section "Binomial Proportion" on page 2719 for more information.

## CHISQ

requests exact tests for the Pearson chi-square, likelihood ratio chi-square, and Mantel-Haenszel chi-square for two-way tables. See the section "Chi-Square Tests and Statistics" on page 2704 for details. The CHISQ option in the TABLES statement provides asymptotic tests for these statistics.

For one-way tables, the CHISQ option requests the exact chi-square goodness-of-fit test. See the section "Chi-Square Test for One-Way Tables" on page 2704 for details.

## COMOR

requests an exact test and exact confidence limits for the common odds ratio for multiway $2 \times 2$ tables. See the section "Exact Confidence Limits for the Common Odds Ratio" on page 2754 for details. The CMH option in the TABLES statement provides Mantel-Haenszel and logit estimates and asymptotic confidence limits for the common odds ratio.

## EQOR

## ZELEN

requests Zelen's exact test for equal odds ratios, which is available for multiway $2 \times 2$ tables. See the section "Zelen's Exact Test for Equal Odds Ratios" on page 2754 for details. The CMH option in the TABLES statement provides the asymptotic Breslow-Day test for homogeneity of odds ratios.

## FISHER

request Fisher's exact test. See the sections "Fisher's Exact Test" on page 2708 and "Exact Statistics" on page 2757 for details. For $2 \times 2$ tables, the CHISQ option in the TABLES statement provides Fisher's exact test. For general $R \times C$ tables, Fisher's exact test is also known as the Freeman-Halton test.
requests the exact Jonckheere-Terpstra test. See the sections "Jonckheere-Terpstra Test" on page 2741 and "Exact Statistics" on page 2757 for details. The JT option in the TABLES statement provides the asymptotic Jonckheere-Terpstra test.

## KAPPA

requests an exact test for the simple kappa coefficient. See the sections "Simple Kappa Coefficient" on page 2744 and "Exact Statistics" on page 2757 for details. The AGREE option in the TABLES statement provides the simple kappa estimate, standard error, and confidence limits. The KAPPA option in the TEST statement provides the asymptotic test for the simple kappa coefficient.
Kappa coefficients are defined only for square tables, where the number of rows equals the number of columns. PROC FREQ does not compute kappa coefficients for tables that are not square.

## KENTB

## TAUB

requests an exact test for Kendall's tau- $b$. See the sections "Kendall's Tau-b" on page 2711 and "Exact Statistics" on page 2757 for details. The MEASURES option in the TABLES statement provides the Kendall's tau- $b$ estimate and standard error. The KENTB option in the TEST statement provides an asymptotic test for Kendall's tau-b.

## LRCHI

requests an exact test for the likelihood ratio chi-square for two-way tables. See the sections "Likelihood Ratio Chi-Square Test" on page 2707 and "Exact Statistics" on page 2757 for details. For one-way tables, the LRCHI option requests an exact likelihood ratio goodness-of-fit test. See the section "Likelihood Ratio Chi-Square Test for One-Way Tables" on page 2706 for details.

The CHISQ option in the TABLES statement provides asymptotic likelihood ratio chi-square tests.

## MCNEM

requests the exact McNemar's test. See the sections "McNemar's Test" on page 2743 and "Exact Statistics" on page 2757 for details. The AGREE option in the TABLES statement provides the asymptotic McNemar's test.

## MEASURES

requests exact tests for the Pearson and Spearman correlations. See the sections "Pearson Correlation Coefficient" on page 2713, "Spearman Rank Correlation Coefficient" on page 2714, and "Exact Statistics" on page 2757 for details. The PCORR and SCORR options in the TEST statement provide asymptotic tests for the Pearson and Spearman correlations, respectively.

This option also requests exact confidence limits for the odds ratio for $2 \times 2$ tables. For more information, see the sections "Odds Ratio" on page 2737 and "Exact Confidence Limits for the Odds Ratio" on page 2738. The MEASURES, OR(CL=), and RELRISK options in the TABLES statement provide asymptotic confidence limits for the odds ratio.

## MHCHI

requests an exact test for the Mantel-Haenszel chi-square. See the sections "Mantel-Haenszel ChiSquare Test" on page 2707 and "Exact Statistics" on page 2757 for details. The CHISQ option in the TABLES statement provides the asymptotic Mantel-Haenszel chi-square test.

## OR

## ODDSRATIO

requests exact confidence limits for the odds ratio for $2 \times 2$ tables. For more information, see the sections "Odds Ratio" on page 2737 and "Exact Confidence Limits for the Odds Ratio" on page 2738. The MEASURES, OR(CL=), and RELRISK options in the TABLES statement provide asymptotic confidence limits for the odds ratio.

You can set the confidence level by specifying the ALPHA= option in the TABLES statement. The default of ALPHA=0.05 produces $95 \%$ confidence limits.

## PCHI

requests an exact test for the Pearson chi-square for two-way tables. See the sections "Pearson ChiSquare Test for Two-Way Tables" on page 2705 and "Exact Statistics" on page 2757 for details. For one-way tables, the PCHI option requests an exact chi-square goodness-of-fit test. See the section "Chi-Square Test for One-Way Tables" on page 2704 for details. The CHISQ option in the TABLES statement provides asymptotic chi-square tests.

## PCORR

requests an exact test for the Pearson correlation coefficient. See the sections "Pearson Correlation Coefficient" on page 2713 and "Exact Statistics" on page 2757 for details. The MEASURES option in the TABLES statement provides the Pearson correlation estimate and standard error. The PCORR option in the TEST statement provides an asymptotic test for the Pearson correlation.

## RELRISK < (options) >

requests exact unconditional confidence limits for the relative risk for $2 \times 2$ tables. PROC FREQ computes the confidence limits by inverting two separate one-sided exact tests (Santner and Snell 1980). By default, this computation uses the unstandardized relative risk as the test statistic. If you specify the RELRISK(METHOD=SCORE) option, the computation uses the score statistic (Chan and Zhang 1999). For more information, see the section "Exact Unconditional Confidence Limits for the Relative Risk" on page 2739.

You can set the confidence level by specifying the ALPHA= option in the TABLES statement. The default of ALPHA=0.05 produces $95 \%$ confidence limits.

The RELRISK and MEASURES options in the TABLES statement provide asymptotic confidence limits for the relative risk. For more information, see the section "Relative Risks" on page 2739.

You can specify the following options:

## COLUMN=1 | 2 | BOTH

specifies the table column for which to compute the relative risk. The default is COLUMN=1, which provides exact confidence limits for the column 1 relative risk. If you specify COLUMN=BOTH, PROC FREQ provides exact confidence limits for both column 1 and column 2 relative risks.

## METHOD=SCORE

requests exact unconditional confidence limits that are based on the score statistic (Chan and Zhang 1999). For more information, see the section "Exact Unconditional Confidence Limits for the Relative Risk" on page 2739. If you do not specify METHOD=SCORE, by default the exact confidence limit computations are based on the unstandardized relative risk.

## RISKDIFF < (options) >

requests exact unconditional confidence limits for the risk difference for $2 \times 2$ tables. PROC FREQ computes the confidence limits by inverting two separate one-sided exact tests (Santner and Snell 1980). By default, this computation uses the unstandardized risk difference as the test statistic. If you specify the RISKDIFF(METHOD=SCORE) option, the computation uses the score statistic (Chan and Zhang 1999). See the section "Exact Unconditional Confidence Limits for the Risk Difference" on page 2734 for more information.

You can set the confidence level by specifying the ALPHA= option in the TABLES statement. The default of ALPHA $=0.05$ produces $95 \%$ confidence limits.

The RISKDIFF option in the TABLES statement provides asymptotic confidence limits for the risk difference, including Wald, Newcombe, and Miettinen-Nurminen (score) confidence limits. See the section "Risk Difference Confidence Limits" on page 2727 for more information.

You can specify the following options:

## COLUMN=1 | 2 | BOTH

specifies the table column for which to compute the risk difference. The default is COLUMN=BOTH, which provides exact confidence limits for both column 1 and column 2 risk differences.

## METHOD=SCORE

requests exact unconditional confidence limits that are based on the score statistic (Chan and Zhang 1999). See the section "Exact Unconditional Confidence Limits for the Risk Difference" on page 2734 for more information. If you do not specify METHOD=SCORE, by default the exact confidence limit computations are based on the unstandardized risk difference.

## SCORR

requests an exact test for the Spearman correlation coefficient. See the sections "Spearman Rank Correlation Coefficient" on page 2714 and "Exact Statistics" on page 2757 for details. The MEASURES option in the TABLES statement provides the Spearman correlation estimate and standard error. The SCORR option in the TEST statement provides an asymptotic test for the Spearman correlation.

## SMDCR

requests an exact test for Somers' $D(C \mid R)$. See the sections "Somers' $D$ " on page 2712 and "Exact Statistics" on page 2757 for details. The MEASURES option in the TABLES statement provides Somers' $D(C \mid R)$ estimate and the standard error. The SMDCR option in the TEST statement provides an asymptotic test for Somers' $D(C \mid R)$.

## SMDRC

requests an exact test for Somers' $D(R \mid C)$. See the sections "Somers' $D$ " on page 2712 and "Exact Statistics" on page 2757 for details. The MEASURES option in the TABLES statement provides Somers' $D(R \mid C)$ estimate and the standard error. The SMDRC option in the TEST statement requests an asymptotic test for Somers' $D(C \mid R)$.

## STUTC

TAUC
requests an exact test for Stuart's tau-c. See the sections "Stuart's Tau-c" on page 2712 and "Exact Statistics" on page 2757 for details. The MEASURES option in the TABLES statement provides Stuart's tau-c estimate and the standard error. The STUTC option in the TEST statement provides an asymptotic test for Stuart's tau-c.

## TREND

requests the exact Cochran-Armitage test for trend. See the sections "Cochran-Armitage Test for Trend" on page 2740 and "Exact Statistics" on page 2757 for details. The TREND option in the TABLES statement provides the asymptotic Cochran-Armitage trend test. The trend test is available for tables of dimensions $2 \times C$ or $R \times 2$.

## WTKAP

## WTKAPPA

requests an exact test for the weighted kappa coefficient. See the sections "Weighted Kappa Coefficient" on page 2745 and "Exact Statistics" on page 2757 for details. The AGREE option in the TABLES statement provides the weighted kappa estimate, standard error, and confidence limits. The WTKAP option in the TEST statement provides the asymptotic test for the weighted kappa coefficient.

Kappa coefficients are defined only for square tables, where the number of rows equals the number of columns. PROC FREQ does not compute kappa coefficients for tables that are not square. For $2 \times 2$ tables, the weighted kappa coefficient equals the simple kappa coefficient, and PROC FREQ does not present separate results for the weighted kappa coefficient.

## Computation Options

The computation-options specify options for computing exact statistics. You can specify the following computation-options in the EXACT statement after a slash (/).

## ALPHA $=\alpha$

specifies the level of the confidence limits for Monte Carlo $p$-value estimates. The value of $\alpha$ must be between 0 and 1, and the default is 0.01 . A confidence level of $\alpha$ produces $100(1-\alpha) \%$ confidence limits. The default of ALPHA=. 01 produces $99 \%$ confidence limits for the Monte Carlo estimates.

The ALPHA= option invokes the MC option.

## MAXTIME=value

specifies the maximum clock time (in seconds) that PROC FREQ can use to compute an exact $p$-value. If the procedure does not complete the computation within the specified time, the computation terminates. The MAXTIME= value must be a positive number. This option is available for Monte Carlo estimation of exact $p$-values as well as for direct exact $p$-value computation. For more information, see the section "Computational Resources" on page 2759.

## MC

requests Monte Carlo estimation of exact $p$-values instead of direct exact $p$-value computation. Monte Carlo estimation can be useful for large problems that require a considerable amount of time and memory for exact computations but for which asymptotic approximations might not be sufficient. For more information, see the section "Monte Carlo Estimation" on page 2760.

This option is available for all EXACT statistic-options except the BINOMIAL option and the following options that apply only to $2 \times 2$ or $h \times 2 \times 2$ tables: BARNARD, COMOR, EQOR, MCNEM, OR, RELRISK, and RISKDIFF. PROC FREQ always computes exact tests or confidence limits (not Monte Carlo estimates) for these statistics.

The ALPHA=, $\mathrm{N}=$, and $\mathrm{SEED}=$ options invoke the MC option.
MIDP
requests exact mid $p$-values for the exact tests. The exact mid $p$-value is defined as the exact $p$-value minus half the exact point probability. For more information, see the section "Definition of $p$-Values" on page 2758.

The MIDP option is available for all EXACT statement statistic-options except the following: BARNARD, EQOR, OR, RELRISK, and RISKDIFF. You cannot specify both the MIDP option and the MC option.

## $\mathrm{N}=n$

specifies the number of samples for Monte Carlo estimation. The value of $n$ must be a positive integer, and the default is 10,000 . Larger values of $n$ produce more precise estimates of exact $p$-values. Because larger values of $n$ generate more samples, the computation time increases.

The $\mathrm{N}=$ option invokes the MC option.

## PFORMAT=format-name | EXACT

specifies the display format for exact $p$-values. PROC FREQ applies this format to one- and two-sided exact $p$-values, exact point probabilities, and exact mid $p$-values. By default, PROC FREQ displays exact $p$-values in the PVALUE6.4 format.

You can provide a format-name or you can specify PFORMAT=EXACT to control the format of exact $p$-values. The value of format-name can be any standard SAS numeric format or a user-defined format. The format length must not exceed 24. For information about formats, see the FORMAT procedure in the Base SAS Procedures Guide and the FORMAT statement and SAS format in SAS Formats and Informats: Reference.

If you specify PFORMAT=EXACT, PROC FREQ uses the 6.4 format to display exact $p$-values that are greater than or equal to 0.001 ; the procedure uses the E10.3 format to display values that are between 0.000 and 0.001 . This is the format that PROC FREQ uses to display exact $p$-values in releases before SAS/STAT 12.3. Beginning in SAS/STAT 12.3, by default PROC FREQ uses the PVALUE6.4 format to display exact $p$-values.

## POINT

requests exact point probabilities for the exact tests. The exact point probability is the exact probability that the test statistic equals the observed value. For more information, see the section "Definition of $p$-Values" on page 2758.

The POINT option is available for all EXACT statement statistic-options except the following: BARNARD, EQOR, OR, RELRISK, and RISKDIFF. You cannot specify both the POINT option and the MC option.

## SEED=number

specifies the initial seed for random number generation for Monte Carlo estimation. The value of the SEED= option must be an integer. If you do not specify the SEED= option or if the SEED= value is negative or zero, PROC FREQ uses the time of day from the computer's clock to obtain the initial seed.

The SEED= option invokes the MC option.

## OUTPUT Statement

## OUTPUT < OUT=SAS-data-set > output-options;

The OUTPUT statement creates a SAS data set that contains statistics that are computed by PROC FREQ. Table 40.7 lists the statistics that can be stored in the output data set. You identify which statistics to include by specifying output-options.

You must use a TABLES statement with the OUTPUT statement. The OUTPUT statement stores statistics for only one table request. If you use multiple TABLES statements, the contents of the output data set correspond to the last TABLES statement. If you use multiple table requests in a single TABLES statement, the contents of the output data set correspond to the last table request. Only one OUTPUT statement is allowed in a single invocation of the procedure.

For a one-way or two-way table, the output data set contains one observation that stores the requested statistics for the table. For a multiway table, the output data set contains an observation for each two-way table (stratum) of the multiway crosstabulation. If you request summary statistics for the multiway table, the output data set also contains an observation that stores the across-strata summary statistics. If you use a BY statement, the output data set contains an observation or set of observations for each BY group. For more information about the contents of the output data set, see the section "Contents of the OUTPUT Statement Output Data Set" on page 2763.

The output data set that is created by the OUTPUT statement is not the same as the output data set that is created by the OUT= option in the TABLES statement. The OUTPUT statement creates a data set that contains statistics (such as the Pearson chi-square and its $p$-value), and the OUT= option in the TABLES statement creates a data set that contains frequency table counts and percentages. See the section "Output Data Sets" on page 2762 for more information.

As an alternative to the OUTPUT statement, you can use the Output Delivery System (ODS) to store statistics that PROC FREQ computes. ODS can create a SAS data set from any table that PROC FREQ produces. See the section "ODS Table Names" on page 2773 for more information.

You can specify the following options in the OUTPUT statement:

## OUT=SAS-data-set

specifies the name of the output data set. When you use an OUTPUT statement but do not use the OUT= option, PROC FREQ creates a data set and names it by using the DATA $n$ convention.

## output-options

specify the statistics to include in the output data set. Table 40.7 lists the output-options that are available in the OUTPUT statement, together with the TABLES statement options that are required to produce the statistics. Descriptions of the output-options follow the table in alphabetical order.

You can specify output-options to request individual statistics, or you can request groups of statistics by using output-options that are identical to the group options in the TABLES statement (for example, the CHISQ, MEASURES, CMH, AGREE, and ALL options).

When you specify an output-option, the output data set includes statistics from the corresponding analysis. In addition to the estimate or test statistic, the output data set includes associated values such as standard errors, confidence limits, $p$-values, and degrees of freedom. See the section "Contents of the OUTPUT Statement Output Data Set" on page 2763 for details.

To store a statistic in the output data set, you must also request computation of that statistic with the appropriate TABLES, EXACT, or TEST statement option. For example, the PCHI output-option includes the Pearson chi-square in the output data set. You must also request computation of the Pearson chi-square by specifying the CHISQ option in the TABLES statement. Or, if you use only one TABLES statement, you can request computation of the Pearson chi-square by specifying the PCHI or CHISQ option in the EXACT statement. Table 40.7 lists the TABLES statement options that are required to produce the OUTPUT data set statistics.

Table 40.7 OUTPUT Statement Output Options

| Output Option | Output Data Set Statistics | Required TABLES Statement Option |
| :---: | :---: | :---: |
| AGREE | McNemar's test ( $2 \times 2$ tables), Bowker's test, simple and weighted kappas; for multiple strata, overall simple and weighted kappas, tests for equal kappas, and Cochran's $Q(h \times 2 \times 2$ tables $)$ | AGREE |
| AJCHI | Continuity-adjusted chi-square ( $2 \times 2$ tables) | CHISQ |
| ALL | CHISQ, MEASURES, and CMH statistics; <br> N (number of nonmissing observations) | ALL |
| BDCHI | Breslow-Day test ( $h \times 2 \times 2$ tables) | CMH, CMH1, or CMH2 |
| BINOMIAL I BIN | Binomial statistics (one-way tables) | BINOMIAL |
| CHISQ | For one-way tables, goodness-of-fit test; for two-way tables, Pearson, likelihood ratio, continuity-adjusted, and Mantel-Haenszel chi-squares, Fisher's exact test ( $2 \times 2$ tables), phi and contingency coefficients, Cramér's $V$ | CHISQ |
| CMH | Cochran-Mantel-Haenszel (CMH) correlation, row mean scores (ANOVA), and general association statistics; for $2 \times 2$ tables, logit and Mantel-Haenszel common odds ratios and relative risks, Breslow-Day test | CMH |
| CMH1 | CMH statistics, except row mean scores (ANOVA) and general association statistics | CMH or CMH1 |
| CMH2 | CMH statistics, except general association statistic | CMH or CMH2 |
| CMHCOR | CMH correlation statistic | CMH, CMH1, or CMH2 |
| CMHGA | CMH general association statistic | CMH |
| CMHRMS | CMH row mean scores (ANOVA) statistic | CMH or CMH2 |
| COCHQ | Cochran's $Q(h \times 2 \times 2$ tables) | AGREE |
| CONTGY | Contingency coefficient | CHISQ |
| CRAMV | Cramér's V | CHISQ |
| EQKAP | Test for equal simple kappas | AGREE |
| EQOR I ZELEN | Zelen's test for equal odds ratios ( $h \times 2 \times 2$ tables) | CMH and EXACT EQOR |
| EQWKP | Test for equal weighted kappas | AGREE |
| FISHER | Fisher's exact test | CHISQ or FISHER ${ }^{1}$ |
| GAMMA | Gamma | MEASURES |
| GS I GAILSIMON | Gail-Simon test | CMH(GAILSIMON) |

[^0]Table 40.7 continued

| Output Option | Output Data Set Statistics | Required TABLES <br> Statement Option |
| :---: | :---: | :---: |
| JT | Jonckheere-Terpstra test | JT |
| KAPPA | Simple kappa coefficient | AGREE |
| KENTB I TAUB | Kendall's tau-b | MEASURES |
| LAMCR | Lambda asymmetric ( $C \mid R$ ) | MEASURES |
| LAMDAS | Lambda symmetric | MEASURES |
| LAMRC | Lambda asymmetric ( $R \mid C$ ) | MEASURES |
| LGOR | Logit common odds ratio | CMH, CMH1, or CMH2 |
| LGRRC1 | Logit common relative risk, column 1 | CMH, CMH1, or CMH2 |
| LGRRC2 | Logit common relative risk, column 2 | CMH, CMH1, or CMH2 |
| LRCHI | Likelihood ratio chi-square | CHISQ |
| MCNEM | McNemar's test ( $2 \times 2$ tables) | AGREE |
| MEASURES | Gamma, Kendall's tau-b, Stuart's tau- $c$, Somers' $D(C \mid R)$ and $D(R \mid C)$, Pearson and Spearman correlations, lambda asymmetric $(C \mid R)$ and $(R \mid C)$, lambda symmetric, uncertainty coefficients $(C \mid R)$ and $(R \mid C)$, symmetric uncertainty coefficient; odds ratio and relative risks ( $2 \times 2$ tables) | MEASURES |
| MHCHI | Mantel-Haenszel chi-square | CHISQ |
| MHOR I COMOR | Mantel-Haenszel common odds ratio | CMH, CMH1, or CMH2 |
| MHRRC1 | Mantel-Haenszel common relative risk, column 1 | CMH, CMH1, or CMH2 |
| MHRRC2 | Mantel-Haenszel common relative risk, column 2 | CMH, CMH1, or CMH2 |
| N | Number of nonmissing observations |  |
| NMISS | Number of missing observations |  |
| OR I ODDSRATIO | Odds ratio ( $2 \times 2$ tables) | MEASURES or RELRISK |
| PCHI | Chi-square goodness-of-fit test (one-way tables), Pearson chi-square (two-way tables) | CHISQ |
| PCORR | Pearson correlation coefficient | MEASURES |
| PHI | Phi coefficient | CHISQ |
| PLCORR | Polychoric correlation coefficient | PLCORR |
| RDIF1 | Column 1 risk difference (row 1 - row 2) | RISKDIFF |
| RDIF2 | Column 2 risk difference (row 1 - row 2 ) | RISKDIFF |
| RELRISK | Odds ratio and relative risks ( $2 \times 2$ tables) | MEASURES or RELRISK |
| RISKDIFF | Risks and risk differences ( $2 \times 2$ tables) | RISKDIFF |
| RISKDIFF1 | Risks and risk difference, column 1 | RISKDIFF |
| RISKDIFF2 | Risks and risk difference, column 2 | RISKDIFF |
| RRC1 \\| RELRISK1 | Relative risk, column 1 | MEASURES or RELRISK |
| RRC2 \| RELRISK2 | Relative risk, column 2 | MEASURES or RELRISK |
| RSK1 I RISK1 | Column 1 overall risk | RISKDIFF |
| RSK11 I RISK11 | Column 1 risk for row 1 | RISKDIFF |
| RSK12 \| RISK12 | Column 2 risk for row 1 | RISKDIFF |
| RSK2 I RISK2 | Column 2 overall risk | RISKDIFF |
| RSK21 I RISK21 | Column 1 risk for row 2 | RISKDIFF |
| RSK22 \| RISK22 | Column 2 risk for row 2 | RISKDIFF |

Table 40.7 continued

| Output Option | Output Data Set Statistics | Required TABLES <br> Statement Option |
| :--- | :--- | :--- |
| SCORR | Spearman correlation coefficient | MEASURES |
| SMDCR | Somers' $D(C \mid R)$ | MEASURES |
| SMDRC | Somers' $D(R \mid C)$ | MEASURES |
| STUTC I TAUC | Stuart's tau-c | MEASURES |
| TREND | Cochran-Armitage test for trend | TREND |
| TSYMM \| BOWKER | Bowker's test of symmetry | AGREE |
| U | Symmetric uncertainty coefficient | MEASURES |
| UCR | Uncertainty coefficient $(C \mid R)$ | MEASURES |
| URC | Uncertainty coefficient $(R \mid C)$ | MEASURES |
| WTKAP \| WTKAPPA | Weighted kappa coefficient | AGREE |

You can specify the following output-options in the OUTPUT statement.

## AGREE

includes the following tests and measures of agreement in the output data set: McNemar's test (for $2 \times 2$ tables), Bowker's test of symmetry, the simple kappa coefficient, and the weighted kappa coefficient. For multiway tables, the AGREE option also includes the following statistics in the output data set: overall simple and weighted kappa coefficients, tests for equal simple and weighted kappa coefficients, and Cochran's $Q$ test.

The AGREE option in the TABLES statement requests computation of tests and measures of agreement. See the section "Tests and Measures of Agreement" on page 2743 for details about these statistics.

AGREE statistics are computed only for square tables, where the number of rows equals the number of columns. PROC FREQ provides Bowker's test of symmetry and weighted kappa coefficients only for tables larger than $2 \times 2$. (For $2 \times 2$ tables, Bowker's test is identical to McNemar's test, and the weighted kappa coefficient equals the simple kappa coefficient.) Cochran's $Q$ is available for multiway $2 \times 2$ tables.

## AJCHI

includes the continuity-adjusted chi-square in the output data set. The continuity-adjusted chi-square is available for $2 \times 2$ tables and is provided by the CHISQ option in the TABLES statement. See the section "Continuity-Adjusted Chi-Square Test" on page 2707 for details.

ALL
includes all statistics that are requested by the CHISQ, MEASURES, and CMH output-options in the output data set. ALL also includes the number of nonmissing observations, which you can request individually by specifying the N output-option.

## BDCHI

includes the Breslow-Day test in the output data set. The Breslow-Day test for homogeneity of odds ratios is computed for multiway $2 \times 2$ tables and is provided by the CMH, CMH1, and CMH2 options in the TABLES statement. See the section "Breslow-Day Test for Homogeneity of the Odds Ratios" on page 2753 for details.

## BINOMIAL

## BIN

includes the binomial proportion estimate, confidence limits, and tests in the output data set. The BINOMIAL option in the TABLES statement requests computation of binomial statistics, which are available for one-way tables. See the section "Binomial Proportion" on page 2719 for details.

## CHISQ

includes the following chi-square tests and measures in the output data set for two-way tables: Pearson chi-square, likelihood ratio chi-square, Mantel-Haenszel chi-square, phi coefficient, contingency coefficient, and Cramér's $V$. For $2 \times 2$ tables, CHISQ also includes Fisher's exact test and the continuityadjusted chi-square in the output data set. See the section "Chi-Square Tests and Statistics" on page 2704 for details. For one-way tables, CHISQ includes the chi-square goodness-of-fit test in the output data set. See the section "Chi-Square Test for One-Way Tables" on page 2704 for details. The CHISQ option in the TABLES statement requests computation of these statistics.

If you specify the CHISQ(WARN=OUTPUT) option in the TABLES statement, the CHISQ option also includes the variable WARN_PCHI in the output data set. This variable indicates the validity warning for the asymptotic Pearson chi-square test.

## CMH

includes the following Cochran-Mantel-Haenszel statistics in the output data set: correlation, row mean scores (ANOVA), and general association. For $2 \times 2$ tables, the CMH option also includes the Mantel-Haenszel and logit estimates of the common odds ratio and relative risks. For multiway (stratified) $2 \times 2$ tables, the CMH option includes the Breslow-Day test for homogeneity of odds ratios. The CMH option in the TABLES statement requests computation of these statistics. See the section "Cochran-Mantel-Haenszel Statistics" on page 2748 for details.

If you specify the $\mathrm{CMH}(\mathrm{MF})$ option in the TABLES statement, the CMH option includes the MantelFleiss analysis in the output data set. The variables MF_CMH and WARN_CMH contain the MantelFleiss criterion and the warning indicator, respectively.

## CMH1

includes the CMH statistics in the output data set, with the exception of the row mean scores (ANOVA) statistic and the general association statistic. The CMH1 option in the TABLES statement requests computation of these statistics. See the section "Cochran-Mantel-Haenszel Statistics" on page 2748 for details.

## CMH2

includes the CMH statistics in the output data set, with the exception of the general association statistic. The CMH2 option in the TABLES statement requests computation of these statistics. See the section "Cochran-Mantel-Haenszel Statistics" on page 2748 for details.

## CMHCOR

includes the Cochran-Mantel-Haenszel correlation statistic in the output data set. The CMH option in the TABLES statement requests computation of this statistic. See the section "Correlation Statistic" on page 2749 for details.

## CMHGA

includes the Cochran-Mantel-Haenszel general association statistic in the output data set. The CMH option in the TABLES statement requests computation of this statistic. See the section "General Association Statistic" on page 2750 for details.

## CMHRMS

includes the Cochran-Mantel-Haenszel row mean scores (ANOVA) statistic in the output data set. The CMH option in the TABLES statement requests computation of this statistic. See the section "ANOVA (Row Mean Scores) Statistic" on page 2750 for details.

## COCHQ

includes Cochran's $Q$ test in the output data set. The AGREE option in the TABLES statement requests computation of this test, which is available for multiway $2 \times 2$ tables. See the section "Cochran's $Q$ Test" on page 2747 for details.

## CONTGY

includes the contingency coefficient in the output data set. The CHISQ option in the TABLES statement requests computation of the contingency coefficient. See the section "Contingency Coefficient" on page 2709 for details.

## CRAMV

includes Cramér's $V$ in the output data set. The CHISQ option in the TABLES statement requests computation of Cramér's $V$. See the section "Cramér's $V$ " on page 2709 for details.

## EQKAP

includes the test for equal simple kappa coefficients in the output data set. The AGREE option in the TABLES statement requests computation of this test, which is available for multiway, square ( $h \times r \times r$ ) tables. See the section "Tests for Equal Kappa Coefficients" on page 2747 for details.

## EQOR

## ZELEN

includes Zelen's exact test for equal odds ratios in the output data set. The EQOR option in the EXACT statement requests computation of this test, which is available for multiway $2 \times 2$ tables. See the section "Zelen's Exact Test for Equal Odds Ratios" on page 2754 for details.

## EQWKP

includes the test for equal weighted kappa coefficients in the output data set. The AGREE option in the TABLES statement requests computation of this test. The test for equal weighted kappas is available for multiway, square ( $h \times r \times r$ ) tables where $r>2$. See the section "Tests for Equal Kappa Coefficients" on page 2747 for details.

## FISHER

includes Fisher's exact test in the output data set. For $2 \times 2$ tables, the CHISQ option in the TABLES statement provides Fisher's exact test. For tables larger than $2 \times 2$, the FISHER option in the EXACT statement provides Fisher's exact test. See the section "Fisher's Exact Test" on page 2708 for details.

GAMMA
includes the gamma statistic in the output data set. The MEASURES option in the TABLES statement requests computation of the gamma statistic. See the section "Gamma" on page 2711 for details.

## GS

## GAILSIMON

includes the Gail-Simon test for qualitative interaction in the output data set. The CMH(GAILSIMON) option in the TABLES statement requests computation of this test. See the section "Gail-Simon Test for Qualitative Interactions" on page 2756 for details.
includes the Jonckheere-Terpstra test in the output data set. The JT option in the TABLES statement requests the Jonckheere-Terpstra test. See the section "Jonckheere-Terpstra Test" on page 2741 for details.

## KAPPA

includes the simple kappa coefficient in the output data set. The AGREE option in the TABLES statement requests computation of kappa, which is available for square tables (where the number of rows equals the number of columns). For multiway square tables, the KAPPA option also includes the overall kappa coefficient in the output data set. See the sections "Simple Kappa Coefficient" on page 2744 and "Overall Kappa Coefficient" on page 2747 for details.

## KENTB

## TAUB

includes Kendall's tau- $b$ in the output data set. The MEASURES option in the TABLES statement requests computation of Kendall's tau- $b$. See the section "Kendall's Tau-b" on page 2711 for details.

## LAMCR

includes the asymmetric lambda $\lambda(C \mid R)$ in the output data set. The MEASURES option in the TABLES statement requests computation of lambda. See the section "Lambda (Asymmetric)" on page 2716 for details.

## LAMDAS

includes the symmetric lambda in the output data set. The MEASURES option in the TABLES statement requests computation of lambda. See the section "Lambda (Symmetric)" on page 2717 for details.

## LAMRC

includes the asymmetric lambda $\lambda(R \mid C)$ in the output data set. The MEASURES option in the TABLES statement requests computation of lambda. See the section "Lambda (Asymmetric)" on page 2716 for details.

## LGOR

includes the logit estimate of the common odds ratio in the output data set. The CMH option in the TABLES statement requests computation of this statistic, which is available for $2 \times 2$ tables. See the section "Adjusted Odds Ratio and Relative Risk Estimates" on page 2751 for details.

## LGRRC1

includes the logit estimate of the common relative risk (column 1) in the output data set. The CMH option in the TABLES statement requests computation of this statistic, which is available for $2 \times 2$ tables. See the section "Adjusted Odds Ratio and Relative Risk Estimates" on page 2751 for details.

## LGRRC2

includes the logit estimate of the common relative risk (column 2) in the output data set. The CMH option in the TABLES statement requests computation of this statistic, which is available for $2 \times 2$ tables. See the section "Adjusted Odds Ratio and Relative Risk Estimates" on page 2751 for details.

## LRCHI

includes the likelihood ratio chi-square in the output data set. The CHISQ option in the TABLES statement requests computation of the likelihood ratio chi-square. See the section "Likelihood Ratio Chi-Square Test" on page 2707 for details.

## MCNEM

includes McNemar's test (for $2 \times 2$ tables) in the output data set. The AGREE option in the TABLES statement requests computation of McNemar's test. See the section "McNemar's Test" on page 2743 for details.

## MEASURES

includes the following measures of association in the output data set: gamma, Kendall's tau- $b$, Stuart's tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, Pearson and Spearman correlation coefficients, lambda (symmetric and asymmetric), and uncertainty coefficients (symmetric and asymmetric). For $2 \times 2$ tables, the MEASURES option also includes the odds ratio, column 1 relative risk, and column 2 relative risk. The MEASURES option in the TABLES statement requests computation of these statistics. See the section "Measures of Association" on page 2709 for details.

## MHCHI

includes the Mantel-Haenszel chi-square in the output data set. The CHISQ option in the TABLES statement requests computation of the Mantel-Haenszel chi-square. See the section "Mantel-Haenszel Chi-Square Test" on page 2707 for details.

## MHOR

## COMOR

includes the Mantel-Haenszel estimate of the common odds ratio in the output data set. The CMH option in the TABLES statement requests computation of this statistic, which is available for $2 \times 2$ tables. See the section "Adjusted Odds Ratio and Relative Risk Estimates" on page 2751 for details.

## MHRRC1

includes the Mantel-Haenszel estimate of the common relative risk (column 1) in the output data set. The CMH option in the TABLES statement requests computation of this statistic, which is available for $2 \times 2$ tables. See the section "Adjusted Odds Ratio and Relative Risk Estimates" on page 2751 for details.

## MHRRC2

includes the Mantel-Haenszel estimate of the common relative risk (column 2) in the output data set. The CMH option in the TABLES statement requests computation of this statistic, which is available for $2 \times 2$ tables. See the section "Adjusted Odds Ratio and Relative Risk Estimates" on page 2751 for details.

## N

includes the number of nonmissing observations in the output data set.

## NMISS

includes the number of missing observations in the output data set. See the section "Missing Values" on page 2699 for details about how PROC FREQ handles missing values.

OR
ODDSRATIO

## RROR

includes the odds ratio (for $2 \times 2$ tables) in the output data set. The MEASURES, OR(CL=), and RELRISK options in the TABLES statement request this statistic. For more information, see the section "Odds Ratio" on page 2737.

## PCHI

includes the Pearson chi-square in the output data set for two-way tables. See the section "Pearson Chi-Square Test for Two-Way Tables" on page 2705 for details. For one-way tables, the PCHI option includes the chi-square goodness-of-fit test in the output data set. See the section "Chi-Square Test for One-Way Tables" on page 2704 for details. The CHISQ option in the TABLES statement requests computation of these statistics.

If you specify the CHISQ(WARN=OUTPUT) option in the TABLES statement, the PCHI option also includes the variable WARN_PCHI in the output data set. This variable indicates the validity warning for the asymptotic Pearson chi-square test.

## PCORR

includes the Pearson correlation coefficient in the output data set. The MEASURES option in the TABLES statement requests computation of the Pearson correlation. See the section "Pearson Correlation Coefficient" on page 2713 for details.

## PHI

includes the phi coefficient in the output data set. The CHISQ option in the TABLES statement requests computation of the phi coefficient. See the section "Phi Coefficient" on page 2709 for details.

## PLCORR

includes the polychoric correlation coefficient in the output data set. For $2 \times 2$ tables, this statistic is known as the tetrachoric correlation coefficient. The PLCORR option in the TABLES statement requests computation of the polychoric correlation. See the section "Polychoric Correlation" on page 2715 for details.

## RDIF1

includes the column 1 risk difference (row 1 - row 2) in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences, which are available for $2 \times 2$ tables. See the section "Risks and Risk Differences" on page 2725 for details.

## RDIF2

includes the column 2 risk difference (row 1 - row 2) in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences, which are available for $2 \times 2$ tables. See the section "Risks and Risk Differences" on page 2725 for details.

## RELRISK

includes the column 1 and column 2 relative risks (for $2 \times 2$ tables) in the output data set. The MEASURES and RELRISK options in the TABLES statement request these statistics. For more information, see the section "Relative Risks" on page 2739.

## RISKDIFF

includes risks (binomial proportions) and risk differences for $2 \times 2$ tables in the output data set. These statistics include the row 1 risk, row 2 risk, total (overall) risk, and risk difference (row 1 - row 2 ) for column 1 and column 2. The RISKDIFF option in the TABLES statement requests computation of these statistics. See the section "Risks and Risk Differences" on page 2725 for details.

## RISKDIFF1

includes column 1 risks (binomial proportions) and risk differences for $2 \times 2$ tables in the output data set. These statistics include the row 1 risk, row 2 risk, total (overall) risk, and risk difference (row 1 row 2). The RISKDIFF option in the TABLES statement requests computation of these statistics. See the section "Risks and Risk Differences" on page 2725 for details.

## RISKDIFF2

includes column 2 risks (binomial proportions) and risk differences for $2 \times 2$ tables in the output data set. These statistics include the row 1 risk, row 2 risk, total (overall) risk, and risk difference (row $1-$ row 2). The RISKDIFF option in the TABLES statement requests computation of these statistics. See the section "Risks and Risk Differences" on page 2725 for details.

## RRC1

## RELRISK1

includes the column 1 relative risk in the output data set. The MEASURES and RELRISK options in the TABLES statement request relative risks, which are available for $2 \times 2$ tables. See the section "Odds Ratio and Relative Risks for $2 \times 2$ Tables" on page 2737 for details.

RRC2

## RELRISK2

includes the column 2 relative risk in the output data set. The MEASURES and RELRISK options in the TABLES statement request relative risks, which are available for $2 \times 2$ tables. See the section "Odds Ratio and Relative Risks for $2 \times 2$ Tables" on page 2737 for details.

## RSK1

## RISK1

includes the overall column 1 risk in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences, which are available for $2 \times 2$ tables. See the section "Risks and Risk Differences" on page 2725 for details.

## RSK11

## RISK11

includes the column 1 risk for row 1 in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences, which are available for $2 \times 2$ tables. See the section "Risks and Risk Differences" on page 2725 for details.

## RSK12

## RISK12

includes the column 2 risk for row 1 in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences, which are available for $2 \times 2$ tables. See the section "Risks and Risk Differences" on page 2725 for details.

## RSK2

RISK2
includes the overall column 2 risk in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences. See the section "Risks and Risk Differences" on page 2725 for details.

## RSK21

RISK21
includes the column 1 risk for row 2 in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences, which are available for $2 \times 2$ tables. See the section "Risks and Risk Differences" on page 2725 for details.

## RSK22

RISK22
includes the column 2 risk for row 2 in the output data set. The RISKDIFF option in the TABLES statement requests computation of risks and risk differences, which are available for $2 \times 2$ tables. See the section "Risks and Risk Differences" on page 2725 for details.

## SCORR

includes the Spearman correlation coefficient in the output data set. The MEASURES option in the TABLES statement requests computation of the Spearman correlation. See the section "Spearman Rank Correlation Coefficient" on page 2714 for details.

## SMDCR

includes Somers' $D(C \mid R)$ in the output data set. The MEASURES option in the TABLES statement requests computation of Somers' $D$. See the section "Somers' $D$ " on page 2712 for details.

## SMDRC

includes Somers' $D(R \mid C)$ in the output data set. The MEASURES option in the TABLES statement requests computation of Somers' $D$. See the section "Somers' $D$ " on page 2712 for details.

## STUTC

TAUC
includes Stuart's tau- $c$ in the output data set. The MEASURES option in the TABLES statement requests computation of tau-c. See the section "Stuart's Tau-c" on page 2712 for details.

## TREND

includes the Cochran-Armitage test for trend in the output data set. The TREND option in the TABLES statement requests computation of the trend test. This test is available for tables of dimension $2 \times C$ or $R \times 2$. See the section "Cochran-Armitage Test for Trend" on page 2740 for details.

## TSYMM

## BOWKER

includes Bowker's test of symmetry in the output data set. The AGREE option in the TABLES statement requests computation of Bowker's test. See the section "Bowker's Test of Symmetry" on page 2744 for details.

U
includes the uncertainty coefficient (symmetric) in the output data set. The MEASURES option in the TABLES statement requests computation of the uncertainty coefficient. See the section "Uncertainty Coefficient (Symmetric)" on page 2718 for details.

## UCR

includes the asymmetric uncertainty coefficient $U(C \mid R)$ in the output data set. The MEASURES option in the TABLES statement requests computation of the uncertainty coefficient. See the section "Uncertainty Coefficients (Asymmetric)" on page 2718 for details.

## URC

includes the asymmetric uncertainty coefficient $U(R \mid C)$ in the output data set. The MEASURES option in the TABLES statement requests computation of the uncertainty coefficient. See the section "Uncertainty Coefficients (Asymmetric)" on page 2718 for details.

## WTKAP <br> WTKAPPA

includes the weighted kappa coefficient in the output data set. The AGREE option in the TABLES statement requests computation of weighted kappa, which is available for square tables larger than $2 \times 2$. For multiway tables, the WTKAP option also includes the overall weighted kappa coefficient in the output data set. See the sections "Weighted Kappa Coefficient" on page 2745 and "Overall Kappa Coefficient" on page 2747 for details.

## TABLES Statement

TABLES requests < / options > ;
The TABLES statement requests one-way to $n$-way frequency and crosstabulation tables and statistics for those tables.

If you omit the TABLES statement, PROC FREQ generates one-way frequency tables for all data set variables that are not listed in the other statements.

The following argument is required in the TABLES statement.

## requests

specify the frequency and crosstabulation tables to produce. A request is composed of one variable name or several variable names separated by asterisks. To request a one-way frequency table, use a single variable. To request a two-way crosstabulation table, use an asterisk between two variables. To request a multiway table (an $n$-way table, where $n>2$ ), separate the desired variables with asterisks. The unique values of these variables form the rows, columns, and strata of the table. You can include up to 50 variables in a single multiway table request.

For two-way to multiway tables, the values of the last variable form the crosstabulation table columns, while the values of the next-to-last variable form the rows. Each level (or combination of levels) of the other variables forms one stratum. PROC FREQ produces a separate crosstabulation table for each stratum. For example, a specification of $\mathrm{A} * \mathrm{~B} * \mathrm{C} * \mathrm{D}$ in a TABLES statement produces $k$ tables, where $k$ is the number of different combinations of values for $A$ and $B$. Each table lists the values for $C$ down the side and the values for D across the top.

You can use multiple TABLES statements in the PROC FREQ step. PROC FREQ builds all the table requests in one pass of the data, so that there is essentially no loss of efficiency. You can also specify any number of table requests in a single TABLES statement. To specify multiple table requests quickly, use a grouping syntax by placing parentheses around several variables and joining other variables or variable combinations. For example, the statements shown in Table 40.8 illustrate grouping syntax.

Table 40.8 Grouping Syntax

| TABLES Request | Equivalent to |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $A *(B C)$ | $A * B$ | $A * C$ |  |  |
| $(A B) *(C D)$ | $A * C$ | $B * C$ | $A * D$ | $B * D$ |
| $(A B C) * D$ | $A * D$ | $B * D$ | $C * D$ |  |
| $A--C$ | $A$ | $B$ | $C$ |  |
| $(A--C) * D$ | $A * D$ | $B * D$ | $C * D$ |  |

The TABLES statement variables are one or more variables from the DATA= input data set. These variables can be either character or numeric, but the procedure treats them as categorical variables. PROC FREQ uses the formatted values of the TABLES variable to determine the categorical variable levels. So if you assign a format to a variable with a FORMAT statement, PROC FREQ formats the values before dividing observations into the levels of a frequency or crosstabulation table. See the FORMAT procedure in the Base SAS Procedures Guide and the FORMAT statement and SAS formats in SAS Formats and Informats: Reference.

If you use PROC FORMAT to create a user-written format that combines missing and nonmissing values into one category, PROC FREQ treats the entire category of formatted values as missing. See the discussion in the section "Grouping with Formats" on page 2698 for more information.

By default, the frequency or crosstabulation table lists the values of both character and numeric variables in ascending order based on internal (unformatted) variable values. You can change the order of the values in the table by specifying the ORDER= option in the PROC FREQ statement. To list the values in ascending order by formatted value, use ORDER=FORMATTED.

## Without Options

If you request a one-way frequency table for a variable without specifying options, PROC FREQ produces frequencies, cumulative frequencies, percentages of the total frequency, and cumulative percentages for each value of the variable. If you request a two-way or an $n$-way crosstabulation table without specifying options, PROC FREQ produces crosstabulation tables that include cell frequencies, cell percentages of the total frequency, cell percentages of row frequencies, and cell percentages of column frequencies. The procedure excludes observations with missing values from the table but displays the total frequency of missing observations below each table.

## Options

Table 40.9 lists the options available in the TABLES statement. Descriptions of the options follow in alphabetical order.

Table 40.9 TABLES Statement Options

| Option | Description |
| :--- | :--- |
| Control Statistical | Analysis <br> AGREE |
| Requests tests and measures of classification agreement |  |
| ALL | Requests tests and measures of association produced by the <br> CHISQ, MEASURES, and CMH options |
| ALPHA $=$ | Sets confidence level for confidence limits <br> BINOMIAL I BIN <br> Requests binomial proportions, confidence limits, and tests <br> for one-way tables |
| CHISQ | Requests chi-square tests and measures based on chi-square <br> Requests confidence limits for MEASURES statistics |
| CL | Requests all Cochran-Mantel-Haenszel statistics <br> CMH |
| CMH1 | Requests CMH correlation statistic, adjusted odds ratios, <br> and adjusted relative risks |
| CMH2 | Requests CMH correlation and row mean scores (ANOVA) <br> statistics, adjusted odds ratios, and adjusted relative risks |
| FISHER | Requests Fisher's exact test for tables larger than $2 \times 2$ <br> Requests Gail-Simon test for qualitative interactions |
| GAILSIMON |  |

Table 40.9 continued

| Option | Description |
| :---: | :---: |
| JT | Requests Jonckheere-Terpstra test |
| MEASURES | Requests measures of association |
| MISSING | Treats missing values as nonmissing |
| PLCORR | Requests polychoric correlation |
| RELRISK | Requests odds ratio and relative risks for $2 \times 2$ tables |
| RISKDIFF | Requests risks and risk differences for $2 \times 2$ tables |
| SCORES $=$ | Specifies type of row and column scores |
| TREND | Requests Cochran-Armitage test for trend |
| Control Additional Table Information |  |
| CELLCHI2 | Displays cell contributions to the Pearson chi-square statistic |
| CUMCOL | Displays cumulative column percentages |
| DEVIATION | Displays deviations of cell frequencies from expected values |
| EXPECTED | Displays expected cell frequencies |
| MISSPRINT | Displays missing value frequencies |
| PEARSONRES | Displays Pearson residuals in the CROSSLIST table |
| PRINTKWTS | Displays kappa coefficient weights |
| SCOROUT | Displays row and column scores |
| SPARSE | Includes all possible combinations of variable levels in the LIST table and OUT= data set |
| STDRES | Displays standardized residuals in the CROSSLIST table |
| TOTPCT | Displays percentages of total frequency for $n$-way tables ( $n>2$ ) |
| Control Displayed Output |  |
| CONTENTS= | Specifies contents label for crosstabulation tables |
| CROSSLIST | Displays crosstabulation tables in ODS column format |
| FORMAT= | Formats frequencies in crosstabulation tables |
| LIST | Displays two-way to $n$-way tables in list format |
| MAXLEVELS $=$ | Specifies maximum number of levels to display in one-way tables |
| NOCOL | Suppresses display of column percentages |
| NOCUM | Suppresses display of cumulative frequencies and percentages |
| NOFREQ | Suppresses display of frequencies |
| NOPERCENT | Suppresses display of percentages |
| NOPRINT | Suppresses display of crosstabulation tables but displays statistics |
| NOROW | Suppresses display of row percentages |
| NOSPARSE | Suppresses zero frequency levels in the CROSSLIST table, LIST table, and OUT = data set |
| NOWARN | Suppresses log warning message for the chi-square test |
| Produce Statistical Graphics |  |
| PLOTS= | Requests plots from ODS Graphics |
| Create an Output Data Set |  |
| OUT= | Names an output data set to contain frequency counts |
| OUTCUM | Includes cumulative frequencies and percentages in the output data set for one-way tables |

Table 40.9 continued

| Option | Description |
| :--- | :--- |
| OUTEXPECT | Includes expected frequencies in the output data set |
| OUTPCT | Includes row, column, and two-way table percentages in the |
|  | output data set |

You can specify the following options in a TABLES statement.

## AGREE < (options) >

requests tests and measures of classification agreement for square tables. This option provides the simple and weighted kappa coefficients along with their standard errors and confidence limits. For multiway tables, the AGREE option also produces the overall simple and weighted kappa coefficients (along with their standard errors and confidence limits) and tests for equal kappas among strata. For $2 \times 2$ tables, this option provides McNemar's test; for square tables that have more than two response categories (levels), this option provides Bowker's test of symmetry. For multiway tables that have two response categories, the AGREE option also produces Cochran's $Q$ test. For more information, see the section "Tests and Measures of Agreement" on page 2743.

Measures of agreement can be computed only for square tables, where the number of rows equals the number of columns. If your table is not square because of observations that have zero weights, you can specify the ZEROS option in the WEIGHT statement to include these observations in the analysis. For more information, see the section "Tables with Zero Rows and Columns" on page 2747.

You can set the level for agreement confidence limits by specifying the ALPHA= option in the TABLES statement. The default of ALPHA=0.05 produces $95 \%$ confidence limits.

You can specify the TEST statement to request asymptotic tests for the simple and weighted kappa coefficients. You can specify the EXACT statement to request McNemar's exact test (for $2 \times 2$ tables) and exact tests for the simple and weighted kappa coefficients. For more information, see the section "Exact Statistics" on page 2757.

The weighted kappa coefficient is computed by using agreement weights that reflect the relative agreement between pairs of variable levels. To specify the type of agreement weights and to display the agreement weights, you can specify the following options:

## PRINTKWTS

displays the agreement weights that PROC FREQ uses to compute the weighted kappa coefficient. Agreement weights reflect the relative agreement between pairs of variable levels. By default, PROC FREQ uses the Cicchetti-Allison form of agreement weights. If you specify the WT=FC option, the procedure uses the Fleiss-Cohen form of agreement weights. For more information, see the section "Weighted Kappa Coefficient" on page 2745.

## WT=FC

requests Fleiss-Cohen agreement weights in the weighted kappa computation. By default, PROC FREQ uses Cicchetti-Allison agreement weights to compute the weighted kappa coefficient. Agreement weights reflect the relative agreement between pairs of variable levels. For more information, see the section "Weighted Kappa Coefficient" on page 2745.

## ALL

requests all tests and measures that are produced by the CHISQ, MEASURES, and CMH options. You can control the number of CMH statistics to compute by specifying the CMH1 or CMH2 option.

## ALPHA $=\alpha$

specifies the level of confidence limits. The value of $\alpha$ must be between 0 and 1 , and the default is 0.05 . A confidence level of $\alpha$ produces $100(1-\alpha) \%$ confidence limits. The default of ALPHA=0.05 produces $95 \%$ confidence limits.

ALPHA= applies to confidence limits that are requested by TABLES statement options. There is a separate ALPHA= option in the EXACT statement that sets the level of confidence limits for Monte Carlo estimates of exact $p$-values, which you request by specifying the MC option in the EXACT statement.

## BINOMIAL < (binomial-options) >

BIN < (binomial-options) >
requests the binomial proportion for one-way tables. When you specify the BINOMIAL option, by default PROC FREQ also provides the asymptotic standard error, asymptotic Wald and exact (Clopper-Pearson) confidence limits, and the asymptotic equality test for the binomial proportion.

You can specify binomial-options in parentheses after the BINOMIAL option. The LEVEL= binomialoption identifies the variable level for which to compute the proportion. If you do not specify LEVEL=, PROC FREQ computes the proportion for the first level that appears in the output. The $\mathrm{P}=$ binomialoption specifies the null proportion for the binomial tests. If you do not specify $\mathrm{P}=, \mathrm{PROC}$ FREQ uses $\mathrm{P}=0.5$ by default.

You can also specify binomial-options to request additional tests and confidence limits for the binomial proportion. The EQUIV, NONINF, and SUP binomial-options request tests of equivalence, noninferiority, and superiority, respectively. The $\mathrm{CL}=$ binomial-option requests confidence limits for the binomial proportion. Table 40.10 summarizes the binomial-options.

Available confidence limits for the binomial proportion include Agresti-Coull, exact (Clopper-Pearson), Jeffreys, Wald, and Wilson (score) confidence limits. You can specify more than one type of binomial confidence limits in the same analysis. If you do not specify any confidence limit requests, PROC FREQ computes Wald asymptotic confidence limits and exact (Clopper-Pearson) confidence limits by default. The ALPHA= option determines the confidence level, and the default of ALPHA=0.05 produces $95 \%$ confidence limits for the binomial proportion.

As part of the noninferiority, superiority, and equivalence analyses, PROC FREQ provides test-based confidence limits that have a confidence coefficient of $100(1-2 \alpha) \%$ (Schuirmann 1999). The ALPHA= option determines the confidence level, and the default of ALPHA=0.05 produces $90 \%$ confidence limits. See the sections "Noninferiority Test" on page 2722 and "Equivalence Test" on page 2724 for details.

To request exact tests for the binomial proportion, specify the BINOMIAL option in the EXACT statement. PROC FREQ then computes exact $p$-values for all binomial tests that you request with binomial-options, which can include tests of noninferiority, superiority, and equivalence, in addition to the test of equality.

See the section "Binomial Proportion" on page 2719 for details.

Table 40.10 BINOMIAL Options

| Option | Description |
| :--- | :--- |
| LEVEL= | Specifies the variable level |
| P= | Specifies the null proportion |
| CORRECT | Requests continuity correction |
| Request Confidence Limits |  |
| CL=AGRESTICOULL \| AC | Requests Agresti-Coull confidence limits |
| CL=ALL | Requests all confidence limits |
| CL=EXACT I CLOPPERPEARSON | Requests Clopper-Pearson confidence limits |
| CL=JEFFREYS | Requests Jeffreys confidence limits |
| CL=WALD | Requests Wald confidence limits |
| CL=WILSON I SCORE | Requests Wilson (score) confidence limits |
| Request Tests |  |
| EQUIV I EQUIVALENCE | Requests an equivalence test |
| NONINF I NONINFERIORITY | Requests a noninferiority test |
| SUP I SUPERIORITY | Requests a superiority test |
| MARGIN= | Specifies the test margin |
| VAR=SAMPLE I NULL | Specifies the test variance |

You can specify the following binomial-options:
CL=type | (types)
requests confidence limits for the binomial proportion. You can specify one or more types of confidence limits. When you specify only one type, you can omit the parentheses around the request.

PROC FREQ displays the confidence limits in the "Binomial Confidence Limits" table. The ALPHA $=$ option determines the level of the confidence limits that the $\mathrm{CL}=$ binomial-option provides. The default of ALPHA=0.05 produces $95 \%$ confidence limits for the binomial proportion. This differs from the test-based confidence limits that are provided with the equivalence, noninferiority, and superiority tests (EQUIV, NONINF, and SUP), which have a confidence coefficient of $100(1-2 \alpha) \%$ (Schuirmann 1999). See the sections "Equivalence Test" on page 2724 and "Noninferiority Test" on page 2722 for details.

You can specify the CL= binomial-option with or without requests for binomial tests. The confidence limits that $\mathrm{CL}=$ produces do not depend on the tests that you request and do not use the value of the test margin (which you specify by using the MARGIN= binomial-option).

The following types of binomial confidence limits are available:

## AGRESTICOULL

AC
requests Agresti-Coull confidence limits for the binomial proportion. See the section "Agresti-Coull Confidence Limits" on page 2719 for details.

## ALL

requests all available types of confidence limits for the binomial proportion. These include the following: Agresti-Coull, exact (Clopper-Pearson), Jeffreys, Wald, and Wilson (score) confidence limits.

## EXACT

## CLOPPERPEARSON

requests exact (Clopper-Pearson) confidence limits for the binomial proportion. See the section "Exact (Clopper-Pearson) Confidence Limits" on page 2720 for details. If you do not request any binomial confidence limits by specifying the CL= binomial-option, PROC FREQ produces Wald and exact (Clopper-Pearson) confidence limits by default. To request exact tests for the binomial proportion, specify the BINOMIAL option in the EXACT statement.

## JEFFREYS

requests Jeffreys confidence limits for the binomial proportion. See the section "Jeffreys Confidence Limits" on page 2720 for details.

## WALD < (CORRECT) >

requests Wald confidence limits for the binomial proportion. See the section "Wald Confidence Limits" on page 2719 for details. If you do not request any binomial confidence limits by specifying the CL= binomial-option, PROC FREQ produces Wald and exact (Clopper-Pearson) confidence limits by default.

If you specify $\mathrm{CL}=\mathrm{WALD}$ (CORRECT), the Wald confidence limits include a continuity correction. If you specify the CORRECT binomial-option, the Wald confidence limits and the Wald tests include continuity corrections.

## WILSON < (CORRECT) > <br> SCORE < (CORRECT) >

requests Wilson confidence limits for the binomial proportion. These are also known as score confidence limits. See the section "Wilson (Score) Confidence Limits" on page 2720 for details.

If you specify CL=WILSON(CORRECT) or the CORRECT binomial-option, the Wilson confidence limits include a continuity correction.

## CORRECT

includes a continuity correction in the Wald confidence limits, Wald tests, and Wilson confidence limits.

You can request continuity corrections individually for Wald or Wilson confidence limits by specifying the $\mathrm{CL}=\mathrm{WALD}(\mathrm{CORRECT}$ ) or $\mathrm{CL}=\mathrm{WILSON}(\mathrm{CORRECT}$ ) binomial-option, respectively.

## EQUIV

## EQUIVALENCE

requests a test of equivalence for the binomial proportion. See the section "Equivalence Test" on page 2724 for details. You can specify the equivalence test margins, the null proportion, and the variance type with the MARGIN $=, \mathrm{P}=$, and $\mathrm{VAR}=$ binomial-options, respectively. To request an exact equivalence test, specify the BINOMIAL option in the EXACT statement.

## LEVEL=level-number | ‘level-value’

specifies the variable level for the binomial proportion. By default, PROC FREQ computes the proportion of observations for the first variable level that appears in the output. To request a different level, use LEVEL=level-number or LEVEL='level-value', where level-number is the variable level's number or order in the output, and level-value is the formatted value of the variable level. The value of level-number must be a positive integer. You must enclose level-value in single quotes.

## MARGIN=value | (lower, upper)

specifies the margin for the noninferiority, superiority, and equivalence tests, which you request with the NONINF, SUP, and EQUIV binomial-options, respectively. If you do not specify this option, PROC FREQ uses a margin of 0.2 by default.

For noninferiority and superiority tests, specify a single value for the MARGIN=option. The MARGIN= value must be a positive number. You can specify value as a number between 0 and 1 . Or you can specify value in percentage form as a number between 1 and 100, and PROC FREQ converts that number to a proportion. The procedure treats the value 1 as $1 \%$.

For noninferiority and superiority tests, the test limits must be between 0 and 1 . The limits are determined by the null proportion value (which you can specify with the $\mathrm{P}=$ binomial-option) and by the margin value. The noninferiority limit equals the null proportion minus the margin. By default, the null proportion equals 0.5 and the margin equals 0.2 , which gives a noninferiority limit of 0.3 . The superiority limit equals the null proportion plus the margin, which is 0.7 by default.

For an equivalence test, you can specify a single MARGIN= value, or you can specify both lower and upper values. If you specify a single MARGIN= value, it must be a positive number, as described previously. If you specify a single MARGIN= value for an equivalence test, PROC FREQ uses -value as the lower margin and value as the upper margin for the test. If you specify both lower and upper values for an equivalence test, you can specify them in proportion form as numbers between -1 or 1 . Or you can specify them in percentage form as numbers between -100 and 100 , and PROC FREQ converts the numbers to proportions. The value of lower must be less than the value of upper.

The equivalence limits must be between 0 and 1 . The equivalence limits are determined by the null proportion value (which you can specify with the $\mathrm{P}=$ binomial-option) and by the margin values. The lower equivalence limit equals the null proportion plus the lower margin. By default, the null proportion equals 0.5 and the lower margin equals -0.2 , which gives a lower equivalence limit of 0.3 . The upper equivalence limit equals the null proportion plus the upper margin, which is 0.7 by default.

See the sections "Noninferiority Test" on page 2722 and "Equivalence Test" on page 2724 for details.

## NONINF

## NONINFERIORITY

requests a test of noninferiority for the binomial proportion. See the section "Noninferiority Test" on page 2722 for details. You can specify the noninferiority test margin, the null proportion, and the variance type with the MARGIN=, $\mathrm{P}=$, and VAR= binomial-options, respectively. To request an exact noninferiority test, specify the BINOMIAL option in the EXACT statement.

## $\mathbf{P}=$ value

specifies the null hypothesis proportion for the binomial tests. If you omit this option, PROC FREQ uses 0.5 as the null proportion. The null proportion value must be a positive number. You can specify value as a number between 0 and 1 . Or you can specify value in percentage form as a number between 1 and 100, and PROC FREQ converts that number to a proportion. The procedure treats the value 1 as $1 \%$.

## SUP

## SUPERIORITY

requests a test of superiority for the binomial proportion. See the section "Superiority Test" on page 2723 for details. You can specify the superiority test margin, the null proportion, and the variance type with the MARGIN=, $\mathrm{P}=$, and $\mathrm{VAR}=$ binomial-options, respectively. To request an exact superiority test, specify the BINOMIAL option in the EXACT statement.

## VAR=SAMPLE | NULL

specifies the type of variance estimate to use in the Wald tests of noninferiority, superiority, and equivalence. The default is VAR=SAMPLE, which estimates the variance from the sample proportion. VAR=NULL uses a test-based variance that is computed from the null hypothesis proportion (which is specified by the $\mathrm{P}=$ binomial-option). See the sections "Noninferiority Test" on page 2722 and "Equivalence Test" on page 2724 for details.

## CELLCHI2

displays each table cell's contribution to the Pearson chi-square statistic in the crosstabulation table. The cell chi-square is computed as (frequency - expected) $)^{2} /$ expected, where frequency is the table cell frequency (count) and expected is the expected cell frequency, which is computed under the null hypothesis that the row and column variables are independent. For more information, see the section "Pearson Chi-Square Test for Two-Way Tables" on page 2705. This option has no effect for one-way tables or for tables that are displayed in list format (which you can request by specifying the LIST option).

## CHISQ < (chisq-options) >

requests chi-square tests of homogeneity or independence and measures of association that are based on the chi-square statistic. For two-way tables, the chi-square tests include the Pearson chi-square, likelihood ratio chi-square, and Mantel-Haenszel chi-square tests. The chi-square measures include the phi coefficient, contingency coefficient, and Cramér's $V$. For $2 \times 2$ tables, the CHISQ option also provides Fisher's exact test and the continuity-adjusted chi-square test. See the section "Chi-Square Tests and Statistics" on page 2704 for details.

For one-way tables, the CHISQ option provides the Pearson chi-square goodness-of-fit test. You can also request the likelihood ratio goodness-of-fit test for one-way tables by specifying the LRCHI chisq-option in parentheses after the CHISQ option. By default, the one-way chi-square tests are based on the null hypothesis of equal proportions. Alternatively, you can provide null hypothesis proportions or frequencies by specifying the TESTP= or TESTF= chisq-option, respectively. See the section "Chi-Square Test for One-Way Tables" on page 2704 for more information.
To request Fisher's exact test for tables larger than $2 \times 2$, specify the FISHER option in the EXACT statement. Exact $p$-values are also available for the Pearson, likelihood ratio, and Mantel-Haenszel chi-square tests. See the description of the EXACT statement for more information.

You can specify the following chisq-options:

## DF=df

specifies the degrees of freedom for the chi-square tests. The value of $d f$ must not be zero. If the value of $d f$ is positive, PROC FREQ uses $d f$ as the degrees of freedom for the chi-square tests. If the value of $d f$ is negative, PROC FREQ uses $d f$ to adjust the default degrees of freedom for the chi-square tests.

By default for one-way tables, the value of $d f$ is $(n-1)$, where $n$ is the number of variable levels in the table. By default for two-way tables, the value of $d f$ is $(r-1)(c-1)$, where $r$ is the number of rows in the table and $c$ is the number of columns. See the sections "Chi-Square Test for One-Way Tables" on page 2704 and "Chi-Square Tests and Statistics" on page 2704 for more information.

If you specify a negative value of $d f$, PROC FREQ adjusts the default degrees of freedom by adding the (negative) value of $d f$ to the default value to produce the adjusted degrees of freedom. The adjusted degrees of freedom must be positive.

The $\mathrm{DF}=$ chisq-option specifies or adjusts the degrees of freedom for the following chi-square tests: the Pearson and likelihood ratio goodness-of-fit tests for one-way tables; and the Pearson, likelihood ratio, and Mantel-Haenszel chi-square tests for two-way tables.

## LRCHI

requests the likelihood ratio goodness-of-fit test for one-way tables. See the section "Likelihood Ratio Chi-Square Test for One-Way Tables" on page 2706 for more information.
By default, this test is based on the null hypothesis of equal proportions. You can provide null hypothesis proportions or frequencies by specifying the TESTP= or TESTF= chisq-option, respectively. You can request an exact likelihood ratio goodness-of-fit test by specifying the LRCHI option in the EXACT statement.

## TESTF=(values)| SAS-data-set

specifies null hypothesis frequencies for the one-way chi-square goodness-of-fit tests. See the section "Chi-Square Test for One-Way Tables" on page 2704 for details. You can list the null frequencies as values in parentheses after TESTF=. Or you can provide the null frequencies in a secondary input data set by specifying TESTF=SAS-data-set. The TESTF=SAS-data-set cannot be the same data set that you specify in the DATA= option. You can specify only one TESTF= or TESTP $=$ data set in a single invocation of the procedure.

If you list the null frequencies as values, you can separate the values with blanks or commas. The values must be positive numbers. The number of values must equal the number of variable levels in the one-way table. The sum of the values must equal the total frequency for the one-way table. Order the values to match the order in which the corresponding variable levels appear in the one-way frequency table.

If you provide the null frequencies in a secondary input data set (TESTF=SAS-data-set), the variable that contains the null frequencies should be named _TESTF_, TestFrequency, or Frequency. The null frequencies must be positive numbers. The number of frequencies must equal the number of levels in the one-way frequency table, and the sum of the frequencies must equal the total frequency for the one-way table. Order the null frequencies in the data set to match the order in which the corresponding variable levels appear in the one-way frequency table.

## TESTP=(values)| SAS-data-set

specifies null hypothesis proportions for the one-way chi-square goodness-of-fit tests. See the section "Chi-Square Test for One-Way Tables" on page 2704 for details. You can list the null proportions as values in parentheses after TESTP $=$. Or you can provide the null proportions in a secondary input data set by specifying TESTP=SAS-data-set. The TESTP=SAS-data-set cannot be the same data set that you specify in the DATA= option. You can specify only one TESTF= or TESTP $=$ data set in a single invocation of the procedure.

If you list the null proportions as values, you can separate the values with blanks or commas. The values must be positive numbers. The number of values must equal the number of variable levels in the one-way table. Order the values to match the order in which the corresponding variable levels appear in the one-way frequency table. You can specify values in probability form as numbers between 0 and 1 , where the proportions sum to 1 . Or you can specify values in percentage form as numbers between 0 and 100, where the percentages sum to 100 .

If you provide the null proportions in a secondary input data set (TESTP=SAS-data-set), the variable that contains the null proportions should be named _TESTP_, TestPercent, or Percent. The null proportions must be positive numbers. The number of proportions must equal the number of levels in the one-way frequency table. You can provide the proportions in probability form as numbers between 0 and 1, where the proportions sum to 1 . Or you can provide the proportions in percentage form as numbers between 0 and 100 , where the percentages sum to 100. Order the null proportions in the data set to match the order in which the corresponding variable levels appear in the one-way frequency table.

## WARN=type | (types)

controls the warning message for the validity of the asymptotic Pearson chi-square test. By default, PROC FREQ displays a warning message when more than $20 \%$ of the table cells have expected frequencies that are less than 5. If you specify the NOPRINT option in the PROC FREQ statement, the procedure displays the warning in the log; otherwise, the procedure displays the warning as a footnote in the chi-square table. You can use the WARN= option to suppress the warning and to include a warning indicator in the output data set.

You can specify one or more of the following types in the WARN= option. If you specify more than one type value, enclose the values in parentheses after WARN=. For example, warn $=$ (output noprint).

| Value of WARN $=$ | Description |
| :--- | :--- |
| OUTPUT | Adds a warning indicator variable to the output data set |
| NOLOG | Suppresses the chi-square warning message in the log |
| NOPRINT | Suppresses the chi-square warning message in the display |
| NONE | Suppresses the chi-square warning message entirely |

If you specify the WARN=OUTPUT option, the ODS output data set ChiSq contains a variable named Warning that equals 1 for the Pearson chi-square observation when more than $20 \%$ of the table cells have expected frequencies that are less than 5 and equals 0 otherwise. If you specify WARN=OUTPUT and also specify the CHISQ option in the OUTPUT statement, the statistics output data set contains a variable named WARN_PCHI that indicates the warning.

The WARN=NOLOG option has the same effect as the NOWARN option in the TABLES statement.

CL
requests confidence limits for the measures of association, which you can request by specifying the MEASURES option. For more information, see the sections "Measures of Association" on page 2709 and "Confidence Limits" on page 2710. You can set the level of the confidence limits by using the ALPHA $=$ option. The default of ALPHA= 0.05 produces $95 \%$ confidence limits.

If you omit the MEASURES option, the CL option invokes MEASURES. The CL option is equivalent to the MEASURES(CL) option.

## CMH < (cmh-options) >

requests Cochran-Mantel-Haenszel statistics, which test for association between the row and column variables after adjusting for the remaining variables in a multiway table. The Cochran-Mantel-Haenszel statistics include the nonzero correlation statistic, the row mean scores (ANOVA) statistic, and the general association statistic. In addition, for $2 \times 2$ tables, the CMH option provides the adjusted Mantel-Haenszel and logit estimates of the odds ratio and relative risks, together with their confidence limits. For stratified $2 \times 2$ tables, the CMH option provides the Breslow-Day test for homogeneity of odds ratios. (To request Tarone's adjustment for the Breslow-Day test, specify the BDT cmh-option.) See the section "Cochran-Mantel-Haenszel Statistics" on page 2748 for details.

You can use the CMH1 or CMH2 option to control the number of CMH statistics that PROC FREQ computes.
For stratified $2 \times 2$ tables, you can request Zelen's exact test for equal odds ratios by specifying the EQOR option in the EXACT statement. See the section "Zelen's Exact Test for Equal Odds Ratios" on page 2754 for details. You can request exact confidence limits for the common odds ratio by specifying the COMOR option in the EXACT statement. This option also provides a common odds ratio test. See the section "Exact Confidence Limits for the Common Odds Ratio" on page 2754 for details.

You can specify the following cmh-options in parentheses after the CMH option. These cmh-options, which apply to stratified $2 \times 2$ tables, are also available with the CMH1 or CMH2 option.

## BDT

requests Tarone's adjustment in the Breslow-Day test for homogeneity of odds ratios. See the section "Breslow-Day Test for Homogeneity of the Odds Ratios" on page 2753 for details.

## GAILSIMON < (COLUMN=1 | 2$)$ >

## GS < (COLUMN=1 | 2) >

requests the Gail-Simon test for qualitative interaction, which applies to stratified $2 \times 2$ tables. See the section "Gail-Simon Test for Qualitative Interactions" on page 2756 for details.

The COLUMN= option specifies the column of the risk differences to use in computing the GailSimon test. By default, PROC FREQ uses column 1 risk differences. If you specify COLUMN=2, PROC FREQ uses column 2 risk differences.

The GAILSIMON $c m h$-option has the same effect as the GAILSIMON option in the TABLES statement.

## MANTELFLEISS

MF
requests the Mantel-Fleiss criterion for the Mantel-Haenszel statistic for stratified $2 \times 2$ tables. See the section "Mantel-Fleiss Criterion" on page 2751 for details.

## CMH1 < (cmh-options) >

requests the Cochran-Mantel-Haenszel correlation statistic. This option does not provide the CMH row mean scores (ANOVA) statistic or the general association statistic, which are provided by the CMH option. For tables larger than $2 \times 2$, the CMH1 option requires less memory than the CMH option, which can require an enormous amount of memory for large tables.

For $2 \times 2$ tables, the CMH1 option also provides the adjusted Mantel-Haenszel and logit estimates of the odds ratio and relative risks, together with their confidence limits. For stratified $2 \times 2$ tables, the CMH1 option provides the Breslow-Day test for homogeneity of odds ratios.

The cmh-options for CMH1 are the same as the cmh-options that are available with the CMH option. See the description of the CMH option for details.

## CMH2 < (cmh-options) >

requests the Cochran-Mantel-Haenszel correlation statistic and the row mean scores (ANOVA) statistic. This option does not provide the CMH general association statistic, which is provided by the CMH option. For tables larger than $2 \times 2$, the CMH2 option requires less memory than the CMH option, which can require an enormous amount of memory for large tables.

For $2 \times 2$ tables, the CMH1 option also provides the adjusted Mantel-Haenszel and logit estimates of the odds ratio and relative risks, together with their confidence limits. For stratified $2 \times 2$ tables, the CMH1 option provides the Breslow-Day test for homogeneity of odds ratios.

The cmh-options for CMH2 are the same as the cmh-options that are available with the CMH option. See the description of the CMH option for details.

## CONTENTS='string'

specifies the label to use for crosstabulation tables in the contents file, the Results window, and the trace record. For information about output presentation, see the SAS Output Delivery System: User's Guide.

If you omit the CONTENTS= option, the contents label for crosstabulation tables is "Cross-Tabular Freq Table" by default.

Note that contents labels for all crosstabulation tables that are produced by a single TABLES statement use the same text. To specify different contents labels for different crosstabulation tables, request the tables in separate TABLES statements and use the CONTENTS = option in each TABLES statement.

To remove the crosstabulation table entry from the contents file, you can specify a null label with CONTENTS=".

The CONTENTS = option affects only contents labels for crosstabulation tables. It does not affect contents labels for other PROC FREQ tables.

To specify the contents label for any PROC FREQ table, you can use PROC TEMPLATE to create a customized table template. The CONTENTS_LABEL attribute in the DEFINE TABLE statement of PROC TEMPLATE specifies the contents label for the table. See the chapter "The TEMPLATE Procedure" in the SAS Output Delivery System: User's Guide for more information.

## CROSSLIST < (options) >

displays crosstabulation tables by using an ODS column format instead of the default crosstabulation cell format. In the CROSSLIST table display, the rows correspond to the crosstabulation table cells, and the columns correspond to descriptive statistics such as frequencies and percentages. The CROSSLIST
table displays the same information as the default crosstabulation table (but it uses an ODS column format). For more information about the contents of the CROSSLIST table, See the section "Two-Way and Multiway Tables" on page 2767.
You can control the contents of a CROSSLIST table by specifying the same options available for the default crosstabulation table. These include the NOFREQ, NOPERCENT, NOROW, and NOCOL options. You can request additional information in a CROSSLIST table by specifying the CELLCHI2, DEVIATION, EXPECTED, MISSPRINT, and TOTPCT options. You can also display standardized residuals or Pearson residuals in a CROSSLIST table by specifying the CROSSLIST(STDRES) or CROSSLIST(PEARSONRES) option, respectively; these options are not available for the default crosstabulation table. The FORMAT = and CUMCOL options have no effect on CROSSLIST tables. You cannot specify both the LIST option and the CROSSLIST option in the same TABLES statement.

For CROSSLIST tables, you can use the NOSPARSE option to suppress display of variable levels that have zero frequencies. By default, PROC FREQ displays all levels of the column variable within each level of the row variable, including any levels that have zero frequencies. By default for multiway tables that are displayed as CROSSLIST tables, the procedure displays all levels of the row variable for each stratum of the table, including any row levels with zero frequencies in the stratum.

You can specify the following options:

## STDRES

displays the standardized residuals of the table cells in the CROSSLIST table. The standardized residual is the ratio of (frequency - expected) to its standard error, where frequency is the table cell frequency (count) and expected is the expected table cell frequency, which is computed under the null hypothesis that the row and column variables are independent. For more information, see the section "Standardized Residuals" on page 2705. You can display the expected values and deviations by specifying the EXPECTED and DEVIATION options, respectively.

## PEARSONRES

displays the Pearson residuals of the table cells in the CROSSLIST table. The Pearson residual is the square root of the table cell's contribution to the Pearson chi-square statistic. The Pearson residual is computed as (frequency - expected) $/ \sqrt{\text { expected }}$, where frequency is the table cell frequency (count) and expected is the expected table cell frequency, which is computed under the null hypothesis that the row and column variables are independent. For more information, see the section "Pearson Chi-Square Test for Two-Way Tables" on page 2705. You can display the expected values, deviations, and cell chi-squares by specifying the EXPECTED, DEVIATION, and CELLCHI2 options, respectively.

## CUMCOL

displays the cumulative column percentages in the cells of the crosstabulation table. The CUMCOL option does not apply to crosstabulation tables produced with the LIST or CROSSLIST option.

## DEVIATION

displays the deviations of the frequencies from the expected frequencies (frequency - expected) in the crosstabulation table. The expected frequencies are computed under the null hypothesis that the row and column variables are independent. For more information, see the section "Pearson Chi-Square Test for Two-Way Tables" on page 2705. You can display the expected values by specifying the EXPECTED option. This option has no effect for one-way tables or for tables that are displayed in list format (which you can request by specifying the LIST option).

## EXPECTED

displays the expected cell frequencies in the crosstabulation table. The expected frequencies are computed under the null hypothesis that the row and column variables are independent. For more information, see the section "Pearson Chi-Square Test for Two-Way Tables" on page 2705. This option has no effect for one-way tables or for tables that are displayed in list format (which you can request by specifying the LIST option).

## FISHER

requests Fisher's exact test for tables that are larger than $2 \times 2$. (For $2 \times 2$ tables, the CHISQ option provides Fisher's exact test.) This test is also known as the Freeman-Halton test. See the sections "Fisher's Exact Test" on page 2708 and "Exact Statistics" on page 2757 for more information.

If you omit the CHISQ option in the TABLES statement, the FISHER option invokes CHISQ. You can also request Fisher's exact test by specifying the FISHER option in the EXACT statement.

NOTE: PROC FREQ computes exact tests by using fast and efficient algorithms that are superior to direct enumeration. Exact tests are appropriate when a data set is small, sparse, skewed, or heavily tied. For some large problems, computation of exact tests might require a substantial amount of time and memory. Consider using asymptotic tests for such problems. Alternatively, when asymptotic methods might not be sufficient for such large problems, consider using Monte Carlo estimation of exact $p$-values. You can request Monte Carlo estimation by specifying the MC computation-option in the EXACT statement. See the section "Computational Resources" on page 2759 for more information.

## FORMAT=format-name

specifies a format for the following crosstabulation table cell values: frequency, expected frequency, and deviation. PROC FREQ also uses the specified format to display the row and column total frequencies and the overall total frequency in crosstabulation tables.

You can specify any standard SAS numeric format or a numeric format defined with the FORMAT procedure. The format length must not exceed 24. If you omit the FORMAT= option, by default PROC FREQ uses the BEST6. format to display frequencies less than 1E6, and the BEST7. format otherwise.

The FORMAT= option applies only to crosstabulation tables displayed in the default format. It does not apply to crosstabulation tables produced with the LIST or CROSSLIST option.

To change display formats in any FREQ table, you can use PROC TEMPLATE. See the chapter "The TEMPLATE Procedure" in the SAS Output Delivery System: User's Guide for more information.

## GAILSIMON < (COLUMN=1 | 2) >

GS < (COLUMN=1 | 2) >
requests the Gail-Simon test for qualitative interaction, which applies to stratified $2 \times 2$ tables. See the section "Gail-Simon Test for Qualitative Interactions" on page 2756 for details.

The COLUMN= option specifies the column of the risk differences to use in computing the Gail-Simon test. By default, PROC FREQ uses column 1 risk differences. If you specify COLUMN=2, PROC FREQ uses column 2 risk differences.

## JT

requests the Jonckheere-Terpstra test. See the section "Jonckheere-Terpstra Test" on page 2741 for details. To request exact $p$-values for the Jonckheere-Terpstra test, specify the JT option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.
displays two-way and multiway tables by using a list format instead of the default crosstabulation cell format. This option displays an entire multiway table in one table, instead of displaying a separate two-way table for each stratum. For more information, see the section "Two-Way and Multiway Tables" on page 2767.

The LIST option is not available when you request tests and statistics; you must use the standard crosstabulation table display or the CROSSLIST display when you request tests and statistics.

## MAXLEVELS=n

specifies the maximum number of variable levels to display in one-way frequency tables. The value of $n$ must be a positive integer. PROC FREQ displays the first $n$ variable levels, matching the order in which the levels appear in the one-way frequency table. (The ORDER= option controls the order of the variable levels. By default, ORDER=INTERNAL, which orders the variable levels by unformatted value.)

The MAXLEVELS $=$ option also applies to one-way frequency plots, which you can request by specifying the PLOTS=FREQPLOT option when ODS Graphics is enabled.

If you specify the MISSPRINT option to display missing levels in the frequency table, the MAXLEVELS $=$ option displays the first $n$ nonmissing levels.

The MAXLEVELS $=$ option does not apply to the OUT= output data set, which includes all variable levels. The MAXLEVELS= option does not affect the computation of percentages, statistics, or tests for the one-way table; these values are based on the complete table.

## MEASURES < (CL) >

requests measures of association and their asymptotic standard errors. This option provides the following measures: gamma, Kendall's tau- $b$, Stuart's tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, Pearson and Spearman correlation coefficients, lambda (symmetric and asymmetric), and uncertainty coefficients (symmetric and asymmetric). If you specify the CL option in parentheses after the MEASURES option, PROC FREQ provides confidence limits for the measures of association. For more information, see the section "Measures of Association" on page 2709.

For $2 \times 2$ tables, the MEASURES option also provides the odds ratio, column 1 relative risk, column 2 relative risk, and their asymptotic Wald confidence limits. You can request the odds ratio and relative risks separately (without the other measures of association) by specifying the RELRISK option. You can request confidence limits for the odds ratio by specifying the $\mathrm{OR}(\mathrm{CL}=)$ option.

You can use the TEST statement to request asymptotic tests for the following measures of association: gamma, Kendall's tau-b, Stuart's tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, and Pearson and Spearman correlation coefficients. You can use the EXACT statement to request exact confidence limits for the odds ratio, exact unconditional confidence limits for the relative risks, and exact tests for the following measures of association: Kendall's tau-b, Stuart's tau-c, Somers' $D(C \mid R)$ and $D(R \mid C)$, and Pearson and Spearman correlation coefficients. For more information, see the descriptions of the TEST and EXACT statements and the section "Exact Statistics" on page 2757.

## MISSING

treats missing values as a valid nonmissing level for all TABLES variables. The MISSING option displays the missing levels in frequency and crosstabulation tables and includes them in all calculations of percentages, tests, and measures.

By default, if you do not specify the MISSING or MISSPRINT option, an observation is excluded from a table if it has a missing value for any of the variables in the TABLES request. When PROC FREQ excludes observations with missing values, it displays the total frequency of missing observations below the table. See the section "Missing Values" on page 2699 for more information.

## MISSPRINT

displays missing value frequencies in frequency and crosstabulation tables but does not include the missing value frequencies in any computations of percentages, tests, or measures.

By default, if you do not specify the MISSING or MISSPRINT option, an observation is excluded from a table if it has a missing value for any of the variables in the TABLES request. When PROC FREQ excludes observations with missing values, it displays the total frequency of missing observations below the table. See the section "Missing Values" on page 2699 for more information.

## NOCOL

suppresses the display of column percentages in crosstabulation table cells.

## NOCUM

suppresses the display of cumulative frequencies and percentages in one-way frequency tables. The NOCUM option also suppresses the display of cumulative frequencies and percentages in crosstabulation tables in list format, which you request with the LIST option.

## NOFREQ

suppresses the display of cell frequencies in crosstabulation tables. The NOFREQ option also suppresses row total frequencies. This option has no effect for one-way tables or for crosstabulation tables in list format, which you request with the LIST option.

## NOPERCENT

suppresses the display of overall percentages in crosstabulation tables. These percentages include the cell percentages of the total (two-way) table frequency, as well as the row and column percentages of the total table frequency. To suppress the display of cell percentages of row or column totals, use the NOROW or NOCOL option, respectively.

For one-way frequency tables and crosstabulation tables in list format, the NOPERCENT option suppresses the display of percentages and cumulative percentages.

## NOPRINT

suppresses the display of frequency and crosstabulation tables but displays all requested tests and statistics. To suppress the display of all output, including tests and statistics, use the NOPRINT option in the PROC FREQ statement.

## NOROW

suppresses the display of row percentages in crosstabulation table cells.

## NOSPARSE

suppresses the display of cells with a zero frequency count in LIST output and omits them from the OUT= data set. The NOSPARSE option applies when you specify the ZEROS option in the WEIGHT statement to include observations with zero weights. By default, the ZEROS option invokes the SPARSE option, which displays table cells with a zero frequency count in the LIST output and includes them in the OUT= data set. See the description of the ZEROS option for more information.

The NOSPARSE option also suppresses the display of variable levels with zero frequency in CROSSLIST tables. By default for CROSSLIST tables, PROC FREQ displays all levels of the column variable within each level of the row variable, including any column variable levels with zero frequency for that row. For multiway tables displayed with the CROSSLIST option, the procedure displays all levels of the row variable for each stratum of the table by default, including any row variable levels with zero frequency for the stratum.

## NOWARN

suppresses the log warning message for the validity of the asymptotic Pearson chi-square test. By default, PROC FREQ provides a validity warning for the asymptotic Pearson chi-square test when more than 20cells have expected frequencies that are less than 5 . This warning message appears in the log if you specify the NOPRINT option in the PROC FREQ statement,

The NOWARN option is equivalent to the CHISQ(WARN=NOLOG) option. You can also use the CHISQ(WARN=) option to suppress the warning message in the display and to request a warning variable in the chi-square ODS output data set or in the OUTPUT data set.

OR(CL=type | (types))
ODDSRATIO(CL=type | (types))
requests the odds ratio and confidence limits for $2 \times 2$ tables. You can specify one or more types of confidence limits, which include exact, score, and Wald confidence limits. When you specify only one confidence limit type, you can omit the parentheses around the request.

PROC FREQ displays the confidence limits in the "Confidence Limits for the Odds Ratio" table. Specifying the OR option without the CL= option is equivalent to specifying the RELRISK option, which produces the "Odds Ratio and Relative Risks" table. For more information, see the description of the RELRISK option. When you specify the $\mathrm{OR}(\mathrm{CL}=)$ option, PROC FREQ does not produce the "Odds Ratio and Relative Risks" table unless you also specify the RELRISK or MEASURES option.

The ALPHA = option determines the confidence level, and the default of ALPHA=0.05 produces $95 \%$ confidence limits for the odds ratio.

You can specify the following types:

## EXACT

displays exact confidence limits for the odds ratio in the "Confidence Limits for the Odds Ratio" table. You must also request computation of the exact confidence limits by specifying the OR option in the EXACT statement. For more information, see the section "Exact Confidence Limits for the Odds Ratio" on page 2738.

## SCORE < (CORRECT=NO) >

requests score confidence limits for the odds ratio. For more information, see the section "Score Confidence Limits for the Odds Ratio" on page 2738. If you specify CORRECT=NO, PROC FREQ provides the uncorrected form of the confidence limits.

## WALD

requests asymptotic Wald confidence limits for the odds ratio. For more information, see the section "Odds Ratio and Relative Risks for $2 \times 2$ Tables" on page 2737.

## OUT=SAS-data-set

names an output data set that contains frequency or crosstabulation table counts and percentages. If more than one table request appears in the TABLES statement, the contents of the OUT= data set correspond to the last table request in the TABLES statement. The OUT= data set variable COUNT contains the frequencies and the variable PERCENT contains the percentages. See the section "Output Data Sets" on page 2762 for details. You can specify the following options to include additional information in the OUT= data set: OUTCUM, OUTEXPECT, and OUTPCT.

## OUTCUM

includes cumulative frequencies and cumulative percentages in the OUT= data set for one-way tables. The variable CUM_FREQ contains the cumulative frequencies, and the variable CUM_PCT contains the cumulative percentages. See the section "Output Data Sets" on page 2762 for details. The OUTCUM option has no effect for two-way or multiway tables.

## OUTEXPECT

includes expected cell frequencies in the OUT= data set for crosstabulation tables. The variable EXPECTED contains the expected cell frequencies. See the section "Output Data Sets" on page 2762 for details. The EXPECTED option has no effect for one-way tables.

## OUTPCT

includes the following additional variables in the OUT= data set for crosstabulation tables:
PCT_COL percentage of column frequency
PCT_ROW percentage of row frequency
PCT_TABL percentage of stratum (two-way table) frequency, for $n$-way tables where $n>2$
See the section "Output Data Sets" on page 2762 for details. The OUTPCT option has no effect for one-way tables.

## PLCORR < (options) >

requests the polychoric correlation coefficient and its asymptotic standard error. For $2 \times 2$ tables, this statistic is more commonly known as the tetrachoric correlation coefficient, and it is labeled as such in the displayed output. For more information, see the section "Polychoric Correlation" on page 2715.

If you also specify the CL or MEASURES(CL) option, PROC FREQ provides confidence limits for the polychoric correlation. If you specify the PLCORR option in the TEST statement, the procedure provides Wald and likelihood ratio tests for the polychoric correlation. The PLCORR option invokes the MEASURES option.

You can specify the following options:

## CONVERGE=value

specifies the convergence criterion for computing the polychoric correlation. The convergence criterion value must be a positive number. By default, CONVERGE $=0.0001$. Iterative computation of the polychoric correlation stops when the convergence measure falls below value or when the number of iterations exceeds the MAXITER= number, whichever happens first. For parameter values that are less than 0.01, PROC FREQ evaluates convergence by using the absolute difference instead of the relative difference. For more information, see the section "Polychoric Correlation" on page 2715.

## MAXITER=number

specifies the maximum number of iterations for computing the polychoric correlation. The value of number must be a positive integer. By default, MAXITER=20. Iterative computation of the polychoric correlation stops when the number of iterations exceeds the maximum number or when the convergence measure falls below the CONVERGE= value, whichever happens first. For more information, see the section "Polychoric Correlation" on page 2715.

PLOTS $<$ (global-plot-options) $><=$ plot-request $<$ (plot-options) $\gg$
PLOTS $<$ (global-plot-options) >
< =(plot-request < (plot-options) > < . .. plot-request < (plot-options) \gg )>
controls the plots that are produced through ODS Graphics. Plot-requests identify the plots, and plot-options control the appearance and content of the plots. You can specify plot-options in parentheses after a plot-request. A global-plot-option applies to all plots for which it is available unless it is altered by a specific plot-option. You can specify global-plot-options in parentheses after the PLOTS option.

When you specify only one plot-request, you can omit the parentheses around the request. For example:

```
plots=all
plots=freqplot
plots=(freqplot oddsratioplot)
plots(only)=(cumfreqplot deviationplot)
```

ODS Graphics must be enabled before plots can be requested. For example:

```
ods graphics on;
proc freq;
    tables treatment*response / chisq plots=freqplot;
    weight wt;
run;
ods graphics off;
```

For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" on page 606 in Chapter 21, "Statistical Graphics Using ODS."

If ODS Graphics is enabled but you do not specify the PLOTS= option, PROC FREQ produces all plots that are associated with the analyses that you request, with the exception of the frequency, cumulative frequency, and mosaic plots. To produce a frequency plot or cumulative frequency plot when ODS Graphics is enabled, you must specify the FREQPLOT or CUMFREQPLOT plot-request, respectively, in the PLOTS = option, or you must specify the PLOTS=ALL option. To produce a mosaic plot when ODS Graphics is enabled, you must specify the MOSAICPLOT plot-request in the PLOTS= option, or you must specify the PLOTS=ALL option.

PROC FREQ produces the remaining plots (listed in Table 40.11) by default when you request the corresponding TABLES statement options. You can suppress default plots and request specific plots by using the PLOTS $(\mathrm{ONLY})=$ option; $\operatorname{PLOTS}(\mathrm{ONLY})=($ plot-requests $)$ produces only the plots that are specified as plot-requests. You can suppress all plots by specifying the PLOTS=NONE option. The PLOTS option has no effect when you specify the NOPRINT option in the PROC FREQ statement.

## Plot Requests

Table 40.11 lists the available plot-requests together with their required TABLES statement options. Descriptions of the plot-requests follow the table in alphabetical order.

Table 40.11 Plot Requests

| Plot Request | Description | Required TABLES Statement Option |
| :--- | :--- | :--- |
| AGREEPLOT | Agreement plot | AGREE $(r \times r$ table $)$ |
| ALL | All plots | None |
| CUMFREQPLOT | Cumulative frequency plot | One-way table request |
| DEVIATIONPLOT | Deviation plot | CHISQ (one-way table) |
| FREQPLOT | Frequency plot | Any table request |
| KAPPAPLOT | Kappa plot | AGREE $(h \times r \times r$ table $)$ |
| MOSAICPLOT | Mosaic plot | Two-way or multiway table request |
| NONE | No plots | None |
| ODDSRATIOPLOT | Odds ratio plot | MEASURES or RELRISK $(h \times 2 \times 2$ table) |
| RELRISKPLOT | Relative risk plot | MEASURES or RELRISK $(h \times 2 \times 2$ table) |
| RISKDIFFPLOT | Risk difference plot | RISKDIFF $(h \times 2 \times 2$ table $)$ |
| WTKAPPAPLOT | Weighted kappa plot | AGREE $(h \times r \times r$ table, $r>2)$ |

The following plot-requests are available with the PLOTS= option:

## AGREEPLOT < (plot-options) >

requests an agreement plot (Bangdiwala and Bryan 1987), An agreement plot displays the strength of agreement in a two-way table, where the row and column variables represent two independent ratings of $n$ subjects. For information about agreement plots, see Bangdiwala (1988); Bangdiwala et al. (2008), and Friendly (2000, Section 3.7.2).

To produce an agreement plot, you must also specify the AGREE option in the TABLES statement. Agreement statistics and plots are available for two-way square tables, where the number of rows equals the number of columns.

Table 40.12 lists the plot-options that are available for agreement plots. See the subsection "Plot Options" for descriptions of the plot-options.

Table 40.12 Plot Options for AGREEPLOT

| Plot Option | Description | Values |
| :--- | :--- | :--- |
| LEGEND $=$ | Legend | YES $^{*}$ or NO |
| PARTIAL $=$ | Partial agreement | YES* $^{*}$ or NO |
| SHOWSCALE= | Frequency scale | YES* or NO $^{*}$ NTS |
| STATS | Statistics | None |

[^1]
## ALL

requests all plots that are associated with the specified analyses. Table 40.11 lists the available plot-requests and the corresponding analysis options. If you specify the PLOTS=ALL option, PROC FREQ produces the frequency, cumulative frequency, and mosaic plots that are associated with the tables that you request. (These plots are not produced by default when ODS Graphics is enabled.)

## CUMFREQPLOT < (plot-options) >

requests a plot of cumulative frequencies. Cumulative frequency plots are available for one-way frequency tables.

To produce a cumulative frequency plot, you must specify the CUMFREQPLOT plot-request in the PLOTS = option, or you must specify the PLOTS=ALL option. PROC FREQ does not produce cumulative frequency plots by default when ODS Graphics is enabled.

Table 40.13 lists the plot-options that are available for cumulative frequency plots. See the subsection "Plot Options" for descriptions of the plot-options.

Table 40.13 Plot Options for CUMFREQPLOT

| Plot Option | Description | Values |
| :--- | :--- | :--- |
| ORIENT $=$ | Orientation | VERTICAL* or HORIZONTAL |
| SCALE $=$ | Scale | FREQ* $^{*}$ or PERCENT |
| TYPE $=$ | Type | BARCHART $^{*}$ or DOTPLOT |

*Default

DEVIATIONPLOT < (plot-options) >
requests a plot of relative deviations from expected frequencies. Deviation plots are available for chi-square analysis of one-way frequency tables. To produce a deviation plot, you must also specify the CHISQ option in the TABLES statement for a one-way frequency table.

Table 40.14 lists the plot-options that are available for deviation plots. See the subsection "Plot Options" for descriptions of the plot-options.

Table 40.14 Plot Options for DEVIATIONPLOT

| Plot Option | Description | Values |
| :--- | :--- | :--- |
| NOSTAT | No statistic | None |
| ORIENT $=$ | Orientation | VERTICAL* or HORIZONTAL |
| TYPE $=$ | Type | BARCHART* or DOTPLOT |

*Default

FREQPLOT < (plot-options) >
requests a frequency plot. Frequency plots are available for frequency and crosstabulation tables. For multiway crosstabulation tables, PROC FREQ provides a two-way frequency plot for each stratum (two-way table).

To produce a frequency plot, you must specify the FREQPLOT plot-request in the PLOTS= option, or you must specify the PLOTS=ALL option. PROC FREQ does not produce frequency plots by default when ODS Graphics is enabled.

By default, PROC FREQ displays frequency plots as bar charts. You can specify the TYPE=DOTPLOT plot-option to display frequency plots as dot plots. You can plot percentages instead of frequencies by specifying the SCALE=PERCENT plot-option. There are four frequency plot layouts available, which you can request by specifying the TWOWAY= plot-option. See the subsection "Plot Options" for more information.

By default, the primary grouping of graph cells in a two-way layout is by column variable. Row variable levels are then displayed within column variable levels. You can specify the GROUPBY=ROW plot-option to group first by row variable.

Table 40.15 lists the plot-options that are available for frequency plots. See the subsection "Plot Options" for descriptions of the plot-options.

The following plot-options are available for all frequency plots: ORIENT=, SCALE=, and TYPE=. The following plot-options are available for frequency plots of two-way (and multiway) tables: GROUPBY=, NPANELPOS=, and TWOWAY=. The NPANELPOS= plot-option is not available with the TWOWAY=CLUSTER or TWOWAY=STACKED layout, which is always displayed in a single panel.

Table 40.15 Plot Options for FREQPLOT

| Plot Option | Description | Values |
| :---: | :---: | :---: |
| GROUPBY=** | Primary group | COLUMN* or ROW |
| NPANELPOS=** | Sections per panel | Number (4*) |
| ORIENT= | Orientation | VERTICAL* or HORIZONTAL |
| SCALE= | Scale | FREQ*, GROUPPERCENT**, |
|  |  | LOG, PERCENT, SQRT |
| TWOWAY=** | Two-way layout | GROUPVERTICAL* CLUSTER, GROUPHORIZONTAL, or STACKED |
| TYPE= | Type | BARCHART* or DOTPLOT |

*Default
${ }^{* *}$ For two-way tables

## KAPPAPLOT < (plot-options) >

requests a plot of kappa statistics with confidence limits. Kappa plots are available for multiway square tables and display the kappa statistic (with confidence limits) for each two-way table (stratum). Kappa plots also display the overall kappa statistic unless you specify the COMMON=NO plot-option. To produce a kappa plot, you must specify the AGREE option in the TABLES statement to compute kappa statistics.

Table 40.16 lists the plot-options that are available for kappa plots. See the subsection "Plot Options" for descriptions of the plot-options.

Table 40.16 Plot Options for KAPPAPLOT and WTKAPPAPLOT

| Plot Option | Description | Values |
| :--- | :--- | :--- |
| CLDISPLAY $=$ | Error bar type | SERIF*, SERIFARROW, |
|  |  | LINE, LINEARROW, or BAR |
| COMMON $=$ | Overall kappa | YES* or NO |
| NPANELPOS $=$ | Statistics per graphic | Number (All*) |
| ORDER $=$ | Order of two-way levels | ASCENDING or DESCENDING |
| RANGE $=$ | Range to display | Values or CLIP |
| STATS | Statistic values | None |

* Default


## MOSAICPLOT < (plot-options) >

requests a mosaic plot. Mosaic plots are available for two-way and multiway crosstabulation tables; for multiway tables, PROC FREQ provides a mosaic plot for each two-way table (stratum).

To produce a mosaic plot, you must specify the MOSAICPLOT plot-request in the PLOTS= option, or you must specify the PLOTS=ALL option. PROC FREQ does not produce mosaic plots by default when ODS Graphics is enabled.

Mosaic plots display tiles that correspond to the crosstabulation table cells. The areas of the tiles are proportional to the frequencies of the table cells. The column variable is displayed on the X axis, and the tile widths are proportional to the relative frequencies of the column variable levels. The row variable is displayed on the Y axis, and the tile heights are proportional to the relative frequencies of the row levels within column levels. For more information, see Friendly (2000).

By default, the colors of the tiles correspond to the row variable levels. If you specify the COLORSTAT $=$ plot-option, the tiles are colored according to the values of the Pearson or standardized residuals.

You can specify the following plot-options:

## COLORSTAT < =PEARSONRES | STDRES >

colors the mosaic plot tiles according to the values of residuals. If you specify COLORSTAT=PEARSONRES, the tiles are colored according to the Pearson residuals of the corresponding table cells. For more information, see the section "Pearson Chi-Square Test for Two-Way Tables" on page 2705. If you specify COLORSTAT=STDRES, the tiles are colored according to the standardized residuals of the corresponding table cells. For more information, see the section "Standardized Residuals" on page 2705. You can display the Pearson or standardized residuals in the CROSSLIST table by specifying the CROSSLIST(PEARSONRES) or CROSSLIST(STDRES) option, respectively.

## SQUARE

produces a square mosaic plot, where the height of the $Y$ axis equals the width of the $X$ axis. In a square mosaic plot, the scale of the relative frequencies is the same on both axes. By default, PROC FREQ produces a rectangular mosaic plot.

## NONE

suppresses all plots.

## ODDSRATIOPLOT < (plot-options) >

requests a plot of odds ratios with confidence limits. Odds ratio plots are available for multiway $2 \times 2$ tables and display the odds ratio (with confidence limits) for each $2 \times 2$ table (stratum). To produce an odds ratio plot, you must also specify the MEASURES, OR(CL=), or RELRISK option in the TABLES statement to compute odds ratios.

By default, the odds ratio plot displays asymptotic Wald confidence limits for the odds ratios ( $\mathrm{CL}=\mathrm{WALD}$ ). You can specify the CL=EXACT or CL=SCORE plot-option to display exact or score confidence limits, respectively. To display exact or score confidence limits, you must also request their computation. You can specify the OR option in the EXACT statement to request exact confidence limits for the odds ratio; you can specify the OR(CL=SCORE) option in the TABLES statement to request score confidence limits for the odds ratio.

By default, the odds ratio plot displays the common odds ratio when the procedure computes the common value. You can suppress display of the common odds ratio by specifying the COMMON=NO plot-option. To compute the common odds ratio for a multiway table, you can specify the CMH option in the TABLES statement. To display the common odds ratio when the plot displays exact confidence limits ( $\mathrm{CL}=\mathrm{EXACT}$ ), you must request computation of exact confidence limits for the common odds ratio by specifying the COMOR option in the EXACT statement. When you specify the CL=SCORE plot-option, the odds ratio plot does not display the common odds ratio.

Table 40.17 lists the plot-options that are available for odds ratio plots. See the subsection "Plot Options" for descriptions of the plot-options.

Table 40.17 Plot Options for ODDSRATIOPLOT, RELRISKPLOT, and RISKDIFFPLOT

| Plot Option | Description | Values |
| :--- | :--- | :--- |
| CL= | Confidence limit type | Type |
| CLDISPLAY $=$ | Error bar type | SERIF*, SERIFARROW, |
|  |  | LINE, LINEARROW, or BAR |
| COMMON $=$ | Common value | YES* or NO |
| COLUMN=** | Risk column | $\mathbf{1 *}^{*}$ or 2 |
| LOGBASE=*** | Axis scale | 2, E, or 10 |
| NPANELPOS $=$ | Statistics per graphic | Number (All*) |
| ORDER $=$ | Order of two-way levels | ASCENDING or DESCENDING |
| RANGE $=$ | Range to display | Values or CLIP |
| STATS | Statistic values | None |

* Default
${ }^{* *}$ Available for RELRISKPLOT and RISKDIFFPLOT
*** Available for ODDSRATIOPLOT and RELRISKPLOT


## RELRISKPLOT < (plot-options) >

requests a plot of relative risks with confidence limits. Relative risk plots are available for multiway $2 \times 2$ tables and display the relative risk (with confidence limits) for each $2 \times 2$ table
(stratum). To produce a relative risk plot, you must also specify the MEASURES or RELRISK option in the TABLES statement to compute relative risks.

If you specify the CMH option in the TABLES statement to compute the common relative risk for the multiway table, by default the plot displays the common relative risk. The COMMON=NO plot-option suppresses display of the common relative risk. If you do not specify the CMH option, the common relative risk is not available to be displayed in the plot.

By default, relative risk plots display asymptotic confidence limits. You can specify the CL=EXACT plot-option to display exact confidence limits for the relative risks. You must also request computation of exact confidence limits by specifying the RELRISK option in the EXACT statement. When you specify the CL=EXACT plot-option, the relative risk plot does not display the common relative risk.

Table 40.17 lists the plot-options that are available for relative risk plots. See the subsection "Plot Options" for descriptions of the plot-options.

## RISKDIFFPLOT < (plot-options) >

requests a plot of risk (proportion) differences with confidence limits. Risk difference plots are available for multiway $2 \times 2$ tables and display the risk difference (with confidence limits) for each $2 \times 2$ table (stratum). To produce a risk difference plot, you must also specify the RISKDIFF option in the TABLES statement to compute risk differences.

By default, risk difference plots display asymptotic Wald confidence limits. You can specify the CL=EXACT plot-option to display exact confidence limits for the risk differences. You must also request computation of exact confidence limits by specifying the RISKDIFF option in the EXACT statement.

In addition to Wald and exact confidence limits, you can display the following confidence limit types in a risk difference plot: Agresti-Caffo, Hauck-Anderson, Miettinen-Nurminen (score), and Newcombe. See the descriptions of the CL=plot-option and the RISKDIFF option for more information.

By default or when you specify CL=WALD, the risk difference plot displays the common risk difference when PROC FREQ computes the common value. To compute the common risk difference for a multiway table, you can specify the RISKDIFF(COMMON) option in the TABLES statement. When you specify other confidence limit types, the risk difference plot does not display the common risk difference. You can suppress display of the common risk difference by specifying the COMMON=NO plot-option.

Table 40.17 lists the plot-options that are available for risk difference plots. See the subsection "Plot Options" for descriptions of the plot-options.

## WTKAPPAPLOT < (plot-options) >

requests a plot of weighted kappa coefficients with confidence limits. Weighted kappa plots are available for multiway square tables and display the weighted kappa coefficient (with confidence limits) for each two-way table (stratum). Weighted kappa plots also display the overall weighted kappa coefficient unless you specify the COMMON=NO plot-option.

To produce a weighted kappa plot, you must specify the AGREE option in the TABLES statement to compute weighted kappa coefficients, and the table dimension must be greater than 1.

Table 40.16 lists the plot-options that are available for weighted kappa plots. See the subsection "Plot Options" for descriptions of the plot-options.

## Global Plot Options

A global-plot-option applies to all plots for which the option is available unless it is altered by an individual plot-option. You can specify global-plot-options in parentheses after the PLOTS option. For example:

```
plots(order=ascending stats)=(riskdiffplot oddsratioplot)
plots(only)=freqplot
```

The following plot-options are available as global-plot-options: CLDISPLAY=, COLUMN=, COMMON=, EXACT, LOGBASE=, NPANELPOS=, ORDER=, ORIENT=, RANGE=, SCALE=, STATS, and TYPE=. See the subsection "Plot Options" for description of these plot-options.

In addition to these plot-options, you can specify the following global-plot-option:

## ONLY

suppresses the default plots and requests only the plots that are specified as plot-requests.

## Plot Options

You can specify the following plot-options in parentheses after a plot-request:

## CL=type

specifies the type of confidence limits to display. The $\mathrm{CL}=$ plot-option is available for the following plots: ODDSRATIOPLOT, RELRISKPLOT, and RISKDIFFPLOT.

For odds ratio plots, the available confidence limit types include exact, score, and Wald, which you can request by specifying $C L=E X A C T, C L=S C O R E$, and $C L=W A L D$, respectively. The default confidence limit type for odds ratio plots is CL=WALD. When you specify CL=EXACT to display exact confidence limits, you must also request computation of exact confidence limits by specifying the OR option in the EXACT statement. When you specify CL=SCORE, you must request computation of score confidence limits by specifying the OR(CL=SCORE) option in the TABLES statement.

For relative risk plots, the available confidence limit types include exact and Wald, which you can request by specifying $C L=E X A C T$ and $C L=W A L D$, respectively. The default is $C L=W A L D$. When you specify $\mathrm{CL}=\mathrm{EXACT}$ to display exact confidence limits, you must also request computation of exact confidence limits by specifying the RELRISK option in the EXACT statement.

For risk difference plots, the available confidence limit types include the following: $\mathrm{CL}=\mathrm{AC}$ (Agresti-Caffo), CL=EXACT (exact unconditional), CL=HA (Hauck-Anderson), CL=MN (Miettinen-Nurminen (score)), CL=NEWCOMBE (Newcombe), and CL=WALD (Wald). For more information, see the description of the $\mathrm{CL}=$ riskdiff-option and the section "Risk Difference Confidence Limits" on page 2727 . If you specify CL=EXACT to display exact confidence limits in the plot, you must also request computation of exact confidence limits by specifying the RISKDIFF option in the EXACT statement.

## CLDISPLAY=SERIF | SERIFARROW | LINE | LINEARROW | BAR < width >

controls the appearance of the confidence limit error bars. This plot-option is available for the following plots: KAPPAPLOT, ODDSRATIOPLOT, RELRISKPLOT, RISKDIFFPLOT, and WTKAPPAPLOT.

The default is CLDISPLAY=SERIF, which displays the confidence limits as lines with serifs. CLDISPLAY=LINE displays the confidence limits as plain lines without serifs. The CLDISPLAY=SERIFARROW and CLDISPLAY=LINEARROW plot-options display arrowheads on any error bars that are clipped by the RANGE= plot-option; if an entire error bar is cut from the plot, the plot displays an arrowhead that points toward the statistic.

CLDISPLAY=BAR displays the confidence limits as bars. By default, the width of the bars equals the size of the marker for the estimate. You can control the width of the bars and the size of the marker by specifying the value of width as a percentage of the distance between bars, $0<$ width $\leq 1$. The bar might disappear when the value of width is very small.

## COLUMN=1 | 2

specifies the table column to use to compute the risks (proportion) for the relative risk plot (RELRISKPLOT) and the risk difference plot (RISKDIFFPLOT). If you specify COLUMN=1, the plot displays the column 1 relative risks or the column 1 risk differences. Similarly, if you specify COLUMN=2, the plot displays the column 2 relative risks or risk differences.

For relative risk plots, the default is COLUMN=1. For risk difference plots, the default is COLUMN $=1$ if you request computation of both column 1 and column 2 risk differences by specifying the RISKDIFF option. If you request computation of only the column 1 (or column 2) risk differences by specifying the RISKDIFF(COLUMN=1) (or RISKDIFF(COLUMN=2)) option, by default the risk difference plot displays the risk differences for the column that you specify.

## COMMON=YES | NO

controls the display of the common (overall) statistic in plots that display stratum (two-way table) statistics for a multiway table. The COMMON= plot-option is available for the following plots: KAPPAPLOT, ODDSRATIOPLOT, RELRISKPLOT, RISKDIFFPLOT, and WTKAPPAPLOT.

By default, COMMON=YES and plots display the common value when PROC FREQ computes it. When you specify the AGREE option in the TABLES statement for a multiway table, PROC FREQ computes the common simple and weighted kappa coefficients, which can be displayed in the simple and weighted kappa plots, respectively. When you specify the MEASURES or RELRISK option in the TABLES statement, PROC FREQ computes only stratum-level odds ratios and relative risks; to compute common odds ratios and relative risks, you must also specify the CMH option in the TABLES statement. To compute the common risk difference, you must specify the RISKDIFF(COMMON) option. For more information, see the descriptions of the plot-requests.

## EXACT

requests display of exact confidence limits instead of asymptotic confidence limits. The EXACT plot-option is available for the odds ratio plot (ODDSRATIOPLOT), relative risk plot (RELRISKPLOT), and risk difference plot (RISKDIFFPLOT). The EXACT plot-option is equivalent to the CL=EXACT plot-option.

When you specify the EXACT plot-option, you must also request computation of exact confidence limits by specifying the appropriate statistic-option in the EXACT statement.

## GROUPBY=COLUMN | ROW

specifies the primary grouping for two-way frequency plots, which you can request by specifying the FREQPLOT plot=request. The default is GROUPBY=COLUMN, which groups graph cells first by column variable and displays row variable levels within column variable levels. You can specify GROUPBY=ROW to group first by row variable. In two-way and multiway table requests, the column variable is the last variable specified and forms the columns of the crosstabulation table. The row variable is the next-to-last variable specified and forms the rows of the table.

By default for a bar chart that is displayed in the TWOWAY=STACKED layout, bars correspond to the column variable levels, and row levels are displayed (stacked) within each column bar. By default for a bar chart that is displayed in the TWOWAY=CLUSTER layout, bars are first grouped by column variable levels, and row levels are displayed as adjacent bars within each column-level group. You can reverse the default row and column variable grouping by specifying GROUPBY=ROW.

## LOGBASE=2 | E | 10

applies to the odds ratio plot (ODDSRATIOPLOT) and the relative risk plot (RELRISKPLOT). This plot-option displays the odds ratio or relative risk axis on the log scale that you specify.

## LEGEND=YES | NO

applies to the agreement plot (AGREEPLOT). LEGEND=NO suppresses the legend that identifies the areas of exact and partial agreement. The default is LEGEND=YES.

## NOSTAT

applies to the deviation plot (DEVIATIONPLOT). NOSTAT suppresses the chi-square $p$-value that deviation plot displays by default.

## NPANELPOS=n

divides the plot into multiple panels that display at most $|n|$ statistics or sections.
If $n$ is positive, the number of statistics or sections per panel is balanced; if $n$ is negative, the number of statistics per panel is not balanced. For example, suppose you want to display 21 odds ratios. NPANELPOS=20 displays two panels, the first with 11 odds ratios and the second with 10 odds ratios; NPANELPOS $=-20$ displays 20 odds ratios in the first panel but only 1 odds ratio in the second panel. This plot-option is available for all plots except mosaic plots and one-way weighted frequency plots.

For two-way frequency plots (FREQPLOT), NPANELPOS $=n$ requests that panels display at most $|n|$ sections, where sections correspond to row or column variable levels, depending on the type of plot and the grouping. By default, $n=4$ and each panel includes at most four sections. This plot-option applies to two-way plots that are displayed in the TWOWAY=GROUPVERTICAL or TWOWAY=GROUPHORIZONTAL layout. The NPANELPOS= plot-option does not apply to the TWOWAY=CLUSTER and TWOWAY=STACKED layouts, which are always displayed in a single panel.

For plots that display statistics along with confidence limits, NPANELPOS=n requests that panels display at most $|n|$ statistics. By default, $n=0$ and all statistics are displayed in a single panel. This plot-option applies to the following plots: KAPPAPLOT, ORPLOT, RELRISKPLOT, RISKDIFFPLOT, and WTKAPPAPLOT.

## ORDER=ASCENDING | DESCENDING

displays the two-way table (strata) statistics in order of the statistic value. If you specify ORDER=ASCENDING or ORDER=DESCENDING, the plot displays the statistics in ascending or descending order, respectively. By default, the order of the statistics in the plot matches the order that the two-way table strata appear in the multiway table display.

This plot-option applies to the following plots: KAPPAPLOT, ODDSRATIOPLOT, RELRISKPLOT, RISKDIFFPLOT, and WTKAPPAPLOT.

## ORIENT=HORIZONTAL | VERTICAL

controls the orientation of the plot. The ORIENT= plot-option applies to the following plots: CUMFREQPLOT, DEVIATIONPLOT, and FREQPLOT.

ORIENT=HORIZONTAL places the variable levels on the Y axis and the frequencies, percentages, or statistic values on the X axis. ORIENT=VERTICAL places the variable levels on the X axis. The default orientation is ORIENT=VERTICAL for bar charts (TYPE=BARCHART) and ORIENT=HORIZONTAL for dot plots (TYPE=DOTPLOT).

## PARTIAL=YES | NO

controls the display of partial agreement in the agreement plot (AGREEPLOT). PARTIAL=NO suppresses the display of partial agreement. When you specify PARTIAL=NO, the agreement plot displays only exact agreement. Exact agreement includes the diagonal cells of the square table, where the row and column variable levels are the same. Partial agreement includes the adjacent off-diagonal table cells, where the row and column values are within one level of exact agreement. The default is PARTIAL=YES.

## RANGE=( < min > < , max > )| CLIP

specifies the range of values to display. If you specify RANGE=CLIP, the confidence limits are clipped and the display range is determined by the minimum and maximum values of the statistics. By default, the display range includes all confidence limits.

This plot-option is available for the following plots: KAPPAPLOT, ODDSRATIOPLOT, RELRISKPLOT, RISKDIFFPLOT, and WTKAPPAPLOT.

## SCALE=FREQ | GROUPPERCENT | LOG | PERCENT | SQRT

specifies the scale of the frequencies to display. This plot-option is available for frequency plots (FREQPLOT) and cumulative frequency plots (CUMFREQPLOT).

The default is SCALE=FREQ, which displays unscaled frequencies. SCALE=PERCENT displays percentages (relative frequencies) of the total frequency. SCALE=LOG displays log (base 10) frequencies. SCALE=SQRT displays square roots of the frequencies, producing a plot known as a rootogram.

SCALE=GROUPPERCENT is available for two-way frequency plots. This option displays the row or column percentages instead of the overall percentages (of the table frequency). By default (or when you specify the GROUPBY=COLUMN plot-option), SCALE=GROUPPERCENT displays the column percentages. If you specify the GROUPBY=ROW plot-option, the primary grouping of graph cells is by row variable level and the plot displays row percentages. For more information, see the description of the GROUPBY= plot-option.

## SHOWSCALE=YES | NO

controls the display of the cumulative frequency scale on the right side of the agreement plot (AGREEPLOT). SHOWSCALE=NO suppresses the display of the scale. The default is SHOWSCALE=YES.

## STATS

displays statistic values in the plot. For the following plots, the STATS plot-option displays the statistics and their confidence limits on the right side of the plot: KAPPAPLOT, ODDSRATIOPLOT, RELRISKPLOT, RISKDIFFPLOT, and WTKAPPAPLOT.

For the agreement plot (AGREEPLOT), STATS displays the values of the kappa statistic, the weighted kappa statistic, and the $B_{n}$ measure (Bangdiwala and Bryan 1987).

If you do not request the STATS plot-option, these plots do not display the statistic values.

## TWOWAY=CLUSTER | GROUPHORIZONTAL | GROUPVERTICAL | STACKED

specifies the layout for two-way frequency plots.
All TWOWAY= layouts are available for bar charts (TYPE=BARCHART). All TWOWAY= layouts except TWOWAY=CLUSTER are available for dot plots (TYPE=DOTPLOT). The ORIENT $=$ and GROUPBY = plot-options are available for all TWOWAY= layouts.

The default two-way layout is TWOWAY=GROUPVERTICAL, which produces a grouped plot that has a vertical common baseline. By default for bar charts (TYPE=BARCHART, ORIENT=VERTICAL), the X axis displays column variable levels, and the Y axis displays frequencies. The plot includes a vertical (Y-axis) block for each row variable level. The relative positions of the graph cells in this plot layout are the same as the relative positions of the table cells in the crosstabulation table. You can reverse the default row and column grouping by specifying the GROUPBY=ROW plot-option.
The TWOWAY=GROUPHORIZONTAL layout produces a grouped plot that has a horizontal common baseline. By default (GROUPBY=COLUMN), the plot displays a block on the X axis for each column variable level. Within each column-level block, the plot displays row variable levels.

The TWOWAY=STACKED layout produces stacked displays of frequencies. By default (GROUPBY=COLUMN) in a stacked bar chart, the bars correspond to column variable levels, and row levels are stacked within each column level. By default in a stacked dot plot, the dotted lines correspond to column levels, and cell frequencies are plotted as data dots on the corresponding column line. The dot color identifies the row level.
The TWOWAY=CLUSTER layout, which is available only for bar charts, displays groups of adjacent bars. By default, the primary grouping is by column variable level, and row levels are displayed within each column level.

You can reverse the default row and column grouping in any layout by specifying the GROUPBY=ROW plot-option. The default is GROUPBY=COLUMN, which groups first by column variable.

## TYPE=BARCHART | DOTPLOT

specifies the plot type (format) of the frequency (FREQPLOT), cumulative frequency (CUMFREQPLOT), and deviation plots (DEVIATIONPLOT). TYPE=BARCHART produces a bar chart and TYPE=DOTPLOT produces a dot plot. The default is TYPE=BARCHART.

## PRINTKWTS

displays the agreement weights that PROC FREQ uses to compute the weighted kappa coefficient. Agreement weights reflect the relative agreement between pairs of variable levels. By default, PROC FREQ uses the Cicchetti-Allison form of agreement weights. If you specify the AGREE(WT=FC) option, the procedure uses the Fleiss-Cohen form of agreement weights. For more information, see the section "Weighted Kappa Coefficient" on page 2745.

This option has no effect unless you also specify the AGREE option to compute the weighted kappa coefficient. The PRINTKWTS option is equivalent to the AGREE(PRINTKWTS) option.

## RELRISK

## OR

requests relative risk measures and their asymptotic Wald confidence limits for $2 \times 2$ tables. These measures include the odds ratio, the column 1 relative risk, and the column 2 relative risk. PROC FREQ displays the statistics and confidence limits in the "Odds Ratio and Relative Risks" table. For more information, see the section "Odds Ratio and Relative Risks for $2 \times 2$ Tables" on page 2737.

You can request exact confidence limits for the odds ratio by specifying the OR option in the EXACT statement. You can request exact unconditional confidence limits for the relative risks by specifying the RELRISK option in the EXACT statement. For more information, see the sections "Exact Confidence Limits for the Odds Ratio" on page 2738 and "Exact Unconditional Confidence Limits for the Relative Risk" on page 2739.

You can request score confidence limits for the odds ratio by specifying the $\mathrm{OR}(\mathrm{CL}=)$ option. This option produces the "Confidence Limits for the Odds Ratio" table, which can also display Wald and exact confidence limits. For more information, see the section "Score Confidence Limits for the Odds Ratio" on page 2738.

You can also obtain the relative risk measures by specifying the MEASURES option, which produces other measures of association in addition to the relative risks.

## RISKDIFF < (riskdiff-options) >

requests risks (binomial proportions) and risk differences for $2 \times 2$ tables. When you specify the RISKDIFF option, PROC FREQ provides the row 1 risk, row 2 risk, total (overall) risk, and risk difference (row 1 - row 2 ), together with their asymptotic standard errors and Wald confidence limits. PROC FREQ also provides exact (Clopper-Pearson) confidence limits for the row 1, row 2, and total risks by default. You can request exact unconditional confidence limits for the risk difference by specifying the RISKDIFF option in the EXACT statement. For more information, see the section "Risks and Risk Differences" on page 2725. PROC FREQ displays these results in the column 1 and column 2 "Risk Estimates" tables.

You can specify riskdiff-options in parentheses after the RISKDIFF option to request tests and additional confidence limits for the risk difference, as well as estimates of the common risk difference for multiway $2 \times 2$ tables. Table 40.18 summarizes the riskdiff-options.

The $\mathrm{CL}=$ riskdiff-option requests confidence limits for the risk difference. Available confidence limit types include Agresti-Caffo, exact unconditional, Hauck-Anderson, Miettinen-Nurminen (score), Newcombe, and Wald. Continuity-corrected Newcombe and Wald confidence limits are also available. You can request more than one type of confidence limits in the same analysis. PROC FREQ displays the confidence limits in the "Confidence Limits for the Proportion (Risk) Difference" table.

The CL=EXACT riskdiff-option displays exact unconditional confidence limits in the "Proportion (Risk) Difference Confidence Limits" table. When you use CL=EXACT, you must also request computation of the exact confidence limits by specifying the RISKDIFF option in the EXACT statement.
The EQUIV, NONINF, and SUP riskdiff-options request tests of equivalence, noninferiority, and superiority, respectively, for the risk difference. Available test methods include Farrington-Manning (score), Hauck-Anderson, and Newcombe (hybrid-score), in addition to the Wald test.

As part of the noninferiority, superiority, and equivalence analyses, PROC FREQ provides null-based equivalence limits that have a confidence coefficient of $100(1-2 \alpha) \%$ (Schuirmann 1999). The ALPHA $=$ option determines the confidence level, and the default of ALPHA $=0.05$ produces $90 \%$ equivalence limits. See the sections "Noninferiority Tests" on page 2731 and "Equivalence Tests" on page 2733 for details.

Table 40.18 RISKDIFF (Proportion Difference) Options

| Option | Description |
| :--- | :--- |
| COLUMN=1।2 | Specifies the risk column |
| COMMON | Requests common risk difference |
| CORRECT | Requests continuity correction |
| NORISKS | Suppresses default risk tables |
| Request Confidence Limits |  |
| CL=AC | Requests Agresti-Caffo confidence limits |
| CL=EXACT | Displays exact confidence limits |
| CL=HA | Requests Hauck-Anderson confidence limits |
| CL=MN I SCORE | Requests Miettinen-Nurminen confidence limits |
| CL=NEWCOMBE | Requests Newcombe confidence limits |
| CL=WALD | Requests Wald confidence limits |
| Request Tests |  |
| EQUAL | Requests an equality test |
| EQUIV I EQUIVALENCE | Requests an equivalence test |
| NONINF I NONINFERIORITY | Requests a noninferiority test |
| SUP I SUPERIORITY | Requests a superiority test |
| MARGIN= | Specifies the test margin |
| METHOD= | Specifies the test method |
| VAR=SAMPLE I NULL | Specifies the test variance |

You can specify the following riskdiff-options in parentheses after the RISKDIFF option:

## CL=type | (types)

requests confidence limits for the risk difference. You can specify one or more types of confidence limits. When you specify only one type, you can omit the parentheses around the request. PROC FREQ displays the confidence limits in the "Proportion (Risk) Difference Confidence Limits" table.

The ALPHA= option determines the confidence level, and the default of ALPHA $=0.05$ produces $95 \%$ confidence limits for the risk difference. This differs from the test-based confidence limits that are provided with the equivalence, noninferiority, and superiority tests (EQUIV, NONINF, and SUP), which have a confidence coefficient of $100(1-2 \alpha) \%$ (Schuirmann 1999).

You can specify $\mathrm{CL}=$ with or without requests for risk difference tests. The confidence limits produced by CL= do not depend on the tests that you request and do not use the value of the test margin (which you specify by using MARGIN= riskdiff-option).

You can control the risk column for the confidence limits by specifying the COLUMN= riskdiffoption. If you do not specify COLUMN=, PROC FREQ provides confidence limits for the column 1 risk difference by default.

The following types of confidence limits are available:

## AGRESTICAFFO

AC
requests Agresti-Caffo confidence limits for the risk difference. See the subsection "AgrestiCaffo Confidence Limits" in the section "Risk Difference Confidence Limits" on page 2727 for details.

## EXACT

displays exact unconditional confidence limits for the risk difference in the "Proportion (Risk) Difference Confidence Limits" table. You must also request computation of the exact confidence limits by specifying the RISKDIFF option in the EXACT statement.

PROC FREQ computes the confidence limits by inverting two separate one-sided exact tests (tail method), where the tests are based on the unstandardized risk difference by default. If you specify the RISKDIFF(METHOD=SCORE) option in the EXACT statement, the tests are based on the score statistic. See the RISKDIFF option in the EXACT statement and the section "Exact Unconditional Confidence Limits for the Risk Difference" on page 2734 for more information.

By default, PROC FREQ also displays these exact confidence limits in the "Risk Estimates" table. You can suppress this table by specifying the NORISKS riskdiff-option.

## HA

requests Hauck-Anderson confidence limits for the risk difference. See the subsection "Hauck-Anderson Confidence Limits" in the section "Risk Difference Confidence Limits" on page 2727 for details.

## MN < (CORRECT=NO | MEE) >

SCORE < (CORRECT=NO | MEE) >
requests Miettinen-Nurminen (score) confidence limits for the risk difference. For more information, see the subsection "Miettinen-Nurminen (Score) Confidence Limits" in the section "Risk Difference Confidence Limits" on page 2727. By default, the MiettinenNurminen confidence limits include a bias correction factor (Miettinen and Nurminen 1985; Newcombe and Nurminen 2011). If you specify CL=MN(CORRECT=NO), PROC FREQ provides the uncorrected form of the confidence limits (Mee 1984).

NEWCOMBE < (CORRECT) >
requests Newcombe hybrid-score confidence limits for the risk difference. If you specify CL=NEWCOMBE(CORRECT) or the CORRECT riskdiff-option, the Newcombe confidence limits include a continuity correction. See the subsection "Newcombe Confidence Limits" in the section "Risk Difference Confidence Limits" on page 2727 for details.

WALD $<$ (CORRECT) $>$
requests Wald confidence limits for the risk difference. If you specify CL=WALD(CORRECT) or the CORRECT riskdiff-option, the Wald confidence limits include a continuity correction. See the subsection "Wald Confidence Limits" in the section "Risk Difference Confidence Limits" on page 2727 for details.

## COLUMN=1 | 2 | BOTH

specifies the table column for which to compute the risk difference tests (EQUAL, EQUIV, NONINF, and SUP) and the risk difference confidence limits (which are requested by the CL= riskdiff-option).

If you do not specify COLUMN=, PROC FREQ provides the risk difference tests and confidence limits for column 1 by default. The COLUMN= option has no effect on the "Risk Estimates" table, which is produced for both column 1 and column 2. You can suppress the "Risk Estimates" table by specifying the NORISKS riskdiff-option.

## COMMON

requests estimates of the common (overall) risk difference for multiway $2 \times 2$ tables. PROC FREQ produces Mantel-Haenszel and summary score estimates for the common risk difference, together with their confidence limits. For more information, see the section "Common Risk Difference" on page 2736.

If you do not specify the COLUMN= riskdiff-option, PROC FREQ provides the common risk difference for column 1 by default. If you specify COLUMN=2, PROC FREQ provides the common risk difference for column 2. COLUMN=BOTH does not apply to the common risk difference.

## CORRECT

includes a continuity correction in the Wald confidence limits, Wald tests, and Newcombe confidence limits. See the section "Risks and Risk Differences" on page 2725 for details.

## EQUAL

requests a test of the null hypothesis that the risk difference equals zero. PROC FREQ provides an asymptotic Wald test of equality. If you specify the CORRECT riskdiff-option, the Wald test includes a continuity correction. If you specify the VAR=NULL riskdiff-option, the test uses the null (test-based) variance instead of the sample variance. See the section "Equality Test" on page 2730 for details.

## EQUIV

## EQUIVALENCE

requests a test of equivalence for the risk difference. See the section "Equivalence Tests" on page 2733 for details. You can specify the equivalence test margins with the MARGIN= riskdiff-option and the test method with the METHOD= riskdiff-option. PROC FREQ uses METHOD=WALD by default.

## MARGIN=value | (lower, upper)

specifies the margin for the noninferiority, superiority, and equivalence tests, which you request with the NONINF, SUP, and EQUIV riskdiff-options, respectively. If you do not specify MARGIN=, PROC FREQ uses a margin of 0.2 by default.

For noninferiority and superiority tests, specify a single value for MARGIN=. The MARGIN= value must be a positive number. You can specify value as a number between 0 and 1 . Or you can specify value in percentage form as a number between 1 and 100, and PROC FREQ converts that number to a proportion. The procedure treats the value 1 as $1 \%$.

For an equivalence test, you can specify a single MARGIN= value, or you can specify both lower and upper values. If you specify a single MARGIN= value, it must be a positive number, as described previously. If you specify a single MARGIN= value for an equivalence test, PROC FREQ uses -value as the lower margin and value as the upper margin for the test. If you specify both lower and upper values for an equivalence test, you can specify them in proportion form as numbers between -1 or 1 . Or you can specify them in percentage form as numbers between -100 and 100, and PROC FREQ converts the numbers to proportions. The value of lower must be less than the value of upper.

## METHOD=method

specifies the method for the noninferiority, superiority, and equivalence analyses, which you request with the NONINF, SUP, and EQUIV riskdiff-options, respectively. If you do not specify the METHOD= riskdiff-option, PROC FREQ uses METHOD=WALD by default.

The following methods are available:

## FM

SCORE
requests Farrington-Manning (score) tests and equivalence limits for the equivalence, noninferiority, and superiority analyses. See the subsection "Farrington-Manning (Score) Test" in the section "Noninferiority Tests" on page 2731 for details.

HA
requests Hauck-Anderson tests and confidence limits for the equivalence, noninferiority, and superiority analyses. See the subsection "Hauck-Anderson Test" in the section "Noninferiority Tests" on page 2731 for details.

## NEWCOMBE

requests Newcombe (hybrid-score) confidence limits for the equivalence, noninferiority, and superiority analyses. If you specify the CORRECT riskdiff-option, the Newcombe confidence limits include a continuity correction. See the subsection "Newcombe Noninferiority Analysis" in the section "Noninferiority Tests" on page 2731 for details.

## WALD

requests Wald tests and confidence limits for the equivalence, noninferiority, and superiority analyses. If you specify the CORRECT riskdiff-option, the Wald confidence limits include a continuity correction. If you specify the VAR=NULL riskdiff-option, the tests and confidence limits use the null (test-based) variance instead of the sample variance. See the subsection "Wald Test" in the section "Noninferiority Tests" on page 2731 for details.

## NONINF

## NONINFERIORITY

requests a test of noninferiority for the risk difference. See the section "Noninferiority Tests" on page 2731 for details. You can specify the test margin with the MARGIN= riskdiff-option and the test method with the METHOD= riskdiff-option. PROC FREQ uses METHOD=WALD by default.

## NORISKS

suppresses display of the "Risk Estimates" tables, which the RISKDIFF option produces by default for column 1 and column 2. The "Risk Estimates" tables contain the risks and risk differences, together with their asymptotic standard errors, Wald confidence limits, and exact confidence limits.

## SUP

## SUPERIORITY

requests a test of superiority for the binomial proportion. See the section "Superiority Test" on page 2733 for details. You can specify the test margin with the MARGIN= riskdiff-option and the test method with the METHOD= riskdiff-option. PROC FREQ uses METHOD=WALD by default.

## VAR=SAMPLE | NULL

specifies the type of variance estimate to use in the Wald tests of noninferiority, superiority, equivalence, and equality. The default is VAR=SAMPLE, which uses the sample variance. VAR=NULL uses a test-based variance that is computed by using the null hypothesis risk difference value. See the sections "Equality Test" on page 2730 and "Noninferiority Tests" on page 2731 for details.

## SCORES=type

specifies the type of row and column scores that PROC FREQ uses to compute the following statistics: Mantel-Haenszel chi-square, Pearson correlation, Cochran-Armitage test for trend, weighted kappa coefficient, and Cochran-Mantel-Haenszel statistics. The value of type can be one of the following:

## - MODRIDIT

- RANK
- RIDIT
- TABLE

See the section "Scores" on page 2703 for descriptions of these score types.
If you do not specify the SCORES= option, PROC FREQ uses SCORES=TABLE by default. For character variables, the row and column TABLE scores are the row and column numbers. That is, the TABLE score is 1 for row 1,2 for row 2 , and so on. For numeric variables, the row and column TABLE scores equal the variable values. See the section "Scores" on page 2703 for details. Using MODRIDIT, RANK, or RIDIT scores yields nonparametric analyses.

You can use the SCOROUT option to display the row and column scores.

## SCOROUT

displays the row and column scores that PROC FREQ uses to compute score-based tests and statistics. You can specify the score type with the SCORES= option. See the section "Scores" on page 2703 for details.

The scores are computed and displayed only when PROC FREQ computes statistics for two-way tables. You can use ODS to store the scores in an output data set. See the section "ODS Table Names" on page 2773 for more information.

## SPARSE

reports all possible combinations of the variable values for an $n$-way table when $n>1$, even if a combination does not occur in the data. The SPARSE option applies only to crosstabulation tables displayed in LIST format and to the OUT= output data set. If you do not use the LIST or OUT= option, the SPARSE option has no effect.

When you specify the SPARSE and LIST options, PROC FREQ displays all combinations of variable values in the table listing, including those with a frequency count of zero. By default, without the SPARSE option, PROC FREQ does not display zero-frequency levels in LIST output. When you use the SPARSE and OUT= options, PROC FREQ includes empty crosstabulation table cells in the output data set. By default, PROC FREQ does not include zero-frequency table cells in the output data set.

See the section "Missing Values" on page 2699 for more information.

## TOTPCT

displays the percentage of the total multiway table frequency in crosstabulation tables for $n$-way tables, where $n>2$. By default, PROC FREQ displays the percentage of the individual two-way table frequency but does not display the percentage of the total frequency for multiway crosstabulation tables. See the section "Two-Way and Multiway Tables" on page 2767 for more information.

The percentage of total multiway table frequency is displayed by default when you specify the LIST option. It is also provided by default in the PERCENT variable in the OUT= output data set.

## TREND

requests the Cochran-Armitage test for trend. The table must be $2 \times C$ or $R \times 2$ to compute the trend test. See the section "Cochran-Armitage Test for Trend" on page 2740 for details. To request exact $p$-values for the trend test, specify the TREND option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## TEST Statement

## TEST test-options ;

The TEST statement requests asymptotic tests for measures of association and measures of agreement. The test-options identify which tests to compute. Table 40.19 lists the available test-options, together with their corresponding TABLES statement options. Descriptions of the test-options follow the table in alphabetical order.

For each measure of association or agreement that you request in the TEST statement, PROC FREQ provides an asymptotic test that the measure equals zero. The procedure displays the asymptotic standard error under the null hypothesis, the test statistic, and the one-sided and two-sided $p$-values. Additionally, PROC FREQ reports the confidence limits for the measure. The ALPHA= option in the TABLES statement determines the confidence level, which by default equals 0.05 and provides $95 \%$ confidence limits. See the sections "Asymptotic Tests" on page 2710 and "Confidence Limits" on page 2710 for details. Also see the sections "Measures of Association" on page 2709 and "Tests and Measures of Agreement" on page 2743 for information about the individual measures.

You can also request exact tests for selected measures of association and agreement by using the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Using the TEST Statement with the TABLES Statement

You must use a TABLES statement with the TEST statement. If you use only one TABLES statement, you do not need to specify the same options in both the TABLES and TEST statements; when you specify an option in the TEST statement, PROC FREQ automatically invokes the corresponding TABLES statement option. However, when you use the TEST statement with multiple TABLES statements, you must specify options in the TABLES statements to request statistics. PROC FREQ then provides asymptotic tests for those statistics that you also specify in the TEST statement.

Table 40.19 TEST Statement Options

| Test Option | Asymptotic Tests | Required TABLES <br> Statement Option |
| :--- | :--- | :--- |
| AGREE | Simple and weighted kappa coefficients | AGREE |
| GAMMA | Gamma | ALL or MEASURES |
| KAPPA | Simple kappa coefficient | AGREE |
| KENTB \| TAUB | Kendall's tau- $b$ | ALL or MEASURES |
| MEASURES | Gamma, Kendall's tau- $b$, Stuart's tau- $c$, | ALL or MEASURES |
|  | Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, |  |
|  | Pearson and Spearman correlations |  |
| PCORR | Pearson correlation coefficient | ALL or MEASURES |
| PLCORR | Polychoric correlation | PLCORR |
| SCORR | Spearman correlation coefficient | ALL or MEASURES |
| SMDCR | Somers' $D(C \mid R)$ | ALL or MEASURES |
| SMDRC | Somers' $D(R \mid C)$ | ALL or MEASURES |
| STUTC I TAUC | Stuart's tau- $c$ | ALL or MEASURES |
| WTKAP \| WTKAPPA | Weighted kappa coefficient | AGREE |

You can specify the following test-options in the TEST statement.

## AGREE

requests asymptotic tests for the simple kappa coefficient and the weighted kappa coefficient. See the sections "Simple Kappa Coefficient" on page 2744 and "Weighted Kappa Coefficient" on page 2745 for details.

The AGREE option in the TABLES statement provides estimates, standard errors, and confidence limits for kappa coefficients. You can request exact tests for kappa coefficients by using the EXACT statement.

Kappa coefficients are defined only for square tables, where the number of rows equals the number of columns. Kappa coefficients are not computed for tables that are not square. For $2 \times 2$ tables, the weighted kappa coefficient is identical to the simple kappa coefficient, and PROC FREQ presents only the simple kappa coefficient.

## GAMMA

requests an asymptotic test for the gamma statistic. See the section "Gamma" on page 2711 for details. The MEASURES option in the TABLES statement provides the gamma statistic and its asymptotic standard error.

## KAPPA

requests an asymptotic test for the simple kappa coefficient. See the section "Simple Kappa Coefficient" on page 2744 for details.

The AGREE option in the TABLES statement provides the kappa statistic, its standard error, and its confidence limits. You can request an exact test for the simple kappa coefficient by specifying the KAPPA option in the EXACT statement.

Kappa coefficients are defined only for square tables, where the number of rows equals the number of columns. PROC FREQ does not compute kappa coefficients for tables that are not square.

## KENTB

TAUB
requests an asymptotic test for Kendall's tau-b. See the section "Kendall's Tau-b" on page 2711 for details.

The MEASURES option in the TABLES statement provides Kendall's tau- $b$ and its standard error. You can request an exact test for Kendall's tau-b by specifying the KENTB option in the EXACT statement.

## MEASURES

requests asymptotic tests for the following measures of association: gamma, Kendall's tau- $b$, Pearson correlation coefficient, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, Spearman correlation coefficient, and Stuart's tau-c. See the section "Measures of Association" on page 2709 for details.

The MEASURES option in the TABLES statement provides measures of association and their asymptotic standard errors. You can request exact tests for selected measures by using the EXACT statement.

## PCORR

requests an asymptotic test for the Pearson correlation coefficient. See the section "Pearson Correlation Coefficient" on page 2713 for details.

The MEASURES option in the TABLES statement provides the Pearson correlation and its standard error. You can request an exact test for the Pearson correlation by specifying the PCORR option in the EXACT statement.

## PLCORR

requests Wald and likelihood ratio tests for the polychoric correlation coefficient. See the section "Polychoric Correlation" on page 2715 for details.

The PLCORR option in the TABLES statement provides the polychoric correlation and its standard error.

## SCORR

requests an asymptotic test for the Spearman correlation coefficient. See the section "Spearman Rank Correlation Coefficient" on page 2714 for details.

The MEASURES option in the TABLES statement provides the Spearman correlation and its standard error. You can request an exact test for the Spearman correlation by specifying the SCORR option in the EXACT statement.

## SMDCR

requests an asymptotic test for Somers' $D(C \mid R)$. See the section "Somers' $D$ " on page 2712 for details.

The MEASURES option in the TABLES statement provides Somers' $D(C \mid R)$ and its standard error. You can request an exact test for Somers' $D(C \mid R)$ by specifying the SMDCR option in the EXACT statement.

## SMDRC

requests an asymptotic test for Somers' $D(R \mid C)$. See the section "Somers' $D$ " on page 2712 for details.

The MEASURES option in the TABLES statement provides Somers' $D(R \mid C)$ and its standard error. You can request an exact test for Somers' $D(R \mid C)$ by specifying the SMDRC option in the EXACT statement.

## STUTC

TAUC
requests an asymptotic test for Stuart's tau-c. See the section "Stuart's Tau-c" on page 2712 for details.
The MEASURES option in the TABLES statement provides Stuart's tau-c and its standard error. You can request an exact test for Stuart's tau- $c$ by specifying the STUTC option in the EXACT statement.

## WTKAP

WTKAPPA
requests an asymptotic test for the weighted kappa coefficient. See the section "Weighted Kappa Coefficient" on page 2745 for details.

The AGREE option in the TABLES statement provides the weighted kappa coefficient, its standard error, and confidence limits. You can request an exact test for the weighted kappa by specifying the WTKAP option in the EXACT statement.

Kappa coefficients are defined only for square tables, where the number of rows equals the number of columns. PROC FREQ does not compute kappa coefficients for tables that are not square. For $2 \times 2$ tables, the weighted kappa coefficient is identical to the simple kappa coefficient, and PROC FREQ presents only the simple kappa coefficient.

## WEIGHT Statement

WEIGHT variable < / option > ;
The WEIGHT statement names a numeric variable that provides a weight for each observation in the input data set. The WEIGHT statement is most commonly used to input cell count data. See the section "Inputting Frequency Counts" on page 2697 for more information. If you use a WEIGHT statement, PROC FREQ assumes that an observation represents $n$ observations, where $n$ is the value of variable. The value of the WEIGHT variable is not required to be an integer.
If the value of the WEIGHT variable is missing, PROC FREQ does not use that observation in the analysis. If the value of the WEIGHT variable is zero, PROC FREQ ignores the observation unless you specify the ZEROS option, which includes observations with zero weights. If you do not specify a WEIGHT statement,

PROC FREQ assigns a weight of one to each observation. The sum of the WEIGHT variable values represents the total number of observations.

If any value of the WEIGHT variable is negative, PROC FREQ displays the frequencies computed from the weighted values but does not compute percentages and statistics. If you create an output data set by using the OUT= option in the TABLES statement, PROC FREQ assigns missing values to the PERCENT variable. PROC FREQ also assigns missing values to the variables that the OUTEXPECT and OUTPCT options provide. If any value of the WEIGHT variable is negative, you cannot create an output data set by using the OUTPUT statement because statistics are not computed when there are negative weights.

You can specify the following option in the WEIGHT statement:

## ZEROS

includes observations with zero weight values. By default, PROC FREQ ignores observations with zero weights.
If you specify the ZEROS option, frequency and and crosstabulation tables display any levels corresponding to observations with zero weights. Without the ZEROS option, PROC FREQ does not process observations with zero weights, and so does not display levels that contain only observations with zero weights.
With the ZEROS option, PROC FREQ includes levels with zero weights in the chi-square goodness-of-fit test for one-way tables. Also, PROC FREQ includes any levels with zero weights in binomial computations for one-way tables. This makes it possible to compute binomial tests and estimates when the specified level contains no observations with positive weights.

For two-way tables, the ZEROS option enables computation of kappa statistics when there are levels that contain no observations with positive weight. For more information, see the section "Tables with Zero Rows and Columns" on page 2747.

Note that even with the ZEROS option, PROC FREQ does not compute the CHISQ or MEASURES statistics for two-way tables when the table has a zero row or zero column because most of these statistics are undefined in this case.

The ZEROS option invokes the SPARSE option in the TABLES statement, which includes table cells with a zero frequency count in the LIST output and in the OUT= data set. By default, without the SPARSE option, PROC FREQ does not include zero frequency cells in the LIST output or in the OUT= data set. If you specify the ZEROS option in the WEIGHT statement but do not want the SPARSE option, you can specify the NOSPARSE option in the TABLES statement.

## Details: FREQ Procedure

## Inputting Frequency Counts

PROC FREQ can use either raw data or cell count data to produce frequency and crosstabulation tables. Raw data, also known as case-record data, report the data as one record for each subject or sample member. Cell count data report the data as a table, listing all possible combinations of data values along with the frequency counts. This way of presenting data often appears in published results.

The following DATA step statements store raw data in a SAS data set:

```
data Raw;
    input Subject $ R C @@;
    datalines;
01 1.1}1
06 1 2 2 07 1 2 0 08 1 2 0 09 2 1 1 10
```



```
;
```

You can store the same data as cell counts by using the following DATA step statements:

```
data CellCounts;
    input R C Count @@;
    datalines;
1 1 5 1 2 3
2 14 2 2 3
;
```

The variable $R$ contains the values for the rows, and the variable $C$ contains the values for the columns. The variable Count contains the cell count for each row and column combination.

Both the Raw data set and the CellCounts data set produce identical frequency counts, two-way tables, and statistics. When using the CellCounts data set, you must include a WEIGHT statement to specify that the variable Count contains cell counts. For example, the following PROC FREQ statements create a two-way crosstabulation table by using the CellCounts data set:

```
proc freq data=CellCounts;
    tables R*C;
    weight Count;
run;
```


## Grouping with Formats

PROC FREQ groups a variable's values according to its formatted values. If you assign a format to a variable with a FORMAT statement, PROC FREQ formats the variable values before dividing observations into the levels of a frequency or crosstabulation table.

For example, suppose that variable $X$ has the values $1.1,1.4,1.7,2.1$, and 2.3. Each of these values appears as a level in the frequency table. If you decide to round each value to a single digit, include the following statement in the PROC FREQ step:

```
format X 1.;
```

Now the table lists the frequency count for formatted level 1 as two and for formatted level 2 as three.
PROC FREQ treats formatted character variables in the same way. The formatted values are used to group the observations into the levels of a frequency table or crosstabulation table. PROC FREQ uses the entire value of a character format to classify an observation.

You can also use the FORMAT statement to assign formats that were created with the FORMAT procedure to the variables. User-written formats determine the number of levels for a variable and provide labels for a
table. If you use the same data with different formats, then you can produce frequency counts and statistics for different classifications of the variable values.

When you use PROC FORMAT to create a user-written format that combines missing and nonmissing values into one category, PROC FREQ treats the entire category of formatted values as missing. For example, a questionnaire codes 1 as yes, 2 as no, and 8 as a no answer. The following PROC FORMAT statements create a user-written format:

```
proc format;
    value Questfmt 1 ='Yes'
        2 ='NO'
        8,. ='Missing';
run;
```

When you use a FORMAT statement to assign Questfmt. to a variable, the variable's frequency table no longer includes a frequency count for the response of 8 . You must use the MISSING or MISSPRINT option in the TABLES statement to list the frequency for no answer. The frequency count for this level includes observations with either a value of 8 or a missing value (.).

The frequency or crosstabulation table lists the values of both character and numeric variables in ascending order based on internal (unformatted) variable values unless you change the order with the ORDER= option. To list the values in ascending order by formatted values, use ORDER=FORMATTED in the PROC FREQ statement.

For more information about the FORMAT statement, see SAS Formats and Informats: Reference.

## Missing Values

When the value of the WEIGHT variable is missing, PROC FREQ does not include that observation in the analysis.

PROC FREQ treats missing BY variable values like any other BY variable value. The missing values form a separate BY group.
If an observation has a missing value for a variable in a TABLES request, by default PROC FREQ does not include that observation in the frequency or crosstabulation table. Also by default, PROC FREQ does not include observations with missing values in the computation of percentages and statistics. The procedure displays the number of missing observations below each table.

PROC FREQ also reports the number of missing values in output data sets. The TABLES statement OUT= data set includes an observation that contains the missing value frequency. The NMISS option in the OUTPUT statement provides an output data set variable that contains the missing value frequency.
The following options change the way in which PROC FREQ handles missing values of TABLES variables:
MISSPRINT displays missing value frequencies in frequency or crosstabulation tables but does not include them in computations of percentages or statistics.
MISSING treats missing values as a valid nonmissing level for all TABLES variables. Displays missing levels in frequency and crosstabulation tables and includes them in computations of percentages and statistics.

This example shows the three ways that PROC FREQ can handle missing values of TABLES variables. The following DATA step statements create a data set with a missing value for the variable A:

```
data one;
    input A Freq;
    datalines;
12
2 2
-2
;
```

The following PROC FREQ statements request a one-way frequency table for the variable $A$. The first request does not specify a missing value option. The second request specifies the MISSPRINT option in the TABLES statement. The third request specifies the MISSING option in the TABLES statement.

```
proc freq data=one;
    tables A;
    weight Freq;
    title 'Default';
run;
proc freq data=one;
    tables A / missprint;
    weight Freq;
    title 'MISSPRINT Option';
run;
proc freq data=one;
    tables A / missing;
    weight Freq;
    title 'MISSING Option';
run;
```

Figure 40.12 displays the frequency tables produced by this example. The first table shows PROC FREQ's default behavior for handling missing values. The observation with a missing value of the TABLES variable A is not included in the table, and the frequency of missing values is displayed below the table. The second table, for which the MISSPRINT option is specified, displays the missing observation but does not include its frequency when computing the total frequency and percentages. The third table shows that PROC FREQ treats the missing level as a valid nonmissing level when the MISSING option is specified. The table displays the missing level, and PROC FREQ includes this level when computing frequencies and percentages.

Figure 40.12 Missing Values in Frequency Tables


Figure 40.12 continued


When a combination of variable values for a two-way table is missing, PROC FREQ assigns zero to the frequency count for the table cell. By default, PROC FREQ does not display missing combinations in LIST format. Also, PROC FREQ does not include missing combinations in the OUT= output data set by default. To include missing combinations, you can specify the SPARSE option with the LIST or OUT= option in the TABLES statement.

## In-Database Computation

The FREQ procedure can use in-database computation to construct frequency and crosstabulation tables when the DATA= input data set is stored as a table in a supported database management system (DBMS). Supported databases include Teradata, DB2 under UNIX, and Oracle. In-database computation can provide the advantages of faster processing and reduced data transfer between the database and SAS software. For information about in-database computation, see the section "In-Database Procedures" in SAS/ACCESS for Relational Databases: Reference.

PROC FREQ performs in-database computation by using SQL implicit pass-through. The procedure generates SQL queries that are based on the tables that you request in the TABLES statement. The database executes these SQL queries to construct initial summary tables, which are then transmitted to PROC FREQ. The procedure uses this summary information to perform the remaining analyses and tasks in the usual way (out of the database). So instead of transferring the entire data set over the network between the database and SAS software, the in-database method transfers only the summary tables. This can substantially reduce processing time when the dimensions of the summary tables (in terms of rows and columns) are much smaller than
the dimensions of the entire database table (in terms of individual observations). Additionally, in-database summarization uses efficient parallel processing, which can also provide performance advantages.

In-database computation is controlled by the SQLGENERATION option, which you can specify in either a LIBNAME statement or an OPTIONS statement. See the section "In-Database Procedures" in SAS/ACCESS for Relational Databases: Reference for details about the SQLGENERATION option and other options that affect in-database computation. By default, PROC FREQ uses in-database computation when possible. There are no FREQ procedure options that control in-database computation.

PROC FREQ uses formatted values to group observations into the levels of frequency and crosstabulation tables. See the section "Grouping with Formats" on page 2698 for more information. If formats are available in the database, then in-database summarization uses the formats. If formats are not available in the database, then in-database summarization is based on the raw data values, and PROC FREQ performs the final, formatted classification (out of the database). For more information, see the section "Deploying and Using SAS Formats in Teradata" in SAS/ACCESS for Relational Databases: Reference.

The order of observations is not inherently defined for DBMS tables. The following options relate to the order of observations and therefore should not be specified for PROC FREQ in-database computation:

- If you specify the FIRSTOBS = or OBS= data set option, PROC FREQ does not perform in-database computation.
- If you specify the NOTSORTED option in the BY statement, PROC FREQ in-database computation ignores it and uses the default ASCENDING order for BY variables.
- If you specify the ORDER=DATA option for input data in a DBMS table, PROC FREQ computation might produce different results for separate runs of the same analysis. In addition to determining the order of variable levels in crosstabulation table displays, the ORDER= option can also affect the values of many of the test statistics and measures that PROC FREQ computes.


## Statistical Computations

## Definitions and Notation

A two-way table represents the crosstabulation of row variable X and column variable Y . Let the table row values or levels be denoted by $X_{i}, i=1,2, \ldots, R$, and the column values by $Y_{j}, j=1,2, \ldots, C$. Let $n_{i j}$ denote the frequency of the table cell in the $i$ th row and $j$ th column and define the following notation:

$$
\begin{aligned}
n_{i \cdot} & =\sum_{j} n_{i j} & & (\text { row totals }) \\
n_{\cdot j} & =\sum_{i} n_{i j} & & (\text { column totals }) \\
n & =\sum_{i} \sum_{j} n_{i j} & & \text { (overall total) } \\
p_{i j} & =n_{i j} / n & & \text { (cell percentages) } \\
p_{i \cdot} & =n_{i \cdot / n} & & \text { (row percentages of total) } \\
p \cdot j & =n \cdot j / n & & \text { (column percentages of total) }
\end{aligned}
$$

$$
\begin{aligned}
R_{i} & =\text { score for row } i \\
C_{j} & =\text { score for column } j \\
\bar{R} & =\sum_{i} n_{i} \cdot R_{i} / n \quad \text { (average row score) } \\
\bar{C} & =\sum_{j} n_{\cdot j} C_{j} / n \quad \text { (average column score) } \\
A_{i j} & =\sum_{k>i} \sum_{l>j} n_{k l}+\sum_{k<i} \sum_{l<j} n_{k l} \\
D_{i j} & =\sum_{k>i} \sum_{l<j} n_{k l}+\sum_{k<i} \sum_{l>j} n_{k l} \\
P & =\sum_{i} \sum_{j} n_{i j} A_{i j} \quad \text { (twice the number of concordances) } \\
Q & =\sum_{i} \sum_{j} n_{i j} D_{i j} \quad \text { (twice the number of discordances) }
\end{aligned}
$$

## Scores

PROC FREQ uses scores of the variable values to compute the Mantel-Haenszel chi-square, Pearson correlation, Cochran-Armitage test for trend, weighted kappa coefficient, and Cochran-Mantel-Haenszel statistics. The SCORES = option in the TABLES statement specifies the score type that PROC FREQ uses. The available score types are TABLE, RANK, RIDIT, and MODRIDIT scores. The default score type is TABLE. Using MODRIDIT, RANK, or RIDIT scores yields nonparametric analyses.

For numeric variables, table scores are the values of the row and column levels. If the row or column variable is formatted, then the table score is the internal numeric value corresponding to that level. If two or more numeric values are classified into the same formatted level, then the internal numeric value for that level is the smallest of these values. For character variables, table scores are defined as the row numbers and column numbers (that is, 1 for the first row, 2 for the second row, and so on).
Rank scores, which you request with the SCORES=RANK option, are defined as

$$
\begin{aligned}
R_{i}^{1} & =\sum_{k<i} n_{k} \cdot+\left(n_{i} \cdot+1\right) / 2
\end{aligned} \quad i=1,2, \ldots, R, \text { } \quad j=1,2, \ldots, C
$$

where $R_{i}^{1}$ is the rank score of row $i$, and $C_{j}^{1}$ is the rank score of column $j$. Note that rank scores yield midranks for tied values.

Ridit scores, which you request with the SCORES=RIDIT option, are defined as rank scores standardized by the sample size (Bross 1958; Mack and Skillings 1980). Ridit scores are derived from the rank scores as

$$
\begin{array}{ll}
R_{i}^{2}=R_{i}^{1} / n & i=1,2, \ldots, R \\
C_{j}^{2}=C_{j}^{1} / n & j=1,2, \ldots, C
\end{array}
$$

Modified ridit scores (SCORES=MODRIDIT) represent the expected values of the order statistics of the uniform distribution on ( 0,1 ) (van Elteren 1960; Lehmann and D'Abrera 2006). Modified ridit scores are
derived from rank scores as

$$
\begin{array}{ll}
R_{i}^{3}=R_{i}^{1} /(n+1) & i=1,2, \ldots, R \\
C_{j}^{3}=C_{j}^{1} /(n+1) & j=1,2, \ldots, C
\end{array}
$$

## Chi-Square Tests and Statistics

The CHISQ option provides chi-square tests of homogeneity or independence and measures of association that are based on the chi-square statistic. When you specify the CHISQ option in the TABLES statement, PROC FREQ computes the following chi-square tests for each two-way table: Pearson chi-square, likelihood ratio chi-square, and Mantel-Haenszel chi-square tests. PROC FREQ provides the following measures of association that are based on the Pearson chi-square statistic: phi coefficient, contingency coefficient, and Cramér's $V$. For $2 \times 2$ tables, the CHISQ option also provides Fisher's exact test and the continuity-adjusted chi-square statistic. You can request Fisher's exact test for general $R \times C$ tables by specifying the FISHER option in the TABLES or EXACT statement.

If you specify the CHISQ option for one-way tables, PROC FREQ provides a one-way Pearson chi-square goodness-of-fit test. If you specify the CHISQ(LRCHI) option for one-way tables, PROC FREQ also provides a one-way likelihood ratio chi-square test. The other tests and statistics that the CHISQ option produces are available only for two-way tables.

For two-way tables, the null hypothesis for the chi-square tests is no association between the row variable and the column variable. When the sample size $n$ is large, the test statistics have asymptotic chi-square distributions under the null hypothesis. When the sample size is not large, or when the data set is sparse or heavily tied, exact tests might be more appropriate than asymptotic tests. PROC FREQ provides exact $p$-values for the Pearson chi-square, likelihood ratio chi-square, and Mantel-Haenszel chi-square tests, in addition to Fisher's exact test. For one-way tables, PROC FREQ provides exact p-values for the Pearson and likelihood ratio chi-square goodness-of-fit tests. You can request these exact tests by specifying the corresponding options in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

The Mantel-Haenszel chi-square statistic is appropriate only when both variables lie on an ordinal scale. The other chi-square tests and statistics in this section are appropriate for either nominal or ordinal variables. The following sections give the formulas that PROC FREQ uses to compute the chi-square tests and statistics. For more information about these statistics, see Agresti (2007) and Stokes, Davis, and Koch (2012), and the other references cited.

## Chi-Square Test for One-Way Tables

For one-way frequency tables, the CHISQ option in the TABLES statement provides a chi-square goodness-of-fit test. Let $C$ denote the number of classes, or levels, in the one-way table. Let $f_{i}$ denote the frequency of class $i$ (or the number of observations in class $i$ ) for $i=1,2, \ldots, C$. Then PROC FREQ computes the one-way chi-square statistic as

$$
Q_{P}=\sum_{i=1}^{C}\left(f_{i}-e_{i}\right)^{2} / e_{i}
$$

where $e_{i}$ is the expected frequency for class $i$ under the null hypothesis.
In the test for equal proportions, which is the default for the CHISQ option, the null hypothesis specifies equal proportions of the total sample size for each class. Under this null hypothesis, the expected frequency
for each class equals the total sample size divided by the number of classes,

$$
e_{i}=n / C \quad \text { for } i=1,2, \ldots, C
$$

In the test for specified frequencies, which PROC FREQ computes when you input null hypothesis frequencies by using the TESTF= option, the expected frequencies are the TESTF= values that you specify. In the test for specified proportions, which PROC FREQ computes when you input null hypothesis proportions by using the TESTP $=$ option, the expected frequencies are determined from the specified TESTP= proportions $p_{i}$ as

$$
e_{i}=p_{i} \times n \quad \text { for } i=1,2, \ldots, C
$$

Under the null hypothesis (of equal proportions, specified frequencies, or specified proportions), $Q_{P}$ has an asymptotic chi-square distribution with $C-1$ degrees of freedom.
In addition to the asymptotic test, you can request an exact one-way chi-square test by specifying the CHISQ option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Pearson Chi-Square Test for Two-Way Tables

The Pearson chi-square for two-way tables involves the differences between the observed and expected frequencies, where the expected frequencies are computed under the null hypothesis of independence. The Pearson chi-square statistic is computed as

$$
Q_{P}=\sum_{i} \sum_{j}\left(n_{i j}-e_{i j}\right)^{2} / e_{i j}
$$

where $n_{i j}$ is the observed frequency in table cell $(i, j)$ and $e_{i j}$ is the expected frequency for table cell ( $i$, $j$ ). The expected frequency is computed under the null hypothesis that the row and column variables are independent,

$$
e_{i j}=\left(n_{i} \cdot n \cdot j\right) / n
$$

When the row and column variables are independent, $Q_{P}$ has an asymptotic chi-square distribution with $(R-1)(C-1)$ degrees of freedom. For large values of $Q_{P}$, this test rejects the null hypothesis in favor of the alternative hypothesis of general association.

In addition to the asymptotic test, you can request an exact Pearson chi-square test by specifying the PCHI or CHISQ option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

For $2 \times 2$ tables, the Pearson chi-square is also appropriate for testing the equality of two binomial proportions. For $R \times 2$ and $2 \times C$ tables, the Pearson chi-square tests the homogeneity of proportions. See Fienberg (1980) for details.

## Standardized Residuals

When you specify the CROSSLIST(STDRES) option in the TABLES statement for two-way or multiway tables, PROC FREQ displays the standardized residuals in the CROSSLIST table.

The standardized residual of a crosstabulation table cell is the ratio of (frequency - expected) to its standard error, where frequency is the table cell frequency and expected is the estimated expected cell frequency. The expected frequency is computed under the null hypothesis that the row and column variables are independent. See the section "Pearson Chi-Square Test for Two-Way Tables" on page 2705 for more information.

PROC FREQ computes the standardized residual of table cell $(i, j)$ as

$$
\left(n_{i j}-e_{i j}\right) / \sqrt{e_{i j}\left(1-p_{i}\right)\left(1-p_{\cdot j}\right)}
$$

where $n_{i j}$ is the observed frequency of table cell $(i, j), e_{i j}$ is the expected frequency of the table cell, $p_{i}$. is the proportion in row $i\left(n_{i} \cdot / n\right)$, and $p \cdot j$ is the proportion in column $j\left(n_{\cdot j} / n\right)$. The expected frequency of table cell $(i, j)$ is computed as

$$
e_{i j}=\left(n_{i} \cdot n \cdot j\right) / n
$$

Under the null hypothesis of independence, each standardized residual has an asymptotic standard normal distribution. See section 2.4.5 of Agresti (2007) for more information.

## Likelihood Ratio Chi-Square Test for One-Way Tables

For one-way frequency tables, the CHISQ(LRCHI) option in the TABLES statement provides a likelihood ratio chi-square goodness-of-fit test. By default, the likelihood ratio test is based on the null hypothesis of equal proportions in the $C$ classes (levels) of the one-way table. If you specify null hypothesis proportions or frequencies by using the CHISQ(TESTP=) or CHISQ(TESTF=) option, respectively, the likelihood ratio test is based on the null hypothesis values that you specify.
PROC FREQ computes the one-way likelihood ratio test as

$$
G^{2}=2 \sum_{i=1}^{C} f_{i} \ln \left(f_{i} / e_{i}\right)
$$

where $f_{i}$ is the observed frequency of class $i$, and $e_{i}$ is the expected frequency of class $i$ under the null hypothesis.

For the null hypothesis of equal proportions, the expected frequency of each class equals the total sample size divided by the number of classes,

$$
e_{i}=n / C \quad \text { for } \quad i=1,2, \ldots, C
$$

If you provide null hypothesis frequencies by specifying the CHISQ(TESTF=) option in the TABLES statement, the expected frequencies are the $\mathrm{TESTF}=$ values that you specify. If you provide null hypothesis proportions by specifying the CHISQ(TESTP=) option in the TABLES statement, PROC FREQ computes the expected frequencies as

$$
e_{i}=p_{i} \times n \quad \text { for } \quad i=1,2, \ldots, C
$$

where the proportions $p_{i}$ are the TESTP= values that you specify.
Under the null hypothesis (of equal proportions, specified frequencies, or specified proportions), the likelihood ratio statistic $G^{2}$ has an asymptotic chi-square distribution with $C-1$ degrees of freedom.

In addition to the asymptotic test, you can request an exact one-way likelihood ratio chi-square test by specifying the LRCHI option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Likelihood Ratio Chi-Square Test

The likelihood ratio chi-square involves the ratios between the observed and expected frequencies. The likelihood ratio chi-square statistic is computed as

$$
G^{2}=2 \sum_{i} \sum_{j} n_{i j} \ln \left(n_{i j} / e_{i j}\right)
$$

where $n_{i j}$ is the observed frequency in table cell $(i, j)$ and $e_{i j}$ is the expected frequency for table cell $(i, j)$.
When the row and column variables are independent, $G^{2}$ has an asymptotic chi-square distribution with $(R-1)(C-1)$ degrees of freedom.

In addition to the asymptotic test, you can request an exact likelihood ratio chi-square test by specifying the LRCHI or CHISQ option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Continuity-Adjusted Chi-Square Test

The continuity-adjusted chi-square for $2 \times 2$ tables is similar to the Pearson chi-square, but it is adjusted for the continuity of the chi-square distribution. The continuity-adjusted chi-square is most useful for small sample sizes. The use of the continuity adjustment is somewhat controversial; this chi-square test is more conservative (and more like Fisher's exact test) when the sample size is small. As the sample size increases, the continuity-adjusted chi-square becomes more like the Pearson chi-square.

The continuity-adjusted chi-square statistic is computed as

$$
Q_{C}=\sum_{i} \sum_{j}\left(\max \left(0,\left|n_{i j}-e_{i j}\right|-0.5\right)\right)^{2} / e_{i j}
$$

Under the null hypothesis of independence, $Q_{C}$ has an asymptotic chi-square distribution with $(R-1)(C-1)$ degrees of freedom.

## Mantel-Haenszel Chi-Square Test

The Mantel-Haenszel chi-square statistic tests the alternative hypothesis that there is a linear association between the row variable and the column variable. Both variables must lie on an ordinal scale. The Mantel-Haenszel chi-square statistic is computed as

$$
Q_{M H}=(n-1) r^{2}
$$

where $r$ is the Pearson correlation between the row variable and the column variable. For a description of the Pearson correlation, see the "Pearson Correlation Coefficient" on page 2713. The Pearson correlation and thus the Mantel-Haenszel chi-square statistic use the scores that you specify in the SCORES= option in the TABLES statement. See Mantel and Haenszel (1959) and Landis, Heyman, and Koch (1978) for more information.

Under the null hypothesis of no association, $Q_{M H}$ has an asymptotic chi-square distribution with one degree of freedom.

In addition to the asymptotic test, you can request an exact Mantel-Haenszel chi-square test by specifying the MHCHI or CHISQ option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Fisher's Exact Test

Fisher's exact test is another test of association between the row and column variables. This test assumes that the row and column totals are fixed, and then uses the hypergeometric distribution to compute probabilities of possible tables conditional on the observed row and column totals. Fisher's exact test does not depend on any large-sample distribution assumptions, and so it is appropriate even for small sample sizes and for sparse tables.
$\mathbf{2 \times 2}$ Tables For $2 \times 2$ tables, PROC FREQ gives the following information for Fisher's exact test: table probability, two-sided $p$-value, left-sided $p$-value, and right-sided $p$-value. The table probability equals the hypergeometric probability of the observed table, and is in fact the value of the test statistic for Fisher's exact test.

Where $p$ is the hypergeometric probability of a specific table with the observed row and column totals, Fisher's exact $p$-values are computed by summing probabilities $p$ over defined sets of tables,

$$
\operatorname{Prob}=\sum_{A} p
$$

The two-sided $p$-value is the sum of all possible table probabilities (conditional on the observed row and column totals) that are less than or equal to the observed table probability. For the two-sided $p$-value, the set $A$ includes all possible tables with hypergeometric probabilities less than or equal to the probability of the observed table. A small two-sided $p$-value supports the alternative hypothesis of association between the row and column variables.

For $2 \times 2$ tables, one-sided $p$-values for Fisher's exact test are defined in terms of the frequency of the cell in the first row and first column of the table, the $(1,1)$ cell. Denoting the observed $(1,1)$ cell frequency by $n_{11}$, the left-sided $p$-value for Fisher's exact test is the probability that the $(1,1)$ cell frequency is less than or equal to $n_{11}$. For the left-sided $p$-value, the set $A$ includes those tables with a $(1,1)$ cell frequency less than or equal to $n_{11}$. A small left-sided $p$-value supports the alternative hypothesis that the probability of an observation being in the first cell is actually less than expected under the null hypothesis of independent row and column variables.

Similarly, for a right-sided alternative hypothesis, $A$ is the set of tables where the frequency of the $(1,1)$ cell is greater than or equal to that in the observed table. A small right-sided $p$-value supports the alternative that the probability of the first cell is actually greater than that expected under the null hypothesis.

Because the ( 1,1 ) cell frequency completely determines the $2 \times 2$ table when the marginal row and column sums are fixed, these one-sided alternatives can be stated equivalently in terms of other cell probabilities or ratios of cell probabilities. The left-sided alternative is equivalent to an odds ratio less than 1 , where the odds ratio equals ( $n_{11} n_{22} / n_{12} n_{21}$ ). Additionally, the left-sided alternative is equivalent to the column 1 risk for row 1 being less than the column 1 risk for row $2, p_{1 \mid 1}<p_{1 \mid 2}$. Similarly, the right-sided alternative is equivalent to the column 1 risk for row 1 being greater than the column 1 risk for row $2, p_{1 \mid 1}>p_{1 \mid 2}$. See Agresti (2007) for details.
$\mathbf{R} \times \mathbf{C}$ Tables Fisher's exact test was extended to general $R \times C$ tables by Freeman and Halton (1951), and this test is also known as the Freeman-Halton test. For $R \times C$ tables, the two-sided $p$-value definition is the same as for $2 \times 2$ tables. The set $A$ contains all tables with $p$ less than or equal to the probability of the observed table. A small $p$-value supports the alternative hypothesis of association between the row and column variables. For $R \times C$ tables, Fisher's exact test is inherently two-sided. The alternative hypothesis is defined only in terms of general, and not linear, association. Therefore, Fisher's exact test does not have right-sided or left-sided $p$-values for general $R \times C$ tables.

For $R \times C$ tables, PROC FREQ computes Fisher's exact test by using the network algorithm of Mehta and Patel (1983), which provides a faster and more efficient solution than direct enumeration. See the section "Exact Statistics" on page 2757 for more details.

## Phi Coefficient

The phi coefficient is a measure of association derived from the Pearson chi-square. The range of the phi coefficient is $-1 \leq \phi \leq 1$ for $2 \times 2$ tables. For tables larger than $2 \times 2$, the range is $0 \leq \phi \leq$ $\min (\sqrt{R-1}, \sqrt{C-1})$ (Liebetrau 1983). The phi coefficient is computed as

$$
\begin{aligned}
& \phi=\left(n_{11} n_{22}-n_{12} n_{21}\right) / \sqrt{n_{1} \cdot n_{2} \cdot n \cdot 1 n \cdot 2} \quad \text { for } 2 \times 2 \text { tables } \\
& \phi=\sqrt{Q_{P} / n} \quad \text { otherwise }
\end{aligned}
$$

See Fleiss, Levin, and Paik (2003, pp. 98-99) for more information.

## Contingency Coefficient

The contingency coefficient is a measure of association derived from the Pearson chi-square. The range of the contingency coefficient is $0 \leq P \leq \sqrt{(m-1) / m}$, where $m=\min (R, C)$ (Liebetrau 1983). The contingency coefficient is computed as

$$
P=\sqrt{Q_{P} /\left(Q_{P}+n\right)}
$$

See Kendall and Stuart (1979, pp. 587-588) for more information.

## Cramér's V

Cramér's $V$ is a measure of association derived from the Pearson chi-square. It is designed so that the attainable upper bound is always 1 . The range of Cramér's $V$ is $-1 \leq V \leq 1$ for $2 \times 2$ tables; for tables larger than $2 \times 2$, the range is $0 \leq V \leq 1$. Cramér's $V$ is computed as

$$
\begin{aligned}
& V=\phi \quad \text { for } 2 \times 2 \text { tables } \\
& V=\sqrt{\frac{Q_{P} / n}{\min (R-1, C-1)}} \quad \text { otherwise }
\end{aligned}
$$

See Kendall and Stuart (1979, p. 588) for more information.

## Measures of Association

When you specify the MEASURES option in the TABLES statement, PROC FREQ computes several statistics that describe the association between the row and column variables of the contingency table. The following are measures of ordinal association that consider whether the column variable Y tends to increase as the row variable X increases: gamma, Kendall's tau- $b$, Stuart's tau- $c$, and Somers' $D$. These measures are appropriate for ordinal variables, and they classify pairs of observations as concordant or discordant. A pair is concordant if the observation with the larger value of $X$ also has the larger value of $Y$. A pair is discordant if the observation with the larger value of $X$ has the smaller value of $Y$. See Agresti (2007) and the other references cited for the individual measures of association.

The Pearson correlation coefficient and the Spearman rank correlation coefficient are also appropriate for ordinal variables. The Pearson correlation describes the strength of the linear association between the row and column variables, and it is computed by using the row and column scores specified by the SCORES= option in the TABLES statement. The Spearman correlation is computed with rank scores. The polychoric correlation (requested by the PLCORR option) also requires ordinal variables and assumes that the variables
have an underlying bivariate normal distribution. The following measures of association do not require ordinal variables and are appropriate for nominal variables: lambda asymmetric, lambda symmetric, and the uncertainty coefficients.

PROC FREQ computes estimates of the measures according to the formulas given in the following sections. For each measure, PROC FREQ computes an asymptotic standard error (ASE), which is the square root of the asymptotic variance denoted by Var in the following sections.

## Confidence Limits

If you specify the CL option in the TABLES statement, PROC FREQ computes asymptotic confidence limits for all MEASURES statistics. The confidence coefficient is determined according to the value of the ALPHA $=$ option, which, by default, equals 0.05 and produces $95 \%$ confidence limits.

The confidence limits are computed as

$$
\text { Est } \pm\left(z_{\alpha / 2} \times \text { ASE }\right)
$$

where Est is the estimate of the measure, $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution, and ASE is the asymptotic standard error of the estimate.

## Asymptotic Tests

For each measure that you specify in the TEST statement, PROC FREQ computes an asymptotic test of the null hypothesis that the measure equals zero. Asymptotic tests are available for the following measures of association: gamma, Kendall's tau- $b$, Stuart's tau-c, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, the Pearson correlation coefficient, and the Spearman rank correlation coefficient. To compute an asymptotic test, PROC FREQ uses a standardized test statistic $z$, which has an asymptotic standard normal distribution under the null hypothesis. The test statistic is computed as

$$
z=\text { Est } / \sqrt{\operatorname{Var}_{0}(\text { Est })}
$$

where Est is the estimate of the measure and $\operatorname{Var}_{0}$ (Est) is the variance of the estimate under the null hypothesis. Formulas for $\operatorname{Var}_{0}$ (Est) for the individual measures of association are given in the following sections.

Note that the ratio of Est to $\sqrt{\operatorname{Var}_{0}(E s t)}$ is the same for the following measures: gamma, Kendall's tau- $b$, Stuart's tau- $c$, Somers' $D(C \mid R)$, and Somers' $D(R \mid C)$. Therefore, the tests for these measures are identical. For example, the $p$-values for the test of $H_{0}$ : gamma $=0$ equal the $p$-values for the test of $H_{0}$ : tau $-b=0$.

PROC FREQ computes one-sided and two-sided $p$-values for each of these tests. When the test statistic $z$ is greater than its null hypothesis expected value of zero, PROC FREQ displays the right-sided $p$-value, which is the probability of a larger value of the statistic occurring under the null hypothesis. A small right-sided $p$-value supports the alternative hypothesis that the true value of the measure is greater than zero. When the test statistic is less than or equal to zero, PROC FREQ displays the left-sided $p$-value, which is the probability of a smaller value of the statistic occurring under the null hypothesis. A small left-sided $p$-value supports the alternative hypothesis that the true value of the measure is less than zero. The one-sided $p$-value $P_{1}$ can be expressed as

$$
P_{1}= \begin{cases}\operatorname{Prob}(Z>z) & \text { if } \quad z>0 \\ \operatorname{Prob}(Z<z) & \text { if } \quad z \leq 0\end{cases}
$$

where $Z$ has a standard normal distribution. The two-sided $p$-value $P_{2}$ is computed as

$$
P_{2}=\operatorname{Prob}(|Z|>|z|)
$$

## Exact Tests

Exact tests are available for the following measures of association: Kendall's tau-b, Stuart's tau- $c$, Somers' $D(C \mid R)$ and $(R \mid C)$, the Pearson correlation coefficient, and the Spearman rank correlation coefficient. If you request an exact test for a measure of association in the EXACT statement, PROC FREQ computes the exact test of the hypothesis that the measure equals zero. See the section "Exact Statistics" on page 2757 for details.

## Gamma

The gamma ( $\Gamma$ ) statistic is based only on the number of concordant and discordant pairs of observations. It ignores tied pairs (that is, pairs of observations that have equal values of $X$ or equal values of $Y$ ). Gamma is appropriate only when both variables lie on an ordinal scale. The range of gamma is $-1 \leq \Gamma \leq 1$. If the row and column variables are independent, then gamma tends to be close to zero. Gamma is computed as

$$
G=(P-Q) /(P+Q)
$$

and the asymptotic variance is

$$
\operatorname{Var}(G)=\frac{16}{(P+Q)^{4}} \sum_{i} \sum_{j} n_{i j}\left(Q A_{i j}-P D_{i j}\right)^{2}
$$

For $2 \times 2$ tables, gamma is equivalent to Yule's $Q$. See Goodman and Kruskal (1979) and Agresti (2002) for more information.
The variance under the null hypothesis that gamma equals zero is computed as

$$
\operatorname{Var}_{0}(G)=\frac{4}{(P+Q)^{2}}\left(\sum_{i} \sum_{j} n_{i j}\left(A_{i j}-D_{i j}\right)^{2}-(P-Q)^{2} / n\right)
$$

See Brown and Benedetti (1977) for details.

## Kendall's Tau-b

Kendall's tau-b $\left(\tau_{b}\right)$ is similar to gamma except that tau- $b$ uses a correction for ties. Tau- $b$ is appropriate only when both variables lie on an ordinal scale. The range of tau- $b$ is $-1 \leq \tau_{b} \leq 1$. Kendall's tau- $b$ is computed as

$$
t_{b}=(P-Q) / \sqrt{w_{r} w_{c}}
$$

and the asymptotic variance is

$$
\operatorname{Var}\left(t_{b}\right)=\frac{1}{w^{4}}\left(\sum_{i} \sum_{j} n_{i j}\left(2 w d_{i j}+t_{b} v_{i j}\right)^{2}-n^{3} t_{b}^{2}\left(w_{r}+w_{c}\right)^{2}\right)
$$

where

$$
\begin{aligned}
w & =\sqrt{w_{r} w_{c}} \\
w_{r} & =n^{2}-\sum_{i} n_{i \cdot}^{2} \\
w_{c} & =n^{2}-\sum_{j} n_{\cdot j}^{2} \\
d_{i j} & =A_{i j}-D_{i j} \\
v_{i j} & =n_{i} \cdot w_{c}+n \cdot{ }^{2} w_{r}
\end{aligned}
$$

See Kendall (1955) for more information.
The variance under the null hypothesis that tau-b equals zero is computed as

$$
\operatorname{Var}_{0}\left(t_{b}\right)=\frac{4}{w_{r} w_{c}}\left(\sum_{i} \sum_{j} n_{i j}\left(A_{i j}-D_{i j}\right)^{2}-(P-Q)^{2} / n\right)
$$

See Brown and Benedetti (1977) for details.
PROC FREQ also provides an exact test for the Kendall's tau-b. You can request this test by specifying the KENTB option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Stuart's Tau-c

Stuart's tau- $c\left(\tau_{c}\right)$ makes an adjustment for table size in addition to a correction for ties. Tau- $c$ is appropriate only when both variables lie on an ordinal scale. The range of tau- $c$ is $-1 \leq \tau_{c} \leq 1$. Stuart's tau-c is computed as

$$
t_{c}=m(P-Q) / n^{2}(m-1)
$$

and the asymptotic variance is

$$
\operatorname{Var}\left(t_{c}\right)=\frac{4 m^{2}}{(m-1)^{2} n^{4}}\left(\sum_{i} \sum_{j} n_{i j} d_{i j}^{2}-(P-Q)^{2} / n\right)
$$

where $m=\min (R, C)$ and $d_{i j}=A_{i j}-D_{i j}$. The variance under the null hypothesis that tau-c equals zero is the same as the asymptotic variance Var,

$$
\operatorname{Var}_{0}\left(t_{c}\right)=\operatorname{Var}\left(t_{c}\right)
$$

See Brown and Benedetti (1977) for details.
PROC FREQ also provides an exact test for the Stuart's tau- $c$. You can request this test by specifying the STUTC option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Somers' D

Somers' $D(C \mid R)$ and Somers' $D(R \mid C)$ are asymmetric modifications of tau- $b . C \mid R$ indicates that the row variable X is regarded as the independent variable and the column variable Y is regarded as dependent. Similarly, $R \mid C$ indicates that the column variable Y is regarded as the independent variable and the row variable X is regarded as dependent. Somers' $D$ differs from tau- $b$ in that it uses a correction only for pairs that are tied on the independent variable. Somers' $D$ is appropriate only when both variables lie on an ordinal scale. The range of Somers' $D$ is $-1 \leq D \leq 1$. Somers' $D(C \mid R)$ is computed as

$$
D(C \mid R)=(P-Q) / w_{r}
$$

and its asymptotic variance is

$$
\operatorname{Var}(D(C \mid R))=\frac{4}{w_{r}^{4}} \sum_{i} \sum_{j} n_{i j}\left(w_{r} d_{i j}-(P-Q)\left(n-n_{i} .\right)\right)^{2}
$$

where $d_{i j}=A_{i j}-D_{i j}$ and

$$
w_{r}=n^{2}-\sum_{i} n_{i}^{2}
$$

For more information, see Somers (1962); Goodman and Kruskal (1979); Liebetrau (1983).
The variance under the null hypothesis that $D(C \mid R)$ equals zero is computed as

$$
\operatorname{Var}_{0}(D(C \mid R))=\frac{4}{w_{r}^{2}}\left(\sum_{i} \sum_{j} n_{i j}\left(A_{i j}-D_{i j}\right)^{2}-(P-Q)^{2} / n\right)
$$

See Brown and Benedetti (1977) for details.
Formulas for Somers' $D(R \mid C)$ are obtained by interchanging the indices.
PROC FREQ also provides exact tests for Somers' $D(C \mid R)$ and $(R \mid C)$. You can request these tests by specifying the SMDCR and SMDCR options in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Pearson Correlation Coefficient

The Pearson correlation coefficient $(\rho)$ is computed by using the scores specified in the SCORES= option. This measure is appropriate only when both variables lie on an ordinal scale. The range of the Pearson correlation is $-1 \leq \rho \leq 1$. The Pearson correlation coefficient is computed as

$$
r=v / w=s s_{r c} / \sqrt{s s_{r} s s_{c}}
$$

and its asymptotic variance is

$$
\operatorname{Var}(r)=\frac{1}{w^{4}} \sum_{i} \sum_{j} n_{i j}\left(w\left(R_{i}-\bar{R}\right)\left(C_{j}-\bar{C}\right)-\frac{b_{i j} v}{2 w}\right)^{2}
$$

where $R_{i}$ and $C_{j}$ are the row and column scores and

$$
\begin{aligned}
s_{r} & =\sum_{i} \sum_{j} n_{i j}\left(R_{i}-\bar{R}\right)^{2} \\
s s_{c} & =\sum_{i} \sum_{j} n_{i j}\left(C_{j}-\bar{C}\right)^{2} \\
s s_{r c} & =\sum_{i} \sum_{j} n_{i j}\left(R_{i}-\bar{R}\right)\left(C_{j}-\bar{C}\right) \\
b_{i j} & =\left(R_{i}-\bar{R}\right)^{2} s s_{c}+\left(C_{j}-\bar{C}\right)^{2} s s_{r} \\
v & =s s_{r c} \\
w & =\sqrt{s s_{r} s s_{c}}
\end{aligned}
$$

See Snedecor and Cochran (1989) for more information.

The SCORES= option in the TABLES statement determines the type of row and column scores used to compute the Pearson correlation (and other score-based statistics). The default is SCORES=TABLE. See the section "Scores" on page 2703 for details about the available score types and how they are computed.
The variance under the null hypothesis that the correlation equals zero is computed as

$$
\operatorname{Var}_{0}(r)=\left(\sum_{i} \sum_{j} n_{i j}\left(R_{i}-\bar{R}\right)^{2}\left(C_{j}-\bar{C}\right)^{2}-s s_{r c}^{2} / n\right) / s s_{r} s s_{c}
$$

Note that this expression for the variance is derived for multinomial sampling in a contingency table framework, and it differs from the form obtained under the assumption that both variables are continuous and normally distributed. See Brown and Benedetti (1977) for details.

PROC FREQ also provides an exact test for the Pearson correlation coefficient. You can request this test by specifying the PCORR option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Spearman Rank Correlation Coefficient

The Spearman correlation coefficient $\left(\rho_{s}\right)$ is computed by using rank scores, which are defined in the section "Scores" on page 2703. This measure is appropriate only when both variables lie on an ordinal scale. The range of the Spearman correlation is $-1 \leq \rho_{s} \leq 1$. The Spearman correlation coefficient is computed as

$$
r_{s}=v / w
$$

and its asymptotic variance is

$$
\operatorname{Var}\left(r_{s}\right)=\frac{1}{n^{2} w^{4}} \sum_{i} \sum_{j} n_{i j}\left(z_{i j}-\bar{z}\right)^{2}
$$

where $R_{i}^{1}$ and $C_{j}^{1}$ are the row and column rank scores and

$$
\begin{aligned}
v & =\sum_{i} \sum_{j} n_{i j} R(i) C(j) \\
w & =\frac{1}{12} \sqrt{F G} \\
F & =n^{3}-\sum_{i} n_{i \cdot}^{3} \\
G & =n^{3}-\sum_{j} n_{\cdot j}^{3} \\
R(i) & =R_{i}^{1}-n / 2 \\
C(j) & =C_{j}^{1}-n / 2 \\
\bar{z} & =\frac{1}{n} \sum_{i} \sum_{j} n_{i j} z_{i j} \\
z_{i j} & =w v_{i j}-v w_{i j}
\end{aligned}
$$

$$
\begin{aligned}
v_{i j}= & n\left(R(i) C(j)+\frac{1}{2} \sum_{l} n_{i l} C(l)+\frac{1}{2} \sum_{k} n_{k j} R(k)+\right. \\
& \left.\sum_{l} \sum_{k>i} n_{k l} C(l)+\sum_{k} \sum_{l>j} n_{k l} R(k)\right) \\
w_{i j}= & \frac{-n}{96 w}\left(F n_{\cdot j}^{2}+G n_{i \cdot}^{2}\right)
\end{aligned}
$$

See Snedecor and Cochran (1989) for more information.
The variance under the null hypothesis that the correlation equals zero is computed as

$$
\operatorname{Var}_{0}\left(r_{s}\right)=\frac{1}{n^{2} w^{2}} \sum_{i} \sum_{j} n_{i j}\left(v_{i j}-\bar{v}\right)^{2}
$$

where

$$
\bar{v}=\sum_{i} \sum_{j} n_{i j} v_{i j} / n
$$

Note that the asymptotic variance is derived for multinomial sampling in a contingency table framework, and it differs from the form obtained under the assumption that both variables are continuous and normally distributed. See Brown and Benedetti (1977) for details.

PROC FREQ also provides an exact test for the Spearman correlation coefficient. You can request this test by specifying the SCORR option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Polychoric Correlation

When you specify the PLCORR option in the TABLES statement, PROC FREQ computes the polychoric correlation and its standard error. The polychoric correlation is based on the assumption that the two ordinal, categorical variables of the frequency table have an underlying bivariate normal distribution. The polychoric correlation coefficient is the maximum likelihood estimate of the product-moment correlation between the underlying normal variables. The range of the polychoric correlation is from -1 to 1 . For $2 \times 2$ tables, the polychoric correlation is also known as the tetrachoric correlation (and it is labeled as such in the displayed output). See Drasgow (1986) for an overview of polychoric correlation coefficient.

Olsson (1979) gives the likelihood equations and the asymptotic standard errors for estimating the polychoric correlation. The underlying continuous variables relate to the observed crosstabulation table through thresholds, which define a range of numeric values that correspond to each categorical (table) level. PROC FREQ uses Olsson's maximum likelihood method for simultaneous estimation of the polychoric correlation and the thresholds. (Olsson also presents a two-step method that estimates the thresholds first.)

PROC FREQ iteratively solves the likelihood equations by using a Newton-Raphson algorithm. The initial estimates of the thresholds are computed from the inverse of the normal distribution function at the cumulative marginal proportions of the table. Iterative computation of the polychoric correlation stops when the convergence measure falls below the convergence criterion or when the maximum number of iterations is reached, whichever occurs first. For parameter values that are less than 0.01 , the procedure evaluates convergence by using the absolute difference instead of the relative difference. The PLCORR(CONVERGE=)
option specifies the convergence criterion, which is 0.0001 by default. The PLCORR(MAXITER=) option specifies the maximum number of iterations, which is 20 by default.

If you specify the CL option in the TABLES statement, PROC FREQ provides confidence limits for the polychoric correlation. The confidence limits are computed as

$$
\hat{\rho} \pm\left(z_{\alpha / 2} \times \mathrm{SE}(\hat{\rho})\right)
$$

where $\hat{\rho}$ is the estimate of the polychoric correlation, $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution, and $\operatorname{SE}(\hat{\rho})$ is the standard error of the polychoric correlation estimate.

If you specify the PLCORR option in the TEST statement, PROC FREQ provides Wald and likelihood ratio tests of the null hypothesis that the polychoric correlation equals 0 . The Wald test statistic is computed as

$$
z=\hat{\rho} / \mathrm{SE}(\hat{\rho})
$$

which has a standard normal distribution under the null hypothesis. PROC FREQ computes one-sided and two-sided $p$-values for the Wald test. When the test statistic $z$ is greater than its null expected value of 0 , PROC FREQ displays the right-sided $p$-value. When the test statistic is less than or equal to 0 , PROC FREQ displays the left-sided $p$-value.

The likelihood ratio statistic for the polychoric correlation is computed as

$$
G^{2}=-2 \ln \left(L_{0} / L_{1}\right)
$$

where $L_{0}$ is the value of the likelihood function (Olsson 1979) when the polychoric correlation is 0 , and $L_{1}$ is the value of the likelihood function at the maximum (where all parameters are replaced by their maximum likelihood estimates). Under the null hypothesis, the likelihood ratio statistic has an asymptotic chi-square distribution with one degree of freedom.

## Lambda (Asymmetric)

Asymmetric lambda, $\lambda(C \mid R)$, is interpreted as the probable improvement in predicting the column variable Y given knowledge of the row variable X . The range of asymmetric lambda is $0 \leq \lambda(C \mid R) \leq 1$. Asymmetric lambda $(C \mid R)$ is computed as

$$
\lambda(C \mid R)=\frac{\sum_{i} r_{i}-r}{n-r}
$$

and its asymptotic variance is

$$
\operatorname{Var}(\lambda(C \mid R))=\frac{n-\sum_{i} r_{i}}{(n-r)^{3}}\left(\sum_{i} r_{i}+r-2 \sum_{i}\left(r_{i} \mid l_{i}=l\right)\right)
$$

where

$$
\begin{aligned}
r_{i} & =\max _{j}\left(n_{i j}\right) \\
r & =\max _{j}\left(n_{\cdot j}\right) \\
c_{j} & =\max _{i}\left(n_{i j}\right) \\
c & =\max _{i}\left(n_{i}\right)
\end{aligned}
$$

The values of $l_{i}$ and $l$ are determined as follows. Denote by $l_{i}$ the unique value of $j$ such that $r_{i}=n_{i j}$, and let $l$ be the unique value of $j$ such that $r=n \cdot j$. Because of the uniqueness assumptions, ties in the frequencies or in the marginal totals must be broken in an arbitrary but consistent manner. In case of ties, $l$ is defined as the smallest value of $j$ such that $r=n \cdot j$.

For those columns containing a cell $(i, j)$ for which $n_{i j}=r_{i}=c_{j}, c s_{j}$ records the row in which $c_{j}$ is assumed to occur. Initially $c s_{j}$ is set equal to -1 for all $j$. Beginning with $i=1$, if there is at least one value $j$ such that $n_{i j}=r_{i}=c_{j}$, and if $c s_{j}=-1$, then $l_{i}$ is defined to be the smallest such value of $j$, and $c s_{j}$ is set equal to $i$. Otherwise, if $n_{i l}=r_{i}$, then $l_{i}$ is defined to be equal to $l$. If neither condition is true, then $l_{i}$ is taken to be the smallest value of $j$ such that $n_{i j}=r_{i}$.
The formulas for lambda asymmetric $(R \mid C)$ can be obtained by interchanging the indices.
See Goodman and Kruskal (1979) for more information.

## Lambda (Symmetric)

The nondirectional lambda is the average of the two asymmetric lambdas, $\lambda(C \mid R)$ and $\lambda(R \mid C)$. Its range is $0 \leq \lambda \leq 1$. Lambda symmetric is computed as

$$
\lambda=\frac{\sum_{i} r_{i}+\sum_{j} c_{j}-r-c}{2 n-r-c}=\frac{w-v}{w}
$$

and its asymptotic variance is computed as

$$
\operatorname{Var}(\lambda)=\frac{1}{w^{4}}\left(w v y-2 w^{2}\left(n-\sum_{i} \sum_{j}\left(n_{i j} \mid j=l_{i}, i=k_{j}\right)\right)-2 v^{2}\left(n-n_{k l}\right)\right)
$$

where

$$
\begin{aligned}
r_{i} & =\max _{j}\left(n_{i j}\right) \\
r & =\max _{j}\left(n_{\cdot j}\right) \\
c_{j} & =\max _{i}\left(n_{i j}\right) \\
c & =\max _{i}\left(n_{i} \cdot\right) \\
w & =2 n-r-c \\
v & =2 n-\sum_{i} r_{i}-\sum_{j} c_{j} \\
x & =\sum_{i}\left(r_{i} \mid l_{i}=l\right)+\sum_{j}\left(c_{j} \mid k_{j}=k\right)+r_{k}+c_{l} \\
y & =8 n-w-v-2 x
\end{aligned}
$$

The definitions of $l_{i}$ and $l$ are given in the previous section. The values $k_{j}$ and $k$ are defined in a similar way for lambda asymmetric ( $R \mid C$ ).

See Goodman and Kruskal (1979) for more information.

## Uncertainty Coefficients (Asymmetric)

The uncertainty coefficient $U(C \mid R)$ measures the proportion of uncertainty (entropy) in the column variable Y that is explained by the row variable X . Its range is $0 \leq U(C \mid R) \leq 1$. The uncertainty coefficient is computed as

$$
U(C \mid R)=(H(X)+H(Y)-H(X Y)) / H(Y)=v / w
$$

and its asymptotic variance is

$$
\operatorname{Var}(U(C \mid R))=\frac{1}{n^{2} w^{4}} \sum_{i} \sum_{j} n_{i j}\left(H(Y) \ln \left(\frac{n_{i j}}{n_{i \cdot}}\right)+(H(X)-H(X Y)) \ln \left(\frac{n \cdot j}{n}\right)\right)^{2}
$$

where

$$
\begin{aligned}
v & =H(X)+H(Y)-H(X Y) \\
w & =H(Y) \\
H(X) & =-\sum_{i}\left(\frac{n_{i \cdot}}{n}\right) \ln \left(\frac{n_{i \cdot}}{n}\right) \\
H(Y) & =-\sum_{j}\left(\frac{n \cdot j}{n}\right) \ln \left(\frac{n \cdot j}{n}\right) \\
H(X Y) & =-\sum_{i} \sum_{j}\left(\frac{n_{i j}}{n}\right) \ln \left(\frac{n_{i j}}{n}\right)
\end{aligned}
$$

The formulas for the uncertainty coefficient $U(R \mid C)$ can be obtained by interchanging the indices.
See Theil (1972, pp. 115-120) and Goodman and Kruskal (1979) for more information.

## Uncertainty Coefficient (Symmetric)

The uncertainty coefficient $U$ is the symmetric version of the two asymmetric uncertainty coefficients. Its range is $0 \leq U \leq 1$. The uncertainty coefficient is computed as

$$
U=2(H(X)+H(Y)-H(X Y)) /(H(X)+H(Y))
$$

and its asymptotic variance is

$$
\operatorname{Var}(U)=4 \sum_{i} \sum_{j} \frac{n_{i j}\left(H(X Y) \ln \left(\frac{n_{i} \cdot n^{2} \cdot}{n^{2}}\right)-(H(X)+H(Y)) \ln \left(\frac{n_{i j}}{n}\right)\right)^{2}}{n^{2}(H(X)+H(Y))^{4}}
$$

where $H(X), H(Y)$, and $H(X Y)$ are defined in the previous section. See Goodman and Kruskal (1979) for more information.

## Binomial Proportion

If you specify the BINOMIAL option in the TABLES statement, PROC FREQ computes the binomial proportion for one-way tables. By default, this is the proportion of observations in the first variable level that appears in the output. (You can use the LEVEL= option to specify a different level for the proportion.) The binomial proportion is computed as

$$
\hat{p}=n_{1} / n
$$

where $n_{1}$ is the frequency of the first (or designated) level and $n$ is the total frequency of the one-way table. The standard error of the binomial proportion is computed as

$$
\operatorname{se}(\hat{p})=\sqrt{\hat{p}(1-\hat{p}) / n}
$$

## Binomial Confidence Limits

By default, PROC FREQ provides Wald and exact (Clopper-Pearson) confidence limits for the binomial proportion. If you do not specify any confidence limit requests with the $\mathrm{CL}=$ binomial-option, PROC FREQ computes Wald asymptotic confidence limits. You can also request Agresti-Coull, Jeffreys, and Wilson (score) confidence limits for the binomial proportion. For details about these binomial confidence limits, including comparisons of their performance, see Brown, Cai, and DasGupta (2001); Agresti and Coull (1998); Newcombe (1998b).

Wald Confidence Limits Wald asymptotic confidence limits are based on the normal approximation to the binomial distribution. PROC FREQ computes the Wald confidence limits for the binomial proportion as

$$
\hat{p} \pm\left(z_{\alpha / 2} \times \operatorname{se}(\hat{p})\right)
$$

where $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. The confidence level $\alpha$ is determined by the ALPHA= option, which, by default, equals 0.05 and produces $95 \%$ confidence limits.

If you specify CL=WALD(CORRECT) or the CORRECT binomial-option, PROC FREQ includes a continuity correction of $1 / 2 n$ in the Wald asymptotic confidence limits. The purpose of this correction is to adjust for the difference between the normal approximation and the discrete binomial distribution. See Fleiss, Levin, and Paik (2003) for more information. The continuity-corrected Wald confidence limits for the binomial proportion are computed as

$$
\hat{p} \pm\left(z_{\alpha / 2} \times \operatorname{se}(\hat{p})+(1 / 2 n)\right)
$$

Agresti-Coull Confidence Limits If you specify the CL=AGRESTICOULL binomial-option, PROC FREQ computes Agresti-Coull confidence limits for the binomial proportion as

$$
\tilde{p} \pm\left(z_{\alpha / 2} \times \sqrt{\tilde{p}(1-\tilde{p}) / \tilde{n}}\right)
$$

where

$$
\begin{aligned}
\tilde{n}_{1} & =n_{1}+z_{\alpha / 2}^{2} / 2 \\
\tilde{n} & =n+z_{\alpha / 2}^{2} \\
\tilde{p} & =\tilde{n}_{1} / \tilde{n}
\end{aligned}
$$

The Agresti-Coull confidence interval has the same general form as the standard Wald interval but uses $\tilde{p}$ in place of $\hat{p}$. For $\alpha=0.05$, the value of $z_{\alpha / 2}$ is close to 2 , and this interval is the "add 2 successes and 2 failures" adjusted Wald interval of Agresti and Coull (1998).

Jeffreys Confidence Limits If you specify the CL=JEFFREYS binomial-option, PROC FREQ computes Jeffreys confidence limits for the binomial proportion as

$$
\left(\beta\left(\alpha / 2, n_{1}+1 / 2, n-n_{1}+1 / 2\right), \quad \beta\left(1-\alpha / 2, n_{1}+1 / 2, n-n_{1}+1 / 2\right)\right)
$$

where $\beta(\alpha, b, c)$ is the $\alpha$ percentile of the beta distribution with shape parameters $b$ and $c$. The lower confidence limit is set to 0 when $n_{1}=0$, and the upper confidence limit is set to 1 when $n_{1}=n$. This is an equal-tailed interval based on the noninformative Jeffreys prior for a binomial proportion. See Brown, Cai, and DasGupta (2001) for details. See Berger (1985) for information about using beta priors for inference on the binomial proportion.

Wilson (Score) Confidence Limits If you specify the CL=WILSON binomial-option, PROC FREQ computes Wilson confidence limits for the binomial proportion. These are also known as score confidence limits (Wilson 1927). The confidence limits are based on inverting the normal test that uses the null proportion in the variance (the score test). Wilson confidence limits are the roots of

$$
|p-\hat{p}|=z_{\alpha / 2} \sqrt{p(1-p) / n}
$$

and are computed as

$$
\left(\hat{p}+z_{\alpha / 2}^{2} / 2 n \pm z_{\alpha / 2} \sqrt{\left(\hat{p}(1-\hat{p})+z_{\alpha / 2}^{2} / 4 n\right) / n}\right) /\left(1+z_{\alpha / 2}^{2} / n\right)
$$

If you specify $\mathrm{CL}=\mathrm{WILSON}(\mathrm{CORRECT})$ or the CORRECT binomial-option, PROC FREQ provides continuity-corrected Wilson confidence limits, which are computed as the roots of

$$
|p-\hat{p}|-1 / 2 n=z_{\alpha / 2} \sqrt{p(1-p) / n}
$$

The Wilson interval has been shown to have better performance than the Wald interval and the exact (ClopperPearson) interval. For more information, see Agresti and Coull (1998); Brown, Cai, and DasGupta (2001); Newcombe (1998b).

Exact (Clopper-Pearson) Confidence Limits Exact (Clopper-Pearson) confidence limits for the binomial proportion are constructed by inverting the equal-tailed test based on the binomial distribution. This method is attributed to Clopper and Pearson (1934). The exact confidence limits $p_{L}$ and $p_{U}$ satisfy the following equations, for $n_{1}=1,2, \ldots n-1$ :

$$
\begin{aligned}
& \sum_{x=n_{1}}^{n}\binom{n}{x} p_{L}^{x}\left(1-p_{L}\right)^{n-x}=\alpha / 2 \\
& \sum_{x=0}^{n_{1}}\binom{n}{x} p_{U}^{x}\left(1-p_{U}\right)^{n-x}=\alpha / 2
\end{aligned}
$$

The lower confidence limit equals 0 when $n_{1}=0$, and the upper confidence limit equals 1 when $n_{1}=n$.

PROC FREQ computes the exact (Clopper-Pearson) confidence limits by using the $F$ distribution as

$$
\begin{aligned}
& p_{L}=\left(1+\frac{n-n_{1}+1}{n_{1} F\left(\alpha / 2,2 n_{1}, 2\left(n-n_{1}+1\right)\right)}\right)^{-1} \\
& p_{U}=\left(1+\frac{n-n_{1}}{\left(n_{1}+1\right) F\left(1-\alpha / 2,2\left(n_{1}+1\right), 2\left(n-n_{1}\right)\right)}\right)^{-1}
\end{aligned}
$$

where $F(\alpha / 2, b, c)$ is the $\alpha / 2$ percentile of the $F$ distribution with $b$ and $c$ degrees of freedom. See Leemis and Trivedi (1996) for a derivation of this expression. Also see Collett (1991) for more information about exact binomial confidence limits.

Because this is a discrete problem, the confidence coefficient (coverage probability) of the exact (ClopperPearson) interval is not exactly $(1-\alpha)$ but is at least $(1-\alpha)$. Thus, this confidence interval is conservative. Unless the sample size is large, the actual coverage probability can be much larger than the target value. For more information about the performance of these confidence limits, see Agresti and Coull (1998); Brown, Cai, and DasGupta (2001); Leemis and Trivedi (1996).

## Binomial Tests

The BINOMIAL option provides an asymptotic equality test for the binomial proportion by default. You can also specify binomial-options to request tests of noninferiority, superiority, and equivalence for the binomial proportion. If you specify the BINOMIAL option in the EXACT statement, PROC FREQ also computes exact $p$-values for the tests that you request with the binomial-options.

Equality Test PROC FREQ computes an asymptotic test of the hypothesis that the binomial proportion equals $p_{0}$, where you can specify the value of $p_{0}$ with the $\mathrm{P}=$ binomial-option. If you do not specify a null value with $\mathrm{P}=$, PROC FREQ uses $p_{0}=0.5$ by default. The binomial test statistic is computed as

$$
z=\left(\hat{p}-p_{0}\right) / \mathrm{se}
$$

By default, the standard error is based on the null hypothesis proportion as

$$
\text { se }=\sqrt{p_{0}\left(1-p_{0}\right) / n}
$$

If you specify the VAR=SAMPLE binomial-option, the standard error is computed from the sample proportion as

$$
\mathrm{se}=\sqrt{\hat{p}(1-\hat{p}) / n}
$$

If you specify the CORRECT binomial-option, PROC FREQ includes a continuity correction in the asymptotic test statistic, towards adjusting for the difference between the normal approximation and the discrete binomial distribution. See Fleiss, Levin, and Paik (2003) for details. The continuity correction of $(1 / 2 n)$ is subtracted from the numerator of the test statistic if $\left(\hat{p}-p_{0}\right)$ is positive; otherwise, the continuity correction is added to the numerator.

PROC FREQ computes one-sided and two-sided $p$-values for this test. When the test statistic $z$ is greater than zero (its expected value under the null hypothesis), PROC FREQ computes the right-sided $p$-value, which is the probability of a larger value of the statistic occurring under the null hypothesis. A small right-sided $p$-value supports the alternative hypothesis that the true value of the proportion is greater than $p_{0}$. When
the test statistic is less than or equal to zero, PROC FREQ computes the left-sided $p$-value, which is the probability of a smaller value of the statistic occurring under the null hypothesis. A small left-sided $p$-value supports the alternative hypothesis that the true value of the proportion is less than $p_{0}$. The one-sided $p$-value $P_{1}$ can be expressed as

$$
P_{1}=\left\{\begin{array}{lll}
\operatorname{Prob}(Z>z) & \text { if } & z>0 \\
\operatorname{Prob}(Z<z) & \text { if } & z \leq 0
\end{array}\right.
$$

where $Z$ has a standard normal distribution. The two-sided $p$-value is computed as $P_{2}=2 \times P_{1}$.
If you specify the BINOMIAL option in the EXACT statement, PROC FREQ also computes an exact test of the null hypothesis $H_{0}: p=p_{0}$. To compute the exact test, PROC FREQ uses the binomial probability function,

$$
\operatorname{Prob}\left(X=x \mid p_{0}\right)=\binom{n}{x} p_{0}^{x}\left(1-p_{0}\right)^{(n-x)} \quad \text { for } \quad x=0,1,2, \ldots, n
$$

where the variable $X$ has a binomial distribution with parameters $n$ and $p_{0}$. To compute the left-sided $p$-value, $\operatorname{Prob}\left(X \leq n_{1}\right)$, PROC FREQ sums the binomial probabilities over $x$ from zero to $n_{1}$. To compute the right-sided $p$-value, $\operatorname{Prob}\left(X \geq n_{1}\right)$, PROC FREQ sums the binomial probabilities over $x$ from $n_{1}$ to $n$. The exact one-sided $p$-value is the minimum of the left-sided and right-sided $p$-values,

$$
P_{1}=\min \left(\operatorname{Prob}\left(X \leq n_{1} \mid p_{0}\right), \operatorname{Prob}\left(X \geq n_{1} \mid p_{0}\right)\right)
$$

and the exact two-sided $p$-value is computed as $P_{2}=2 \times P_{1}$.
Noninferiority Test If you specify the NONINF binomial-option, PROC FREQ provides a noninferiority test for the binomial proportion. The null hypothesis for the noninferiority test is

$$
H_{0}: p-p_{0} \leq-\delta
$$

versus the alternative

$$
H_{a}: p-p_{0}>-\delta
$$

where $\delta$ is the noninferiority margin and $p_{0}$ is the null proportion. Rejection of the null hypothesis indicates that the binomial proportion is not inferior to the null value. See Chow, Shao, and Wang (2003) for more information.
You can specify the value of $\delta$ with the MARGIN = binomial-option, and you can specify $p_{0}$ with the $\mathrm{P}=$ binomial-option. By default, $\delta=0.2$ and $p_{0}=0.5$.

PROC FREQ provides an asymptotic Wald test for noninferiority. The test statistic is computed as

$$
z=\left(\hat{p}-p_{0}^{*}\right) / \mathrm{se}
$$

where $p_{0}^{*}$ is the noninferiority limit,

$$
p_{0}^{*}=p_{0}-\delta
$$

By default, the standard error is computed from the sample proportion as

$$
\mathrm{se}=\sqrt{\hat{p}(1-\hat{p}) / n}
$$

If you specify the VAR=NULL binomial-option, the standard error is based on the noninferiority limit (determined by the null proportion and the margin) as

$$
\mathrm{se}=\sqrt{p_{0}^{*}\left(1-p_{0}^{*}\right) / n}
$$

If you specify the CORRECT binomial-option, PROC FREQ includes a continuity correction in the asymptotic test statistic $z$. The continuity correction of $(1 / 2 n)$ is subtracted from the numerator of the test statistic if $\left(\hat{p}-p_{0}^{*}\right)$ is positive; otherwise, the continuity correction is added to the numerator.

The $p$-value for the noninferiority test is

$$
P_{z}=\operatorname{Prob}(Z>z)
$$

where $Z$ has a standard normal distribution.
As part of the noninferiority analysis, PROC FREQ provides asymptotic Wald confidence limits for the binomial proportion. These confidence limits are computed as described in the section "Wald Confidence Limits" on page 2719 but use the same standard error (VAR=NULL or VAR=SAMPLE) as the noninferiority test statistic $z$. The confidence coefficient is $100(1-2 \alpha) \%$ (Schuirmann 1999). By default, if you do not specify the ALPHA= option, the noninferiority confidence limits are $90 \%$ confidence limits. You can compare the confidence limits to the noninferiority limit, $p_{0}^{*}=p_{0}-\delta$.
If you specify the BINOMIAL option in the EXACT statement, PROC FREQ provides an exact noninferiority test for the binomial proportion. The exact $p$-value is computed by using the binomial probability function with parameters $p_{0}^{*}$ and $n$,

$$
P_{x}=\sum_{k=n_{1}}^{k=n}\binom{n}{k}\left(p_{0}^{*}\right)^{k}\left(1-p_{0}^{*}\right)^{(n-k)}
$$

See Chow, Shao, and Wang (2003, p. 116) for details. If you request exact binomial statistics, PROC FREQ also includes exact (Clopper-Pearson) confidence limits for the binomial proportion in the equivalence analysis display. See the section "Exact (Clopper-Pearson) Confidence Limits" on page 2720 for details.

Superiority Test If you specify the SUP binomial-option, PROC FREQ provides a superiority test for the binomial proportion. The null hypothesis for the superiority test is

$$
H_{0}: p-p_{0} \leq \delta
$$

versus the alternative

$$
H_{a}: p-p_{0}>\delta
$$

where $\delta$ is the superiority margin and $p_{0}$ is the null proportion. Rejection of the null hypothesis indicates that the binomial proportion is superior to the null value. You can specify the value of $\delta$ with the MARGIN= binomial-option, and you can specify the value of $p_{0}$ with the $\mathrm{P}=$ binomial-option. By default, $\delta=0.2$ and $p_{0}=0.5$.

The superiority analysis is identical to the noninferiority analysis but uses a positive value of the margin $\delta$ in the null hypothesis. The superiority limit equals $p_{0}+\delta$. The superiority computations follow those in the section "Noninferiority Test" on page 2722 but replace $-\delta$ with $\delta$. See Chow, Shao, and Wang (2003) for more information.

Equivalence Test If you specify the EQUIV binomial-option, PROC FREQ provides an equivalence test for the binomial proportion. The null hypothesis for the equivalence test is

$$
H_{0}: p-p_{0} \leq \delta_{L} \quad \text { or } \quad p-p_{0} \geq \delta_{U}
$$

versus the alternative

$$
H_{a}: \delta_{L}<p-p_{0}<\delta_{U}
$$

where $\delta_{L}$ is the lower margin, $\delta_{U}$ is the upper margin, and $p_{0}$ is the null proportion. Rejection of the null hypothesis indicates that the binomial proportion is equivalent to the null value. See Chow, Shao, and Wang (2003) for more information.

You can specify the value of the margins $\delta_{L}$ and $\delta_{U}$ with the MARGIN= binomial-option. If you do not specify MARGIN=, PROC FREQ uses lower and upper margins of -0.2 and 0.2 by default. If you specify a single margin value $\delta$, PROC FREQ uses lower and upper margins of $-\delta$ and $\delta$. You can specify the null proportion $p_{0}$ with the $\mathrm{P}=$ binomial-option. By default, $p_{0}=0.5$.

PROC FREQ computes two one-sided tests (TOST) for equivalence analysis (Schuirmann 1987). The TOST approach includes a right-sided test for the lower margin and a left-sided test for the upper margin. The overall $p$-value is taken to be the larger of the two $p$-values from the lower and upper tests.

For the lower margin, the asymptotic Wald test statistic is computed as

$$
z_{L}=\left(\hat{p}-p_{L}^{*}\right) / \mathrm{se}
$$

where the lower equivalence limit is

$$
p_{L}^{*}=p_{0}+\delta_{L}
$$

By default, the standard error is computed from the sample proportion as

$$
\mathrm{se}=\sqrt{\hat{p}(1-\hat{p}) / n}
$$

If you specify the VAR=NULL binomial-option, the standard error is based on the lower equivalence limit (determined by the null proportion and the lower margin) as

$$
\mathrm{se}=\sqrt{p_{L}^{*}\left(1-p_{L}^{*}\right) / n}
$$

If you specify the CORRECT binomial-option, PROC FREQ includes a continuity correction in the asymptotic test statistic $z_{L}$. The continuity correction of $(1 / 2 n)$ is subtracted from the numerator of the test statistic ( $\hat{p}-p_{L}^{*}$ ) if the numerator is positive; otherwise, the continuity correction is added to the numerator.
The $p$-value for the lower margin test is

$$
P_{z, L}=\operatorname{Prob}\left(Z>z_{L}\right)
$$

The asymptotic test for the upper margin is computed similarly. The Wald test statistic is

$$
z_{U}=\left(\hat{p}-p_{U}^{*}\right) / \mathrm{se}
$$

where the upper equivalence limit is

$$
p_{U}^{*}=p_{0}+\delta_{U}
$$

By default, the standard error is computed from the sample proportion. If you specify the VAR=NULL binomial-option, the standard error is based on the upper equivalence limit as

$$
\mathrm{se}=\sqrt{p_{U}^{*}\left(1-p_{U}^{*}\right) / n}
$$

If you specify the CORRECT binomial-option, PROC FREQ includes a continuity correction of $(1 / 2 n)$ in the asymptotic test statistic $z_{U}$.

The $p$-value for the upper margin test is

$$
P_{z, U}=\operatorname{Prob}\left(Z<z_{U}\right)
$$

Based on the two one-sided tests (TOST), the overall $p$-value for the test of equivalence equals the larger $p$-value from the lower and upper margin tests, which can be expressed as

$$
P_{z}=\max \left(P_{z, L}, P_{z, U}\right)
$$

As part of the equivalence analysis, PROC FREQ provides asymptotic Wald confidence limits for the binomial proportion. These confidence limits are computed as described in the section "Wald Confidence Limits" on page 2719 , but use the same standard error (VAR=NULL or VAR=SAMPLE) as the equivalence test statistics and have a confidence coefficient of $100(1-2 \alpha) \%$ (Schuirmann 1999). By default, if you do not specify the ALPHA= option, the equivalence confidence limits are $90 \%$ limits. If you specify VAR=NULL, separate standard errors are computed for the lower and upper margin tests, each based on the null proportion and the corresponding (lower or upper) margin. The confidence limits are computed by using the maximum of these two standard errors. You can compare the confidence limits to the equivalence limits, $\left(p_{0}+\delta_{L}, p_{0}+\delta_{U}\right)$.

If you specify the BINOMIAL option in the EXACT statement, PROC FREQ also provides an exact equivalence test by using two one-sided exact tests (TOST). The procedure computes lower and upper margin exact tests by using the binomial probability function as described in the section "Noninferiority Test" on page 2722. The overall exact $p$-value for the equivalence test is taken to be the larger $p$-value from the lower and upper margin exact tests. If you request exact statistics, PROC FREQ also includes exact (Clopper-Pearson) confidence limits in the equivalence analysis display. The confidence coefficient is $100(1-2 \alpha) \%$ (Schuirmann 1999). See the section "Exact (Clopper-Pearson) Confidence Limits" on page 2720 for details.

## Risks and Risk Differences

The RISKDIFF option in the TABLES statement provides estimates of risks (binomial proportions) and risk differences for $2 \times 2$ tables. This analysis might be appropriate when comparing the proportion of some characteristic for two groups, where row 1 and row 2 correspond to the two groups, and the columns correspond to two possible characteristics or outcomes. For example, the row variable might be a treatment or dose, and the column variable might be the response. For more information, see Collett (1991); Fleiss, Levin, and Paik (2003); Stokes, Davis, and Koch (2012).

Let the frequencies of the $2 \times 2$ table be represented as follows.

|  | Column 1 | Column 2 | Total |
| :--- | :---: | :---: | :--- |
| Row 1 | $n_{11}$ | $n_{12}$ | $n_{1} \cdot$ |
| Row 2 | $n_{21}$ | $n_{22}$ | $n_{2 \cdot}$ |
| Total | $n \cdot 1$ | $n_{\cdot 2}$ | $n$ |

By default when you specify the RISKDIFF option, PROC FREQ provides estimates of the row 1 risk (proportion), the row 2 risk, the overall risk, and the risk difference for column 1 and for column 2 of the $2 \times 2$ table. The risk difference is defined as the row 1 risk minus the row 2 risk. The risks are binomial proportions of their rows (row 1, row 2, or overall), and the computation of their standard errors and Wald confidence limits follow the binomial proportion computations, which are described in the section "Binomial Proportion" on page 2719.

The column 1 risk for row 1 is the proportion of row 1 observations classified in column 1 ,

$$
\hat{p}_{1}=n_{11} / n_{1} .
$$

which estimates the conditional probability of the column 1 response, given the first level of the row variable. The column 1 risk for row 2 is the proportion of row 2 observations classified in column 1 ,

$$
\hat{p}_{2}=n_{21} / n_{2} .
$$

The overall column 1 risk is the proportion of all observations classified in column 1 ,

$$
\hat{p}=n \cdot 1 / n
$$

The column 1 risk difference compares the risks for the two rows, and it is computed as the column 1 risk for row 1 minus the column 1 risk for row 2 ,

$$
\hat{d}=\hat{p}_{1}-\hat{p}_{2}
$$

The standard error of the column 1 risk for row $i$ is computed as

$$
\operatorname{se}\left(\hat{p}_{i}\right)=\sqrt{\hat{p}_{i}\left(1-\hat{p}_{i}\right) / n_{i}} .
$$

The standard error of the overall column 1 risk is computed as

$$
\operatorname{se}(\hat{p})=\sqrt{\hat{p}(1-\hat{p}) / n}
$$

Where the two rows represent independent binomial samples, the standard error of the column 1 risk difference is computed as

$$
\operatorname{se}(\hat{d})=\sqrt{\hat{p}_{1}\left(1-\hat{p}_{1}\right) / n_{1} \cdot+\hat{p}_{2}\left(1-\hat{p}_{2}\right) / n_{2}}
$$

The computations are similar for the column 2 risks and risk difference.

## Confidence Limits

By default, the RISKDIFF option provides Wald asymptotic confidence limits for the risks (row 1, row 2, and overall) and the risk difference. By default, the RISKDIFF option also provides exact (Clopper-Pearson) confidence limits for the risks. You can suppress the display of this information by specifying the NORISKS riskdiff-option. You can specify riskdiff-options to request tests and other types of confidence limits for the risk difference. See the sections "Risk Difference Confidence Limits" on page 2727 and "Risk Difference Tests" on page 2730 for more information.

The risks are equivalent to the binomial proportions of their corresponding rows. This section describes the Wald confidence limits that are provided by default when you specify the RISKDIFF option. The BINOMIAL option provides additional confidence limit types and tests for risks (binomial proportions). See the sections "Binomial Confidence Limits" on page 2719 and "Binomial Tests" on page 2721 for details.

The Wald confidence limits are based on the normal approximation to the binomial distribution. PROC FREQ computes the Wald confidence limits for the risks and risk differences as

$$
\text { Est } \pm\left(z_{\alpha / 2} \times \operatorname{se}(\text { Est })\right)
$$

where Est is the estimate, $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution, and se(Est) is the standard error of the estimate. The confidence level $\alpha$ is determined by the value of the ALPHA= option; the default of ALPHA=0.05 produces $95 \%$ confidence limits.

If you specify the CORRECT riskdiff-option, PROC FREQ includes continuity corrections in the Wald confidence limits for the risks and risk differences. The purpose of a continuity correction is to adjust for the difference between the normal approximation and the binomial distribution, which is discrete. See Fleiss, Levin, and Paik (2003) for more information. The continuity-corrected Wald confidence limits are computed as

$$
\text { Est } \pm\left(z_{\alpha / 2} \times \mathrm{se}(\text { Est })+c c\right)
$$

where $c c$ is the continuity correction. For the row 1 risk, $c c=\left(1 / 2 n_{1}\right)$; for the row 2 risk, $c c=\left(1 / 2 n_{2}\right)$; for the overall risk, $c c=(1 / 2 n)$; and for the risk difference, $c c=\left(\left(1 / n_{1}+1 / n_{2}.\right) / 2\right)$. The column 1 and column 2 risks use the same continuity corrections.
By default when you specify the RISKDIFF option, PROC FREQ also provides exact (Clopper-Pearson) confidence limits for the column 1, column 2, and overall risks. These confidence limits are constructed by inverting the equal-tailed test that is based on the binomial distribution. See the section "Exact (ClopperPearson) Confidence Limits" on page 2720 for details.

Risk Difference Confidence Limits You can request additional confidence limits for the risk difference by specifying the CL= riskdiff-option. Available confidence limit types include Agresti-Caffo, exact unconditional, Hauck-Anderson, Miettinen-Nurminen (score), Newcombe (hybrid-score), and Wald confidence limits. Continuity-corrected Newcombe and Wald confidence limits are also available.

The confidence coefficient for the confidence limits produced by the CL= riskdiff-option is $100(1-\alpha) \%$, where the value of $\alpha$ is determined by the ALPHA= option. The default of ALPHA=0.05 produces $95 \%$ confidence limits. This differs from the test-based confidence limits that are provided with the equivalence, noninferiority, and superiority tests, which have a confidence coefficient of $100(1-2 \alpha) \%$ (Schuirmann 1999). See the section "Risk Difference Tests" on page 2730 for details.

The section "Exact Unconditional Confidence Limits for the Risk Difference" on page 2734 describes the computation of the exact confidence limits. The confidence limits are constructed by inverting two separate one-sided exact tests (tail method). By default, the tests are based on the unstandardized risk difference. If you specify the RISKDIFF(METHOD=SCORE) option, the tests are based on the score statistic.

The following sections describe the computation of the Agresti-Coull, Hauck-Anderson, Miettinen-Nurminen (score), Newcombe (hybrid-score), and Wald confidence limits for the risk difference.

Agresti-Caffo Confidence Limits The Agresti-Caffo confidence limits for the risk difference are computed as

$$
\tilde{d} \pm\left(z_{\alpha / 2} \times \operatorname{se}(\tilde{d})\right)
$$

where $\tilde{d}=\tilde{p}_{1}-\tilde{p}_{2}, \tilde{p}_{i}=\left(n_{i 1}+1\right) /\left(n_{i} .+2\right)$,

$$
\operatorname{se}(\tilde{d})=\sqrt{\tilde{p}_{1}\left(1-\tilde{p}_{2}\right) /\left(n_{1} \cdot+2\right)+\tilde{p}_{2}\left(1-\tilde{p}_{2}\right) /\left(n_{2}+2\right)}
$$

and $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution.

The Agresti-Caffo interval adjusts the Wald interval for the risk difference by adding a pseudo-observation of each type (success and failure) to each sample. See Agresti and Caffo (2000) and Agresti and Coull (1998) for more information.
Hauck-Anderson Confidence Limits The Hauck-Anderson confidence limits for the risk difference are computed as

$$
\hat{d} \pm\left(c c+z_{\alpha / 2} \times \operatorname{se}(\hat{d})\right)
$$

where $\hat{d}=\hat{p}_{1}-\hat{p}_{2}$ and $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. The standard error is computed from the sample proportions as

$$
\operatorname{se}(\hat{d})=\sqrt{\hat{p}_{1}\left(1-\hat{p}_{1}\right) /\left(n_{1 .}-1\right)+\hat{p}_{2}\left(1-\hat{p}_{2}\right) /\left(n_{2 .}-1\right)}
$$

The Hauck-Anderson continuity correction $c c$ is computed as

$$
c c=1 /\left(2 \min \left(n_{1} ., n_{2} \cdot\right)\right)
$$

See Hauck and Anderson (1986) for more information. The subsection "Hauck-Anderson Test" in the section "Noninferiority Tests" on page 2731 describes the corresponding noninferiority test.

Miettinen-Nurminen (Score) Confidence Limits The Miettinen-Nurminen (score) confidence limits for the risk difference (Miettinen and Nurminen 1985) are computed by inverting score tests for the risk difference. A score-based test statistic for the null hypothesis that the risk difference equals $\delta$ can be expressed as

$$
T(\delta)=(\hat{d}-\delta) / \sqrt{\widetilde{\operatorname{Var}}(\delta)}
$$

where $\hat{d}$ is the observed value of the risk difference ( $\hat{p}_{1}-\hat{p}_{2}$ ),

$$
\widetilde{\operatorname{Var}}(\delta)=(n /(n-1))\left(\tilde{p}_{1}(\delta)\left(1-\tilde{p}_{1}(\delta)\right) / n_{1}+\tilde{p}_{2}(\delta)\left(1-\tilde{p}_{2}(\delta)\right) / n_{2}\right)
$$

and $\tilde{p}_{1}(\delta)$ and $\tilde{p}_{2}(\delta)$ are the maximum likelihood estimates of the row 1 and row 2 risks (proportions) under the restriction that the risk difference is $\delta$. For more information, see Miettinen and Nurminen (1985, pp. 215-216) and Miettinen (1985, chapter 12).

The $100(1-\alpha) \%$ confidence interval for the risk difference consists of all values of $\delta$ for which the score test statistic $T(\delta)$ falls in the acceptance region,

$$
\left\{\delta: T(\delta)<z_{\alpha / 2}\right\}
$$

where $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. PROC FREQ finds the confidence limits by iterative computation, which stops when the iteration increment falls below the convergence criterion or when the maximum number of iterations is reached, whichever occurs first. By default, the convergence criterion is 0.00000001 and the maximum number of iterations is 100 .

By default, the Miettinen-Nurminen confidence limits include the bias correction factor $n /(n-1)$ in the computation of $\widetilde{\operatorname{Var}}(\delta)$ (Miettinen and Nurminen 1985, p. 216). For more information, see Newcombe and Nurminen (2011). If you specify the $\mathrm{CL}=\mathrm{MN}(\mathrm{CORRECT}=\mathrm{NO})$ riskdiff-option, PROC FREQ does not include the bias correction factor in this computation (Mee 1984). See also Agresti (2002, p. 77). The uncorrected confidence limits are labeled as "Miettinen-Nurminen-Mee" confidence limits in the displayed output.

The maximum likelihood estimates of $p_{1}$ and $p_{2}$, subject to the constraint that the risk difference is $\delta$, are computed as

$$
\tilde{p}_{1}=2 u \cos (w)-b / 3 a \quad \text { and } \quad \tilde{p}_{2}=\tilde{p}_{1}+\delta
$$

where

$$
\begin{aligned}
w & =\left(\pi+\cos ^{-1}\left(v / u^{3}\right)\right) / 3 \\
v & =b^{3} /(3 a)^{3}-b c / 6 a^{2}+d / 2 a \\
u & =\operatorname{sign}(v) \sqrt{b^{2} /(3 a)^{2}-c / 3 a} \\
a & =1+\theta \\
b & =-\left(1+\theta+\hat{p}_{1}+\theta \hat{p}_{2}+\delta(\theta+2)\right) \\
c & =\delta^{2}+\delta\left(2 \hat{p}_{1}+\theta+1\right)+\hat{p}_{1}+\theta \hat{p}_{2} \\
d & =-\hat{p}_{1} \delta(1+\delta) \\
\theta & =n_{2} \cdot / n_{1} .
\end{aligned}
$$

For more information, see Farrington and Manning (1990, p. 1453).
Newcombe Confidence Limits Newcombe (hybrid-score) confidence limits for the risk difference are constructed from the Wilson score confidence limits for each of the two individual proportions. The confidence limits for the individual proportions are used in the standard error terms of the Wald confidence limits for the proportion difference. See Newcombe (1998a) and Barker et al. (2001) for more information.

Wilson score confidence limits for $p_{1}$ and $p_{2}$ are the roots of

$$
\left|p_{i}-\hat{p}_{i}\right|=z_{\alpha / 2} \sqrt{p_{i}\left(1-p_{i}\right) / n_{i}}
$$

for $i=1,2$. The confidence limits are computed as

$$
\left(\hat{p}_{i}+z_{\alpha / 2}^{2} / 2 n_{i} . \pm z_{\alpha / 2} \sqrt{\left(\hat{p}_{i}\left(1-\hat{p}_{i}\right)+z_{\alpha}^{2} / 4 n_{i} .\right) / n_{i} .}\right) /\left(1+z_{\alpha / 2}^{2} / n_{i} .\right)
$$

See the section "Wilson (Score) Confidence Limits" on page 2720 for details.
Denote the lower and upper Wilson score confidence limits for $p_{1}$ as $L_{1}$ and $U_{1}$, and denote the lower and upper confidence limits for $p_{2}$ as $L_{2}$ and $U_{2}$. The Newcombe confidence limits for the proportion difference ( $d=p_{1}-p_{2}$ ) are computed as

$$
\begin{aligned}
& d_{L}=\left(\hat{p}_{1}-\hat{p}_{2}\right)-\sqrt{\left(\hat{p}_{1}-L_{1}\right)^{2}+\left(U_{2}-\hat{p}_{2}\right)^{2}} \\
& d_{U}=\left(\hat{p}_{1}-\hat{p}_{2}\right)+\sqrt{\left(U_{1}-\hat{p}_{1}\right)^{2}+\left(\hat{p}_{2}-L_{2}\right)^{2}}
\end{aligned}
$$

If you specify the CORRECT riskdiff-option, PROC FREQ provides continuity-corrected Newcombe confidence limits. By including a continuity correction of $1 / 2 n_{i}$., the Wilson score confidence limits for the individual proportions are computed as the roots of

$$
\left|p_{i}-\hat{p}_{i}\right|-1 / 2 n_{i} .=z_{\alpha / 2} \sqrt{p_{i}\left(1-p_{i}\right) / n_{i}} .
$$

The continuity-corrected confidence limits for the individual proportions are then used to compute the proportion difference confidence limits $d_{L}$ and $d_{U}$.

Wald Confidence Limits The Wald confidence limits for the risk difference are computed as

$$
\hat{d} \pm\left(z_{\alpha / 2} \times \operatorname{se}(\hat{d})\right)
$$

where $\hat{d}=\hat{p}_{1}-\hat{p}_{2}, z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. and the standard error is computed from the sample proportions as

$$
\operatorname{se}(\hat{d})=\sqrt{\hat{p}_{1}\left(1-\hat{p}_{1}\right) / n_{1}+\hat{p}_{2}\left(1-\hat{p}_{2}\right) / n_{2}}
$$

If you specify the CORRECT riskdiff-option, the Wald confidence limits include a continuity correction $c c$,

$$
\hat{d} \pm\left(c c+z_{\alpha / 2} \times \operatorname{se}(\hat{d})\right)
$$

where $c c=\left(1 / n_{1}+1 / n_{2}.\right) / 2$.
The subsection "Wald Test" in the section "Noninferiority Tests" on page 2731 describes the corresponding noninferiority test.

## Risk Difference Tests

You can specify riskdiff-options to request tests of the risk (proportion) difference. You can request tests of equality, noninferiority, superiority, and equivalence for the risk difference. The test of equality is a standard Wald asymptotic test, available with or without a continuity correction. For noninferiority, superiority, and equivalence tests of the risk difference, the following test methods are provided: Wald (with and without continuity correction), Hauck-Anderson, Farrington-Manning (score), and Newcombe (with and without continuity correction). You can specify the test method with the METHOD= riskdiff-option. By default, PROC FREQ uses METHOD=WALD.

Equality Test If you specify the EQUAL riskdiff-option, PROC FREQ computes a test of equality, or a test of the null hypothesis that the risk difference equals zero. For the column 1 (or 2 ) risk difference, this test can be expressed as $H_{0}: d=0$ versus the alternative $H_{a}: d \neq 0$, where $d=p_{1}-p_{2}$ denotes the column 1 (or 2) risk difference. PROC FREQ provides a Wald asymptotic test of equality. The test statistic is computed as

$$
z=\hat{d} / \operatorname{se}(\hat{d})
$$

By default, the standard error is computed from the sample proportions as

$$
\operatorname{se}(\hat{d})=\sqrt{\hat{p}_{1}\left(1-\hat{p}_{1}\right) / n_{1}+\hat{p}_{2}\left(1-\hat{p}_{2}\right) / n_{2}}
$$

If you specify the VAR=NULL riskdiff-option, the standard error is based on the null hypothesis that the row 1 and row 2 risks are equal,

$$
\operatorname{se}(\hat{d})=\sqrt{\hat{p}(1-\hat{p}) \times\left(1 / n_{1} \cdot+1 / n_{2} \cdot\right)}
$$

where $\hat{p}=n_{\cdot 1} / n$ estimates the overall column 1 risk.
If you specify the CORRECT riskdiff-option, PROC FREQ includes a continuity correction in the test statistic. If $\hat{d}>0$, the continuity correction is subtracted from $\hat{d}$ in the numerator of the test statistic; otherwise, the continuity correction is added to the numerator. The value of the continuity correction is $\left(1 / n_{1}+1 / n_{2}\right) / 2$.
PROC FREQ computes one-sided and two-sided $p$-values for this test. When the test statistic $z$ is greater than 0 , PROC FREQ displays the right-sided $p$-value, which is the probability of a larger value occurring under the null hypothesis. The one-sided $p$-value can be expressed as

$$
P_{1}=\left\{\begin{array}{lll}
\operatorname{Prob}(Z>z) & \text { if } & z>0 \\
\operatorname{Prob}(Z<z) & \text { if } & z \leq 0
\end{array}\right.
$$

where $Z$ has a standard normal distribution. The two-sided $p$-value is computed as $P_{2}=2 \times P_{1}$.

Noninferiority Tests If you specify the NONINF riskdiff-option, PROC FREQ provides a noninferiority test for the risk difference, or the difference between two proportions. The null hypothesis for the noninferiority test is

$$
H_{0}: p_{1}-p_{2} \leq-\delta
$$

versus the alternative

$$
H_{a}: p_{1}-p_{2}>-\delta
$$

where $\delta$ is the noninferiority margin. Rejection of the null hypothesis indicates that the row 1 risk is not inferior to the row 2 risk. See Chow, Shao, and Wang (2003) for more information.

You can specify the value of $\delta$ with the MARGIN= riskdiff-option. By default, $\delta=0.2$. You can specify the test method with the METHOD= riskdiff-option. The following methods are available for the risk difference noninferiority analysis: Wald (with and without continuity correction), Hauck-Anderson, Farrington-Manning (score), and Newcombe (with and without continuity correction). The Wald, Hauck-Anderson, and FarringtonManning methods provide tests and corresponding test-based confidence limits; the Newcombe method provides only confidence limits. If you do not specify METHOD=, PROC FREQ uses the Wald test by default.

The confidence coefficient for the test-based confidence limits is $100(1-2 \alpha) \%$ (Schuirmann 1999). By default, if you do not specify the ALPHA= option, these are $90 \%$ confidence limits. You can compare the confidence limits to the noninferiority limit, $-\delta$.

The following sections describe the noninferiority analysis methods for the risk difference.
Wald Test If you specify the METHOD=WALD riskdiff-option, PROC FREQ provides an asymptotic Wald test of noninferiority for the risk difference. This is also the default method. The Wald test statistic is computed as

$$
z=(\hat{d}+\delta) / \operatorname{se}(\hat{d})
$$

where ( $\hat{d}=\hat{p}_{1}-\hat{p}_{2}$ ) estimates the risk difference and $\delta$ is the noninferiority margin.
By default, the standard error for the Wald test is computed from the sample proportions as

$$
\operatorname{se}(\hat{d})=\sqrt{\hat{p}_{1}\left(1-\hat{p}_{1}\right) / n_{1}+\hat{p}_{2}\left(1-\hat{p}_{2}\right) / n_{2}} .
$$

If you specify the VAR=NULL riskdiff-option, the standard error is based on the null hypothesis that the risk difference equals $-\delta$ (Dunnett and Gent 1977). The standard error is computed as

$$
\operatorname{se}(\hat{d})=\sqrt{\tilde{p}(1-\tilde{p}) / n_{2}+(\tilde{p}-\delta)(1-\tilde{p}+\delta) / n_{1}} .
$$

where

$$
\tilde{p}=\left(n_{11}+n_{21}+\delta n_{1} \cdot\right) / n
$$

If you specify the CORRECT riskdiff-option, the test statistic includes a continuity correction. The continuity correction is subtracted from the numerator of the test statistic if the numerator is greater than zero; otherwise, the continuity correction is added to the numerator. The value of the continuity correction is $\left(1 / n_{1} .+\right.$ $1 / n_{2}$.)/2.

The $p$-value for the Wald noninferiority test is $P_{z}=\operatorname{Prob}(Z>z)$, where $Z$ has a standard normal distribution.

Hauck-Anderson Test If you specify the METHOD=HA riskdiff-option, PROC FREQ provides the HauckAnderson test for noninferiority. The Hauck-Anderson test statistic is computed as

$$
z=(\hat{d}+\delta \pm c c) / \operatorname{se}(\hat{d})
$$

where $\hat{d}=\hat{p}_{1}-\hat{p}_{2}$ and the standard error is computed from the sample proportions as

$$
\operatorname{se}(\hat{d})=\sqrt{\hat{p}_{1}\left(1-\hat{p}_{1}\right) /\left(n_{1 .}-1\right)+\hat{p}_{2}\left(1-\hat{p}_{2}\right) /\left(n_{2 \cdot}-1\right)}
$$

The Hauck-Anderson continuity correction $c c$ is computed as

$$
c c=1 /\left(2 \min \left(n_{1} ., n_{2} \cdot\right)\right)
$$

The $p$-value for the Hauck-Anderson noninferiority test is $P_{z}=\operatorname{Prob}(Z>z)$, where $Z$ has a standard normal distribution. See Hauck and Anderson (1986) and Schuirmann (1999) for more information.
Farrington-Manning (Score) Test If you specify the METHOD=FM riskdiff-option, PROC FREQ provides the Farrington-Manning (score) test of noninferiority for the risk difference. A score test statistic for the null hypothesis that the risk difference equals $-\delta$ can be expressed as

$$
z=(\hat{d}+\delta) / \operatorname{se}(\hat{d})
$$

where $\hat{d}$ is the observed value of the risk difference ( $\hat{p}_{1}-\hat{p}_{2}$ ),

$$
\operatorname{se}(\hat{d})=\sqrt{\tilde{p}_{1}\left(1-\tilde{p}_{1}\right) / n_{1} \cdot+\tilde{p}_{2}\left(1-\tilde{p}_{2}\right) / n_{2}} .
$$

and $\tilde{p}_{1}$ and $\tilde{p}_{2}$ are the maximum likelihood estimates of the row 1 and row 2 risks (proportions) under the restriction that the risk difference is $-\delta$. The $p$-value for the noninferiority test is $P_{z}=\operatorname{Prob}(Z>z)$, where $Z$ has a standard normal distribution. For more information, see Miettinen and Nurminen (1985); Miettinen (1985); Farrington and Manning (1990); Dann and Koch (2005).

The maximum likelihood estimates of $p_{1}$ and $p_{1}$, subject to the constraint that the risk difference is $-\delta$, are computed as

$$
\tilde{p}_{1}=2 u \cos (w)-b / 3 a \quad \text { and } \quad \tilde{p}_{2}=\tilde{p}_{1}+\delta
$$

where

$$
\begin{aligned}
w & =\left(\pi+\cos ^{-1}\left(v / u^{3}\right)\right) / 3 \\
v & =b^{3} /(3 a)^{3}-b c / 6 a^{2}+d / 2 a \\
u & =\operatorname{sign}(v) \sqrt{b^{2} /(3 a)^{2}-c / 3 a} \\
a & =1+\theta \\
b & =-\left(1+\theta+\hat{p}_{1}+\theta \hat{p}_{2}-\delta(\theta+2)\right) \\
c & =\delta^{2}-\delta\left(2 \hat{p}_{1}+\theta+1\right)+\hat{p}_{1}+\theta \hat{p}_{2} \\
d & =\hat{p}_{1} \delta(1-\delta) \\
\theta & =n_{2} \cdot / n_{1} .
\end{aligned}
$$

For more information, see Farrington and Manning (1990, p. 1453).
Newcombe Noninferiority Analysis If you specify the METHOD=NEWCOMBE riskdiff-option, PROC FREQ provides a noninferiority analysis that is based on Newcombe hybrid-score confidence limits for the
risk difference. The confidence coefficient for the confidence limits is $100(1-2 \alpha) \%$ (Schuirmann 1999). By default, if you do not specify the ALPHA= option, these are $90 \%$ confidence limits. You can compare the confidence limits with the noninferiority limit, $-\delta$. If you specify the CORRECT riskdiff-option, the confidence limits includes a continuity correction. See the subsection "Newcombe Confidence Limits" in the section "Risk Difference Confidence Limits" on page 2727 for more information.

Superiority Test If you specify the SUP riskdiff-option, PROC FREQ provides a superiority test for the risk difference. The null hypothesis is

$$
H_{0}:: p_{1}-p_{2} \leq \delta
$$

versus the alternative

$$
H_{a}: p_{1}-p_{2}>\delta
$$

where $\delta$ is the superiority margin. Rejection of the null hypothesis indicates that the row 1 proportion is superior to the row 2 proportion. You can specify the value of $\delta$ with the MARGIN= riskdiff-option. By default, $\delta=0.2$.

The superiority analysis is identical to the noninferiority analysis but uses a positive value of the margin $\delta$ in the null hypothesis. The superiority computations follow those in the section "Noninferiority Tests" on page 2731 by replacing $-\delta$ by $\delta$. See Chow, Shao, and Wang (2003) for more information.

Equivalence Tests If you specify the EQUIV riskdiff-option, PROC FREQ provides an equivalence test for the risk difference, or the difference between two proportions. The null hypothesis for the equivalence test is

$$
H_{0}: p_{1}-p_{2} \leq-\delta_{L} \quad \text { or } \quad p_{1}-p_{2} \geq \delta_{U}
$$

versus the alternative

$$
H_{a}: \delta_{L}<p_{1}-p_{2}<\delta_{U}
$$

where $\delta_{L}$ is the lower margin and $\delta_{U}$ is the upper margin. Rejection of the null hypothesis indicates that the two binomial proportions are equivalent. See Chow, Shao, and Wang (2003) for more information.

You can specify the value of the margins $\delta_{L}$ and $\delta_{U}$ with the MARGIN= riskdiff-option. If you do not specify MARGIN=, PROC FREQ uses lower and upper margins of -0.2 and 0.2 by default. If you specify a single margin value $\delta$, PROC FREQ uses lower and upper margins of $-\delta$ and $\delta$. You can specify the test method with the METHOD= riskdiff-option. The following methods are available for the risk difference equivalence analysis: Wald (with and without continuity correction), Hauck-Anderson, Farrington-Manning (score), and Newcombe (with and without continuity correction). The Wald, Hauck-Anderson, and Farrington-Manning methods provide tests and corresponding test-based confidence limits; the Newcombe method provides only confidence limits. If you do not specify METHOD=, PROC FREQ uses the Wald test by default.

PROC FREQ computes two one-sided tests (TOST) for equivalence analysis (Schuirmann 1987). The TOST approach includes a right-sided test for the lower margin $\delta_{L}$ and a left-sided test for the upper margin $\delta_{U}$. The overall $p$-value is taken to be the larger of the two $p$-values from the lower and upper tests.

The section "Noninferiority Tests" on page 2731 gives details about the Wald, Hauck-Anderson, FarringtonManning (score), and Newcombe methods for the risk difference. The lower margin equivalence test statistic
takes the same form as the noninferiority test statistic but uses the lower margin value $\delta_{L}$ in place of $-\delta$. The upper margin equivalence test statistic take the same form as the noninferiority test statistic but uses the upper margin value $\delta_{U}$ in place of $-\delta$.

The test-based confidence limits for the risk difference are computed according to the equivalence test method that you select. If you specify METHOD=WALD with VAR=NULL, or METHOD=FM, separate standard errors are computed for the lower and upper margin tests. In this case, the test-based confidence limits are computed by using the maximum of these two standard errors. These confidence limits have a confidence coefficient of $100(1-2 \alpha) \%$ (Schuirmann 1999). By default, if you do not specify the ALPHA= option, these are $90 \%$ confidence limits. You can compare the test-based confidence limits to the equivalence limits, $\left(\delta_{L}, \delta_{U}\right)$.

## Exact Unconditional Confidence Limits for the Risk Difference

If you specify the RISKDIFF option in the EXACT statement, PROC FREQ provides exact unconditional confidence limits for the risk difference. PROC FREQ computes the confidence limits by inverting two separate one-sided tests (tail method), where the size of each test is at most $\alpha / 2$ and the confidence coefficient is at least $(1-\alpha)$. Exact conditional methods, described in the section "Exact Statistics" on page 2757, do not apply to the risk difference due to the presence of a nuisance parameter (Agresti 1992). The unconditional approach eliminates the nuisance parameter by maximizing the $p$-value over all possible values of the parameter (Santner and Snell 1980).

By default, PROC FREQ uses the unstandardized risk difference as the test statistic in the confidence limit computations. If you specify the RISKDIFF(METHOD=SCORE) option, the procedure uses the score statistic (Chan and Zhang 1999). The score statistic is a less discrete statistic than the raw risk difference and produces less conservative confidence limits (Agresti and Min 2001). See also Santner et al. (2007) for comparisons of methods for computing exact confidence limits for the risk difference.

PROC FREQ computes the confidence limits as follows. The risk difference is defined as the difference between the row 1 and row 2 risks (proportions), $d=p_{1}-p_{2}$, and $n_{1}$ and $n_{2}$ denote the row totals of the $2 \times 2$ table. The joint probability function for the table can be expressed in terms of the table cell frequencies, the risk difference, and the nuisance parameter $p_{2}$ as

$$
f\left(n_{11}, n_{21} ; n_{1}, n_{2}, d, p_{2}\right)=\binom{n_{1}}{n_{11}}\left(d+p_{2}\right)^{n_{11}}\left(1-d-p_{2}\right)^{n_{1}-n_{11}} \times\binom{ n_{2}}{n_{21}} p_{2}^{n_{21}}\left(1-p_{2}\right)^{n_{2}-n_{21}}
$$

The $100(1-\alpha / 2) \%$ confidence limits for the risk difference are computed as

$$
\begin{aligned}
d_{L} & =\sup \left(d_{*}: P_{U}\left(d_{*}\right)>\alpha / 2\right) \\
d_{U} & =\inf \left(d_{*}: P_{L}\left(d_{*}\right)>\alpha / 2\right)
\end{aligned}
$$

where

$$
\begin{aligned}
& P_{U}\left(d_{*}\right)=\sup _{p_{2}}\left(\sum_{A, T(a) \geq t_{0}} f\left(n_{11}, n_{21} ; n_{1}, n_{2}, d_{*}, p_{2}\right)\right) \\
& P_{L}\left(d_{*}\right)=\sup _{p_{2}}\left(\sum_{A, T(a) \leq t_{0}} f\left(n_{11}, n_{21} ; n_{1}, n_{2}, d_{*}, p_{2}\right)\right)
\end{aligned}
$$

The set $A$ includes all $2 \times 2$ tables with row sums equal to $n_{1}$ and $n_{2}$, and $T(a)$ denotes the value of the test statistic for table $a$ in $A$. To compute $P_{U}\left(d_{*}\right)$, the sum includes probabilities of those tables for which
$\left(T(a) \geq t_{0}\right)$, where $t_{0}$ is the value of the test statistic for the observed table. For a fixed value of $d_{*}, P_{U}\left(d_{*}\right)$ is taken to be the maximum sum over all possible values of $p_{2}$.

By default, PROC FREQ uses the unstandardized risk difference as the test statistic $T$. If you specify the RISKDIFF(METHOD=SCORE) option, the procedure uses the risk difference score statistic as the test statistic (Chan and Zhang 1999). For information about the computation of the score statistic, see the section """ on page 2728. For more information, see Miettinen and Nurminen (1985) and Farrington and Manning (1990).

## Barnard's Unconditional Exact Test

The BARNARD option in the EXACT statement provides an unconditional exact test for the risk (proportion) difference for $2 \times 2$ tables. The reference set for the unconditional exact test consists of all $2 \times 2$ tables that have the same row sums as the observed table (Barnard 1945, 1947, 1949). This differs from the reference set for exact conditional inference, which is restricted to the set of tables that have the same row sums and the same column sums as the observed table. See the sections "Fisher's Exact Test" on page 2708 and "Exact Statistics" on page 2757 for more information.

The test statistic is the standardized risk difference, which is computed as

$$
T=d / \sqrt{p \cdot 1(1-p \cdot 1)\left(1 / n_{1}+1 / n_{2}\right)}
$$

where the risk difference $d$ is defined as the difference between the row 1 and row 2 risks (proportions), $d=\left(n_{11} / n_{1}-n_{21} / n_{2}\right) ; n_{1}$ and $n_{2}$ are the row 1 and row 2 totals, respectively; and $p \cdot 1$ is the overall proportion in column $1,\left(n_{11}+n_{21}\right) / n$.

Under the null hypothesis that the risk difference is 0 , the joint probability function for a table can be expressed in terms of the table cell frequencies, the row totals, and the unknown parameter $\pi$ as

$$
f\left(n_{11}, n_{21} ; n_{1}, n_{2}, \pi\right)=\binom{n_{1}}{n_{11}}\binom{n_{2}}{n_{21}} \pi^{n_{11}+n_{21}}(1-\pi)^{n-n_{11}-n_{21}}
$$

where $\pi$ is the common value of the risk (proportion).
PROC FREQ sums the table probabilities over the reference set for those tables where the test statistic is greater than or equal to the observed value of the test statistic. This sum can be expressed as

$$
\operatorname{Prob}(\pi)=\sum_{A, T(a) \geq t_{0}} f\left(n_{11}, n_{21} ; n_{1}, n_{2}, \pi\right)
$$

where the set $A$ contains all $2 \times 2$ tables with row sums equal to $n_{1}$ and $n_{2}$, and $T(a)$ denotes the value of the test statistic for table $a$ in $A$. The sum includes probabilities of those tables for which $\left(T(a) \geq t_{0}\right)$, where $t_{0}$ is the value of the test statistic for the observed table.

The sum $\operatorname{Prob}(\pi)$ depends on the unknown value of $\pi$. To compute the exact $p$-value, PROC FREQ eliminates the nuisance parameter $\pi$ by taking the maximum value of $\operatorname{Prob}(\pi)$ over all possible values of $\pi$,

$$
\operatorname{Prob}=\sup _{(0 \leq \pi \leq 1)}(\operatorname{Prob}(\pi))
$$

See Suissa and Shuster (1985) and Mehta and Senchaudhuri (2003).

## Common Risk Difference

PROC FREQ provides Mantel-Haenszel and summary score estimates for the common risk (proportion) difference for multiway $2 \times 2$ tables.

## Mantel-Haenszel Estimate of the Common Risk Difference

PROC FREQ computes the Mantel-Haenszel (Mantel and Haenszel 1959) estimate of the common risk difference as

$$
\hat{d}_{M H}=\left(\sum_{h} \hat{d}_{h} w_{h}\right) /\left(\sum_{h} w_{h}\right)
$$

where $\hat{d}_{h}$ is the risk difference in stratum $h$ and

$$
w_{h}=n_{h 1} \cdot n_{h 2} \cdot / n_{h}
$$

The column 1 risk difference in stratum $h$ is computed as

$$
\hat{d}_{h}=\hat{p}_{h 1}-\hat{p}_{h 2}=\left(n_{h 11} / n_{h 1} .\right)-\left(n_{h 21} / n_{h 2} .\right)
$$

where $\hat{p}_{h 1}$ is the proportion of row 1 observations that are classified in column 1 and $\hat{p}_{h 2}$ is the proportion or row 2 observations that are classified in column 1. The column 2 risk is computed in the same way. For more information, see Agresti (2013, p. 231).
PROC FREQ computes the variance of $\hat{d}_{M H}$ (Sato 1989) as

$$
\hat{\sigma}^{2}\left(\hat{d}_{M H}\right)=\left(\hat{d}_{M H} \sum_{h} P_{h}+\sum_{h} Q_{h}\right) /\left(\sum_{h} w_{h}\right)^{2}
$$

where

$$
\begin{aligned}
& P_{h}=\left(n_{h 1}^{2} \cdot n_{h 21}-n_{h 2}^{2} \cdot n_{h 11}+n_{h 1} \cdot n_{h 2} \cdot\left(n_{h 2} \cdot-n_{h 1} \cdot\right) / 2\right) / n_{h}^{2} \\
& Q_{h}=\left(n_{h 11}\left(n_{h 2} \cdot-n_{h 21}\right)+n_{h 21}\left(n_{h 1} \cdot-n_{h 11}\right)\right) / 2 n_{h}
\end{aligned}
$$

The $100(1-\alpha / 2) \%$ confidence limits for the common risk difference are

$$
\hat{d}_{M H} \pm\left(z_{\alpha / 2} \times \hat{\sigma}\left(\hat{d}_{M H}\right)\right)
$$

## Summary Score Estimate of the Common Risk Difference

PROC FREQ computes the summary score estimate of the common risk difference as described in Agresti (2013, p. 231). This estimate is computed from Miettinen-Nurminen (score) confidence limits for the stratum risk differences. For more information, see the section "Miettinen-Nurminen (Score) Confidence Limits." The score confidence interval for the risk difference in stratum $h$ can be expressed as $\hat{d}_{h}^{\prime} \pm z_{\alpha / 2} s_{h}^{\prime}$, where $\hat{d}_{h}^{\prime}$ is the midpoint of the score confidence interval and $s_{h}^{\prime}$ is the width of the confidence interval divided by $2 z_{\alpha / 2}$. The summary score estimate of the common risk difference is computed as

$$
\hat{d}_{S}=\sum_{h} \hat{d}_{h}^{\prime} w_{h}^{\prime}
$$

where

$$
w_{h}^{\prime}=\left(1 / s_{h}^{\prime 2}\right) / \sum_{i}\left(1 / s_{i}^{\prime 2}\right)
$$

The variance of $\hat{d}_{S}$ is computed as

$$
\hat{\sigma}^{2}\left(\hat{d}_{S}\right)=1 / \sum_{h}\left(1 / s_{h}^{\prime}{ }^{2}\right)
$$

The $100(1-\alpha / 2) \%$ summary score confidence limits for the common risk difference are

$$
\hat{d}_{S} \pm\left(z_{\alpha / 2} \times \hat{\sigma}\left(\hat{d}_{S}\right)\right)
$$

## Odds Ratio and Relative Risks for $2 \times 2$ Tables

## Odds Ratio

The odds ratio is a useful measure of association for a variety of study designs. For a retrospective design called a case-control study, the odds ratio can be used to estimate the relative risk when the probability of positive response is small (Agresti 2002). In a case-control study, two independent samples are identified based on a binary (yes-no) response variable, and the conditional distribution of a binary explanatory variable is examined, within fixed levels of the response variable. See Stokes, Davis, and Koch (2012) and Agresti (2007).

The odds of a positive response (column 1) in row 1 is $n_{11} / n_{12}$. Similarly, the odds of a positive response in row 2 is $n_{21} / n_{22}$. The odds ratio is formed as the ratio of the row 1 odds to the row 2 odds. The odds ratio for a $2 \times 2$ table is defined as

$$
O R=\frac{n_{11} / n_{12}}{n_{21} / n_{22}}=\frac{n_{11} n_{22}}{n_{12} n_{21}}
$$

The odds ratio can be any nonnegative number. When the row and column variables are independent, the true value of the odds ratio equals 1 . An odds ratio greater than 1 indicates that the odds of a positive response are higher in row 1 than in row 2 . Values less than 1 indicate the odds of positive response are higher in row 2. The strength of association increases with the deviation from 1.

The transformation $G=(O R-1) /(O R+1)$ transforms the odds ratio to the range $(-1,1)$ with $G=0$ when $O R=1 ; G=-1$ when $O R=0$; and $G$ approaches 1 as $O R$ approaches infinity. $G$ is the gamma statistic, which PROC FREQ computes when you specify the MEASURES option.

The asymptotic $100(1-\alpha) \%$ confidence limits for the odds ratio are

$$
(O R \times \exp (-z \sqrt{v}), \quad O R \times \exp (z \sqrt{v}))
$$

where

$$
v=\operatorname{Var}(\ln O R)=\frac{1}{n_{11}}+\frac{1}{n_{12}}+\frac{1}{n_{21}}+\frac{1}{n_{22}}
$$

and $z$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. If any of the four cell frequencies are zero, the estimates are not computed.

Score Confidence Limits for the Odds Ratio Score confidence limits for the odds ratio (Miettinen and Nurminen 1985) are computed by inverting score tests for the odds ratio. A score-based chi-square test statistic for the null hypothesis that the odds ratio equals $\theta$ can be expressed as

$$
Q(\theta)=\left\{n_{1} \cdot\left(\hat{p}_{1}-\tilde{p}_{1}\right)\right\}^{2} /\{n /(n-1)\}\left\{1 /\left(n_{1} \cdot \tilde{p}_{1}\left(1-\tilde{p}_{1}\right)\right)+1 /\left(n_{2} \cdot \tilde{p}_{2}\left(1-\tilde{p}_{2}\right)\right)\right\}^{-1}
$$

where $\hat{p}_{1}$ is the observed row 1 risk (proportion), and $\tilde{p}_{1}$ and $\tilde{p}_{2}$ are the maximum likelihood estimates of the row 1 and row 2 risks under the restriction that the odds ratio $\left(n_{11} n_{22} / n_{12} n_{21}\right)$ is $\theta$. For more information, see Miettinen and Nurminen (1985) and Miettinen (1985, chapter 14).

The $100(1-\alpha) \%$ score confidence interval for the odds ratio consists of all values of $\theta$ for which the test statistic $Q(\theta)$ falls in the acceptance region,

$$
\left\{\theta: Q(\theta)<\chi_{1, \alpha}^{2}\right\}
$$

where $\chi_{1, \alpha}^{2}$ is the $100(1-\alpha)$ percentile of the chi-square distribution with 1 degree of freedom. PROC FREQ finds the confidence limits by iterative computation. For more information about score confidence limits, see Agresti (2013).
By default, the score confidence limits include the bias correction factor $n /(n-1)$ in the denominator of $Q(\theta)$ (Miettinen and Nurminen 1985, p. 217). If you specify the $\mathrm{CL}=\operatorname{SCORE}(\mathrm{CORRECT}=\mathrm{NO}$ ) option, PROC FREQ does not include this factor in the computation.
The maximum likelihood estimates of $p_{1}$ and $p_{2}$, subject to the constraint that the odds ratio is $\theta$, are computed as

$$
\tilde{p_{2}}=\left(-b+\left(b^{2}-4 a c\right)\right) / 2 a \quad \text { and } \quad \tilde{p_{1}}=\tilde{p}_{2} \theta /\left(1+\tilde{p}_{2}(\theta-1)\right)
$$

where

$$
\begin{aligned}
a & =n_{2} \cdot(\theta-1) \\
b & =n_{1} \cdot \theta+n_{2} \cdot-\hat{p} \cdot 1(\theta-1) \\
c & =-\hat{p} \cdot 1
\end{aligned}
$$

For more information, see Miettinen and Nurminen (1985, pp. 217-218) and Miettinen (1985, chapter 14).

Exact Confidence Limits for the Odds Ratio When you specify the OR option in the EXACT statement, PROC FREQ computes exact confidence limits for the odds ratio. Because this is a discrete problem, the confidence coefficient for the exact confidence interval is not exactly $(1-\alpha)$ but is at least $(1-\alpha)$. Thus, these confidence limits are conservative. See Agresti (1992) for more information.

PROC FREQ computes exact confidence limits for the odds ratio by using an algorithm based on Thomas (1971). See also Gart (1971). The following two equations are solved iteratively to determine the lower and upper confidence limits, $\phi_{1}$ and $\phi_{2}$ :

$$
\begin{aligned}
& \sum_{i=n_{11}}^{n_{1}}\binom{n_{1} \cdot}{i}\binom{n_{2} \cdot}{n \cdot 1-i} \phi_{1}^{i} / \sum_{i=0}^{n \cdot 1}\binom{n_{1} \cdot}{i}\binom{n_{2} \cdot}{n \cdot 1-i} \phi_{1}^{i}=\alpha / 2 \\
& \sum_{i=0}^{n_{11}}\binom{n_{1} \cdot}{i}\binom{n_{2 \cdot}}{n \cdot 1-i} \phi_{2}^{i} / \sum_{i=0}^{n \cdot 1}\binom{n_{1} \cdot}{i}\binom{n_{2} \cdot}{n \cdot 1-i} \phi_{2}^{i}=\alpha / 2
\end{aligned}
$$

When the odds ratio equals zero, which occurs when either $n_{11}=0$ or $n_{22}=0$, PROC FREQ sets the lower exact confidence limit to zero and determines the upper limit with level $\alpha$. Similarly, when the odds ratio equals infinity, which occurs when either $n_{12}=0$ or $n_{21}=0$, PROC FREQ sets the upper exact confidence limit to infinity and determines the lower limit with level $\alpha$.

## Relative Risks

These measures of relative risk are useful in cohort (prospective) study designs, where two samples are identified based on the presence or absence of an explanatory factor. The two samples are observed in future time for the binary (yes-no) response variable under study. Relative risk measures are also useful in cross-sectional studies, where two variables are observed simultaneously. See Stokes, Davis, and Koch (2012) and Agresti (2007) for more information.

The column 1 relative risk is the ratio of the column 1 risk for row 1 to row 2 . The column 1 risk for row 1 is the proportion of the row 1 observations classified in column 1 ,

$$
p_{1}=n_{11} / n_{1} .
$$

Similarly, the column 1 risk for row 2 is

$$
p_{2}=n_{21} / n_{2} .
$$

The column 1 relative risk is computed as

$$
R R_{1}=p_{1} / p_{2}
$$

A relative risk greater than 1 indicates that the probability of positive response is greater in row 1 than in row 2. Similarly, a relative risk less than 1 indicates that the probability of positive response is less in row 1 than in row 2 . The strength of association increases with the deviation from 1.

Asymptotic $100(1-\alpha) \%$ confidence limits for the column 1 relative risk are computed as

$$
\left(R R_{1} \times \exp (-z \sqrt{v}), \quad R R_{1} \times \exp (z \sqrt{v})\right)
$$

where

$$
v=\operatorname{Var}\left(\ln R R_{1}\right)=\left(\left(1-p_{1}\right) / n_{11}\right)+\left(\left(1-p_{2}\right) / n_{21}\right)
$$

and $z$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. If either $n_{11}$ or $n_{21}$ is zero, the estimates are not computed.

PROC FREQ computes the column 2 relative risks in the same way.
Exact Unconditional Confidence Limits for the Relative Risk If you specify the RELRISK option in the EXACT statement, PROC FREQ provides exact unconditional confidence limits for the relative risk. PROC FREQ computes the confidence limits by inverting two separate one-sided tests (tail method), where the size of each test is at most $\alpha / 2$ and the confidence coefficient is at least $(1-\alpha)$. Exact conditional methods, described in the section "Exact Statistics" on page 2757, do not apply to the relative risk due to the presence of a nuisance parameter (Agresti 1992). The unconditional approach eliminates the nuisance parameter by maximizing the $p$-value over all possible values of the parameter (Santner and Snell 1980).

By default, PROC FREQ uses the unstandardized relative risk as the test statistic in the confidence limit computations. If you specify the RELRISK(METHOD=SCORE) option, the procedure uses the relative risk
score statistic (Chan and Zhang 1999). The score statistic is a less discrete statistic than the raw relative risk and produces less conservative confidence limits (Agresti and Min 2001). See also Santner et al. (2007) for comparisons of methods for computing exact confidence limits.
See the section "Exact Unconditional Confidence Limits for the Risk Difference" on page 2734 for a description of the method that PROC FREQ uses to compute confidence limits for the relative risk. The test statistic for the relative risk computation is either the unstandardized relative risk (by default) or the relative risk score statistic (if you specify the RELRISK(METHOD=SCORE) option). PROC FREQ uses the following form of the unstandardized relative risk, which adds 0.05 to each frequency, to ensure that the statistic is defined when there are zero table cells (Gart and Nam 1988):

$$
r r=\frac{\left(n_{11}+0.5\right) /\left(n_{1}+0.5\right)}{\left(n_{21}+0.5\right) /\left(n_{2}+0.5\right)}
$$

If you specify the RELRISK(METHOD=SCORE) option, PROC FREQ uses the relative risk score statistic (Miettinen and Nurminen 1985; Farrington and Manning 1990). This test statistic is computed as

$$
z=\left(\hat{p_{1}}-R_{0} \hat{p_{2}}\right) / \operatorname{se}(r r)
$$

where

$$
\operatorname{se}(r r)=\sqrt{\tilde{p}_{1}\left(1-\tilde{p}_{1}\right) / n_{1}+R_{0}^{2} \tilde{p}_{2}\left(1-\tilde{p}_{2}\right) / n_{2}}
$$

where $\tilde{p}_{1}$ and $\tilde{p}_{2}$ are the maximum likelihood estimators of $p_{1}$ and $p_{2}$ under the null hypothesis that the relative risk equals $R_{0}$. From Farrington and Manning (1990, p. 1454), the maximum likelihood solution is

$$
\tilde{p}_{1}=\left(-b-\sqrt{b^{2}-4 a c}\right) / 2 a \quad \text { and } \quad \tilde{p}_{2}=\tilde{p}_{1} / R_{0}
$$

where

$$
\begin{aligned}
a & =1+\theta \\
b & =-\left(R_{0}\left(1+\theta \hat{p}_{2}\right)+\theta+\hat{p}_{1}\right) \\
c & =R_{0}\left(\hat{p}_{1}+\theta \hat{p}_{2}\right) \\
\theta & =n_{2} \cdot / n_{1} .
\end{aligned}
$$

## Cochran-Armitage Test for Trend

The TREND option in the TABLES statement provides the Cochran-Armitage test for trend, which tests for trend in binomial proportions across levels of a single factor or covariate. This test is appropriate for a two-way table where one variable has two levels and the other variable is ordinal. The two-level variable represents the response, and the other variable represents an explanatory variable with ordered levels. When the two-way has two columns and $R$ rows, PROC FREQ tests for trend across the $R$ levels of the row variable, and the binomial proportion is computed as the proportion of observations in the first column. When the table has two rows and $C$ columns, PROC FREQ tests for trend across the $C$ levels of the column variable, and the binomial proportion is computed as the proportion of observations in the first row.

The trend test is based on the regression coefficient for the weighted linear regression of the binomial proportions on the scores of the explanatory variable levels. See Margolin (1988) and Agresti (2002) for details. If the table has two columns and $R$ rows, the trend test statistic is computed as

$$
T=\sum_{i=1}^{R} n_{i 1}\left(R_{i}-\bar{R}\right) / \sqrt{p \cdot 1(1-p \cdot 1) s^{2}}
$$

where $R_{i}$ is the score of row $i, \bar{R}$ is the average row score, and

$$
s^{2}=\sum_{i=1}^{R} n_{i} \cdot\left(R_{i}-\bar{R}\right)^{2}
$$

The SCORES= option in the TABLES statement determines the type of row scores used in computing the trend test (and other score-based statistics). The default is SCORES=TABLE. See the section "Scores" on page 2703 for details. For character variables, the table scores for the row variable are the row numbers (for example, 1 for the first row, 2 for the second row, and so on). For numeric variables, the table score for each row is the numeric value of the row level. When you perform the trend test, the explanatory variable might be numeric (for example, dose of a test substance), and the variable values might be appropriate scores. If the explanatory variable has ordinal levels that are not numeric, you can assign meaningful scores to the variable levels. Sometimes equidistant scores, such as the table scores for a character variable, might be appropriate. For more information on choosing scores for the trend test, see Margolin (1988).

The null hypothesis for the Cochran-Armitage test is no trend, which means that the binomial proportion $p_{i 1}=n_{i 1} / n_{i}$. is the same for all levels of the explanatory variable. Under the null hypothesis, the trend statistic has an asymptotic standard normal distribution.

PROC FREQ computes one-sided and two-sided $p$-values for the trend test. When the test statistic is greater than its null hypothesis expected value of zero, PROC FREQ displays the right-sided $p$-value, which is the probability of a larger value of the statistic occurring under the null hypothesis. A small right-sided $p$-value supports the alternative hypothesis of increasing trend in proportions from row 1 to row $R$. When the test statistic is less than or equal to zero, PROC FREQ displays the left-sided $p$-value. A small left-sided $p$-value supports the alternative of decreasing trend.

The one-sided $p$-value for the trend test is computed as

$$
P_{1}= \begin{cases}\operatorname{Prob}(Z>T) & \text { if } \quad T>0 \\ \operatorname{Prob}(Z<T) & \text { if } T \leq 0\end{cases}
$$

where $Z$ has a standard normal distribution. The two-sided $p$-value is computed as

$$
P_{2}=\operatorname{Prob}(|Z|>|T|)
$$

PROC FREQ also provides exact $p$-values for the Cochran-Armitage trend test. You can request the exact test by specifying the TREND option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Jonckheere-Terpstra Test

The JT option in the TABLES statement provides the Jonckheere-Terpstra test, which is a nonparametric test for ordered differences among classes. It tests the null hypothesis that the distribution of the response variable does not differ among classes. It is designed to detect alternatives of ordered class differences, which can be expressed as $\tau_{1} \leq \tau_{2} \leq \cdots \leq \tau_{R}$ (or $\tau_{1} \geq \tau_{2} \geq \cdots \geq \tau_{R}$ ), with at least one of the inequalities being strict, where $\tau_{i}$ denotes the effect of class $i$. For such ordered alternatives, the Jonckheere-Terpstra test can be preferable to tests of more general class difference alternatives, such as the Kruskal-Wallis test (produced by the WILCOXON option in the NPAR1WAY procedure). See Pirie (1983) and Hollander and Wolfe (1999) for more information about the Jonckheere-Terpstra test.

The Jonckheere-Terpstra test is appropriate for a two-way table in which an ordinal column variable represents the response. The row variable, which can be nominal or ordinal, represents the classification variable. The levels of the row variable should be ordered according to the ordering you want the test to detect. The order of variable levels is determined by the ORDER = option in the PROC FREQ statement. The default is ORDER=INTERNAL, which orders by unformatted values. If you specify ORDER=DATA, PROC FREQ orders values according to their order in the input data set. For more information about how to order variable levels, see the ORDER= option.

The Jonckheere-Terpstra test statistic is computed by first forming $R(R-1) / 2$ Mann-Whitney counts $M_{i, i^{\prime}}$, where $i<i^{\prime}$, for pairs of rows in the contingency table,

$$
\begin{aligned}
M_{i, i^{\prime}} & =\left\{\text { number of times } X_{i, j}<X_{i^{\prime}, j^{\prime}}, \quad j=1, \ldots, n_{i .} ; \quad j^{\prime}=1, \ldots, n_{i^{\prime} .}\right\} \\
& +\frac{1}{2}\left\{\text { number of times } X_{i, j}=X_{i^{\prime}, j^{\prime}}, \quad j=1, \ldots, n_{i .} ; \quad j^{\prime}=1, \ldots, n_{i^{\prime}} .\right\}
\end{aligned}
$$

where $X_{i, j}$ is response $j$ in row $i$. The Jonckheere-Terpstra test statistic is computed as

$$
J=\sum_{1 \leq i<i^{\prime} \leq R} M_{i, i^{\prime}}
$$

This test rejects the null hypothesis of no difference among classes for large values of $J$. Asymptotic $p$-values for the Jonckheere-Terpstra test are obtained by using the normal approximation for the distribution of the standardized test statistic. The standardized test statistic is computed as

$$
J^{*}=\left(J-\mathrm{E}_{0}(J)\right) / \sqrt{\operatorname{Var}_{0}(J)}
$$

where $\mathrm{E}_{0}(J)$ and $\operatorname{Var}_{0}(J)$ are the expected value and variance of the test statistic under the null hypothesis,

$$
\begin{aligned}
& \mathrm{E}_{0}(J)=\left(n^{2}-\sum_{i} n_{i \cdot}^{2}\right) / 4 \\
& \operatorname{Var}_{0}(J)=A / 72+B /(36 n(n-1)(n-2))+C /(8 n(n-1))
\end{aligned}
$$

where

$$
\begin{aligned}
A & =n(n-1)(2 n+5)-\sum_{i} n_{i} \cdot\left(n_{i \cdot}-1\right)\left(2 n_{i}+5\right)-\sum_{j} n \cdot j(n \cdot j-1)(2 n \cdot j+5) \\
B & =\left(\sum_{i} n_{i \cdot}\left(n_{i} \cdot-1\right)\left(n_{i} \cdot-2\right)\right)\left(\sum_{j} n \cdot j(n \cdot j-1)(n \cdot j-2)\right) \\
C & =\left(\sum_{i} n_{i} \cdot\left(n_{i} \cdot-1\right)\right)\left(\sum_{j} n \cdot j(n \cdot j-1)\right)
\end{aligned}
$$

PROC FREQ computes one-sided and two-sided $p$-values for the Jonckheere-Terpstra test. When the standardized test statistic is greater than its null hypothesis expected value of zero, PROC FREQ displays the right-sided $p$-value, which is the probability of a larger value of the statistic occurring under the null hypothesis. A small right-sided $p$-value supports the alternative hypothesis of increasing order from row 1 to row $R$. When the standardized test statistic is less than or equal to zero, PROC FREQ displays the left-sided $p$-value. A small left-sided $p$-value supports the alternative of decreasing order from row 1 to row $R$.

The one-sided $p$-value for the Jonckheere-Terpstra test, $P_{1}$, is computed as

$$
P_{1}=\left\{\begin{array}{lll}
\operatorname{Prob}\left(Z>J^{*}\right) & \text { if } & J^{*}>0 \\
\operatorname{Prob}\left(Z<J^{*}\right) & \text { if } & J^{*} \leq 0
\end{array}\right.
$$

where $Z$ has a standard normal distribution. The two-sided $p$-value, $P_{2}$, is computed as

$$
P_{2}=\operatorname{Prob}\left(|Z|>\left|J^{*}\right|\right)
$$

PROC FREQ also provides exact $p$-values for the Jonckheere-Terpstra test. You can request the exact test by specifying the JT option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Tests and Measures of Agreement

When you specify the AGREE option in the TABLES statement, PROC FREQ computes tests and measures of agreement for square tables (that is, for tables where the number of rows equals the number of columns). For two-way tables, these tests and measures include McNemar's test for $2 \times 2$ tables, Bowker's test of symmetry, the simple kappa coefficient, and the weighted kappa coefficient. For multiple strata ( $n$-way tables, where $n>2$ ), PROC FREQ also computes the overall simple kappa coefficient and the overall weighted kappa coefficient, as well as tests for equal kappas (simple and weighted) among strata. Cochran's $Q$ is computed for multiway tables when each variable has two levels, that is, for $h \times 2 \times 2$ tables.

PROC FREQ computes the kappa coefficients (simple and weighted), their asymptotic standard errors, and their confidence limits when you specify the AGREE option in the TABLES statement. If you also specify the KAPPA option in the TEST statement, then PROC FREQ computes the asymptotic test of the hypothesis that simple kappa equals zero. Similarly, if you specify the WTKAP option in the TEST statement, PROC FREQ computes the asymptotic test for weighted kappa.

In addition to the asymptotic tests described in this section, PROC FREQ provides exact $p$-values for McNemar's test, the simple kappa coefficient test, and the weighted kappa coefficient test. You can request these exact tests by specifying the corresponding options in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

The following sections provide the formulas that PROC FREQ uses to compute the AGREE statistics. For information about the use and interpretation of these statistics, see Agresti (2002, 2007); Fleiss, Levin, and Paik (2003), and the other references cited for each statistic.

## McNemar's Test

PROC FREQ computes McNemar's test for $2 \times 2$ tables when you specify the AGREE option. McNemar's test is appropriate when you are analyzing data from matched pairs of subjects with a dichotomous (yes-no) response. It tests the null hypothesis of marginal homogeneity, or $p_{1} .=p \cdot 1$. McNemar's test is computed as

$$
Q_{M}=\left(n_{12}-n_{21}\right)^{2} /\left(n_{12}+n_{21}\right)
$$

Under the null hypothesis, $Q_{M}$ has an asymptotic chi-square distribution with one degree of freedom. See McNemar (1947), as well as the general references cited in the preceding section. In addition to the asymptotic test, PROC FREQ also computes the exact $p$-value for McNemar's test when you specify the MCNEM option in the EXACT statement.

## Bowker's Test of Symmetry

For Bowker's test of symmetry, the null hypothesis is that the cell proportions are symmetric, or that $p_{i j}=p_{j i}$ for all pairs of table cells. For $2 \times 2$ tables, Bowker's test is identical to McNemar's test, and so PROC FREQ provides Bowker's test for square tables larger than $2 \times 2$.
Bowker's test of symmetry is computed as

$$
Q_{B}=\sum_{i<j} \sum_{\left.\left(n_{i j}-n_{j i}\right)^{2} /\left(n_{i j}+n_{j i}\right), ~\right)}
$$

For large samples, $Q_{B}$ has an asymptotic chi-square distribution with $R(R-1) / 2$ degrees of freedom under the null hypothesis of symmetry. See Bowker (1948) for details.

## Simple Kappa Coefficient

The simple kappa coefficient, introduced by Cohen (1960), is a measure of interrater agreement. PROC FREQ computes the simple kappa coefficient as

$$
\hat{\kappa}=\left(P_{o}-P_{e}\right) /\left(1-P_{e}\right)
$$

where $P_{o}=\sum_{i} p_{i i}$ and $P_{e}=\sum_{i} p_{i . p . i}$. If the two response variables are viewed as two independent ratings of the $n$ subjects, the kappa coefficient equals +1 when there is complete agreement of the raters. When the observed agreement exceeds chance agreement, kappa is positive, with its magnitude reflecting the strength of agreement. Although this is unusual in practice, kappa is negative when the observed agreement is less than chance agreement. The minimum value of kappa is between -1 and 0 , depending on the marginal proportions.
The asymptotic variance of the simple kappa coefficient is computed as

$$
\operatorname{Var}(\hat{\kappa})=(A+B-C) /\left(1-P_{e}\right)^{2} n
$$

where

$$
\begin{aligned}
A & =\sum_{i} p_{i i}\left(1-\left(p_{i}+p_{\cdot i}\right)(1-\hat{\kappa})\right)^{2} \\
B & =(1-\hat{\kappa})^{2} \sum_{i \neq j} \sum_{i j} p_{i j}\left(p \cdot i+p_{j} \cdot\right)^{2} \\
C & =\left(\hat{\kappa}-P_{e}(1-\hat{\kappa})\right)^{2}
\end{aligned}
$$

See Fleiss, Cohen, and Everitt (1969) for details.
PROC FREQ computes confidence limits for the simple kappa coefficient as

$$
\hat{\kappa} \pm\left(z_{\alpha / 2} \times \sqrt{\operatorname{Var}(\hat{\kappa})}\right)
$$

where $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. The value of $\alpha$ is determined by the value of the ALPHA= option, which, by default, equals 0.05 and produces $95 \%$ confidence limits.
To compute an asymptotic test for the kappa coefficient, PROC FREQ uses the standardized test statistic $\hat{\kappa}^{*}$, which has an asymptotic standard normal distribution under the null hypothesis that kappa equals zero. The standardized test statistic is computed as

$$
\hat{\kappa}^{*}=\hat{\kappa} / \sqrt{\operatorname{Var}_{0}(\hat{\kappa})}
$$

where $\operatorname{Var}_{0}(\hat{\kappa})$ is the variance of the kappa coefficient under the null hypothesis,

$$
\operatorname{Var}_{0}(\hat{\kappa})=\left(P_{e}+P_{e}^{2}-\sum_{i} p_{i \cdot p \cdot i}\left(p_{i} \cdot+p_{\cdot i}\right)\right) /\left(1-P_{e}\right)^{2} n
$$

See Fleiss, Levin, and Paik (2003) for details.
PROC FREQ also provides an exact test for the simple kappa coefficient. You can request the exact test by specifying the KAPPA or AGREE option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

## Weighted Kappa Coefficient

The weighted kappa coefficient is a generalization of the simple kappa coefficient that uses weights to quantify the relative difference between categories. For $2 \times 2$ tables, the weighted kappa coefficient equals the simple kappa coefficient. PROC FREQ displays the weighted kappa coefficient only for tables larger than $2 \times 2$. PROC FREQ computes the kappa weights from the column scores, by using either Cicchetti-Allison weights or Fleiss-Cohen weights, both of which are described in the following section. The weights $w_{i j}$ are constructed so that $0 \leq w_{i j}<1$ for all $i \neq j, w_{i i}=1$ for all $i$, and $w_{i j}=w_{j i}$. The weighted kappa coefficient is computed as

$$
\hat{\kappa}_{w}=\left(P_{o(w)}-P_{e(w)}\right) /\left(1-P_{e(w)}\right)
$$

where

$$
\begin{aligned}
& P_{o(w)}=\sum_{i} \sum_{j} w_{i j} p_{i j} \\
& P_{e(w)}=\sum_{i} \sum_{j} w_{i j} p_{i \cdot p \cdot j}
\end{aligned}
$$

The asymptotic variance of the weighted kappa coefficient is

$$
\operatorname{Var}\left(\hat{\kappa}_{w}\right)=\left(\sum_{i} \sum_{j} p_{i j}\left(w_{i j}-\left(\bar{w}_{i} \cdot+\bar{w}_{\cdot j}\right)\left(1-\hat{\kappa}_{w}\right)\right)^{2}-\left(\hat{\kappa}_{w}-P_{e(w)}\left(1-\hat{\kappa}_{w}\right)\right)^{2}\right) /\left(1-P_{e(w)}\right)^{2} n
$$

where

$$
\begin{aligned}
& \bar{w}_{i} \cdot=\sum_{j} p_{\cdot j} w_{i j} \\
& \bar{w}_{\cdot j}=\sum_{i} p_{i} \cdot w_{i j}
\end{aligned}
$$

See Fleiss, Cohen, and Everitt (1969) for details.
PROC FREQ computes confidence limits for the weighted kappa coefficient as

$$
\hat{\kappa}_{w} \pm\left(z_{\alpha / 2} \times \sqrt{\operatorname{Var}\left(\hat{\kappa}_{w}\right)}\right)
$$

where $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution. The value of $\alpha$ is determined by the value of the ALPHA= option, which, by default, equals 0.05 and produces $95 \%$ confidence limits.

To compute an asymptotic test for the weighted kappa coefficient, PROC FREQ uses the standardized test statistic $\hat{\kappa}_{w}^{*}$, which has an asymptotic standard normal distribution under the null hypothesis that weighted kappa equals zero. The standardized test statistic is computed as

$$
\hat{\kappa}_{w}^{*}=\hat{\kappa}_{w} / \sqrt{\operatorname{Var}_{0}\left(\hat{\kappa}_{w}\right)}
$$

where $\operatorname{Var}_{0}\left(\hat{\kappa}_{w}\right)$ is the variance of the weighted kappa coefficient under the null hypothesis,

$$
\operatorname{Var}_{0}\left(\hat{\kappa}_{w}\right)=\left(\sum_{i} \sum_{j} p_{i \cdot p \cdot j}\left(w_{i j}-\left(\bar{w}_{i} \cdot+\bar{w}_{\cdot j}\right)\right)^{2}-P_{e(w)}^{2}\right) /\left(1-P_{e(w)}\right)^{2} n
$$

See Fleiss, Levin, and Paik (2003) for details.
PROC FREQ also provides an exact test for the weighted kappa coefficient. You can request the exact test by specifying the WTKAPPA or AGREE option in the EXACT statement. See the section "Exact Statistics" on page 2757 for more information.

Weights PROC FREQ computes kappa coefficient weights by using the column scores and one of the two available weight types. The column scores are determined by the SCORES= option in the TABLES statement. The two available types of kappa weights are Cicchetti-Allison and Fleiss-Cohen weights. By default, PROC FREQ uses Cicchetti-Allison weights. If you specify ( $\mathrm{WT}=\mathrm{FC}$ ) with the AGREE option, then PROC FREQ uses Fleiss-Cohen weights to compute the weighted kappa coefficient.

PROC FREQ computes Cicchetti-Allison kappa coefficient weights as

$$
w_{i j}=1-\frac{\left|C_{i}-C_{j}\right|}{C_{C}-C_{1}}
$$

where $C_{i}$ is the score for column $i$ and $C$ is the number of categories or columns. See Cicchetti and Allison (1971) for details.

The SCORES= option in the TABLES statement determines the type of column scores used to compute the kappa weights (and other score-based statistics). The default is SCORES=TABLE. See the section "Scores" on page 2703 for details. For numeric variables, table scores are the values of the variable levels. You can assign numeric values to the levels in a way that reflects their level of similarity. For example, suppose you have four levels and order them according to similarity. If you assign them values of $0,2,4$, and 10 , the Cicchetti-Allison kappa weights take the following values: $w_{12}=0.8, w_{13}=0.6, w_{14}=0, w_{23}=0.8, w_{24}$ $=0.2$, and $w_{34}=0.4$. Note that when there are only two categories (that is, $C=2$ ), the weighted kappa coefficient is identical to the simple kappa coefficient.

If you specify ( $\mathrm{WT}=\mathrm{FC}$ ) with the AGREE option in the TABLES statement, PROC FREQ computes Fleiss-Cohen kappa coefficient weights as

$$
w_{i j}=1-\frac{\left(C_{i}-C_{j}\right)^{2}}{\left(C_{C}-C_{1}\right)^{2}}
$$

See Fleiss and Cohen (1973) for details.
For the preceding example, the Fleiss-Cohen kappa weights are: $w_{12}=0.96, w_{13}=0.84, w_{14}=0, w_{23}=$ $0.96, w_{24}=0.36$, and $w_{34}=0.64$.

## Overall Kappa Coefficient

When there are multiple strata, PROC FREQ combines the stratum-level estimates of kappa into an overall estimate of the supposed common value of kappa. Assume there are $q$ strata, indexed by $h=1,2, \ldots, q$, and let $\operatorname{Var}\left(\hat{\kappa}_{h}\right)$ denote the variance of $\hat{\kappa}_{h}$. The estimate of the overall kappa coefficient is computed as

$$
\hat{\kappa}_{T}=\sum_{h=1}^{q} \frac{\hat{\kappa}_{h}}{\operatorname{Var}\left(\hat{\kappa}_{h}\right)} / \sum_{h=1}^{q} \frac{1}{\operatorname{Var}\left(\hat{\kappa}_{h}\right)}
$$

See Fleiss, Levin, and Paik (2003) for details.
PROC FREQ computes an estimate of the overall weighted kappa in the same way.

## Tests for Equal Kappa Coefficients

When there are multiple strata, the following chi-square statistic tests whether the stratum-level values of kappa are equal:

$$
Q_{K}=\sum_{h=1}^{q}\left(\hat{\kappa}_{h}-\hat{\kappa}_{T}\right)^{2} / \operatorname{Var}\left(\hat{\kappa}_{h}\right)
$$

Under the null hypothesis of equal kappas for the $q$ strata, $Q_{K}$ has an asymptotic chi-square distribution with $q-1$ degrees of freedom. See Fleiss, Levin, and Paik (2003) for more information. PROC FREQ computes a test for equal weighted kappa coefficients in the same way.

## Cochran's $Q$ Test

Cochran's $Q$ is computed for multiway tables when each variable has two levels, that is, for $2 \times 2 \cdots \times 2$ tables. Cochran's $Q$ statistic is used to test the homogeneity of the one-dimensional margins. Let $m$ denote the number of variables and $N$ denote the total number of subjects. Cochran's $Q$ statistic is computed as

$$
Q_{C}=m(m-1)\left(\sum_{j=1}^{m} T_{j}^{2}-T^{2}\right) /\left(m T-\sum_{k=1}^{N} S_{k}^{2}\right)
$$

where $T_{j}$ is the number of positive responses for variable $j, T$ is the total number of positive responses over all variables, and $S_{k}$ is the number of positive responses for subject $k$. Under the null hypothesis, Cochran's $Q$ has an asymptotic chi-square distribution with $m-1$ degrees of freedom. See Cochran (1950) for details. When there are only two binary response variables ( $m=2$ ), Cochran's $Q$ simplifies to McNemar's test. When there are more than two response categories, you can test for marginal homogeneity by using the repeated measures capabilities of the CATMOD procedure.

## Tables with Zero Rows and Columns

The AGREE statistics are defined only for square tables, where the number of rows equals the number of columns. If the table is not square, PROC FREQ does not compute AGREE statistics. In the kappa statistic framework, where two independent raters assign ratings to each of $n$ subjects, suppose one of the raters does not use all possible $r$ rating levels. If the corresponding table has $r$ rows but only $r-1$ columns, then the table is not square and PROC FREQ does not compute AGREE statistics. To create a square table in this situation, use the ZEROS option in the WEIGHT statement, which requests that PROC FREQ include observations with zero weights in the analysis. Include zero-weight observations in the input data set to
represent any rating levels that are not used by a rater, so that the input data set has at least one observation for each possible rater and rating combination. The analysis then includes all rating levels, even when all levels are not actually assigned by both raters. The resulting table (of rater 1 by rater 2 ) is a square table, and AGREE statistics can be computed.
For more information, see the description of the ZEROS option. By default, PROC FREQ does not process observations that have zero weights, because these observations do not contribute to the total frequency count, and because any resulting zero-weight row or column causes many of the tests and measures of association to be undefined. However, kappa statistics are defined for tables with a zero-weight row or column, and the ZEROS option makes it possible to input zero-weight observations and construct the tables needed to compute kappas.

## Cochran-Mantel-Haenszel Statistics

The CMH option in the TABLES statement gives a stratified statistical analysis of the relationship between the row and column variables after controlling for the strata variables in a multiway table. For example, for the table request $\mathrm{A} * \mathrm{~B} * \mathrm{C} * \mathrm{D}$, the CMH option provides an analysis of the relationship between C and D , after controlling for $A$ and $B$. The stratified analysis provides a way to adjust for the possible confounding effects of $A$ and $B$ without being forced to estimate parameters for them.

The CMH analysis produces Cochran-Mantel-Haenszel statistics, which include the correlation statistic, the ANOVA (row mean scores) statistic, and the general association statistic. For $2 \times 2$ tables, the CMH option also provides Mantel-Haenszel and logit estimates of the common odds ratio and the common relative risks, as well as the Breslow-Day test for homogeneity of the odds ratios.

Exact statistics are also available for stratified $2 \times 2$ tables. If you specify the EQOR option in the EXACT statement, PROC FREQ provides Zelen's exact test for equal odds ratios. If you specify the COMOR option in the EXACT statement, PROC FREQ provides exact confidence limits for the common odds ratio and an exact test that the common odds ratio equals one.

Let the number of strata be denoted by $q$, indexing the strata by $h=1,2, \ldots, q$. Each stratum contains a contingency table with X representing the row variable and Y representing the column variable. For table $h$, denote the cell frequency in row $i$ and column $j$ by $n_{h i j}$, with corresponding row and column marginal totals denoted by $n_{h i}$. and $n_{h \cdot j}$, and the overall stratum total by $n_{h}$.

Because the formulas for the Cochran-Mantel-Haenszel statistics are more easily defined in terms of matrices, the following notation is used. Vectors are presumed to be column vectors unless they are transposed (').

$$
\begin{aligned}
\mathbf{n}_{h i}^{\prime} & =\left(n_{h i 1}, n_{h i 2}, \ldots, n_{h i C}\right) & & (1 \times C) \\
\mathbf{n}_{h}^{\prime} & =\left(\mathbf{n}_{h 1}^{\prime}, \mathbf{n}_{h 2}^{\prime}, \ldots, \mathbf{n}_{h R}^{\prime}\right) & & (1 \times R C) \\
p_{h i} \cdot & =n_{h i} / n_{h} & & (1 \times 1) \\
p_{h \cdot j} & =n_{h \cdot j} / n_{h} & & (1 \times 1) \\
\mathbf{P}_{h * \cdot}^{\prime} & =\left(p_{h 1}, p_{h 2 \cdot}, \ldots, p_{h R} \cdot\right) & & (1 \times R) \\
\mathbf{P}_{h \cdot *}^{\prime} & =\left(p_{h \cdot 1}, p_{h \cdot 2}, \ldots, p_{h \cdot C}\right) & & (1 \times C)
\end{aligned}
$$

Assume that the strata are independent and that the marginal totals of each stratum are fixed. The null hypothesis, $H_{0}$, is that there is no association between X and Y in any of the strata. The corresponding model
is the multiple hypergeometric; this implies that, under $H_{0}$, the expected value and covariance matrix of the frequencies are, respectively,

$$
\begin{aligned}
& \mathbf{m}_{h}=\mathbf{E}\left[\mathbf{n}_{h} \mid H_{0}\right]=n_{h}\left(\mathbf{P}_{h \cdot *} \otimes \mathbf{P}_{h *} .\right) \\
& \operatorname{Var}\left[\mathbf{n}_{h} \mid H_{0}\right]=c\left(\left(\mathbf{D}_{\mathbf{P} h \cdot *}-\mathbf{P}_{h \cdot *} \mathbf{P}_{h \cdot *}^{\prime}\right) \otimes\left(\mathbf{D}_{\mathbf{P} h * \cdot}-\mathbf{P}_{h *} . \mathbf{P}_{h *}^{\prime}\right)\right)
\end{aligned}
$$

where

$$
c=n_{h}^{2} /\left(n_{h}-1\right)
$$

and where $\otimes$ denotes Kronecker product multiplication and $\mathbf{D}_{\mathbf{a}}$ is a diagonal matrix with the elements of a on the main diagonal.
The generalized CMH statistic (Landis, Heyman, and Koch 1978) is defined as

$$
Q_{C M H}=\mathbf{G}^{\prime} \mathbf{V}_{\mathbf{G}}{ }^{-1} \mathbf{G}
$$

where

$$
\begin{aligned}
\mathbf{G} & =\sum_{h} \mathbf{B}_{h}\left(\mathbf{n}_{h}-\mathbf{m}_{h}\right) \\
\mathbf{V}_{\mathbf{G}} & =\sum_{h} \mathbf{B}_{h}\left(\operatorname{Var}\left[\mathbf{n}_{h} \mid H_{0}\right]\right) \mathbf{B}_{h}^{\prime}
\end{aligned}
$$

and where

$$
\mathbf{B}_{h}=\mathbf{C}_{h} \otimes \mathbf{R}_{h}
$$

is a matrix of fixed constants based on column scores $\mathbf{C}_{h}$ and row scores $\mathbf{R}_{h}$. When the null hypothesis is true, the CMH statistic has an asymptotic chi-square distribution with degrees of freedom equal to the rank of $\mathbf{B}_{h}$. If $\mathbf{V}_{\mathbf{G}}$ is found to be singular, PROC FREQ prints a message and sets the value of the CMH statistic to missing.
PROC FREQ computes three CMH statistics by using this formula for the generalized CMH statistic, with different row and column score definitions for each statistic. The CMH statistics that PROC FREQ computes are the correlation statistic, the ANOVA (row mean scores) statistic, and the general association statistic. These statistics test the null hypothesis of no association against different alternative hypotheses. The following sections describe the computation of these CMH statistics.
Caution: The CMH statistics have low power for detecting an association in which the patterns of association for some of the strata are in the opposite direction of the patterns displayed by other strata. Thus, a nonsignificant CMH statistic suggests either that there is no association or that no pattern of association has enough strength or consistency to dominate any other pattern.

## Correlation Statistic

The correlation statistic, popularized by Mantel and Haenszel, has one degree of freedom and is known as the Mantel-Haenszel statistic (Mantel and Haenszel 1959; Mantel 1963).
The alternative hypothesis for the correlation statistic is that there is a linear association between X and Y in at least one stratum. If either $X$ or $Y$ does not lie on an ordinal (or interval) scale, then this statistic is not meaningful.

To compute the correlation statistic, PROC FREQ uses the formula for the generalized CMH statistic with the row and column scores determined by the SCORES= option in the TABLES statement. See the section "Scores" on page 2703 for more information about the available score types. The matrix of row scores $\mathbf{R}_{h}$ has dimension $1 \times R$, and the matrix of column scores $\mathbf{C}_{h}$ has dimension $1 \times C$.
When there is only one stratum, this CMH statistic reduces to $(n-1) r^{2}$, where $r$ is the Pearson correlation coefficient between $X$ and $Y$. When nonparametric (RANK or RIDIT) scores are specified, the statistic reduces to $(n-1) r_{s}^{2}$, where $r_{s}$ is the Spearman rank correlation coefficient between X and Y . When there is more than one stratum, this CMH statistic becomes a stratum-adjusted correlation statistic.

## ANOVA (Row Mean Scores) Statistic

The ANOVA statistic can be used only when the column variable Y lies on an ordinal (or interval) scale so that the mean score of $Y$ is meaningful. For the ANOVA statistic, the mean score is computed for each row of the table, and the alternative hypothesis is that, for at least one stratum, the mean scores of the $R$ rows are unequal. In other words, the statistic is sensitive to location differences among the $R$ distributions of Y .
The matrix of column scores $\mathbf{C}_{h}$ has dimension $1 \times C$, and the column scores are determined by the SCORES= option.

The matrix of row scores $\mathbf{R}_{h}$ has dimension $(R-1) \times R$ and is created internally by PROC FREQ as

$$
\mathbf{R}_{h}=\left[\mathbf{I}_{R-1},-\mathbf{J}_{R-1}\right]
$$

where $\mathbf{I}_{R-1}$ is an identity matrix of rank $R-1$ and $\mathbf{J}_{R-1}$ is an $(R-1) \times 1$ vector of ones. This matrix has the effect of forming $R-1$ independent contrasts of the $R$ mean scores.

When there is only one stratum, this CMH statistic is essentially an analysis of variance (ANOVA) statistic in the sense that it is a function of the variance ratio $F$ statistic that would be obtained from a one-way ANOVA on the dependent variable Y . If nonparametric scores are specified in this case, then the ANOVA statistic is a Kruskal-Wallis test.

If there is more than one stratum, then this CMH statistic corresponds to a stratum-adjusted ANOVA or Kruskal-Wallis test. In the special case where there is one subject per row and one subject per column in the contingency table of each stratum, this CMH statistic is identical to Friedman's chi-square. See Example 40.9 for an illustration.

## General Association Statistic

The alternative hypothesis for the general association statistic is that, for at least one stratum, there is some kind of association between X and Y . This statistic is always interpretable because it does not require an ordinal scale for either X or Y .

For the general association statistic, the matrix $\mathbf{R}_{h}$ is the same as the one used for the ANOVA statistic. The matrix $\mathbf{C}_{h}$ is defined similarly as

$$
\mathbf{C}_{h}=\left[\mathbf{I}_{C-1},-\mathbf{J}_{C-1}\right]
$$

PROC FREQ generates both score matrices internally. When there is only one stratum, then the general association CMH statistic reduces to $Q_{P}(n-1) / n$, where $Q_{P}$ is the Pearson chi-square statistic. When there is more than one stratum, then the CMH statistic becomes a stratum-adjusted Pearson chi-square statistic. Note that a similar adjustment can be made by summing the Pearson chi-squares across the strata. However,
the latter statistic requires a large sample size in each stratum to support the resulting chi-square distribution with $q(R-1)(C-1)$ degrees of freedom. The CMH statistic requires only a large overall sample size because it has only $(R-1)(C-1)$ degrees of freedom.

See Cochran (1954); Mantel and Haenszel (1959); Mantel (1963); Birch (1965); Landis, Heyman, and Koch (1978).

## Mantel-Fleiss Criterion

If you specify the CMH(MANTELFLEISS) option in the TABLES statement, PROC FREQ computes the Mantel-Fleiss criterion for stratified $2 \times 2$ tables. The Mantel-Fleiss criterion can be used to assess the validity of the chi-square approximation for the distribution of the Mantel-Haenszel statistic for $2 \times 2$ tables. For details, see Mantel and Fleiss (1980); Mantel and Haenszel (1959); Stokes, Davis, and Koch (2012); Dmitrienko et al. (2005).

The Mantel-Fleiss criterion is computed as

$$
M F=\min \left(\left[\sum_{h} m_{h 11}-\sum_{h}\left(n_{h 11}\right)_{L}\right],\left[\sum_{h}\left(n_{h 11}\right)_{U}-\sum_{h} m_{h 11}\right]\right)
$$

where $m_{h 11}$ is the expected value of $n_{h 11}$ under the hypothesis of no association between the row and column variables in table $h,\left(n_{h 11}\right)_{L}$ is the minimum possible value of the table cell frequency, and $\left(n_{h 11}\right)_{U}$ is the maximum possible value,

$$
\begin{aligned}
m_{h 11} & =n_{h 1} \cdot n_{h \cdot 1} / n_{h} \\
\left(n_{h 11}\right)_{L} & =\max \left(0, n_{h 1 \cdot}-n_{h \cdot 2}\right) \\
\left(n_{h 11}\right)_{U} & =\min \left(n_{h \cdot 1}, n_{h 1} .\right)
\end{aligned}
$$

The Mantel-Fleiss guideline accepts the validity of the Mantel-Haenszel approximation when the value of the criterion is at least 5 . When the criterion is less than 5, PROC FREQ displays a warning.

## Adjusted Odds Ratio and Relative Risk Estimates

The CMH option provides adjusted odds ratio and relative risk estimates for stratified $2 \times 2$ tables. For each of these measures, PROC FREQ computes a Mantel-Haenszel estimate and a logit estimate. These estimates apply to $n$-way table requests in the TABLES statement, when the row and column variables both have two levels.

For example, for the table request $A * B * C * D$, if the row and column variables $C$ and $D$ both have two levels, PROC FREQ provides odds ratio and relative risk estimates, adjusting for the confounding variables $A$ and $B$.

The choice of an appropriate measure depends on the study design. For case-control (retrospective) studies, the odds ratio is appropriate. For cohort (prospective) or cross-sectional studies, the relative risk is appropriate. See the section "Odds Ratio and Relative Risks for $2 \times 2$ Tables" on page 2737 for more information on these measures.

Throughout this section, $z$ denotes the $100(1-\alpha / 2)$ percentile of the standard normal distribution.

Odds Ratio, Case-Control Studies PROC FREQ provides Mantel-Haenszel and logit estimates for the common odds ratio for stratified $2 \times 2$ tables.

Mantel-Haenszel Estimator The Mantel-Haenszel estimate of the common odds ratio is computed as

$$
O R_{M H}=\left(\sum_{h} n_{h 11} n_{h 22} / n_{h}\right) /\left(\sum_{h} n_{h 12} n_{h 21} / n_{h}\right)
$$

It is always computed unless the denominator is zero. See Mantel and Haenszel (1959) and Agresti (2002) for details.

To compute confidence limits for the common odds ratio, PROC FREQ uses the Robins, Breslow, and Greenland (1986) variance estimate for $\ln \left(O R_{M H}\right)$. The $100(1-\alpha / 2) \%$ confidence limits for the common odds ratio are

$$
\left(O R_{M H} \times \exp (-z \hat{\sigma}), \quad O R_{M H} \times \exp (z \hat{\sigma})\right)
$$

where

$$
\begin{aligned}
\hat{\sigma}^{2}= & \widehat{\operatorname{Var}}\left(\ln \left(O R_{M H}\right)\right) \\
= & \frac{\sum_{h}\left(n_{h 11}+n_{h 22}\right)\left(n_{h 11} n_{h 22}\right) / n_{h}^{2}}{2\left(\sum_{h} n_{h 11} n_{h 22} / n_{h}\right)^{2}} \\
& +\frac{\sum_{h}\left[\left(n_{h 11}+n_{h 22}\right)\left(n_{h 12} n_{h 21}\right)+\left(n_{h 12}+n_{h 21}\right)\left(n_{h 11} n_{h 22}\right)\right] / n_{h}^{2}}{2\left(\sum_{h} n_{h 11} n_{h 22} / n_{h}\right)\left(\sum_{h} n_{h 12} n_{h 21} / n_{h}\right)} \\
& +\frac{\sum_{h}\left(n_{h 12}+n_{h 21}\right)\left(n_{h 12} n_{h 21}\right) / n_{h}^{2}}{2\left(\sum_{h} n_{h 12} n_{h 21} / n_{h}\right)^{2}}
\end{aligned}
$$

Note that the Mantel-Haenszel odds ratio estimator is less sensitive to small $n_{h}$ than the logit estimator.
Logit Estimator The adjusted logit estimate of the common odds ratio (Woolf 1955) is computed as

$$
O R_{L}=\exp \left(\sum_{h} w_{h} \ln \left(O R_{h}\right) / \sum_{h} w_{h}\right)
$$

and the corresponding $100(1-\alpha) \%$ confidence limits are

$$
\left(O R_{L} \times \exp \left(-z / \sqrt{\sum_{h} w_{h}}\right), O R_{L} \times \exp \left(z / \sqrt{\sum_{h} w_{h}}\right)\right)
$$

where $O R_{h}$ is the odds ratio for stratum $h$, and

$$
w_{h}=1 / \operatorname{Var}\left(\ln \left(O R_{h}\right)\right)
$$

If any table cell frequency in a stratum $h$ is zero, PROC FREQ adds 0.5 to each cell of the stratum before computing $O R_{h}$ and $w_{h}$ (Haldane 1955) for the logit estimate. The procedure prints a warning when this occurs.

Relative Risks, Cohort Studies PROC FREQ provides Mantel-Haenszel and logit estimates of the common relative risks for stratified $2 \times 2$ tables.

Mantel-Haenszel Estimator The Mantel-Haenszel estimate of the common relative risk for column 1 is computed as

$$
R R_{M H}=\left(\sum_{h} n_{h 11} n_{h 2 \cdot} / n_{h}\right) /\left(\sum_{h} n_{h 21} n_{h 1} \cdot / n_{h}\right)
$$

It is always computed unless the denominator is zero. See Mantel and Haenszel (1959) and Agresti (2002) for more information.

To compute confidence limits for the common relative risk, PROC FREQ uses the Greenland and Robins (1985) variance estimate for $\log \left(R R_{M H}\right)$. The $100(1-\alpha / 2) \%$ confidence limits for the common relative risk are

$$
\left(R R_{M H} \times \exp (-z \hat{\sigma}), \quad R R_{M H} \times \exp (z \hat{\sigma})\right)
$$

where

$$
\hat{\sigma}^{2}=\widehat{\operatorname{Var}}\left(\ln \left(R R_{M H}\right)\right)=\frac{\sum_{h}\left(n_{h 1} \cdot n_{h 2} \cdot n_{h \cdot 1}-n_{h 11} n_{h 21} n_{h}\right) / n_{h}^{2}}{\left(\sum_{h} n_{h 11} n_{h 2} \cdot / n_{h}\right)\left(\sum_{h} n_{h 21} n_{h 1} \cdot / n_{h}\right)}
$$

Logit Estimator The adjusted logit estimate of the common relative risk for column 1 is computed as

$$
R R_{L}=\exp \left(\sum_{h} w_{h} \ln \left(R R_{h}\right) / \sum w_{h}\right)
$$

and the corresponding $100(1-\alpha) \%$ confidence limits are

$$
\left(R R_{L} \times \exp \left(-z / \sqrt{\sum_{h} w_{h}}\right), \quad R R_{L} \times \exp \left(z / \sqrt{\sum_{h} w_{h}}\right)\right)
$$

where $R R_{h}$ is the column 1 relative risk estimate for stratum $h$ and

$$
w_{h}=1 / \operatorname{Var}\left(\ln \left(R R_{h}\right)\right)
$$

If $n_{h 11}$ or $n_{h 21}$ is zero, then PROC FREQ adds 0.5 to each cell of the stratum before computing $R R_{h}$ and $w_{h}$ for the logit estimate. The procedure prints a warning when this occurs. See Kleinbaum, Kupper, and Morgenstern (1982, Sections 17.4 and 17.5) for details.

## Breslow-Day Test for Homogeneity of the Odds Ratios

When you specify the CMH option, PROC FREQ computes the Breslow-Day test for stratified $2 \times 2$ tables. It tests the null hypothesis that the odds ratios for the $q$ strata are equal. When the null hypothesis is true, the statistic has approximately a chi-square distribution with $q-1$ degrees of freedom. See Breslow and Day (1980) and Agresti (2007) for more information.

The Breslow-Day statistic is computed as

$$
Q_{B D}=\sum_{h}\left(n_{h 11}-\mathrm{E}\left(n_{h 11} \mid O R_{M H}\right)\right)^{2} / \operatorname{Var}\left(n_{h 11} \mid O R_{M H}\right)
$$

where E and Var denote expected value and variance, respectively. The summation does not include any table with a zero row or column. If $O R_{M H}$ equals zero or if it is undefined, then PROC FREQ does not compute the statistic and prints a warning message.

For the Breslow-Day test to be valid, the sample size should be relatively large in each stratum, and at least $80 \%$ of the expected cell counts should be greater than 5 . Note that this is a stricter sample size requirement than the requirement for the Cochran-Mantel-Haenszel test for $q \times 2 \times 2$ tables, in that each stratum sample size (not just the overall sample size) must be relatively large. Even when the Breslow-Day test is valid, it might not be very powerful against certain alternatives, as discussed in Breslow and Day (1980).

If you specify the BDT option, PROC FREQ computes the Breslow-Day test with Tarone's adjustment, which subtracts an adjustment factor from $Q_{B D}$ to make the resulting statistic asymptotically chi-square. The Breslow-Day-Tarone statistic is computed as

$$
Q_{B D T}=Q_{B D}-\left(\sum_{h}\left(n_{h 11}-\mathrm{E}\left(n_{h 11} \mid O R_{M H}\right)\right)\right)^{2} / \sum_{h} \operatorname{Var}\left(n_{h 11} \mid O R_{M H}\right)
$$

See Tarone (1985); Jones et al. (1989); Breslow (1996) for more information.

## Zelen's Exact Test for Equal Odds Ratios

If you specify the EQOR option in the EXACT statement, PROC FREQ computes Zelen's exact test for equal odds ratios for stratified $2 \times 2$ tables. Zelen's test is an exact counterpart to the Breslow-Day asymptotic test for equal odds ratios. The reference set for Zelen's test includes all possible $q \times 2 \times 2$ tables with the same row, column, and stratum totals as the observed multiway table and with the same sum of cell $(1,1)$ frequencies as the observed table. The test statistic is the probability of the observed $q \times 2 \times 2$ table conditional on the fixed margins, which is a product of hypergeometric probabilities.

The $p$-value for Zelen's test is the sum of all table probabilities that are less than or equal to the observed table probability, where the sum is computed over all tables in the reference set determined by the fixed margins and the observed sum of cell $(1,1)$ frequencies. This test is similar to Fisher's exact test for two-way tables. For more information, see Zelen (1971); Hirji (2006); Agresti (1992). PROC FREQ computes Zelen's exact test by using the polynomial multiplication algorithm of Hirji et al. (1996).

## Exact Confidence Limits for the Common Odds Ratio

If you specify the COMOR option in the EXACT statement, PROC FREQ computes exact confidence limits for the common odds ratio for stratified $2 \times 2$ tables. This computation assumes that the odds ratio is constant over all the $2 \times 2$ tables. Exact confidence limits are constructed from the distribution of $S=\sum_{h} n_{h 11}$, conditional on the marginal totals of the $2 \times 2$ tables.
Because this is a discrete problem, the confidence coefficient for these exact confidence limits is not exactly $(1-\alpha)$ but is at least $(1-\alpha)$. Thus, these confidence limits are conservative. See Agresti (1992) for more information.

PROC FREQ computes exact confidence limits for the common odds ratio by using an algorithm based on Vollset, Hirji, and Elashoff (1991). See also Mehta, Patel, and Gray (1985).
Conditional on the marginal totals of $2 \times 2$ table $h$, let the random variable $S_{h}$ denote the frequency of table cell $(1,1)$. Given the row totals $n_{h 1}$. and $n_{h 2}$. and column totals $n_{h \cdot 1}$ and $n_{h \cdot 2}$, the lower and upper bounds for $S_{h}$ are $l_{h}$ and $u_{h}$,

$$
\begin{aligned}
l_{h} & =\max \left(0, n_{h 1 \cdot}-n_{h \cdot 2}\right) \\
u_{h} & =\min \left(n_{h 1}, n_{h \cdot 1}\right)
\end{aligned}
$$

Let $C_{s_{h}}$ denote the hypergeometric coefficient,

$$
C_{s_{h}}=\binom{n_{h \cdot 1}}{s_{h}}\binom{n_{h \cdot 2}}{n_{h 1}-s_{h}}
$$

and let $\phi$ denote the common odds ratio. Then the conditional distribution of $S_{h}$ is

$$
P\left(S_{h}=s_{h} \mid n_{1 .}, n_{\cdot 1}, n_{\cdot 2}\right)=C_{s_{h}} \phi^{s_{h}} / \sum_{x=l_{h}}^{x=u_{h}} C_{x} \phi^{x}
$$

Summing over all the $2 \times 2$ tables, $S=\sum_{h} S_{h}$, and the lower and upper bounds of $S$ are $l$ and $u$,

$$
l=\sum_{h} l_{h} \quad \text { and } \quad u=\sum_{h} u_{h}
$$

The conditional distribution of the sum $S$ is

$$
P\left(S=s \mid n_{h 1}, n_{h \cdot 1}, n_{h \cdot 2} ; h=1, \ldots, q\right)=C_{s} \phi^{s} / \sum_{x=l}^{x=u} C_{x} \phi^{x}
$$

where

$$
C_{s}=\sum_{s_{1}+\ldots .+s_{q}=s}\left(\prod_{h} C_{s_{h}}\right)
$$

Let $s_{0}$ denote the observed sum of cell $(1,1)$ frequencies over the $q$ tables. The following two equations are solved iteratively for lower and upper confidence limits for the common odds ratio, $\phi_{1}$ and $\phi_{2}$ :

$$
\begin{aligned}
& \sum_{x=s_{0}}^{x=u} C_{x} \phi_{1}^{x} / \sum_{x=l}^{x=u} C_{x} \phi_{1}^{x}=\alpha / 2 \\
& \sum_{x=l}^{x=s_{0}} C_{x} \phi_{2}^{x} / \sum_{x=l}^{x=u} C_{x} \phi_{2}^{x}=\alpha / 2
\end{aligned}
$$

When the observed sum $s_{0}$ equals the lower bound $l$, PROC FREQ sets the lower confidence limit to zero and determines the upper limit with level $\alpha$. Similarly, when the observed sum $s_{0}$ equals the upper bound $u$, PROC FREQ sets the upper confidence limit to infinity and determines the lower limit with level $\alpha$.

When you specify the COMOR option in the EXACT statement, PROC FREQ also computes the exact test that the common odds ratio equals one. Setting $\phi=1$, the conditional distribution of the sum $S$ under the null hypothesis becomes

$$
P_{0}\left(S=s \mid n_{h 1}, n_{h \cdot 1}, n_{h \cdot 2} ; h=1, \ldots, q\right)=C_{s} / \sum_{x=l}^{x=u} C_{x}
$$

The point probability for this exact test is the probability of the observed sum $s_{0}$ under the null hypothesis, conditional on the marginals of the stratified $2 \times 2$ tables, and is denoted by $P_{0}\left(s_{0}\right)$. The expected value of $S$ under the null hypothesis is

$$
\mathrm{E}_{0}(S)=\sum_{x=l}^{x=u} x C_{x} / \sum_{x=l}^{x=u} C_{x}
$$

The one-sided exact $p$-value is computed from the conditional distribution as $P_{0}\left(S>=s_{0}\right)$ or $P_{0}\left(S \leq s_{0}\right)$, depending on whether the observed sum $s_{0}$ is greater or less than $\mathrm{E}_{0}(S)$,

$$
\begin{aligned}
& P_{1}=P_{0}\left(S>=s_{0}\right)=\sum_{x=s_{0}}^{x=u} C_{x} / \sum_{x=l}^{x=u} C_{x} \quad \text { if } s_{0}>\mathrm{E}_{0}(S) \\
& P_{1}=P_{0}\left(S<=s_{0}\right)=\sum_{x=l}^{x=s_{0}} C_{x} / \sum_{x=l}^{x=u} C_{x} \quad \text { if } s_{0} \leq \mathrm{E}_{0}(S)
\end{aligned}
$$

PROC FREQ computes two-sided $p$-values for this test according to three different definitions. A two-sided $p$-value is computed as twice the one-sided $p$-value, setting the result equal to one if it exceeds one,

$$
P_{2}^{a}=2 \times P_{1}
$$

Additionally, a two-sided $p$-value is computed as the sum of all probabilities less than or equal to the point probability of the observed sum $s_{0}$, summing over all possible values of $s, l \leq s \leq u$,

$$
P_{2}^{b}=\sum_{l \leq s \leq u: P_{0}(s) \leq P_{0}\left(s_{0}\right)} P_{0}(s)
$$

Also, a two-sided $p$-value is computed as the sum of the one-sided $p$-value and the corresponding area in the opposite tail of the distribution, equidistant from the expected value,

$$
P_{2}^{c}=P_{0}\left(\left|S-\mathrm{E}_{0}(S)\right| \geq\left|s_{0}-\mathrm{E}_{0}(S)\right|\right)
$$

## Gail-Simon Test for Qualitative Interactions

The GAILSIMON option in the TABLES statement provides the Gail-Simon test for qualitative interaction for stratified $2 \times 2$ tables. For details, see Gail and Simon (1985); Silvapulle (2001); Dmitrienko et al. (2005).

The Gail-Simon test is based on the risk differences in stratified $2 \times 2$ tables, where the risk difference is defined as the row 1 risk (proportion in column 1) minus the row 2 risk. See the section "Risks and Risk Differences" on page 2725 for details. By default, the procedure uses column 1 risks to compute the Gail-Simon test. If you specify the GAILSIMON(COLUMN=2) option, the procedure uses column 2 risks.

PROC FREQ computes the Gail-Simon test statistics as described in Gail and Simon (1985),

$$
\begin{aligned}
Q- & =\sum_{h}\left(d_{h} / s_{h}\right)^{2} I\left(d_{h}>0\right) \\
Q+ & =\sum_{h}\left(d_{h} / s_{h}\right)^{2} I\left(d_{h}<0\right) \\
Q & =\min (Q-, Q+)
\end{aligned}
$$

where $d_{h}$ is the risk difference in table $h, s_{h}$ is the standard error of the risk difference, and $I\left(d_{h}>0\right)$ equals 1 if $d_{h}>0$ and 0 otherwise. Similarly, $I\left(d_{h}<0\right)$ equals 1 if $d_{h}<0$ and 0 otherwise. The $q 2 \times 2$ tables (strata) are indexed by $h=1,2, \ldots, q$.

The $p$-values for the Gail-Simon statistics are computed as

$$
\begin{aligned}
p(Q-) & =\sum_{h}\left(1-F_{h}(Q-)\right) B(h ; n=q, p=0.5) \\
p(Q+) & =\sum_{h}\left(1-F_{h}(Q+)\right) B(h ; n=q, p=0.5) \\
p(Q) & =\sum_{h=1}^{q-1}\left(1-F_{h}(Q)\right) B(h ; n=(q-1), p=0.5)
\end{aligned}
$$

where $F_{h}(\cdot)$ is the cumulative chi-square distribution function with $h$ degrees of freedom and $B(h ; n, p)$ is the binomial probability function with parameters $n$ and $p$. The statistic $Q$ tests the null hypothesis of no qualitative interaction. The statistic $Q$ - tests the null hypothesis of positive risk differences. A small $p$-value for $Q$ - indicates negative differences; similarly, a small $p$-value for $Q+$ indicates positive risk differences.

## Exact Statistics

Exact statistics can be useful in situations where the asymptotic assumptions are not met, and so the asymptotic $p$-values are not close approximations for the true $p$-values. Standard asymptotic methods involve the assumption that the test statistic follows a particular distribution when the sample size is sufficiently large. When the sample size is not large, asymptotic results might not be valid, with the asymptotic $p$-values differing perhaps substantially from the exact $p$-values. Asymptotic results might also be unreliable when the distribution of the data is sparse, skewed, or heavily tied. See Agresti (2007) and Bishop, Fienberg, and Holland (1975) for more information. Exact computations are based on the statistical theory of exact conditional inference for contingency tables, reviewed by Agresti (1992).

In addition to computation of exact $p$-values, PROC FREQ provides the option of estimating exact $p$-values by Monte Carlo simulation. This can be useful for problems that are so large that exact computations require a great amount of time and memory, but for which asymptotic approximations might not be sufficient.

Exact statistics are available for many PROC FREQ tests. For one-way tables, PROC FREQ provides exact $p$-values for the binomial proportion tests and the chi-square goodness-of-fit test. Exact (Clopper-Pearson) confidence limits are available for the binomial proportion. For two-way tables, PROC FREQ provides exact $p$-values for the following tests: Pearson chi-square test, likelihood ratio chi-square test, Mantel-Haenszel chisquare test, Fisher's exact test, Jonckheere-Terpstra test, and Cochran-Armitage test for trend. PROC FREQ also computes exact $p$-values for tests of the following statistics: Kendall's tau- $b$, Stuart's tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, Pearson correlation coefficient, Spearman correlation coefficient, simple kappa coefficient, and weighted kappa coefficient. For $2 \times 2$ tables, PROC FREQ provides McNemar's exact test and exact confidence limits for the odds ratio. PROC FREQ also provides exact unconditional confidence limits for the proportion (risk) difference and for the relative risk. For stratified $2 \times 2$ tables, PROC FREQ provides Zelen's exact test for equal odds ratios, exact confidence limits for the common odds ratio, and an exact test for the common odds ratio.

The following sections summarize the exact computational algorithms, define the exact $p$-values that PROC FREQ computes, discuss the computational resource requirements, and describe the Monte Carlo estimation option.

## Computational Algorithms

PROC FREQ computes exact $p$-values for general $R \times C$ tables by using the network algorithm developed by Mehta and Patel (1983). This algorithm provides a substantial advantage over direct enumeration, which can be very time-consuming and feasible only for small problems. See Agresti (1992) for a review of algorithms for computation of exact $p$-values, and see Mehta, Patel, and Tsiatis (1984) and Mehta, Patel, and Senchaudhuri (1991) for information about the performance of the network algorithm.

The reference set for a given contingency table is the set of all contingency tables with the observed marginal row and column sums. Corresponding to this reference set, the network algorithm forms a directed acyclic network consisting of nodes in a number of stages. A path through the network corresponds to a distinct table in the reference set. The distances between nodes are defined so that the total distance of a path through the network is the corresponding value of the test statistic. At each node, the algorithm computes the shortest and longest path distances for all the paths that pass through that node. For statistics that can be expressed as a linear combination of cell frequencies multiplied by increasing row and column scores, PROC FREQ computes shortest and longest path distances by using the algorithm of Agresti, Mehta, and Patel (1990). For statistics of other forms, PROC FREQ computes an upper bound for the longest path and a lower bound for the shortest path by following the approach of Valz and Thompson (1994).

The longest and shortest path distances or bounds for a node are compared to the value of the test statistic to determine whether all paths through the node contribute to the $p$-value, none of the paths through the node contribute to the $p$-value, or neither of these situations occurs. If all paths through the node contribute, the $p$-value is incremented accordingly, and these paths are eliminated from further analysis. If no paths contribute, these paths are eliminated from the analysis. Otherwise, the algorithm continues, still processing this node and the associated paths. The algorithm finishes when all nodes have been accounted for.

In applying the network algorithm, PROC FREQ uses full numerical precision to represent all statistics, row and column scores, and other quantities involved in the computations. Although it is possible to use rounding to improve the speed and memory requirements of the algorithm, PROC FREQ does not do this because it can result in reduced accuracy of the $p$-values.

For one-way tables, PROC FREQ computes the exact chi-square goodness-of-fit test by the method of Radlow and Alf (1975). PROC FREQ generates all possible one-way tables with the observed total sample size and number of categories. For each possible table, PROC FREQ compares its chi-square value with the value for the observed table. If the table's chi-square value is greater than or equal to the observed chi-square, PROC FREQ increments the exact $p$-value by the probability of that table, which is calculated under the null hypothesis by using the multinomial frequency distribution. By default, the null hypothesis states that all categories have equal proportions. If you specify null hypothesis proportions or frequencies by using the TESTP $=$ or TESTF $=$ option in the TABLES statement, then PROC FREQ calculates the exact chi-square test based on that null hypothesis.

Other exact computations are described in sections about the individual statistics. See the section "Binomial Proportion" on page 2719 for details about how PROC FREQ computes exact confidence limits and tests for the binomial proportion. See the section "Odds Ratio and Relative Risks for $2 \times 2$ Tables" on page 2737 for information about computation of exact confidence limits for the odds ratio for $2 \times 2$ tables. Also, see the sections "Exact Unconditional Confidence Limits for the Risk Difference" on page 2734, "Exact Confidence Limits for the Common Odds Ratio" on page 2754, and "Zelen's Exact Test for Equal Odds Ratios" on page 2754.

## Definition of $p$-Values

For several tests in PROC FREQ, the test statistic is nonnegative, and large values of the test statistic indicate a departure from the null hypothesis. Such nondirectional tests include the Pearson chi-square, the likelihood ratio chi-square, the Mantel-Haenszel chi-square, Fisher's exact test for tables larger than $2 \times 2$, McNemar's
test, and the one-way chi-square goodness-of-fit test. The exact $p$-value for a nondirectional test is the sum of probabilities for those tables having a test statistic greater than or equal to the value of the observed test statistic.

There are other tests where it might be appropriate to test against either a one-sided or a two-sided alternative hypothesis. For example, when you test the null hypothesis that the true parameter value equals $0(T=0)$, the alternative of interest might be one-sided $(T \leq 0$, or $T \geq 0)$ or two-sided $(T \neq 0)$. Such tests include the Pearson correlation coefficient, Spearman correlation coefficient, Jonckheere-Terpstra test, Cochran-Armitage test for trend, simple kappa coefficient, and weighted kappa coefficient. For these tests, PROC FREQ displays the right-sided $p$-value when the observed value of the test statistic is greater than its expected value. The right-sided $p$-value is the sum of probabilities for those tables for which the test statistic is greater than or equal to the observed test statistic. Otherwise, when the observed test statistic is less than or equal to the expected value, PROC FREQ displays the left-sided $p$-value. The left-sided $p$-value is the sum of probabilities for those tables for which the test statistic is less than or equal to the one observed. The one-sided $p$-value $P_{1}$ can be expressed as

$$
P_{1}= \begin{cases}\operatorname{Prob}(\text { Test Statistic } \geq t) & \text { if } t>\mathrm{E}_{0}(T) \\ \operatorname{Prob}(\text { Test Statistic } \leq t) & \text { if } t \leq \mathrm{E}_{0}(T)\end{cases}
$$

where $t$ is the observed value of the test statistic and $\mathrm{E}_{0}(T)$ is the expected value of the test statistic under the null hypothesis. PROC FREQ computes the two-sided $p$-value as the sum of the one-sided $p$-value and the corresponding area in the opposite tail of the distribution of the statistic, equidistant from the expected value. The two-sided $p$-value $P_{2}$ can be expressed as

$$
P_{2}=\operatorname{Prob}\left(\mid \text { Test Statistic }-\mathrm{E}_{0}(T)\left|\geq\left|t-\mathrm{E}_{0}(T)\right|\right)\right.
$$

If you specify the POINT option in the EXACT statement, PROC FREQ provides exact point probabilities for the exact tests. The exact point probability is the exact probability that the test statistic equals the observed value.

If you specify the MIDP option in the EXACT statement, PROC FREQ provides exact mid- $p$-values. The exact mid $p$-value is defined as the exact $p$-value minus half the exact point probability, which equals the average of $\operatorname{Prob}($ Test Statistic $\geq t)$ and $\operatorname{Prob}($ Test Statistic $>t$ ) for a right-sided test. The exact mid $p$-value is smaller and less conservative than the non-adjusted exact $p$-value. For more information, see Agresti (2013, section 1.1.4) and Hirji (2006, sections 2.5 and 2.11.1).

## Computational Resources

PROC FREQ uses relatively fast and efficient algorithms for exact computations. These recently developed algorithms, together with improvements in computer power, now make it feasible to perform exact computations for data sets where previously only asymptotic methods could be applied. Nevertheless, there are still large problems that might require a prohibitive amount of time and memory for exact computations, depending on the speed and memory available on your computer. For large problems, consider whether exact methods are really needed or whether asymptotic methods might give results quite close to the exact results, while requiring much less computer time and memory. When asymptotic methods might not be sufficient for such large problems, consider using Monte Carlo estimation of exact p-values, as described in the section "Monte Carlo Estimation" on page 2760.

A formula does not exist that can predict in advance how much time and memory are needed to compute an exact $p$-value for a certain problem. The time and memory required depend on several factors, including which test is being performed, the total sample size, the number of rows and columns, and the specific arrangement of the observations into table cells. Generally, larger problems (in terms of total sample size,
number of rows, and number of columns) tend to require more time and memory. Additionally, for a fixed total sample size, time and memory requirements tend to increase as the number of rows and columns increases, because this corresponds to an increase in the number of tables in the reference set. Also for a fixed sample size, time and memory requirements increase as the marginal row and column totals become more homogeneous. See Agresti, Mehta, and Patel (1990) and Gail and Mantel (1977) for more information.

At any time while PROC FREQ is computing exact $p$-values, you can terminate the computations by pressing the system interrupt key sequence (see the SAS Companion for your system) and choosing to stop computations. After you terminate exact computations, PROC FREQ completes all other remaining tasks. The procedure produces the requested output and reports missing values for any exact $p$-values that were not computed by the time of termination.

You can also use the MAXTIME= option in the EXACT statement to limit the amount of time PROC FREQ uses for exact computations. You specify a MAXTIME= value that is the maximum amount of clock time (in seconds) that PROC FREQ can use to compute an exact $p$-value. If PROC FREQ does not finish computing an exact $p$-value within that time, it terminates the computation and completes all other remaining tasks.

## Monte Carlo Estimation

If you specify the option MC in the EXACT statement, PROC FREQ computes Monte Carlo estimates of the exact $p$-values instead of directly computing the exact $p$-values. Monte Carlo estimation can be useful for large problems that require a great amount of time and memory for exact computations but for which asymptotic approximations might not be sufficient. To describe the precision of each Monte Carlo estimate, PROC FREQ provides the asymptotic standard error and $100(1-\alpha) \%$ confidence limits. The confidence level $\alpha$ is determined by the ALPHA= option in the EXACT statement, which, by default, equals 0.01 and produces $99 \%$ confidence limits. The $\mathrm{N}=n$ option in the EXACT statement specifies the number of samples that PROC FREQ uses for Monte Carlo estimation; the default is 10000 samples. You can specify a larger value for $n$ to improve the precision of the Monte Carlo estimates. Because larger values of $n$ generate more samples, the computation time increases. Alternatively, you can specify a smaller value of $n$ to reduce the computation time.

To compute a Monte Carlo estimate of an exact $p$-value, PROC FREQ generates a random sample of tables with the same total sample size, row totals, and column totals as the observed table. PROC FREQ uses the algorithm of Agresti, Wackerly, and Boyett (1979), which generates tables in proportion to their hypergeometric probabilities conditional on the marginal frequencies. For each sample table, PROC FREQ computes the value of the test statistic and compares it to the value for the observed table. When estimating a right-sided $p$-value, PROC FREQ counts all sample tables for which the test statistic is greater than or equal to the observed test statistic. Then the $p$-value estimate equals the number of these tables divided by the total number of tables sampled.

$$
\begin{aligned}
\hat{P}_{M C} & =M / N \\
M & =\text { number of samples with (Test Statistic } \geq t) \\
N & =\text { total number of samples } \\
t & =\text { observed Test Statistic }
\end{aligned}
$$

PROC FREQ computes left-sided and two-sided $p$-value estimates in a similar manner. For left-sided $p$-values, PROC FREQ evaluates whether the test statistic for each sampled table is less than or equal to the observed test statistic. For two-sided $p$-values, PROC FREQ examines the sample test statistics according to the expression for $P_{2}$ given in the section "Definition of $p$-Values" on page 2758.

The variable $M$ is a binomially distributed variable with $N$ trials and success probability $p$. It follows that the asymptotic standard error of the Monte Carlo estimate is

$$
\operatorname{se}\left(\hat{P}_{M C}\right)=\sqrt{\hat{P}_{M C}\left(1-\hat{P}_{M C}\right) /(N-1)}
$$

PROC FREQ constructs asymptotic confidence limits for the $p$-values according to

$$
\hat{P}_{M C} \pm\left(z_{\alpha / 2} \times \operatorname{se}\left(\hat{P}_{M C}\right)\right)
$$

where $z_{\alpha / 2}$ is the $100(1-\alpha / 2)$ percentile of the standard normal distribution and the confidence level $\alpha$ is determined by the ALPHA= option in the EXACT statement.
When the Monte Carlo estimate $\hat{P}_{M C}$ equals 0 , PROC FREQ computes the confidence limits for the $p$-value as

$$
\left(0, \quad 1-\alpha^{(1 / N)}\right)
$$

When the Monte Carlo estimate $\hat{P}_{M C}$ equals 1, PROC FREQ computes the confidence limits as

$$
\left(\alpha^{(1 / N)}, 1\right)
$$

## Computational Resources

For each variable in a table request, PROC FREQ stores all of the levels in memory. If all variables are numeric and not formatted, this requires about 84 bytes for each variable level. When there are character variables or formatted numeric variables, the memory that is required depends on the formatted variable lengths, with longer formatted lengths requiring more memory. The number of levels for each variable is limited only by the largest integer that your operating environment can store.

For any single crosstabulation table requested, PROC FREQ builds the entire table in memory, regardless of whether the table has zero cell counts. Thus, if the numeric variables A, B, and C each have 10 levels, PROC FREQ requires 2520 bytes to store the variable levels for the table request $A * B * C$, as follows:

```
3 \text { variables * 10 levels/variable * 84 bytes/level}
```

In addition, PROC FREQ requires 8000 bytes to store the table cell frequencies

```
1000 cells * 8 bytes/cell
```

even though there might be only 10 observations.
When the variables have many levels or when there are many multiway tables, your computer might not have enough memory to construct the tables. If PROC FREQ runs out of memory while constructing tables, it stops collecting levels for the variable with the most levels and returns the memory that is used by that variable. The procedure then builds the tables that do not contain the disabled variables.
If there is not enough memory for your table request and if increasing the available memory is impractical, you can reduce the number of multiway tables or variable levels. If you are not using the CMH or AGREE
option in the TABLES statement to compute statistics across strata, reduce the number of multiway tables by using PROC SORT to sort the data set by one or more of the variables or by using the DATA step to create an index for the variables. Then remove the sorted or indexed variables from the TABLES statement and include a BY statement that uses these variables. You can also reduce memory requirements by using a FORMAT statement in the PROC FREQ step to reduce the number of levels. Additionally, reducing the formatted variable lengths reduces the amount of memory that is needed to store the variable levels. For more information about using formats, see the section "Grouping with Formats" on page 2698.

## Output Data Sets

PROC FREQ produces two types of output data sets that you can use with other statistical and reporting procedures. You can request these data sets as follows:

- Specify the OUT= option in a TABLES statement. This creates an output data set that contains frequency or crosstabulation table counts and percentages
- Specify an OUTPUT statement. This creates an output data set that contains statistics.

PROC FREQ does not display the output data sets. Use PROC PRINT, PROC REPORT, or any other SAS reporting tool to display an output data set.

In addition to these two output data sets, you can create a SAS data set from any piece of PROC FREQ output by using the Output Delivery System. See the section "ODS Table Names" on page 2773 for more information.

## Contents of the TABLES Statement Output Data Set

The OUT= option in the TABLES statement creates an output data set that contains one observation for each combination of variable values (or table cell) in the last table request. By default, each observation contains the frequency and percentage for the table cell. When the input data set contains missing values, the output data set also contains an observation with the frequency of missing values. The output data set includes the following variables:

- BY variables
- table request variables, such as $A, B, C$, and $D$ in the table request $A * B * C * D$
- COUNT, which contains the table cell frequency
- PERCENT, which contains the table cell percentage

If you specify the OUTEXPECT option in the TABLES statement for a two-way or multiway table, the output data set also includes expected frequencies. If you specify the OUTPCT option for a two-way or multiway table, the output data set also includes row, column, and table percentages. The additional variables are as follows:

- EXPECTED, which contains the expected frequency
- PCT_TABL, which contains the percentage of two-way table frequency, for $n$-way tables where $n>2$
- PCT_ROW, which contains the percentage of row frequency
- PCT_COL, which contains the percentage of column frequency

If you specify the OUTCUM option in the TABLES statement for a one-way table, the output data set also includes cumulative frequencies and cumulative percentages. The additional variables are as follows:

- CUM_FREQ, which contains the cumulative frequency
- CUM_PCT, which contains the cumulative percentage

The OUTCUM option has no effect for two-way or multiway tables.
The following PROC FREQ statements create an output data set of frequencies and percentages:

```
proc freq;
    tables A A*B / out=D;
run;
```

The output data set $D$ contains frequencies and percentages for the table of $A$ by $B$, which is the last table request listed in the TABLES statement. If $A$ has two levels ( 1 and 2 ), $B$ has three levels ( 1,2 , and 3 ), and no table cell count is zero or missing, then the output data set $D$ includes six observations, one for each combination of $A$ and $B$ levels. The first observation corresponds to $A=1$ and $B=1$; the second observation corresponds to $\mathrm{A}=1$ and $\mathrm{B}=2$; and so on. The data set includes the variables COUNT and PERCENT. The value of COUNT is the number of observations with the given combination of $A$ and $B$ levels. The value of PERCENT is the percentage of the total number of observations with that $A$ and $B$ combination.

When PROC FREQ combines different variable values into the same formatted level, the output data set contains the smallest internal value for the formatted level. For example, suppose a variable X has the values 1.1., 1.4, 1.7, 2.1, and 2.3. When you submit the statement
format X 1.;
in a PROC FREQ step, the formatted levels listed in the frequency table for X are 1 and 2 . If you create an output data set with the frequency counts, the internal values of the levels of X are 1.1 and 1.7. To report the internal values of X when you display the output data set, use a format of 3.1 for X .

## Contents of the OUTPUT Statement Output Data Set

The OUTPUT statement creates a SAS data set that contains statistics computed by PROC FREQ. Table 40.7 lists the statistics that can be stored in the output data set. You identify which statistics to include by specifying output-options. See the description of the OUTPUT statement for details.
If you specify multiple TABLES statements or multiple table requests in a single TABLES statement, the contents of the output data set correspond to the last table request.

For a one-way table or a two-way table, the output data set contains one observation that stores the requested statistics for the table. For a multiway table, the output data set contains an observation for each two-way table (stratum) of the multiway crosstabulation. If you request summary statistics for the multiway table, the output data set also contains an observation that stores the across-strata summary statistics. If you use a BY
statement, the output data set contains an observation (for one-way or two-way tables) or set of observations (for multiway tables) for each BY group.

The OUTPUT data set can include the following variables:

- BY variables
- Variables that identify the stratum for multiway tables, such as $A$ and $B$ in the table request $A * B * C * D$
- Variables that contain the specified statistics

In addition to the specified estimate or test statistic, the output data set includes associated values such as standard errors, confidence limits, $p$-values, and degrees of freedom.

PROC FREQ constructs variable names for the statistics in the output data set by enclosing the output-option names in underscores. Variable names for the corresponding standard errors, confidence limits, $p$-values, and degrees of freedom are formed by combining the output-option names with prefixes that identify the associated values. Table 40.20 lists the prefixes and their descriptions.

Table 40.20 Output Data Set Variable Name Prefixes

| Prefix | Description |
| :--- | :--- |
| $\mathrm{E}_{-}$ | Asymptotic standard error (ASE) |
| $\mathrm{L}_{-}$ | Lower confidence limit |
| $\mathrm{U}_{-}$ | Upper confidence limit |
| $\mathrm{EO}_{-}$ | Null hypothesis ASE |
| $\mathrm{Z}_{-}$ | Standardized value |
| $\mathrm{DF}_{-}$ | Degrees of freedom |
| $\mathrm{P}_{-}$ | $p$-value |
| $\mathrm{P}_{-}$ | Two-sided $p$-value |
| $\mathrm{PL}_{-}$ | Left-sided $p$-value |
| $\mathrm{PR}_{-}$ | Right-sided $p$-value |
| $\mathrm{XP}_{-}$ | Exact $p$-value |
| $\mathrm{XP2}_{-}$ | Exact two-sided $p$-value |
| $\mathrm{XPL}_{-}$ | Exact left-sided $p$-value |
| $\mathrm{XPR}_{-}$ | Exact right-sided $p$-value |
| $\mathrm{XPT}_{-}$ | Exact point probability |
| $\mathrm{XMP}_{-}$ | Exact mid $p$-value |
| $\mathrm{XL}_{-}$ | Exact lower confidence limit |
| $\mathrm{XU}_{-}$ | Exact upper confidence limit |

For example, the PCHI output-option in the OUTPUT statement includes the Pearson chi-square test in the output data set. The variable names for the Pearson chi-square statistic, its degrees of freedom, and the corresponding $p$-value are _PCHI_, DF_PCHI, and P_PCHI, respectively. For variables that were added to the output data set before SAS/STAT 8.2, PROC FREQ truncates the variable name to eight characters when the length of the prefix plus the output-option name exceeds eight characters.

## Displayed Output

## Number of Variable Levels Table

If you specify the NLEVELS option in the PROC FREQ statement, PROC FREQ displays the "Number of Variable Levels" table. This table provides the number of levels for all variables named in the TABLES statements. PROC FREQ determines the variable levels from the formatted variable values. See "Grouping with Formats" on page 2698 for details. The "Number of Variable Levels" table contains the following information:

- Variable name
- Levels, which is the total number of levels of the variable
- Number of Nonmissing Levels, if there are missing levels for any of the variables
- Number of Missing Levels, if there are missing levels for any of the variables


## One-Way Frequency Tables

PROC FREQ displays one-way frequency tables for all one-way table requests in the TABLES statements, unless you specify the NOPRINT option in the PROC FREQ statement or the NOPRINT option in the TABLES statement. For a one-way table showing the frequency distribution of a single variable, PROC FREQ displays the name of the variable and its values. For each variable value or level, PROC FREQ displays the following information:

- Frequency count, which is the number of observations in the level
- Test Frequency count, if you specify the CHISQ and TESTF= options to request a chi-square goodness-of-fit test for specified frequencies
- Percent, which is the percentage of the total number of observations. (The NOPERCENT option suppresses this information.)
- Test Percent, if you specify the CHISQ and TESTP= options to request a chi-square goodness-of-fit test for specified percents. (The NOPERCENT option suppresses this information.)
- Cumulative Frequency count, which is the sum of the frequency counts for that level and all other levels listed above it in the table. The last cumulative frequency is the total number of nonmissing observations. (The NOCUM option suppresses this information.)
- Cumulative Percent, which is the percentage of the total number of observations in that level and in all other levels listed above it in the table. (The NOCUM or the NOPERCENT option suppresses this information.)

The one-way table also displays the Frequency Missing, which is the number of observations with missing values.

## Statistics for One-Way Frequency Tables

For one-way tables, two statistical options are available in the TABLES statement. The CHISQ option provides a chi-square goodness-of-fit test, and the BINOMIAL option provides binomial proportion statistics and tests. PROC FREQ displays the following information, unless you specify the NOPRINT option in the PROC FREQ statement:

- If you specify the CHISQ option for a one-way table, PROC FREQ provides a chi-square goodness-of-fit test, displaying the Chi-Square statistic, the degrees of freedom (DF), and the probability value ( $\mathrm{Pr}>\mathrm{ChiSq}$ ). If you specify the CHISQ option in the EXACT statement, PROC FREQ also displays the exact probability value for this test. If you specify the POINT option with the CHISQ option in the EXACT statement, PROC FREQ displays the exact point probability for the test statistic. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the exact mid $p$-value for the chi-square test.
- If you specify the BINOMIAL option for a one-way table, PROC FREQ displays the estimate of the binomial Proportion, which is the proportion of observations in the first class listed in the one-way table. PROC FREQ also displays the asymptotic standard error (ASE) and the asymptotic (Wald) and exact (Clopper-Pearson) confidence limits by default. For the binomial proportion test, PROC FREQ displays the asymptotic standard error under the null hypothesis (ASE Under H0), the standardized test statistic ( Z ), and the one-sided and two-sided probability values.
If you specify the BINOMIAL option in the EXACT statement, PROC FREQ also displays the exact one-sided and two-sided probability values for this test. If you specify the POINT option with the BINOMIAL option in the EXACT statement, PROC FREQ displays the exact point probability for the test. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the exact mid $p$-value for the binomial proportion test.
- If you request additional binomial confidence limits by specifying binomial-options, PROC FREQ provides a table that displays the lower and upper confidence limits for each type that you request. In addition to the Wald and exact (Clopper-Pearson) confidence limits, you can request Agresti-Coull, Jeffreys, and Wilson (score) confidence limits for the binomial proportion.
- If you request a binomial noninferiority or superiority test by specifying the NONINF or SUP binomialoption, PROC FREQ displays a Noninferiority Analysis or Superiority Analysis table that contains the following information: the binomial Proportion, the test ASE (under H0 or Sample), the test statistic Z, the probability value, the noninferiority or superiority limit, and the test confidence limits. If you specify the BINOMIAL option in the EXACT statement, PROC FREQ also provides the exact probability value for the test, and exact test confidence limits.
- If you request a binomial equivalence test by specifying the EQUIV binomial-option, PROC FREQ displays an Equivalence Analysis table that contains the following information: binomial Proportion and the test ASE (under H0 or Sample). PROC FREQ displays two one-sided tests (TOST) for equivalence, which include test statistics $(Z)$ and probability values for the Lower and Upper tests, together with the Overall probability value. PROC FREQ also displays the equivalence limits and the test-based confidence limits. If you specify the BINOMIAL option in the EXACT statement, PROC FREQ provides exact probability values for the TOST and exact test-based confidence limits.


## Two-Way and Multiway Tables

PROC FREQ displays all multiway table requests in the TABLES statements, unless you specify the NOPRINT option in the PROC FREQ statement or the NOPRINT option in the TABLES statement.

For two-way to multiway crosstabulation tables, the values of the last variable in the table request form the table columns. The values of the next-to-last variable form the rows. Each level (or combination of levels) of the other variables forms one stratum.

There are three ways to display multiway tables in PROC FREQ. By default, PROC FREQ displays multiway tables as separate two-way crosstabulation tables for each stratum of the multiway table. Also by default, PROC FREQ displays these two-way crosstabulation tables in table cell format. Alternatively, if you specify the CROSSLIST option, PROC FREQ displays the two-way crosstabulation tables in ODS column format. If you specify the LIST option, PROC FREQ displays multiway tables in list format, which presents the entire multiway crosstabulation in a single table.

## Crosstabulation Tables

By default, PROC FREQ displays two-way crosstabulation tables in table cell format. The row variable values are listed down the side of the table, the column variable values are listed across the top of the table, and each row and column variable level combination forms a table cell.

Each cell of a crosstabulation table can contain the following information:

- Frequency, which is the number of observations in the table cell. (The NOFREQ option suppresses this information.)
- Expected frequency under the hypothesis of independence, if you specify the EXPECTED option
- Deviation of the cell frequency from the expected value, if you specify the DEVIATION option
- Cell Chi-Square, which is the cell's contribution to the total chi-square statistic, if you specify the CELLCHI2 option
- Tot Pct, which is the cell's percentage of the total multiway table frequency, for $n$-way tables when $n>$ 2 , if you specify the TOTPCT option
- Percent, which is the cell's percentage of the total (two-way table) frequency. (The NOPERCENT option suppresses this information.)
- Row Pct, or the row percentage, which is the cell's percentage of the total frequency for its row. (The NOROW option suppresses this information.)
- Col Pct, or column percentage, which is the cell's percentage of the total frequency for its column. (The NOCOL option suppresses this information.)
- Cumulative $\mathrm{Col} \%$, or cumulative column percentage, if you specify the CUMCOL option

The table also displays the Frequency Missing, which is the number of observations with missing values.

## CROSSLIST Tables

If you specify the CROSSLIST option, PROC FREQ displays two-way crosstabulation tables in ODS column format. The CROSSLIST column format is different from the default crosstabulation table cell format, but the CROSSLIST table provides the same information (frequencies, percentages, and other statistics) as the default crosstabulation table.

In the CROSSLIST table format, the rows of the display correspond to the crosstabulation table cells, and the columns of the display correspond to descriptive statistics such as frequencies and percentages. Each table cell is identified by the values of its TABLES row and column variable levels, with all column variable levels listed within each row variable level. The CROSSLIST table also provides row totals, column totals, and overall table totals.

For a crosstabulation table in CROSSLIST format, PROC FREQ displays the following information:

- the row variable name and values
- the column variable name and values
- Frequency, which is the number of observations in the table cell. (The NOFREQ option suppresses this information.)
- Expected cell frequency under the hypothesis of independence, if you specify the EXPECTED option
- Deviation of the cell frequency from the expected value, if you specify the DEVIATION option
- Standardized Residual, if you specify the CROSSLIST(STDRES) option
- Pearson Residual, if you specify the CROSSLIST(PEARSONRES) option
- Cell Chi-Square, which is the cell's contribution to the total chi-square statistic, if you specify the CELLCHI2 option
- Total Percent, which is the cell's percentage of the total multiway table frequency, for $n$-way tables when $n>2$, if you specify the TOTPCT option
- Percent, which is the cell's percentage of the total (two-way table) frequency. (The NOPERCENT option suppresses this information.)
- Row Percent, which is the cell's percentage of the total frequency for its row. (The NOROW option suppresses this information.)
- Column Percent, the cell's percentage of the total frequency for its column. (The NOCOL option suppresses this information.)

The table also displays the Frequency Missing, which is the number of observations with missing values.

## LIST Tables

If you specify the LIST option in the TABLES statement, PROC FREQ displays multiway tables in a list format rather than as crosstabulation tables. The LIST option displays the entire multiway table in one table, instead of displaying a separate two-way table for each stratum. The LIST option is not available when you also request statistical options. Unlike the default crosstabulation output, the LIST output does not
display row percentages, column percentages, and optional information such as expected frequencies and cell chi-squares.

For a multiway table in list format, PROC FREQ displays the following information:

- the variable names and values
- Frequency, which is the number of observations in the level (with the indicated variable values)
- Percent, which is the level's percentage of the total number of observations. (The NOPERCENT option suppresses this information.)
- Cumulative Frequency, which is the accumulated frequency of the level and all other levels listed above it in the table. The last cumulative frequency in the table is the total number of nonmissing observations. (The NOCUM option suppresses this information.)
- Cumulative Percent, which is the accumulated percentage of the level and all other levels listed above it in the table. (The NOCUM or the NOPERCENT option suppresses this information.)

The table also displays the Frequency Missing, which is the number of observations with missing values.

## Statistics for Two-Way and Multiway Tables

PROC FREQ computes statistical tests and measures for crosstabulation tables, depending on which statements and options you specify. You can suppress the display of these results by specifying the NOPRINT option in the PROC FREQ statement. With any of the following information, PROC FREQ also displays the Sample Size and the Frequency Missing.

- If you specify the SCOROUT option in the TABLES statement, PROC FREQ displays the Row Scores and Column Scores that it uses for statistical computations. The Row Scores table displays the row variable values and the Score corresponding to each value. The Column Scores table displays the column variable values and the corresponding Scores. PROC FREQ also identifies the score type used to compute the row and column scores. You can specify the score type with the SCORES= option in the TABLES statement.
- If you specify the CHISQ option, PROC FREQ displays the following statistics for each two-way table: Pearson Chi-Square, Likelihood Ratio Chi-Square, Continuity-Adjusted Chi-Square (for $2 \times 2$ tables), Mantel-Haenszel Chi-Square, the Phi Coefficient, the Contingency Coefficient, and Cramér's V. For each test statistic, PROC FREQ also displays the degrees of freedom (DF) and the probability value (Prob).
- If you specify the CHISQ option for $2 \times 2$ tables, PROC FREQ also displays Fisher's exact test. The test output includes the cell ( 1,1 ) frequency ( F ), the exact left-sided and right-sided probability values, the table probability $(\mathrm{P})$, and the exact two-sided probability value. If you specify the POINT option in the EXACT statement, PROC FREQ displays the exact point probability for Fisher's exact test. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the Mid $p$-Value for the test.
- If you specify the FISHER option in the TABLES statement (or, equivalently, the FISHER option in the EXACT statement), PROC FREQ displays Fisher's exact test for tables larger than $2 \times 2$. The test output includes the table probability $(\mathrm{P})$ and the probability value. If you specify the POINT option in the EXACT statement, PROC FREQ displays the exact point probability for Fisher's exact test. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the Mid $p$-Value for the test.
- If you specify the PCHI, LRCHI, or MHCHI option in the EXACT statement, PROC FREQ displays the corresponding exact test: Pearson Chi-Square, Likelihood Ratio Chi-Square, or Mantel-Haenszel Chi-Square, respectively. The test output includes the test statistic, the degrees of freedom (DF), and the asymptotic and exact probability values. If you also specify the POINT option in the EXACT statement, PROC FREQ displays the point probability for each exact test requested. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the exact mid p-value for each test. If you specify the CHISQ option in the EXACT statement, PROC FREQ displays exact probability values for all three of these chi-square tests.
- If you specify the MEASURES option, PROC FREQ displays the following statistics and their asymptotic standard errors (ASE) for each two-way table: Gamma, Kendall's Tau- $b$, Stuart's Tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, Pearson Correlation, Spearman Correlation, Lambda Asymmetric $(C \mid R)$, Lambda Asymmetric $(R \mid C)$, Lambda Symmetric, Uncertainty Coefficient $(C \mid R)$, Uncertainty Coefficient $(R \mid C)$, and Uncertainty Coefficient Symmetric. If you specify the CL option, PROC FREQ also displays confidence limits for these measures.
- If you specify the PLCORR option, PROC FREQ displays the tetrachoric correlation for $2 \times 2$ tables or the polychoric correlation for larger tables. In addition, PROC FREQ displays the MEASURES output listed earlier, even if you do not also specify the MEASURES option.
- If you specify the GAMMA, KENTB, STUTC, SMDCR, SMDRC, PCORR, or SCORR option in the TEST statement, PROC FREQ displays asymptotic tests for Gamma, Kendall's Tau-b, Stuart's Tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, the Pearson Correlation, or the Spearman Correlation, respectively. If you specify the MEASURES option in the TEST statement, PROC FREQ displays all these asymptotic tests. The test output includes the statistic, its asymptotic standard error (ASE), Confidence Limits, the ASE under the null hypothesis H0, the standardized test statistic (Z), and the one-sided and two-sided probability values.
- If you specify the KENTB, STUTC, SMDCR, SMDRC, PCORR, or SCORR option in the EXACT statement, PROC FREQ displays asymptotic and exact tests for the corresponding measure of association: Kendall's Tau- $b$, Stuart's Tau- $c$, Somers' $D(C \mid R)$, Somers' $D(R \mid C)$, the Pearson Correlation, or the Spearman correlation, respectively. The test output includes the correlation, its asymptotic standard error (ASE), Confidence Limits, the ASE under the null hypothesis H0, the standardized test statistic (Z), and the asymptotic and exact one-sided and two-sided probability values. If you also specify the POINT option in the EXACT statement, PROC FREQ displays the point probability for each exact test requested. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the exact Mid $p$-Value for each test.
- If you specify the RISKDIFF option for $2 \times 2$ tables, PROC FREQ displays the Column 1 and Column 2 Risk Estimates. For each column, PROC FREQ displays the Row 1 Risk, Row 2 Risk, Total Risk, and Risk Difference, together with their asymptotic standard errors (ASE) and Asymptotic Confidence Limits. PROC FREQ also displays Exact Confidence Limits for the Row 1 Risk, Row 2 Risk, and Total Risk. If you specify the RISKDIFF option in the EXACT statement, PROC FREQ provides unconditional Exact Confidence Limits for the Risk Difference. You can suppress this table by specifying the RISKDIFF(NORISKS) option.
- If you specify the RISKDIFF(CL=) option for $2 \times 2$ tables, PROC FREQ displays the "Confidence Limits for the Proportion (Risk) Difference" table, which includes the Lower and Upper Confidence Limits for each confidence limit Type that you request (Exact, Hauck-Anderson, Miettinen-Nurminen score, Newcombe, or Wald).
- If you request a noninferiority or superiority test for the proportion difference (RISKDIFF) by specifying the NONINF or SUP riskdiff-option, and if you specify METHOD=HA (Hauck-Anderson), METHOD=FM (Farrington-Manning score), or METHOD=WALD (Wald), PROC FREQ displays the following information: the Proportion Difference, the test ASE (H0, Sample, Sample H-A, or FM, depending on the method you specify), the test statistic Z, the probability value, the Noninferiority or Superiority Limit, and the test-based Confidence Limits. If you specify METHOD=NEWCOMBE (Newcombe), PROC FREQ displays the Proportion Difference, the Noninferiority or Superiority Limit, and the Newcombe Confidence Limits.
- If you request an equivalence test for the proportion difference (RISKDIFF) by specifying the EQUIV riskdiff-option, and if you specify METHOD=HA (Hauck-Anderson), METHOD=FM (FarringtonManning score), or METHOD=WALD (Wald), PROC FREQ displays the following information: the Proportion Difference and the test ASE (H0, Sample, Sample H-A, or FM, depending on the method you specify). PROC FREQ displays a two one-sided test (TOST) for equivalence, which includes test statistics (Z) and probability values for the Lower and Upper tests, together with the Overall probability value. PROC FREQ also displays the Equivalence Limits and the test-based Confidence Limits. If you specify METHOD=NEWCOMBE (Newcombe), PROC FREQ displays the Proportion Difference, the Equivalence Limits, and the Newcombe Confidence Limits.
- If you request an equality test for the proportion difference (RISKDIFF) by specifying the EQUAL riskdiff-option, PROC FREQ displays the following information: the Proportion Difference and the test ASE (H0 or Sample), the test statistic Z, the One-Sided probability value ( $\operatorname{Pr}>\mathrm{Z}$ or $\operatorname{Pr}<\mathrm{Z}$ ), and the Two-Sided probability value, $\operatorname{Pr}>|Z|$.
- If you specify the MEASURES option or the RELRISK option for $2 \times 2$ tables, PROC FREQ displays the "Odds Ratio and Relative Risks" table, which includes the following statistics with their confidence limits: Odds Ratio, Relative Risk (Column 1), and Relative Risk (Column 2). If you specify the OR option in the EXACT statement, PROC FREQ also displays the "Exact Confidence Limits for the Odds Ratio" table If you specify the RELRISK option in the EXACT statement, PROC FREQ displays the "Exact Confidence Limits for the Relative Risk" table.
- If you specify the $\mathrm{OR}(\mathrm{CL}=)$ option for $2 \times 2$ tables, PROC FREQ displays the "Confidence Limits for the Odds Ratio" table, which includes the Lower and Upper Confidence Limits for each confidence limit Type that you request (Exact, Score, or Wald).
- If you specify the TREND option, PROC FREQ displays the Cochran-Armitage Trend Test for tables that are $2 \times C$ or $R \times 2$. For this test, PROC FREQ gives the Statistic ( Z ) and the one-sided and two-sided probability values. If you specify the TREND option in the EXACT statement, PROC FREQ also displays the exact one-sided and two-sided probability values for this test. If you specify the POINT option with the TREND option in the EXACT statement, PROC FREQ displays the exact point probability for the test statistic. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the exact Mid $p$-Value for the trend test.
- If you specify the JT option, PROC FREQ displays the Jonckheere-Terpstra Test, showing the Statistic (JT), the standardized test statistic (Z), and the one-sided and two-sided probability values. If you specify the JT option in the EXACT statement, PROC FREQ also displays the exact one-sided and two-sided probability values for this test. If you specify the POINT option with the JT option in the EXACT statement, PROC FREQ displays the exact point probability for the test statistic. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the exact Mid $p$-Value for the Jonckheere-Terpstra test.
- If you specify the AGREE(PRINTKWTS) option, PROC FREQ displays the Kappa Coefficient Weights for square tables of dimension greater than 2 .
- If you specify the AGREE option, for two-way tables PROC FREQ displays McNemar's Test and the Simple Kappa Coefficient for $2 \times 2$ tables. For square tables larger than $2 \times 2$, PROC FREQ displays Bowker's Test of Symmetry, the Simple Kappa Coefficient, and the Weighted Kappa Coefficient. For McNemar's Test and Bowker's Test of Symmetry, PROC FREQ displays the Statistic (S), the degrees of freedom (DF), and For the simple and weighted kappa coefficients, PROC FREQ displays the kappa values, asymptotic standard errors (ASE), and Confidence Limits. the probability value ( $\operatorname{Pr}>\mathrm{S}$ ).
If you specify the MCNEM option in the EXACT statement, PROC FREQ also displays the exact probability value for McNemar's test. If you specify the POINT option with the MCNEM option in the EXACT statement, PROC FREQ displays the exact point probability for the test statistic. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the Exact Mid p-Value for McNemar's test.
- If you specify the KAPPA or WTKAP option in the TEST statement, PROC FREQ displays asymptotic tests for the simple kappa coefficient or the weighted kappa coefficient, respectively. If you specify the AGREE option in the TEST statement, PROC FREQ displays both these asymptotic tests. The test output includes the kappa coefficient, its asymptotic standard error (ASE), Confidence Limits, the ASE under the null hypothesis H 0 , the standardized test statistic ( Z ), and the one-sided and two-sided probability values.
- If you specify the KAPPA or WTKAP option in the EXACT statement, PROC FREQ displays asymptotic and exact tests for the simple kappa coefficient or the weighted kappa coefficient, respectively. The test output includes the kappa coefficient, its asymptotic standard error (ASE), Confidence Limits, the ASE under the null hypothesis H0, the standardized test statistic (Z), and the asymptotic and exact one-sided and two-sided probability values. If you specify the POINT option in the EXACT statement, PROC FREQ displays the point probability for each exact test requested. If you specify the MIDP option in the EXACT statement, PROC FREQ displays the exact Mid p-Value for each exact test.
- If you specify the MC option in the EXACT statement, PROC FREQ displays Monte Carlo estimates for all exact $p$-values requested by statistic-options in the EXACT statement. The Monte Carlo output includes the $p$-value Estimate, its Confidence Limits, the Number of Samples used to compute the Monte Carlo estimate, and the Initial Seed for random number generation.
- If you specify the AGREE option for a multiway square table, PROC FREQ displays Overall Simple and Weighted Kappa Coefficients, with their asymptotic standard errors (ASE) and Confidence Limits. PROC FREQ also displays Tests for Equal Kappa Coefficients, giving the Chi-Squares, degrees of freedom (DF), and probability values ( $\operatorname{Pr}>C h i S q$ ) for the Simple Kappa and Weighted Kappa. For multiple strata of $2 \times 2$ tables, PROC FREQ displays Cochran's $Q$, giving the Statistic ( Q ), the degrees of freedom (DF), and the probability value $(\operatorname{Pr}>\mathrm{Q})$.
- If you specify the RISKDIFF(COMMON) option for a multiway $2 \times 2$ table, PROC FREQ displays the "Common Proportion (Risk) Difference" table, which includes the following information: Method (Mantel-Haenszel or Summary Score), Value of the common risk difference, Standard Error, and Confidence Limits.
- If you specify the CMH option, PROC FREQ displays Cochran-Mantel-Haenszel Statistics for the following three alternative hypotheses: Nonzero Correlation, Row Mean Scores Differ (ANOVA Statistic), and General Association. For each of these statistics, PROC FREQ gives the degrees of
freedom (DF) and the probability value (Prob). If you specify the MANTELFLEISS option, PROC FREQ displays the Mantel-Fleiss Criterion for $2 \times 2$ tables. For $2 \times 2$ tables, PROC FREQ also displays Estimates of the Common Relative Risk for Case-Control and Cohort studies, together with their confidence limits. These include both Mantel-Haenszel and Logit stratum-adjusted estimates of the common Odds Ratio, Column 1 Relative Risk, and Column 2 Relative Risk. Also for $2 \times 2$ tables, PROC FREQ displays the Breslow-Day Test for Homogeneity of the Odds Ratios. For this test, PROC FREQ gives the Chi-Square, the degrees of freedom (DF), and the probability value ( $\mathrm{Pr}>\mathrm{ChiSq}$ ).
- If you specify the CMH option in the TABLES statement and also specify the COMOR option in the EXACT statement for a multiway $2 \times 2$ table, PROC FREQ displays exact confidence limits for the Common Odds Ratio. PROC FREQ also displays the Exact Test of H0: Common Odds Ratio $=1$. The test output includes the Cell (1,1) Sum (S), Mean of S Under H0, One-sided Pr <= S, and Point Pr = S. PROC FREQ also provides exact two-sided probability values for the test, computed according to the following three methods: $2 *$ One-sided, Sum of probabilities $<=$ Point probability, and $\operatorname{Pr}>=\mid S$ - Meanl. If you specify the MIDP option in the EXACT statement, PROC FREQ provides the exact Mid $p$-Value for the common odds ratio test.
- If you specify the CMH option in the TABLES statement and also specify the EQOR option in the EXACT statement for a multiway $2 \times 2$ table, PROC FREQ computes Zelen's exact test for equal odds ratios. PROC FREQ displays Zelen's test along with the asymptotic Breslow-Day test produced by the CMH option. PROC FREQ displays the test statistic, Zelen's Exact Test (P), and the probability value, Exact $\operatorname{Pr}<=\mathrm{P}$.
- If you specify the GAILSIMON option in the TABLES statement for a multiway $2 \times 2$ tables, PROC FREQ displays the Gail-Simon test for qualitative interactions. The display include the following statistics and their $p$-values: Q+ (Positive Risk Differences), Q- (Negative Risk Differences), and Q (Two-Sided).


## ODS Table Names

PROC FREQ assigns a name to each table that it creates. You can use these names to refer to tables when you use the Output Delivery System (ODS) to select tables and create output data sets. For more information about ODS, see Chapter 20, "Using the Output Delivery System."

Table 40.21 lists the ODS table names together with their descriptions and the options required to produce the tables. Note that the ALL option in the TABLES statement invokes the CHISQ, MEASURES, and CMH options.

Table 40.21 ODS Tables Produced by PROC FREQ

| ODS Table Name | Description | Statement | Option |
| :--- | :--- | :--- | :--- |
| BarnardsTest | Barnard's exact test | EXACT | BARNARD |
| BinomialCLs | Binomial confidence limits | TABLES | BINOMIAL(AC I J \\| W) |
| BinomialEquiv | Binomial equivalence analysis | TABLES | BINOMIAL(EQUIV) |
| BinomialEquivLimits | Binomial equivalence limits | TABLES | BINOMIAL(EQUIV) |
| BinomialEquivTest | Binomial equivalence test | TABLES | BINOMIAL(EQUIV) |
| BinomialNoninf | Binomial noninferiority test | TABLES | BINOMIAL(NONINF) |
| Binomial | Binomial proportion | TABLES | BINOMIAL |
| BinomialTest | Binomial proportion test | TABLES | BINOMIAL |

Table 40.21 continued

| ODS Table Name | Description | Statement | Option |
| :---: | :---: | :---: | :---: |
| BinomialSup | Binomial superiority test | TABLES | BINOMIAL(SUP) |
| BreslowDayTest | Breslow-Day test | TABLES | CMH $(h \times 2 \times 2 \text { table })$ |
| CMH | Cochran-Mantel-Haenszel statistics | TABLES | CMH |
| ChiSq | Chi-square tests | TABLES | CHISQ |
| CochransQ | Cochran's $Q$ | TABLES | AGREE $(h \times 2 \times 2 \text { table })$ |
| ColScores | Column scores | TABLES | SCOROUT |
| CommonOdds- <br> RatioCl | Exact confidence limits for the common odds ratio | EXACT | COMOR $(h \times 2 \times 2 \text { table })$ |
| CommonOdds- <br> RatioTest | Common odds ratio exact test | EXACT | COMOR $(h \times 2 \times 2 \text { table })$ |
| CommonPdiff | Common proportion difference | TABLES | RISKDIFF(COMMON) <br> ( $h \times 2 \times 2$ table) |
| CommonRelRisks | Common relative risks | TABLES | CMH <br> ( $h \times 2 \times 2$ table) |
| CrossList | Crosstabulation table in column format | TABLES | CROSSLIST <br> ( $n$-way table, $n>1$ ) |
| CrossTabFreqs | Crosstabulation table | TABLES | ( $n$-way table, $n>1$ ) |
| EqualKappaTest | Test for equal simple kappas | TABLES | AGREE <br> ( $h \times 2 \times 2$ table) |
| EqualKappaTests | Tests for equal kappas | TABLES | AGREE $(h \times r \times r, r>2)$ |
| EqualOddsRatios | Tests for equal odds ratios | EXACT | $\begin{aligned} & \text { EQOR } \\ & (h \times 2 \times 2 \text { table }) \end{aligned}$ |
| GailSimon | Gail-Simon test | TABLES | GAILSIMON $(h \times 2 \times 2 \text { table })$ |
| FishersExact | Fisher's exact test | EXACT <br> or TABLES or TABLES | FISHER <br> FISHER or EXACT <br> CHISQ ( $2 \times 2$ table) |
| FishersExactMC | Monte Carlo estimates for Fisher's exact test | EXACT | FISHER / MC |
| Gamma | Gamma | TEST | GAMMA |
| GammaTest | Gamma test | TEST | GAMMA |
| JTTest | Jonckheere-Terpstra test | TABLES | JT |
| JTTestMC | Monte Carlo estimates for Jonckheere-Terpstra exact test | EXACT | JT / MC |
| Kappa | Simple kappa coefficient | TEST <br> or EXACT | KAPPA KAPPA |
| KappaMC | Monte Carlo exact test for simple kappa coefficient | EXACT | KAPPA / MC |
| KappaStatistics | Kappa statistics | TABLES | AGREE, <br> no TEST or EXACT <br> ( $r \times r$ table, $r>2$ ) |

Table 40.21 continued

| ODS Table Name | Description | Statement | Option |
| :---: | :---: | :---: | :---: |
| KappaTest | Simple kappa test |  |  |
|  |  | or EXACT | KAPPA |
| KappaWeights | Kappa weights | TABLES | AGREE(PRINTKWTS) |
| List | List format multiway table | TABLES | LIST |
| LRChiSq | Likelihood ratio chi-square exact test | EXACT | LRCHI |
| LRChiSqMC | Monte Carlo exact test for likelihood ratio chi-square | EXACT | LRCHI / MC |
| MantelFleiss | Mantel-Fleiss criterion | TABLES | $\begin{aligned} & \mathrm{CMH}(\mathrm{MF}) \\ & (h \times 2 \times 2 \text { table }) \end{aligned}$ |
| McNemarsTest | McNemar's test | TABLES | AGREE <br> ( $2 \times 2$ table) |
| Measures | Measures of association | TABLES | MEASURES |
| MHChiSq | Mantel-Haenszel chi-square exact test | EXACT | MHCHI |
| MHChiSqMC | Monte Carlo exact test for Mantel-Haenszel chi-square | EXACT | MHCHI / MC |
| NLevels | Number of variable levels | PROC | NLEVELS |
| OddsRatioCLs | Odds ratio confidence limits | TABLES | OR(CL=) ( $2 \times 2$ table $)$ |
| OddsRatioExactCL | Exact confidence limits for the odds ratio | EXACT | OR ( $2 \times 2$ table) |
| OneWayChiSq | One-way chi-square test | TABLES | CHISQ <br> (one-way table) |
| OneWayChiSqMC | Monte Carlo exact test for one-way chi-square | EXACT | CHISQ / MC (one-way table) |
| OneWayFreqs | One-way frequencies | PROC or TABLES | (no TABLES stmt) (one-way table) |
| OneWayLRChiSq | One-way likelihood ratio chi-square test | TABLES | CHISQ(LRCHI) <br> (one-way table) |
| OverallKappa | Overall simple kappa | TABLES | AGREE <br> ( $h \times 2 \times 2$ table) |
| OverallKappas | Overall kappa coefficients | TABLES | AGREE $(h \times r \times r, r>2)$ |
| PdiffCLs | Proportion difference confidence limits | TABLES | $\begin{aligned} & \text { RISKDIFF(CL=) } \\ & (2 \times 2 \text { table }) \end{aligned}$ |
| PdiffEquiv | Equivalence analysis for the proportion difference | TABLES | RISKDIFF(EQUIV) <br> ( $2 \times 2$ table) |
| PdiffEquivLimits | Equivalence limits for the proportion difference | TABLES | RISKDIFF(EQUIV) <br> ( $2 \times 2$ table) |
| PdiffEquivTest | Equivalence test for the proportion difference | TABLES | RISKDIFF(EQUIV) <br> ( $2 \times 2$ table) |
| PdiffNoninf | Noninferiority test for the proportion difference | TABLES | RISKDIFF(NONINF) <br> ( $2 \times 2$ table) |
| PdiffSup | Superiority test for the proportion difference | TABLES | RISKDIFF(SUP) <br> ( $2 \times 2$ table) |

Table 40.21 continued

| ODS Table Name | Description | Statement | Option |
| :---: | :---: | :---: | :---: |
| PdiffTest | Proportion difference test | TABLES | RISKDIFF(EQUAL) <br> ( $2 \times 2$ table) |
| PearsonChiSq | Pearson chi-square exact test | EXACT | PCHI |
| PearsonChiSqMC | Monte Carlo exact test for Pearson chi-square | EXACT | PCHI / MC |
| PearsonCorr | Pearson correlation | TEST <br> or EXACT | PCORR PCORR |
| PearsonCorrMC | Monte Carlo exact test for Pearson correlation | EXACT | PCORR / MC |
| PearsonCorrTest | Pearson correlation test | TEST <br> or EXACT | PCORR PCORR |
| PlCorr | Polychoric correlation | TEST | PLCORR |
| PlCorrTest | Polychoric correlation test | TEST | PLCORR |
| RelativeRisks | Relative risk estimates | TABLES | RELRISK or MEASURES ( $2 \times 2$ table) |
| RelRisk1ExactCL | Exact confidence limits for column 1 relative risk | EXACT | RELRISK <br> ( $2 \times 2$ table) |
| RelRisk2ExactCL | Exact confidence limits for column 2 relative risk | EXACT | RELRISK $(2 \times 2 \text { table })$ |
| RiskDiffCol1 | Column 1 risk estimates | TABLES | RISKDIFF <br> ( $2 \times 2$ table) |
| RiskDiffCol2 | Column 2 risk estimates | TABLES | RISKDIFF <br> ( $2 \times 2$ table) |
| RowScores | Row scores | TABLES | SCOROUT |
| SomersDCR | Somers' $D(C \mid R)$ | TEST <br> or EXACT | $\begin{aligned} & \text { SMDCR } \\ & \text { SMDCR } \end{aligned}$ |
| SomersDCRMC | Monte Carlo exact test for Somers' $D(C \mid R)$ | EXACT | SMDCR / MC |
| SomersDCRTest | Somers' $D(C \mid R)$ test | TEST <br> or EXACT | SMDCR <br> SMDCR |
| SomersDRC | Somers' $D(R \mid C)$ | TEST <br> or EXACT | SMDRC SMDRC |
| SomersDRCMC | Monte Carlo exact test for Somers' $D(R \mid C)$ | EXACT | SMDRC / MC |
| SomersDRCTest | Somers' $D(R \mid C)$ test | TEST <br> or EXACT | SMDRC SMDRC |
| SpearmanCorr | Spearman correlation | TEST <br> or EXACT | $\begin{aligned} & \text { SCORR } \\ & \text { SCORR } \end{aligned}$ |
| SpearmanCorrMC | Monte Carlo exact test for Spearman correlation | EXACT | SCORR / MC |
| SpearmanCorrTest | Spearman correlation test | TEST <br> or EXACT | SCORR SCORR |
| SymmetryTest | Test of symmetry | TABLES | AGREE |

Table 40.21 continued

| ODS Table Name | Description | Statement | Option |
| :---: | :---: | :---: | :---: |
| TauB | Kendall's tau-b | TEST | KENTB |
|  |  | or EXACT | KENTB |
| TauBMC | Monte Carlo exact test for | EXACT | KENTB / MC |
|  | Kendall's tau-b |  |  |
| TauBTest | Kendall's tau- $b$ test | TEST | KENTB |
|  |  | or EXACT | KENTB |
| TauC | Stuart's tau-c | TEST | STUTC |
|  |  | or EXACT | STUTC |
| TauCMC | Monte Carlo exact test for | EXACT | STUTC / MC |
|  | Stuart's tau-c |  |  |
| TauCTest | Stuart's tau-c test | TEST <br> or EXACT | STUTC |
|  |  |  | STUTC |
| TrendTest | Cochran-Armitage trend test | TABLES | TREND |
| TrendTestMC | Monte Carlo exact test for trend | EXACT | TREND / MC |
| WtKappa | Weighted kappa coefficient | TEST | WTKAP |
|  |  | or EXACT | WTKAP |
| WtKappaMC | Monte Carlo exact test for weighted kappa coefficient | EXACT | WTKAP / MC |
| WtKappaTest | Weighted kappa test | TEST | WTKAP |
|  |  | or EXACT | WTKAP |

## ODS Graphics

Statistical procedures use ODS Graphics to create graphs as part of their output. ODS Graphics is described in detail in Chapter 21, "Statistical Graphics Using ODS."

Before you create graphs, ODS Graphics must be enabled (for example, by specifying the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" on page 606 in Chapter 21, "Statistical Graphics Using ODS."

The overall appearance of graphs is controlled by ODS styles. Styles and other aspects of using ODS Graphics are discussed in the section "A Primer on ODS Statistical Graphics" on page 605 in Chapter 21, "Statistical Graphics Using ODS."

When ODS Graphics is enabled, you can request specific plots with the PLOTS= option in the TABLES statement. To produce a frequency plot or cumulative frequency plot, you must specify the FREQPLOT or CUMFREQPLOT plot-request, respectively, in the PLOTS= option. To produce a mosaic plot, you must specify the MOSAICPLOT plot-request in the PLOTS= option. You can also produce frequency, cumulative frequency, and mosaic plots by specifying the PLOTS=ALL option. By default, PROC FREQ produces all other plots that are associated with the analyses that you request in the TABLES statement. You can suppress the default plots and request specific plots by using the $\operatorname{PLOTS}(\mathrm{ONLY})=$ option. See the description of the PLOTS $=$ option for details.

PROC FREQ assigns a name to each graph that it creates with ODS Graphics. You can use these names to refer to the graphs. Table 40.22 lists the names of the graphs that PROC FREQ generates together with their descriptions, their PLOTS = options (plot-requests), and the TABLES statement options that are required to produce the graphs.

Table 40.22 Graphs Produced by PROC FREQ

| ODS Graph Name | Description | PLOTS= Option | TABLES Statement Option |
| :--- | :--- | :--- | :--- |
| AgreePlot | Agreement plot | AGREEPLOT | AGREE $(r \times r$ table $)$ |
| CumFreqPlot | Cumulative frequency plot | CUMFREQPLOT | One-way table request |
| DeviationPlot | Deviation plot | DEVIATIONPLOT | CHISQ (one-way table) |
| FreqPlot | Frequency plot | FREQPLOT | Any table request |
| KappaPlot | Kappa plot | KAPPAPLOT | AGREE $(h \times r \times r$ table $)$ |
| MosaicPlot | Mosaic plot | MOSAICPLOT | Two-way or multiway table request |
| ORPlot | Odds ratio plot | ODDSRATIOPLOT | MEASURES or RELRISK |
|  | Relative risk plot | RELRISKPLOT | MEASURES or RELRISK |
| RelRiskPlot | Risk difference plot | RISKDIFFPLOT | RISKDIFF $(h \times 2 \times 2$ table) |
|  | Weighted kappa plot | WTKAPPAPLOT | AGREE |
| RiskDiffPlot |  |  | $(h \times r \times r$ table, $r>2)$ |
| WtKappaPlot |  |  |  |

## Examples: FREQ Procedure

## Example 40.1: Output Data Set of Frequencies

The eye and hair color of children from two different regions of Europe are recorded in the data set Color. Instead of recording one observation per child, the data are recorded as cell counts, where the variable Count contains the number of children exhibiting each of the 15 eye and hair color combinations. The data set does not include missing combinations.

The following DATA step statements create the SAS data set Color:

```
data Color;
    input Region Eyes $ Hair $ Count @@;
    label Eyes ='Eye Color'
            Hair ='Hair Color'
            Region='Geographic Region';
    datalines;
\begin{tabular}{llllllllll} 
blue fair & 23 & 1 & blue red & 7 & 1 & blue medium & 24 \\
blue dark & 11 & 1 & green fair & 19 & 1 & green red & 7 \\
green medium & 18 & 1 & green dark & 14 & 1 & brown fair & 34 \\
brown red & 5 & 1 & brown medium & 41 & 1 & brown dark & 40 \\
brown black & 3 & 2 blue fair & 46 & 2 blue red & 21 \\
blue medium & 44 & 2 blue dark & 40 & 2 & blue black & 6 \\
green fair & 50 & 2 green red & 31 & 2 & green medium & 37 \\
green dark & 23 & 2 brown fair & 56 & 2 brown red & 42 \\
brown medium & 53 & 2 brown dark & 54 & 2 brown black & 13
\end{tabular}
;
```

The following PROC FREQ statements read the Color data set and create an output data set that contains the frequencies, percentages, and expected cell frequencies of the two-way table of Eyes by Hair. The TABLES statement requests three tables: a frequency table for Eyes, a frequency table for Hair, and a crosstabulation table for Eyes by Hair. The OUT= option creates the FreqCount data set, which contains the crosstabulation table frequencies. The OUTEXPECT option outputs the expected table cell frequencies to FreqCount, and the SPARSE option includes zero cell frequencies in the output data set. The WEIGHT statement specifies that the variable Count contains the observation weights. These statements create Output 40.1.1 through Output 40.1.3.

```
proc freq data=Color;
    tables Eyes Hair Eyes*Hair / out=FreqCount outexpect sparse;
    weight Count;
    title 'Eye and Hair Color of European Children';
run;
proc print data=FreqCount noobs;
    title2 'Output Data Set from PROC FREQ';
run;
```

Output 40.1.1 displays the two frequency tables produced by PROC FREQ: one showing the distribution of eye color, and one showing the distribution of hair color. By default, PROC FREQ lists the variables values in alphabetical order. The 'Eyes*Hair' specification produces a crosstabulation table, shown in Output 40.1.2, with eye color defining the table rows and hair color defining the table columns. A zero cell frequency for green eyes and black hair indicates that this eye and hair color combination does not occur in the data.

The output data set FreqCount (Output 40.1.3) contains frequency counts and percentages for the last table requested in the TABLES statement, Eyes by Hair. Because the SPARSE option is specified, the data set includes the observation with a zero frequency. The variable Expected contains the expected frequencies, as requested by the OUTEXPECT option.

## Output 40.1.1 Frequency Tables



Output 40.1.2 Crosstabulation Table


Output 40.1.3 Output Data Set of Frequencies

| Eye and Hair Color of European Children Output Data Set from PROC FREQ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Eyes | Hair | COUNT | EXPECTED | PERCENT |
| blue | black | 6 | 6.409 | 0.7874 |
| blue | dark | 51 | 53.024 | 6.6929 |
| blue | fair | 69 | 66.425 | 9.0551 |
| blue | medium | 68 | 63.220 | 8.9239 |
| blue | red | 28 | 32.921 | 3.6745 |
| brown | black | 16 | 9.845 | 2.0997 |
| brown | dark | 94 | 81.446 | 12.3360 |
| brown | fair | 90 | 102.031 | 11.8110 |
| brown | medium | 94 | 97.109 | 12.3360 |
| brown | red | 47 | 50.568 | 6.1680 |
| green | black | 0 | 5.745 | 0.0000 |
| green | dark | 37 | 47.530 | 4.8556 |
| green | fair | 69 | 59.543 | 9.0551 |
| green | medium | 55 | 56.671 | 7.2178 |
| green | red | 38 | 29.510 | 4.9869 |

## Example 40.2: Frequency Dot Plots

This example produces frequency dot plots for the children's eye and hair color data from Example 40.1.
PROC FREQ produces plots by using ODS Graphics to create graphs as part of the procedure output. Frequency plots are available for any frequency or crosstabulation table request. You can display frequency plots as bar charts or dot plots. You can use plot-options to specify the orientation (vertical or horizontal), scale, and layout of the plots.

The following PROC FREQ statements request frequency tables and dot plots. The first TABLES statement requests a one-way frequency table of Hair and a crosstabulation table of Eyes by Hair. The PLOTS= option requests frequency plots for the tables, and the TYPE=DOTPLOT plot-option specifies dot plots. By default, frequency plots are produced as bar charts. ODS Graphics must be enabled before producing plots.

The second TABLES statement requests a crosstabulation table of Region by Hair and a frequency dot plot for this table. The SCALE=PERCENT plot-option plots percentages instead of frequency counts. SCALE=LOG and SCALE=SQRT plot-options are also available to plot log frequencies and square roots of frequencies, respectively.

The ORDER=FREQ option in the PROC FREQ statement orders the variable levels by frequency. This order applies to the frequency and crosstabulation table displays and also to the corresponding frequency plots.

```
ods graphics on;
proc freq data=Color order=freq;
    tables Hair Hair*Eyes / plots=freqplot(type=dotplot);
    tables Hair*Region / plots=freqplot(type=dotplot scale=percent);
    weight Count;
    title 'Eye and Hair Color of European Children';
run;
ods graphics off;
```

Output 40.2.1, Output 40.2.2, and Output 40.2.3 display the dot plots produced by PROC FREQ. By default, the orientation of dot plots is horizontal, which places the variable levels on the Y axis. You can specify the ORIENT=VERTICAL plot-option to request a vertical orientation. For two-way plots, you can use the TWOWAY = plot-option to specify the plot layout. The default layout (shown in Output 40.2.2 and Output 40.2.3) is GROUPVERTICAL. Two-way layouts STACKED and GROUPHORIZONTAL are also available.

Output 40.2.1 One-Way Frequency Dot Plot


Output 40.2.2 Two-Way Frequency Dot Plot


Output 40.2.3 Two-Way Percent Dot Plot


## Example 40.3: Chi-Square Goodness-of-Fit Tests

This example examines whether the children's hair color (from Example 40.1) has a specified multinomial distribution for the two geographical regions. The hypothesized distribution of hair color is $30 \%$ fair, $12 \%$ red, $30 \%$ medium, $25 \%$ dark, and $3 \%$ black.

In order to test the hypothesis for each region, the data are first sorted by Region. Then the FREQ procedure uses a BY statement to produce a separate table for each BY group (Region). The option ORDER=DATA orders the variable values (hair color) in the frequency table by their order in the input data set. The TABLES statement requests a frequency table for hair color, and the option NOCUM suppresses the display of the cumulative frequencies and percentages.

The CHISQ option requests a chi-square goodness-of-fit test for the frequency table of Hair. The TESTP= option specifies the hypothesized (or test) percentages for the chi-square test; the number of percentages listed equals the number of table levels, and the percentages sum to $100 \%$. The TESTP= percentages are listed in the same order as the corresponding variable levels appear in frequency table.

The PLOTS= option requests a deviation plot, which is associated with the CHISQ option and displays the relative deviations from the test frequencies. The TYPE=DOTPLOT plot-option requests a dot plot instead
of the default type, which is a bar chart. ODS Graphics must be enabled before producing plots. These statements produce Output 40.3.1 through Output 40.3.4.

```
proc sort data=Color;
    by Region;
run;
ods graphics on;
proc freq data=Color order=data;
    tables Hair / nocum chisq testp=(\begin{array}{lllll}{30}&{12}&{30}&{25}&{3}\end{array})
                plots (only)=deviationplot (type=dotplot);
    weight Count;
    by Region;
    title 'Hair Color of European Children';
run;
ods graphics off;
```

Output 40.3.1 Frequency Table and Chi-Square Test for Region 1


Output 40.3 .1 shows the frequency table and chi-square test for Region 1. The frequency table lists the variable values (hair color) in the order in which they appear in the data set. The "Test Percent" column lists the hypothesized percentages for the chi-square test. Always check that you have ordered the TESTP= percentages to correctly match the order of the variable levels.

Output 40.3.2 shows the deviation plot for Region 1, which displays the relative deviations from the hypothesized values. The relative deviation for a level is the difference between the observed and hypothesized (test) percentage divided by the test percentage. You can suppress the chi-square $p$-value that is displayed by default in the deviation plot by specifying the NOSTATS plot-option.

Output 40.3.2 Deviation Plot for Region 1


Output 40.3.3 and Output 40.3 .4 show the results for Region 2. PROC FREQ computes a chi-square statistic for each region. The chi-square statistic is significant at the 0.05 level for Region 2 ( $p=0.0003$ ) but not for Region 1. This indicates a significant departure from the hypothesized percentages in Region 2.

Output 40.3.3 Frequency Table and Chi-Square Test for Region 2


Output 40.3.4 Deviation Plot for Region 2


## Example 40.4: Binomial Proportions

In this example, PROC FREQ computes binomial proportions, confidence limits, and tests. The example uses the eye and hair color data from Example 40.1. By default, PROC FREQ computes the binomial proportion as the proportion of observations in the first level of the one-way table. You can designate a different level by using the LEVEL= binomial-option.

The following PROC FREQ statements compute the proportion of children with brown eyes (from the data set in Example 40.1) and test the null hypothesis that the population proportion equals $50 \%$. These statements also compute an equivalence for the proportion of children with fair hair.
The first TABLES statement requests a one-way frequency table for the variable Eyes. The BINOMIAL option requests the binomial proportion, confidence limits, and test. PROC FREQ computes the proportion with Eyes = 'brown', which is the first level displayed in the table. The AC, WILSON, and EXACT binomial-options request the following confidence limits types: Agresti-Coull, Wilson (score), and exact (Clopper-Pearson). By default, PROC FREQ provides Wald and exact (Clopper-Pearson) confidence limits for the binomial proportion. The BINOMIAL option also produces an asymptotic Wald test that the proportion equals 0.5 . You can specify a different test proportion with the $\mathrm{P}=$ binomial-option. The ALPHA=$=0.1$ option specifies that $\alpha=10 \%$, which produces $90 \%$ confidence limits.

The second TABLES statement requests a one-way frequency table for the variable Hair. The BINOMIAL option requests the proportion for the first level, Hair = 'fair'. The EQUIV binomial-option requests an equivalence test for the binomial proportion. The $\mathrm{P}=.28$ option specifies 0.28 as the null hypothesis proportion, and the MARGIN $=.1$ option specifies 0.1 as the equivalence test margin.

```
proc freq data=Color order=freq;
    tables Eyes / binomial(ac wilson exact) alpha=.1;
    tables Hair / binomial(equiv p=.28 margin=.1);
    weight Count;
    title 'Hair and Eye Color of European Children';
run;
```

Output 40.4.1 displays the results for eye color, and Output 40.4.2 displays the results for hair color.
Output 40.4.1 Binomial Proportion for Eye Color


The frequency table in Output 40.4.1 displays the values of Eyes in order of descending frequency count. PROC FREQ computes the proportion of children in the first level displayed in the frequency table, Eyes = 'brown'. Output 40.4.1 displays the binomial proportion confidence limits and test. The confidence limits are
$90 \%$ confidence limits. If you do not specify the ALPHA= option, PROC FREQ computes $95 \%$ confidence limits by default. Because the value of $Z$ is less than zero, PROC FREQ displays the a left-sided $p$-value ( 0.0019 ). This small $p$-value supports the alternative hypothesis that the true value of the proportion of children with brown eyes is less than $50 \%$.

Output 40.4.2 displays the equivalence test results produced by the second TABLES statement. The null hypothesis proportion is 0.28 and the equivalence margins are -0.1 and 0.1 , which yield equivalence limits of 0.18 and 0.38 . PROC FREQ provides two one-sided tests (TOST) for equivalence. The small $p$-value indicates rejection of the null hypothesis in favor of the alternative that the proportion is equivalent to the null value.

Output 40.4.2 Binomial Proportion for Hair Color


## Example 40.5: Analysis of a 2x2 Contingency Table

This example computes chi-square tests and Fisher's exact test to compare the probability of coronary heart disease for two types of diet. It also estimates the relative risks and computes exact confidence limits for the odds ratio.

The data set FatComp contains hypothetical data for a case-control study of high fat diet and the risk of coronary heart disease. The data are recorded as cell counts, where the variable Count contains the frequencies for each exposure and response combination. The data set is sorted in descending order by the variables Exposure and Response, so that the first cell of the $2 \times 2$ table contains the frequency of positive exposure and positive response. The FORMAT procedure creates formats to identify the type of exposure and response with character values.

```
proc format;
    value ExpFmt 1='High Cholesterol Diet'
                0='Low Cholesterol Diet';
    value RspFmt 1='Yes'
                0='No';
run;
data FatComp;
    input Exposure Response Count;
    label Response='Heart Disease';
    datalines;
0 6
0 1 2
1 0 4
1 111
;
proc sort data=FatComp;
    by descending Exposure descending Response;
run;
```

In the following PROC FREQ statements, ORDER=DATA option orders the contingency table values by their order in the input data set. The TABLES statement requests a two-way table of Exposure by Response. The CHISQ option produces several chi-square tests, while the RELRISK option produces relative risk measures. The EXACT statement requests the exact Pearson chi-square test and exact confidence limits for the odds ratio.

```
proc freq data=FatComp order=data;
    format Exposure ExpFmt. Response RspFmt.;
    tables Exposure*Response / chisq relrisk;
    exact pchi or;
    weight Count;
    title 'Case-Control Study of High Fat/Cholesterol Diet';
run;
```

The contingency table in Output 40.5 .1 displays the variable values so that the first table cell contains the frequency for the first cell in the data set (the frequency of positive exposure and positive response).

Output 40.5.1 Contingency Table


Output 40.5.2 displays the chi-square statistics. Because the expected counts in some of the table cells are small, PROC FREQ gives a warning that the asymptotic chi-square tests might not be appropriate. In this case, the exact tests are appropriate. The alternative hypothesis for this analysis states that coronary heart disease is more likely to be associated with a high fat diet, so a one-sided test is desired. Fisher's exact right-sided test analyzes whether the probability of heart disease in the high fat group exceeds the probability of heart disease in the low fat group; because this $p$-value is small, the alternative hypothesis is supported.

The odds ratio, displayed in Output 40.5.3, provides an estimate of the relative risk when an event is rare. This estimate indicates that the odds of heart disease is 8.25 times higher in the high fat diet group; however, the wide confidence limits indicate that this estimate has low precision.

Output 40.5.2 Chi-Square Statistics


Output 40.5.3 Relative Risk


## Example 40.6: Output Data Set of Chi-Square Statistics

This example uses the Color data from Example 40.1 to output the Pearson chi-square and the likelihood ratio chi-square statistics to a SAS data set. The following PROC FREQ statements create a two-way table of eye color versus hair color.

```
proc freq data=Color order=data;
    tables Eyes*Hair / expected cellchi2 norow nocol chisq;
    output out=ChiSqData n nmiss pchi lrchi;
    weight Count;
    title 'Chi-Square Tests for 3 by 5 Table of Eye and Hair Color';
run;
proc print data=ChiSqData noobs;
    title1 'Chi-Square Statistics for Eye and Hair Color';
    title2 'Output Data Set from the FREQ Procedure';
run;
```

The EXPECTED option displays expected cell frequencies in the crosstabulation table, and the CELLCHI2 option displays the cell contribution to the overall chi-square. The NOROW and NOCOL options suppress the display of row and column percents in the crosstabulation table. The CHISQ option produces chi-square tests.

The OUTPUT statement creates the ChiSqData output data set and specifies the statistics to include. The N option requests the number of nonmissing observations, the NMISS option stores the number of missing observations, and the PCHI and LRCHI options request Pearson and likelihood ratio chi-square statistics, respectively, together with their degrees of freedom and $p$-values.

The preceding statements produce Output 40.6 .1 and Output 40.6.2. The contingency table in Output 40.6.1 displays eye and hair color in the order in which they appear in the Color data set. The Pearson chi-square statistic in Output 40.6.2 provides evidence of an association between eye and hair color ( $p=0.0073$ ). The cell chi-square values show that most of the association is due to more green-eyed children with fair or red hair and fewer with dark or black hair. The opposite occurs with the brown-eyed children.

Output 40.6.3 displays the output data set created by the OUTPUT statement. It includes one observation that contains the sample size, the number of missing values, and the chi-square statistics and corresponding degrees of freedom and $p$-values as in Output 40.6.2.

Output 40.6.1 Contingency Table


Output 40.6.2 Chi-Square Statistics

| Statistic | DF | Value | Prob |
| :---: | :---: | :---: | :---: |
| Chi-Square | 8 | 20.9248 | 0.0073 |
| Likelihood Ratio Chi-Square | 8 | 25.9733 | 0.0011 |
| Mantel-Haenszel Chi-Square | 1 | 3.7838 | 0.0518 |
| Phi Coefficient |  | 0.1657 |  |
| Contingency Coefficient |  | 0.1635 |  |
| Cramer's V |  | 0.1172 |  |

Output 40.6.3 Output Data Set

| N | Chi-Square Statistics for Eye and Hair Color Output Data Set from the FREQ Procedure |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | NMISS | _PCHI_ | DF_PCHI | P_PCHI | _LRCHI_ | DF_LRCHI | P_LRCHI |
| 762 | 0 | 20.9248 | 8 | . 007349898 | 25.9733 | 8 | . 001061424 |

## Example 40.7: Cochran-Mantel-Haenszel Statistics

The data set Migraine contains hypothetical data for a clinical trial of migraine treatment. Subjects of both genders receive either a new drug therapy or a placebo. Their response to treatment is coded as 'Better' or 'Same'. The data are recorded as cell counts, and the number of subjects for each treatment and response combination is recorded in the variable Count.

```
data Migraine;
    input Gender $ Treatment $ Response $ Count @@;
    datalines;
female Active Better 16 female Active Same 11
female Placebo Better 5 female Placebo Same 20
male Active Better 12 male Active Same 16
male Placebo Better 7 male Placebo Same 19
;
```

The following PROC FREQ statements create a multiway table stratified by Gender, where Treatment forms the rows and Response forms the columns. The RELRISK option in the TABLES statement requests the odds ratio and relative risks for the two-way tables of Treatment by Response. The PLOTS=option requests a relative risk plot, which shows the relative risk and its confidence limits for each level of Gender and overall. The CMH option requests Cochran-Mantel-Haenszel statistics for the multiway table. For this stratified $2 \times 2$ table, the CMH option also produces estimates of the common relative risk and the Breslow-Day test for homogeneity of the odds ratios. The NOPRINT option suppresses the display of the crosstabulation tables.

```
ods graphics on;
proc freq data=Migraine;
    tables Gender*Treatment*Response /
            relrisk plots(only)=relriskplot(stats) cmh noprint;
    weight Count;
    title 'Clinical Trial for Treatment of Migraine Headaches';
run;
ods graphics off;
```

Output 40.7.1 through Output 40.7.4 show the results of the analysis. The relative risk plot (Output 40.7.1) displays the relative risks and confidence limits for the two levels of Gender and for the overall (common) relative risk. Output 40.7.2 displays the CMH statistics. For a stratified $2 \times 2$ table, the three CMH statistics test the same hypothesis. The significant $p$-value (0.004) indicates that the association between treatment and response remains strong after adjusting for gender.

The CMH option also produces a table of overall relative risks, as shown in Output 40.7.3. Because this is a prospective study, the relative risk estimate assesses the effectiveness of the new drug; the "Cohort (Col1 Risk)" values are the appropriate estimates for the first column (the risk of improvement). The probability of migraine improvement with the new drug is just over two times the probability of improvement with the placebo.

The large $p$-value for the Breslow-Day test (0.2218) in Output 40.7.4 indicates no significant gender difference in the odds ratios.

Output 40.7.1 Relative Risk Plot


Output 40.7.2 Cochran-Mantel-Haenszel Statistics

| Cochran-Mantel-Haenszel Statistics (Based on Table Scores) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Statistic | Alternative Hypothesis | DF | Value | Prob |
| 1 | Nonzero Correlation | 1 | 8.3052 | 0.0040 |
| 2 | Row Mean Scores Differ | 1 | 8.3052 | 0.0040 |
| 3 | General Association | 1 | 8.3052 | 0.0040 |

Output 40.7.3 CMH Option: Common Relative Risks

| Statistic | Common Odds Ratio and Relative Risks |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Method | Value | 95\% Confi | Limits |
| Odds Ratio |  | Mantel-Haenszel | 3.3132 | 1.4456 | 7.5934 |
|  |  | Logit | 3.2941 | 1.4182 | 7.6515 |
| Relative Risk | (Column 1) | Mantel-Haenszel | 2.1636 | 1.2336 | 3.7948 |
|  |  | Logit | 2.1059 | 1.1951 | 3.7108 |
| Relative Risk | (Column 2) | Mantel-Haenszel | 0.6420 | 0.4705 | 0.8761 |
|  |  | Logit | 0.6613 | 0.4852 | 0.9013 |

Output 40.7.4 CMH Option: Breslow-Day Test

| Breslow-Day Test for |  |
| :---: | :---: |
| Chi-Square | 1.4929 |
| DF | 1 |
| Pr > ChiSq | 0.2218 |

## Example 40.8: Cochran-Armitage Trend Test

The data set Pain contains hypothetical data for a clinical trial of a drug therapy to control pain. The clinical trial investigates whether adverse responses increase with larger drug doses. Subjects receive either a placebo or one of four drug doses. An adverse response is recorded as Adverse='Yes'; otherwise, it is recorded as Adverse='No'. The number of subjects for each drug dose and response combination is contained in the variable Count.

```
data pain;
    input Dose Adverse $ Count @@;
    datalines;
0 No 26 0 Yes 6
1 No 26 1 Yes 7
2 No 23 2 Yes 9
3 No 18 3 Yes 14
4 No 9 4 Yes 23
;
```

The following PROC FREQ statements provide a trend analysis. The TABLES statement requests a table of Adverse by Dose. The MEASURES option produces measures of association, and the CL option produces confidence limits for these measures. The TREND option tests for a trend across the ordinal values of the variable Dose with the Cochran-Armitage test. The PLOTS= option requests a mosaic plot of Adverse by Dose.

The EXACT statement produces exact $p$-values for this test, and the MAXTIME $=$ option terminates the exact computations if they do not complete within 60 seconds. The TEST statement computes an asymptotic test for Somers' $D(R \mid C)$.

```
ods graphics on;
proc freq data=Pain;
    tables Adverse*Dose / trend measures cl
            plots=mosaicplot;
    test smdrc;
    exact trend / maxtime=60;
    weight Count;
    title 'Clinical Trial for Treatment of Pain';
run;
ods graphics off;
```

Output 40.8.1 through Output 40.8.4 display the results of the analysis. The "Col Pct" values in Output 40.8.1 show the expected increasing trend in the proportion of adverse effects with the increasing dosage (from $18.75 \%$ to $71.88 \%$ ). The corresponding mosaic plot (Output 40.8.2) also shows this increasing trend.

Output 40.8.1 Contingency Table


Output 40.8.2 Mosaic Plot


Output 40.8.3 displays the measures of association produced by the MEASURES option. Somers' $D(R \mid C)$ measures the association treating the row variable (Adverse) as the response and the column variable (Dose) as a predictor. Because the asymptotic $95 \%$ confidence limits do not contain zero, this indicates a strong positive association. Similarly, the Pearson and Spearman correlation coefficients show evidence of a strong positive association, as hypothesized.

The Cochran-Armitage test (Output 40.8.4) supports the trend hypothesis. The small left-sided $p$-values for the Cochran-Armitage test indicate that the probability of the Row 1 level (Adverse='No') decreases as Dose increases or, equivalently, that the probability of the Row 2 level (Adverse='Yes') increases as Dose increases. The two-sided $p$-value tests against either an increasing or decreasing alternative. This is an appropriate hypothesis when you want to determine whether the drug has progressive effects on the probability of adverse effects but the direction is unknown.

Output 40.8.3 Measures of Association

| Statistic |  | Value | ASE | Confide | Limits |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Gamma |  | 0.5313 | 0.0935 | 0.3480 | 0.7146 |
| Kendall's Tau-b |  | 0.3373 | 0.0642 | 0.2114 | 0.4631 |
| Stuart's Tau-c |  | 0.4111 | 0.0798 | 0.2547 | 0.5675 |
| Somers' D C\|R |  | 0.4427 | 0.0837 | 0.2786 | 0.6068 |
| Somers' D R\|C |  | 0.2569 | 0.0499 | 0.1592 | 0.3547 |
| Pearson Correlation |  | 0.3776 | 0.0714 | 0.2378 | 0.5175 |
| Spearman Correlation |  | 0.3771 | 0.0718 | 0.2363 | 0.5178 |
| Lambda Asymmetric C\|R |  | 0.1250 | 0.0662 | 0.0000 | 0.2547 |
| Lambda Asymmetric R\|C |  | 0.2373 | 0.0837 | 0.0732 | 0.4014 |
| Lambda Symmetric |  | 0.1604 | 0.0621 | 0.0388 | 0.2821 |
| Uncertainty Coefficient $C \mid R$ <br> Uncertainty Coefficient R\|C <br> Uncertainty Coefficient Symmetric |  | 0.0515 | 0.0191 | 0.0140 | 0.0890 |
|  |  | 0.1261 | 0.0467 | 0.0346 | 0.2175 |
|  |  | 0.0731 | 0.0271 | 0.0199 | 0.1262 |
| Somers' D R\|C |  |  |  |  |  |
| Somers' D R\|C |  |  | 0.2569 |  |  |
| ASE |  |  | 0.0499 |  |  |
| 95\% Lower Conf Limit |  |  | 0.1592 |  |  |
| 95\% Upper Conf Limit |  |  | 0.3547 |  |  |
| Test of H0: Somers' D R\|C $=0$ |  |  |  |  |  |
| ASE under HO |  |  | 0.0499 |  |  |
| Z |  |  | 5.1511 |  |  |
| One-sided $\operatorname{Pr}>\mathrm{Z}$ |  |  | $<.0001$ |  |  |
| Two-sided Pr > \|Z| |  |  | $<.0001$ |  |  |

Output 40.8.4 Trend Test


## Example 40.9: Friedman's Chi-Square Test

Friedman's test is a nonparametric test for treatment differences in a randomized complete block design. Each block of the design might be a subject or a homogeneous group of subjects. If blocks are groups of subjects, the number of subjects in each block must equal the number of treatments. Treatments are randomly assigned to subjects within each block. If there is one subject per block, then the subjects are repeatedly measured once under each treatment. The order of treatments is randomized for each subject.

In this setting, Friedman's test is identical to the ANOVA (row means scores) CMH statistic when the analysis uses rank scores (SCORES=RANK). The three-way table uses subject (or subject group) as the stratifying variable, treatment as the row variable, and response as the column variable. PROC FREQ handles ties by assigning midranks to tied response values. If there are multiple subjects per treatment in each block, the ANOVA CMH statistic is a generalization of Friedman's test.

The data set Hypnosis contains data from a study investigating whether hypnosis has the same effect on skin potential (measured in millivolts) for four emotions (Lehmann and D'Abrera 2006, p. 264). Eight subjects are asked to display fear, joy, sadness, and calmness under hypnosis. The data are recorded as one observation per subject for each emotion.

```
data Hypnosis;
    length Emotion $ 10;
    input Subject Emotion $ SkinResponse @@;
    datalines;
1 fear 23.1 1 joy 22.7 1 sadness 22.5 1 calmness 22.6
2 fear 57.6 2 joy 53.2 2 sadness 53.7 2 calmness 53.1
3 fear 10.5 3 joy 9.7 3 sadness 10.8 3 calmness 8.3
4 fear 23.6 4 joy 19.6 4 sadness 21.1 4 calmness 21.6
5 fear 11.9 5 joy 13.8 5 sadness 13.7 5 calmness 13.3
6 fear 54.6 6 joy 47.1 6 sadness 39.2 6 calmness 37.0
fear 21.0 7 joy 13.6 7 sadness 13.7 7 calmness 14.8
fear 20.3 8 joy 23.6 8 sadness 16.3 8 calmness 14.8
;
```

In the following PROC FREQ statements, the TABLES statement creates a three-way table stratified by Subject and a two-way table; the variables Emotion and SkinResponse form the rows and columns of each table. The CMH2 option produces the first two Cochran-Mantel-Haenszel statistics, the option SCORES=RANK specifies that rank scores are used to compute these statistics, and the NOPRINT option suppresses the contingency tables. These statements produce Output 40.9.1 and Output 40.9.2.

```
proc freq data=Hypnosis;
    tables Subject*Emotion*SkinResponse /
            cmh2 scores=rank noprint;
run;
proc freq data=Hypnosis;
    tables Emotion*SkinResponse /
                cmh2 scores=rank noprint;
run;
```

Because the CMH statistics in Output 40.9 .1 are based on rank scores, the Row Mean Scores Differ statistic is identical to Friedman's chi-square $(Q=6.45)$. The $p$-value of 0.0917 indicates that differences in skin potential response for different emotions are significant at the $10 \%$ level but not at the $5 \%$ level.

When you do not stratify by subject, the Row Mean Scores Differ CMH statistic is identical to a KruskalWallis test and is not significant ( $p=0.9038$ in Output 40.9.2). Thus, adjusting for subject is critical to reducing the background variation due to subject differences.

Output 40.9.1 CMH Statistics: Stratifying by Subject

| ```Clinical Trial for Treatment of Pain The FREQ Procedure Summary Statistics for Emotion by SkinResponse Controlling for Subject Cochran-Mantel-Haenszel Statistics (Based on Rank Scores)``` |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Nonzero Correlation | 1 | 0.2400 | 0.6242 |
| 2 | Row Mean Scores Differ | 3 | 6.4500 | 0.0917 |

Output 40.9.2 CMH Statistics: No Stratification


## Example 40.10: Cochran's Q Test

When a binary response is measured several times or under different conditions, Cochran's $Q$ tests that the marginal probability of a positive response is unchanged across the times or conditions. When there are more than two response categories, you can use the CATMOD procedure to fit a repeated-measures model.

The data set Drugs contains data for a study of three drugs to treat a chronic disease (Agresti 2002). Forty-six subjects receive drugs A, B, and C. The response to each drug is either favorable (' F ') or unfavorable ('U').

```
proc format;
    value $ResponseFmt 'F'='Favorable'
        'U'='Unfavorable';
run;
```

```
data drugs;
    input Drug_A $ Drug_B $ Drug_C $ Count @@;
    datalines;
F F F 6 U F F 2
FFU16 U FU 4
FU F 2 U U F 6
FU U 4 U U U 6
;
```

The following statements create one-way frequency tables of the responses to each drug. The AGREE option produces Cochran's $Q$ and other measures of agreement for the three-way table. These statements produce Output 40.10.1 through Output 40.10.5.

```
proc freq data=Drugs;
    tables Drug_A Drug_B Drug_C / nocum;
    tables Drug_A*Drug_B*Drug_C / agree noprint;
    format Drug_A Drug_B Drug_C $ResponseFmt.;
    weight Count;
    title 'Study of Three Drug Treatments for a Chronic Disease';
run;
```

The one-way frequency tables in Output 40.10 .1 provide the marginal response for each drug. For drugs A and B, $61 \%$ of the subjects reported a favorable response while $35 \%$ of the subjects reported a favorable response to drug C. Output 40.10.2 and Output 40.10.3 display measures of agreement for the 'Favorable' and 'Unfavorable' levels of drug A, respectively. McNemar's test shows a strong discordance between drugs B and C when the response to drug A is favorable.

Output 40.10.1 One-Way Frequency Tables


Output 40.10.2 Measures of Agreement for Drug A Favorable


Output 40.10.3 Measures of Agreement for Drug A Unfavorable

| McNemar's Test |  |
| :---: | :---: |
| Statistic (S) | 000 |
| DF | 1 |
| $\mathrm{Pr}>\mathrm{S}$ | 271 |
| Simple Kappa Coefficient |  |
| Kappa | -0.1538 |
| ASE | 0.2230 |
| 95\% Lower Conf Limit | -0.5909 |
| 95\% Upper Conf Limit | 0.2832 |

Output 40.10.4 displays the overall kappa coefficient. The small negative value of kappa indicates no agreement between drug B response and drug C response.

Output 40.10.4 Overall Measures of Agreement


Cochran's $Q$ is statistically significant ( $p=0.0145$ in Output 40.10.5), which leads to rejection of the hypothesis that the probability of favorable response is the same for the three drugs.

Output 40.10.5 Cochran's $Q$ Test

```
Cochran's Q, for Drug_A
    by Drug_B by Drug_C
-----------------------
DF 2
Pr > Q 0.0145
```


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[^0]:    ${ }^{1}$ CHISQ computes Fisher's exact test for $2 \times 2$ tables. Use the FISHER option to compute Fisher's exact test for general $r \times c$ tables.

[^1]:    *Default

