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Rebalancing and Returns

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Most investors have portfolios that combine multiple asset classes, such as equities and bonds. Maintaining an asset allocation policy that is suitable for the investor's unique investment needs and risk tolerance requires periodic rebalancing. Left unchecked, any multi-asset class portfolio will drift from its target allocations as some classes outperform others. Over time, a non-rebalanced portfolio will tend to become concentrated in higher-return assets, exposing the investor to a very different risk and return profile than that of the intended allocation.

Though the primary motivation for rebalancing is to control risks, there have been a number of recent studies that look at the relation between rebalancing and returns. One way rebalancing may impact returns is through the amount of drift allowed before the portfolio is rebalanced. Since expected returns are a function of the current allocation, any change in weights, due to rebalancing or drift, will have an impact on expected returns. A rebalancing strategy that tolerates a greater amount of drift will typically have higher expected returns and be exposed to greater risk compared to rebalancing strategies that allow less drift.

Rebalancing strategies may also differ in the amount of costly trading involved. The need to understand risk, costs and benefits, and client preferences is extremely important, as the benefits from rebalancing must be weighed against transaction costs and, if

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applicable, tax liabilities that reduce net returns. Reduced deviations from the desired risk profile are a clear and obvious benefit, but a strict adherence to the target allocation may require excessive rebalancing. Leland (1996) and Clark (1999, 2003) provide a well-reasoned rebalancing model in which the presence of costs create a non-trading region around the target weights. The boundary of this region is where the benefits exactly offset the costs from rebalancing, and optimal rebalancing occurs only when the allocation drifts outside the non-trading region.

For studies that use geometric returns, controlling risk through rebalancing will also have a positive impact on geometric returns. The geometric return is important because it links current investment to future wealth. Geometric returns are approximately equal to average returns less half the variance, thus a reduction of variance can be one way that rebalancing affects a portfolio's geometric return.

The weights of asset classes in a portfolio are constantly evolving in a way that depends on past returns. When one rebalances, the weight of assets that have had high returns is decreased in order to increase the weight of assets that have had low returns. In between rebalancing events, the weights on assets that realize high returns increase. Unless asset class returns are strongly and reliably correlated over time, the timing aspect of rebalancing should not be expected to produce any additional returns in excess of the expected return of the allocation. The specifics of the rebalancing strategy should only affect expected returns through the frequency of costly trading and the amount of allowable drift.

Some may argue that rebalancing strategies can be tailored to produce higher expected returns because they take advantage of long-term reversal in asset prices. However, testing for expected return benefits from rebalancing is difficult because expected returns are not observable. Any analysis must use historical realized returns, which are very noisy. Studies that have looked the impact of rebalancing on returns use investment horizons that range from 10 to 20 years. However, it takes over 20 years of returns to statistically distinguish the equity premium from zero. For the size and value premiums, statistical significance requires about 20 and 30 years, respectively.[1] One must be cautious when interpreting reported benefits of rebalancing strategies that are formulated using historical returns because noise in realized returns will make certain strategies look good over certain periods, especially if researchers try hard enough to find good results. How confident should one be that this strategy will continue to be the best performer in all periods and for all allocations? There are multitudes of trading rules that work well in the historical data from which they were mined but fail to continue their performance out of sample. It is unlikely that rebalancing rules are any exception.

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1. Using monthly data starting in July 1926, the t-statistic for the average return of the market in excess of the one-month Treasury bill rate does not reliably exceed 1.65 until August 1950. The t-statistics for the average value and size premiums, measured using the HML and SMB factors from Kenneth French's website, do not reliably exceed 1.65 until May 1945 and January 1955, respectively. A t-statistic of 1.65 corresponds to marginal significance at the 10% level.

In this paper, I attempt to document the relative importance of asset allocation, variance reduction, trading costs, and time-varying weights in the historical performance of several rebalancing strategies. The purpose is not to try to identify optimal rebalancing strategies that maximize portfolio returns, but to improve understanding about the benefits and limitations of rebalancing strategies reported in studies such as Daryanani (2008) and Plaxco and Arnott (2002).

Replicating Daryanani's Opportunistic Rebalancing

In this study, I reexamine the return benefit from Daryanani's (2008) opportunistic rebalancing strategy. In his experiment, one portfolio was rebalanced, using a relatively large rebalancing tolerance (20% for each asset class). It was monitored daily to biweekly, and it had annual returns that were about 45-50 basis points higher on average than the non-rebalanced portfolio. Though this difference is small relative to the volatility of historical returns data, it is still unexpected given that a non-rebalanced portfolio will tend to have greater allocations in high-returning assets and would be expected to have higher returns over a sufficiently long sample. Daryanani's conclusion that a rebalanced portfolio can do better than the non-rebalanced portfolio, even net of trading costs, is surprising and deserves closer inspection.

I replicate his experiment, with some modifications due to data limitations. I consider a target portfolio with an initial investment of $1 million and with allocations of 25% in the S&P 500 Index, 20% in the Russell 2000 Index, 5% in the Dow Jones AIG Commodity Index, 10% in the Dow Jones Wilshire US REIT Total Return Index, and 40% in the Bloomberg US Government 7-10 Year Index. Daily data availability limits the sample to February 1996-June 2008.[2]

The opportunistic rebalancing strategy specifies both the timing and the tolerance band for rebalancing. *Look intervals*, which range from daily to annually, determine how frequently the portfolio is evaluated for rebalancing. The *rebalance band*, which ranges from 0% to 25%, specifies how far each asset class is allowed to stray from the initial target allocation before rebalancing is required. The 0% rebalancing band corresponds to calendar rebalancing where all classes are brought exactly to target at the end of each interval. The 100% rebalancing band is the non-rebalanced case.[3] Asset classes that fall outside of the specified rebalance band are rebalanced to within 50% of the rebalance band. In this paper, I describe strategies using the notation (*d, r%*), where d is the look interval in business days and *r%* is the width of the rebalance band.

The rebalancing strategy is best demonstrated with an example. Consider the strategy (20, 10%), which dictates that the portfolio weights be checked every 20 business days

2. There are two differences between my sample and that used by Daryanani. Daryanani uses the Dow Jones REIT Total Return index and his sample is from January 1992 to December 2004.

3. In theory, rebalancing can still occur with the 100% band (when an asset class swells to more than double its target weight), although that never occurs in the sample used in this paper.

(approximately one month). For the Russell 2000, which has target weight of 20%, rebalancing occurs only if the class is outside the 10% rebalancing band, or when the weight is no longer between 18% and 22%. If the Russell 2000 needs to be rebalanced, it is brought to within the target band of 19% to 21%. Offsetting trades are made in asset classes that also require rebalancing, though sometimes one additional class that is already within the rebalance bands must be traded to ensure the portfolio remains 100% invested.[4] Each trade incurs a flat $20 trading cost.

To ensure comparability of results, I adopt the start-of-month averaging procedure used by Daryanani. He reports that somewhat different results are obtained depending on the month the portfolio is initially formed. Thus, each of the numbers reported in Table 1 is an average of 12 portfolios, each with a different monthly start date. Panel A reports the average annual geometric return for each of the rebalancing strategies over the period February 1996-December 2004. This period is a subsample of the period studied by Daryanani. I reserve the later part of my sample, from January 2005 to June 2008, for independent, out-of-sample testing. The results are very similar to those reported in the Daryanani study. The rebalancing strategies that produce the highest geometric returns are those that use a 20% rebalance band and a look interval of 1, 5, or 10 days.

The geometric returns can be approximated by:

(1) Net Geometric Return = Gross Average Return – ½ Variance – Cost

These three components of the geometric return are displayed in panels B, C, and D. Most of the variation in geometric returns across rebalancing strategies is mirrored in the arithmetic return, displayed in Panel B. Panel C shows the amount by which the geometric return is lower than the average return because of portfolio volatility. All the rebalancing strategies reduce volatility by similar amounts, accounting for an increase of 5 to 7 basis points in geometric returns over the portfolio that is not rebalanced. Trading costs, shown in Panel D, are assumed to be flat at $20 per trade and do not include tax costs. The costs are computed as the difference in the geometric return with a $20 flat trading cost and the geometric return of the same strategy with zero trading costs. Notice that trading costs significantly diminish returns for the strategies that combine a short look interval with narrow rebalance bands. Strategies involving at most 20 trades per year resulted in an annual return loss of at most 3 basis points for this portfolio, assuming an initial investment of $1 million.

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4. If the portfolio cannot be brought to within the tolerance bands by trading only classes that require rebalancing, an offsetting trade will occur in the asset class with the greatest distance, in dollars, to the tolerance band. If a sale is required, the asset class with the greatest distance in dollars to the lower bound of the tolerance band is selected. If a buy is required, the asset class with the greatest distance in dollars to the upper bound if the tolerance band is selected.

Table 1

Rebalanced Returns
February 1996-December 2004

Panel A: 12-Month Average Geometric Net Return

	Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	6.86	8.53	8.69	8.81	8.87	8.86	8.96
5%	8.92	8.92	8.93	8.94	8.93	8.90	8.96
10%	9.04	9.06	9.09	9.06	9.04	8.91	8.96
15%	9.13	9.18	9.17	9.07	8.99	8.93	8.97
20%	9.18	9.19	9.20	9.08	8.98	8.92	8.93
25%	8.99	8.93	8.93	8.87	8.85	8.81	8.84
100%	8.47	8.47	8.47	8.47	8.47	8.47	8.47

Panel B: Gross Arithmetic Returns

	Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	9.35	9.36	9.32	9.34	9.33	9.30	9.39
5%	9.44	9.42	9.41	9.41	9.37	9.33	9.39
10%	9.51	9.54	9.53	9.52	9.48	9.34	9.40
15%	9.59	9.63	9.60	9.50	9.43	9.36	9.40
20%	9.62	9.63	9.64	9.52	9.41	9.35	9.36
25%	9.44	9.39	9.38	9.31	9.29	9.25	9.27
100%	8.96	8.96	8.96	8.96	8.96	8.96	8.96

Panel C: -½ x Variance

	Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	-0.41	-0.41	-0.41	-0.40	-0.39	-0.39	-0.39
5%	-0.41	-0.41	-0.40	-0.40	-0.39	-0.39	-0.39
10%	-0.41	-0.41	-0.40	-0.40	-0.39	-0.39	-0.39
15%	-0.40	-0.40	-0.40	-0.40	-0.40	-0.40	-0.39
20%	-0.40	-0.40	-0.39	-0.40	-0.40	-0.39	-0.39
25%	-0.41	-0.40	-0.40	-0.40	-0.40	-0.40	-0.40
100%	-0.46	-0.46	-0.46	-0.46	-0.46	-0.46	-0.46

Panel D: Annual Returns Lost to Trading Costs

	Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	-2.04	-0.38	-0.19	-0.09	-0.03	-0.01	-0.01
5%	-0.08	-0.06	-0.04	-0.03	-0.02	-0.01	-0.01
10%	-0.03	-0.03	0.00	-0.02	-0.01	-0.01	0.00
15%	-0.02	-0.01	0.01	0.00	-0.01	0.00	0.00
20%	0.00	-0.01	-0.01	-0.01	0.00	0.00	0.00
25%	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
100%	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Because each return in Table 1 is an average of 12 portfolios, noise in the data is reduced, making it difficult to evaluate the robustness of the results. For the sake of space, Table 2 shows the returns and standard deviations for all 12 portfolios for 3 of the 36 strategies. The (60, 10%) strategy, a common rebalancing strategy that corresponds to about once per quarter, is used as the benchmark strategy. The other two strategies were chosen to represent two contrasting approaches to rebalancing. The (1, 20%) strategy produced the highest returns in Table 1 and involves a frequent look interval with a wide rebalance band. Also shown are the 12 portfolios for the (250, 0%) strategy, which is approximately annual rebalancing to exact targets.

Each portfolio is initially formed in one of the 12 months from February 1996 to January 1997 and runs through December 31, 2004. The returns of each strategy vary within a range of about 1% depending on the formation month. The (1, 20%) strategy has higher returns than the benchmark strategy for all but 1 of the 12 monthly starts, while the (250, 0%) strategy has returns less than the benchmark for 8 of the 12 monthly starts. Because the samples for each portfolio overlap, the 12 portfolios are highly correlated, and one must be careful not to interpret the results as if they were 12 independent observations. If one strategy had higher returns when the portfolio was started in February, it is likely that it will also have higher returns if the portfolio is started in March. It should be expected that the differences will not change by much when the portfolio formation is a few months apart, so that having a strategy that performs better in all 12 portfolios is not to be interpreted as much additional evidence that the return relation is robust. In contrast, if there is a large variation in results, it would be an indication that the data is noisy and that it will be difficult to statistically determine if one strategy is better than another. The t-statistics take these correlations into account and show that the return of the (1, 20%) and (250, 0%) strategies are statistically indistinguishable from the return on the benchmark (60, 10%) strategy.

Aside from the noise in historical return data that makes it impossible to say one strategy performs better with any statistical certainty, a more fundamental problem

Table 2

Net Annualized Returns for Selected Rebalancing Strategies

As of December 31, 2004

	Rebalancing Strategy (Look Interval, Rebalancing Band %)					
	(60, 10%)		(1, 20%)		(250, 0%)	
Starting Month	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev**	**Mean**	**Std. Dev.**
2/1996	9.59	8.81	9.67	8.94	9.35	8.68
3/1996	9.47	8.77	9.70	8.92	9.65	8.69
4/1996	9.43	8.87	9.59	8.87	9.81	8.66
5/1996	9.65	8.87	9.66	8.87	9.51	8.75
6/1996	9.49	8.75	9.37	8.87	9.25	8.69
7/1996	9.46	8.90	9.69	8.92	9.12	8.73
8/1996	9.92	8.85	10.02	8.95	9.66	8.85
9/1996	9.89	8.80	10.05	8.93	9.57	8.87
10/1996	9.40	8.99	9.71	9.03	9.57	9.12
11/1996	9.47	8.83	9.48	9.01	9.31	9.08
12/1996	8.82	8.96	9.14	9.17	8.78	8.96
1/1997	8.98	9.06	9.32	9.06	9.02	8.95
Arithmatic Avg.	9.46	8.87	9.62	8.96	9.38	8.83

	Return Differences			
	(1, 20%) – (60, 10%)		(250, 0%) – (60, 10%)	
Starting Month	**Difference**	**t-stat**	**Difference**	**t-stat**
2/1996	0.08	0.52	-0.24	-1.07
3/1996	0.23	1.54	0.19	0.95
4/1996	0.16	0.95	0.37	1.52
5/1996	0.01	0.10	-0.13	-0.42
6/1996	-0.12	-0.71	-0.25	-1.20
7/1996	0.22	1.47	-0.34	-1.70
8/1996	0.10	0.61	-0.26	-1.90
9/1996	0.16	0.93	-0.31	-2.00
10/1996	0.31	2.29	0.16	0.84
11/1996	0.01	0.03	-0.16	-0.84
12/1996	0.31	1.73	-0.05	-0.30
1/1997	0.34	2.26	0.04	0.19
Arithmatic Avg.	0.15	1.38	-0.08	-0.89

exists when comparing average returns of different rebalancing strategies. Depending on how much drift is allowed by the strategy, some return difference is expected due to allocation differences. Portfolios with more asset class drift will tend to have greater allocations in higher-return assets. The evolution of portfolio weights due to drift and rebalancing will also affect the portfolio return. The average return can be broken down into the expected return based on the average allocation and the effects from the time-varying weights:[5]

$$E(\sum wr) = \sum E(w)E(r) + \sum \text{cov}(w, r) \tag{2}$$

The term on the left-hand side is the arithmetic average portfolio return, which is computed by taking the weighted sum of returns each period, and averaging them over all periods. The first term on the right-hand side is the return on a portfolio with weights fixed at the average allocation. It is computed by first averaging the weights and returns on each asset class, then taking the weighted sum of the averages. Notice that if the weights are independent of returns, the average of the products would equal the product of the averages. Because portfolio weights are not constant through time, it is possible for the weights to have nonzero covariance with returns. If this occurs, the portfolio can have higher or lower returns than what would be expected given its average allocation. Proponents of positive rebalancing returns argue that it is possible to formulate rebalancing strategies that exploit long-term price reversals, making this term positive on average. However, there is also the potential for momentum effects to make this term negative on average. In either case, unless asset class returns are strongly correlated across time, this covariance term should be small.

In Table 3, I show how much of the variation in average returns can be expected due to differences in the average allocation. Panel A of Table 3 displays the annualized arithmetic average return for each rebalancing strategy, which is the term on the left-hand side of (2). In Panel B, the expected return given the average allocation, which is the first term on the right-hand side of (2), is computed by first averaging the weights and returns of the individual asset classes, then taking a weighted sum. Once again, each return in Panels A and B is an average of returns on 12 portfolios, each starting in a different month. The variation in expected returns in Panel B is due completely to differences in the amount of drift allowed by the rebalancing algorithm. There is zero drift in the (1, 0%) strategy, as the portfolio is rebalanced to target each day. All the other strategies allow the portfolio weights to stray from target to some extent, with greater drift occurring in strategies with wider rebalancing bands. Therefore, rebalancing with wide bands, such as 20%-25%, is expected to increase returns by a few basis points relative to rebalancing with narrow bands, because the additional drift implies a slightly larger allocation to high-return assets on average.

5. This is the definition of covariance. For any two variables X and Y, cov(X,Y) = E(XY) – E(X)E(Y).

Table 3

Average Rebalanced Returns
February 1996-December 2004

Panel A: Gross Annualized Arithmetic Average Returns

	Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	9.35	9.36	9.32	9.34	9.33	9.30	9.39
5%	9.44	9.42	9.41	9.41	9.37	9.33	9.39
10%	9.51	9.54	9.53	9.52	9.48	9.34	9.40
15%	9.59	9.63	9.60	9.50	9.43	9.36	9.40
20%	9.62	9.63	9.64	9.52	9.41	9.35	9.36
25%	9.44	9.39	9.38	9.31	9.29	9.25	9.27
100%	8.96	8.96	8.96	8.96	8.96	8.96	8.96

Panel B: Expected Returns for Average Allocations

	Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	9.34	9.35	9.35	9.35	9.35	9.36	9.38
5%	9.36	9.36	9.36	9.36	9.36	9.37	9.38
10%	9.39	9.38	9.38	9.38	9.38	9.38	9.39
15%	9.40	9.40	9.40	9.40	9.39	9.39	9.40
20%	9.40	9.39	9.39	9.40	9.38	9.38	9.40
25%	9.42	9.41	9.40	9.40	9.39	9.39	9.40
100%	9.56	9.56	9.56	9.56	9.56	9.56	9.56

Panel C: Return Difference from Covariance of Weights with Returns

	Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	0.00	0.02	-0.03	-0.01	-0.02	-0.06	0.01
5%	0.08	0.06	0.05	0.05	0.01	-0.04	0.01
10%	0.13	0.15	0.15	0.14	0.10	-0.04	0.00
15%	0.19	0.23	0.20	0.11	0.04	-0.03	0.00
20%	0.21	0.23	0.25	0.12	0.03	-0.03	-0.03
25%	0.02	-0.02	-0.03	-0.09	-0.10	-0.14	-0.13
100%	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59	-0.59

Panel C shows the amount by which the portfolio return exceeds the return on a portfolio with weights held constant at the average allocation, computed as the difference between Panel A and Panel B. Rebalancing with frequent monitoring and a 15%-20% band yielded weights that were positively correlated to returns in this sample, boosting realized returns by 15 to 19 basis points relative to a constant-weight portfolio with the same average allocation. While this difference might be due to skillful rebalancing, it is impossible to distinguish it from random noise with just nine years of returns data. I try to address this issue later in this paper by using a longer sample of historical returns.

Components or Core?

Suppose, for a moment, that the (1, 20%) or the (5, 20%) strategies increase expected returns through skillful rebalancing. The natural question that follows is whether one can achieve higher returns by rebalancing across more asset classes. Daryanani argues that the answer is yes, because "rebalancing benefits can be increased by using more uncorrelated assets, [thus increasing] the number of buy-low/sell-high opportunities." If this were the case, a core equity strategy would underperform an optimally rebalanced components strategy. To test this claim, I consider a core strategy portfolio that invests 45% in the Russell 3000 Index and a components strategy that invests 45% in a combination of the Russell 1000 Growth Index, the Russell 1000 Value Index, the Russell 2000 Growth Index, and the Russell 2000 Value Index.[6] The remainder of the portfolio has 40% in bonds, 10% in REITs, and 5% in commodities, as in the previous analysis. Trading costs are still assumed to be flat at $20 per trade and the initial investment is $1 million.

Table 4 displays the net annualized arithmetic average returns for the core and component strategies over the period February 1996-December 2004. For 37 of the 42 rebalancing strategies, the core portfolio has higher returns than the component strategy. Rebalancing across a greater number of asset classes using the components did not deliver higher returns even with the (1, 20%) strategy, which is the strategy that is supposed to produce the greatest rebalancing benefits. Except in the extreme case of rebalancing to target on a daily or weekly basis where the core strategy clearly dominates the components strategy, none of the return differences between the core and component strategies is statistically significant.

Notice that the (1, 20%) strategy no longer produces the highest returns, despite having an identical sampling period and a very similar allocation as the previous analysis. For the core strategy, the best three rebalancing strategies in the sample period are the (5, 25%), the (1, 25%), and the (125, 20%). For the component strategy, the top three are the (20, 25%), the (125, 25%) and the (250, 25%) strategies.

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6. The target weights of the Russell components are set equal to their average weight in the Russell 3000 in the year prior to portfolio formation, from February 1995 to January 1996. Specifically, the Russell 1000 Value weight was 39.61%; Russell 1000 Growth, 45.85%; Russell 2000 Value, 5.81%; and Russell 2000 Growth, 8.73%. These weights are then scaled to sum to 45%.

Table 4

Core vs. Component Strategies

February 1996-December 2004

Panel A: Core Portfolio

Rebalance Band	Net Arithmetic Average Returns Look Interval (market days) 1	5	10	20	60	125	250
0%	8.06	9.30	9.39	9.47	9.49	9.53	9.67
5%	9.61	9.59	9.58	9.60	9.58	9.60	9.73
10%	9.76	9.73	9.74	9.72	9.69	9.70	9.79
15%	9.80	9.77	9.79	9.78	9.78	9.78	9.81
20%	9.76	9.73	9.78	9.79	9.79	9.83	9.76
25%	9.85	9.86	9.71	9.77	9.79	9.66	9.61
100%	9.33	9.33	9.33	9.33	9.33	9.33	9.33

Rebalance Band	Standard Deviation Look Interval (market days) 1	5	10	20	60	125	250
0%	9.14	9.11	9.09	9.06	8.98	8.93	8.90
5%	9.14	9.11	9.08	9.05	8.98	8.94	8.94
10%	9.16	9.13	9.10	9.10	9.04	9.00	8.99
15%	9.17	9.13	9.17	9.15	9.10	9.06	9.08
20%	9.28	9.28	9.25	9.23	9.25	9.16	9.23
25%	9.30	9.29	9.33	9.32	9.30	9.37	9.47
100%	10.10	10.10	10.10	10.10	10.10	10.10	10.10

Panel B: Components Portfolio

Rebalance Band	Net Arithmetic Average Returns Look Interval (market days) 1	5	10	20	60	125	250
0%	6.62	8.94	9.15	9.31	9.40	9.48	9.67
5%	9.43	9.47	9.45	9.46	9.50	9.54	9.71
10%	9.55	9.58	9.57	9.62	9.63	9.67	9.74
15%	9.67	9.71	9.72	9.72	9.70	9.74	9.76
20%	9.74	9.74	9.72	9.77	9.72	9.71	9.74
25%	9.78	9.78	9.76	9.80	9.76	9.79	9.79
100%	9.18	9.18	9.18	9.18	9.18	9.18	9.18

Rebalance Band	Standard Deviation Look Interval (market days) 1	5	10	20	60	125	250
0%	9.25	9.22	9.19	9.15	9.04	8.97	8.95
5%	9.25	9.21	9.17	9.13	9.04	8.98	8.99
10%	9.25	9.21	9.19	9.17	9.07	8.99	9.04
15%	9.20	9.16	9.18	9.14	9.09	9.05	9.13
20%	9.18	9.15	9.16	9.18	9.12	9.13	9.23
25%	9.19	9.19	9.19	9.18	9.20	9.21	9.28
100%	10.21	10.21	10.21	10.21	10.21	10.21	10.21

Panel C: Core – Components

Rebalance Band	Return Difference Look Interval (market days) 1	5	10	20	60	125	250
0%	1.44	0.36	0.24	0.16	0.09	0.06	0.01
5%	0.18	0.12	0.13	0.14	0.09	0.06	0.02
10%	0.20	0.15	0.17	0.10	0.06	0.03	0.05
15%	0.13	0.06	0.06	0.06	0.08	0.04	0.05
20%	0.01	−0.01	0.06	0.01	0.07	0.13	0.02
25%	0.07	0.09	−0.05	−0.03	0.03	−0.14	−0.19
100%	0.15	0.15	0.15	0.15	0.15	0.15	0.15

Rebalance Band	T-Statistic Look Interval (market days) 1	5	10	20	60	125	250
0%	7.12	1.77	1.21	0.82	0.51	0.36	0.04
5%	0.83	0.58	0.62	0.69	0.47	0.36	0.15
10%	0.86	0.68	0.77	0.47	0.33	0.18	0.33
15%	0.56	0.27	0.28	0.27	0.39	0.22	0.31
20%	0.06	−0.03	0.27	0.06	0.34	0.73	0.10
25%	0.34	0.42	−0.24	−0.16	0.13	−0.62	−0.82
100%	0.69	0.69	0.69	0.69	0.69	0.69	0.69

Out-of-Sample Testing

Given that much of the return benefit of the (1, 20%) comes from the covariance term from (2) and thus relies on the autocorrelation structure of the returns, one should be very skeptical about whether the higher returns are robust or simply due to noise in the returns. Table 2 showed that the return difference between the (1, 20%) strategy and a benchmark (60, 10%) strategy is not statistically significant, even in the sample in which the (1, 20%) performs well. It is entirely possible that this strategy got lucky, in which case its performance is unlikely to continue in a different period or for different portfolios. The (1, 20%) strategy already fails the first out-of-sample test in Table 4, which involves only a minor portfolio change that switches one domestic equity index for another.

Table 5

Rebalanced Returns

January 2005-June 30, 2008

Panel A: Annualized Arithmetic Average Net Return

	Arithmetic Average Return Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	3.33	5.15	5.33	5.43	5.47	5.41	5.50
5%	5.52	5.48	5.49	5.49	5.49	5.44	5.46
10%	5.45	5.50	5.50	5.49	5.43	5.42	5.41
15%	5.39	5.39	5.40	5.41	5.38	5.38	5.37
20%	5.38	5.39	5.39	5.39	5.34	5.35	5.30
25%	5.33	5.35	5.35	5.35	5.33	5.30	5.27
100%	5.16	5.16	5.16	5.16	5.16	5.16	5.16

	Standard Deviation Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	8.78	8.76	8.74	8.72	8.66	8.57	8.52
5%	8.75	8.72	8.71	8.69	8.65	8.59	8.61
10%	8.71	8.72	8.69	8.67	8.71	8.67	8.70
15%	8.73	8.72	8.75	8.74	8.75	8.75	8.79
20%	8.79	8.79	8.79	8.77	8.80	8.80	8.88
25%	8.81	8.79	8.80	8.80	8.84	8.88	8.95
100%	9.10	9.10	9.10	9.10	9.10	9.10	9.10

Panel B: Return Benefit, Relative to (60, 10%) Benchmark

	Return Difference Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	-2.10	-0.29	-0.10	-0.01	0.03	-0.03	9.67
5%	0.08	0.04	0.05	0.06	0.05	0.00	9.71
10%	0.02	0.06	0.07	0.06	0.00	-0.02	9.74
15%	-0.04	-0.04	-0.03	-0.03	-0.05	-0.06	9.76
20%	-0.05	-0.05	-0.05	-0.04	-0.09	-0.09	9.74
25%	-0.10	-0.08	-0.09	-0.08	-0.10	-0.14	9.79
100%	-0.28	-0.28	-0.28	-0.28	-0.28	-0.28	9.18

	T-Statistics Look Interval (market days)						
Rebalance Band	**1**	**5**	**10**	**20**	**60**	**125**	**250**
0%	-7.46	-1.05	-0.39	-0.03	0.17	-0.18	0.39
5%	0.35	0.19	0.27	0.30	0.43	0.05	0.19
10%	0.13	0.52	0.61	0.83	N/A	-0.26	-0.24
15%	-0.89	-0.96	-0.67	-0.59	-0.62	-0.52	-0.50
20%	-0.67	-0.55	-0.54	-0.43	-0.69	-0.63	-0.76
25%	-0.76	-0.63	-0.66	-0.61	-0.66	-0.79	-0.75
100%	-0.85	-0.85	-0.85	-0.85	-0.85	-0.85	-0.85

I conduct a second out-of-sample test using data from January 2005 to July 2008. The allocation is the same as in Tables 1-3; and the annualized arithmetic average net returns, averaged over the 12 monthly starts, are displayed in Table 5. The optimal strategies add about 7 basis points over the benchmark (60, 10%) strategy and about 35 basis points over no rebalancing. However, in this sample, the best three strategies are the (1, 5%), the (250, 0%), and the (10, 10%). The previous winner, the (1, 20%) strategy, now ranks only 27th out of 42. Moreover, the best strategies appear almost random, which is what one would expect if return differences across strategies were mainly driven by noise in returns. Panel B, which displays the difference in return relative to the benchmark (60, 10%) strategy, supports this. Except for the (1, 0%) strategy, which has abysmal returns because of frequent, costly trading, none of the strategies has returns that significantly differ from the benchmark strategy.

The returns for the core and component portfolios in this later sample are displayed in Table 6.[7] Once again, the (1, 20%) rebalancing strategy does not produce the highest returns. Comparing the core and component portfolios in Panel C, the core outperformed the component strategies for 9 of the 42 strategies, although, with the exception of the (1, 0%) strategy, these differences are all statistically insignificant. The largest return difference occurred with the (1, 10%) strategy, which gave the component portfolio a 9 basis point advantage over core. However, in the previous period, the return advantage using the (1, 10%) strategy was the opposite, with the core portfolio having a return about 20 basis points higher than the component portfolio. There is simply no empirical evidence to support the idea that breaking asset classes into smaller components can produce reliably higher returns through rebalancing opportunities.

Is There a Rebalancing Return Benefit?

In both of the periods examined so far, the non-rebalanced portfolio has experienced lower returns than any of the rebalanced portfolios. Even if there is no single rebalancing strategy that will produce the highest returns for every portfolio in every period, can this be interpreted as evidence that rebalancing in general produces some return benefit? Again, 11.5 years is a fairly short period of time when dealing with noisy historical returns. Thus, to empirically test whether rebalancing in general has return benefits over not rebalancing, I obtain monthly US stock and bond data from July 1926 to June 2008.

I form a portfolio in July 1926 with 60% in equities and 40% in bonds. The equity portion combines six Fama/French portfolios formed on size and book-to-market, with weights of 12% in each of three large capitalization portfolios (Large Value, Large Neutral, and Large Growth), and 8% in each of three small capitalization portfolios (Small

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7. As in the previous sample, the component weights are determined by their average weights in the year prior to portfolio formation, from January 2004 to December 2004. Specifically, the Russell 1000 Value, the Russell 1000 Growth, the Russell 2000 Value, and the Russell 2000 Growth had weights of 42.30%, 45.75%, 5.00%, and 6.95% respectively. These weights were scaled to sum to 45%.

Table 6

Core vs. Component Strategies

January 2005-June 2008

Panel A: Core Portfolio

Rebalance Band	Net Arithmetic Average Returns Look Interval (market days) 1	5	10	20	60	125	250
0%	4.43	5.86	6.00	6.09	6.12	6.07	6.16
5%	6.17	6.13	6.13	6.14	6.17	6.11	6.14
10%	6.07	6.10	6.10	6.14	6.08	6.06	6.06
15%	6.10	6.06	6.09	6.09	6.04	6.02	6.01
20%	6.08	6.06	6.06	6.07	6.03	5.99	5.95
25%	6.01	6.01	6.01	6.05	6.00	5.97	5.94
100%	5.82	5.82	5.82	5.82	5.82	5.82	5.82

Rebalance Band	Standard Deviation Look Interval (market days) 1	5	10	20	60	125	250
0%	8.01	8.00	7.98	7.96	7.91	7.84	7.80
5%	8.00	7.97	7.97	7.96	7.91	7.89	7.89
10%	8.02	8.01	8.00	7.98	8.02	7.99	8.01
15%	8.00	8.03	8.03	8.03	8.08	8.06	8.10
20%	8.05	8.07	8.06	8.04	8.07	8.09	8.17
25%	8.08	8.08	8.08	8.05	8.11	8.14	8.21
100%	8.38	8.38	8.38	8.38	8.38	8.38	8.38

Panel B: Components Portfolio

Rebalance Band	Net Arithmetic Average Returns Look Interval (market days) 1	5	10	20	60	125	250
0%	2.98	5.61	5.90	6.06	6.14	6.10	6.20
5%	6.17	6.13	6.15	6.15	6.17	6.11	6.16
10%	6.16	6.16	6.18	6.18	6.16	6.12	6.10
15%	6.12	6.11	6.12	6.11	6.06	6.04	6.02
20%	6.07	6.06	6.06	6.05	6.02	6.00	5.96
25%	6.01	6.02	6.02	6.05	6.00	5.97	5.95
100%	5.82	5.82	5.82	5.82	5.82	5.82	5.82

Rebalance Band	Standard Deviation Look Interval (market days) 1	5	10	20	60	125	250
0%	8.12	8.11	8.10	8.07	8.02	7.94	7.91
5%	8.11	8.09	8.08	8.07	8.03	7.99	7.97
10%	8.06	8.06	8.06	8.03	8.05	8.05	8.09
15%	8.10	8.11	8.12	8.13	8.17	8.16	8.20
20%	8.18	8.18	8.19	8.18	8.19	8.20	8.27
25%	8.20	8.20	8.19	8.17	8.23	8.26	8.32
100%	8.49	8.49	8.49	8.49	8.49	8.49	8.49

Panel C: Core – Components

Rebalance Band	Return Difference Look Interval (market days) 1	5	10	20	60	125	250
0%	1.44	0.24	0.10	0.03	-0.02	-0.03	-0.04
5%	0.00	0.00	-0.02	-0.01	0.00	0.00	-0.02
10%	-0.09	-0.06	-0.08	-0.04	-0.08	-0.07	-0.04
15%	-0.02	-0.04	-0.03	-0.02	-0.03	-0.02	-0.01
20%	0.02	0.00	0.00	0.02	0.01	-0.01	-0.01
25%	0.00	-0.01	-0.01	-0.01	0.00	-0.01	-0.01
100%	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01

Rebalance Band	T-Statistic Look Interval (market days) 1	5	10	20	60	125	250
0%	9.61	1.58	0.65	0.20	-0.13	-0.20	-0.25
5%	0.00	-0.01	-0.15	-0.08	-0.02	-0.03	-0.15
10%	-0.61	-0.40	-0.52	-0.29	-0.55	-0.46	-0.29
15%	-0.10	-0.29	-0.19	-0.12	-0.17	-0.10	-0.08
20%	0.10	-0.01	-0.01	0.16	0.06	-0.06	-0.05
25%	-0.03	-0.05	-0.06	-0.04	0.01	-0.04	-0.04
100%	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04

Value, Small Neutral, and Small Growth).[8] The fixed income portion of the portfolio has 10% in one-month US Treasury bills, 10% in five-year US Treasury notes, 10% in long-term government bonds, and 10% in long-term corporate bonds.[9]

If the portfolio is never rebalanced, the allocation drifts from the initial 60/40 combination and becomes increasingly concentrated in stocks, as shown in the top panel of Exhibit 1. The equity allocation increases from 60% in 1926 to 90% by 1955, and reaches 99.8% in 2008. The bottom panel of Exhibit 1 displays the weights in each asset class and shows that even within equities, the portfolio becomes overexposed to the highest-returning asset class. The Small Value portfolio initially had 8% of the portfolio allocation in 1927, but grew to 63% in 2008.

To determine whether rebalancing increases returns, I compare the returns of the non-rebalanced portfolio to one that is simply rebalanced to target annually. Because 82 years is a long time not to rebalance, even for an extremely long-lived and incredibly inattentive investor, I also consider cases where the portfolio is seldom rebalanced, at intervals of 5, 10, or 20 years. Because these rebalancing strategies do not require many trades, the effect of transactions cost would be minimal and are thus ignored in this analysis.

The means and standard deviations for the rebalanced, seldom rebalanced, and never rebalanced portfolios are shown in Table 7. The non-rebalanced portfolio has greater return and greater volatility than the annually rebalanced and seldom rebalanced portfolios. This should come as no surprise, given the significant allocation to stocks in the non-rebalanced portfolio at the end of the period.

Table 7

Summary Statistics of Monthly Returns

July 1926-June 2008

	Rebalancing Frequency (years)				
	1	**5**	**10**	**20**	**Never**
Average Monthly Return	0.88%	0.85%	0.85%	0.89%	1.07%
Standard Deviation	4.04%	3.67%	3.61%	3.72%	4.59%

Clearly, no meaningful comparison of returns can be made without controlling for risk. Fama and French (1993) show that average returns on stocks and bonds are well

8. Kenneth R. French, "US Research Returns Data," in http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research.

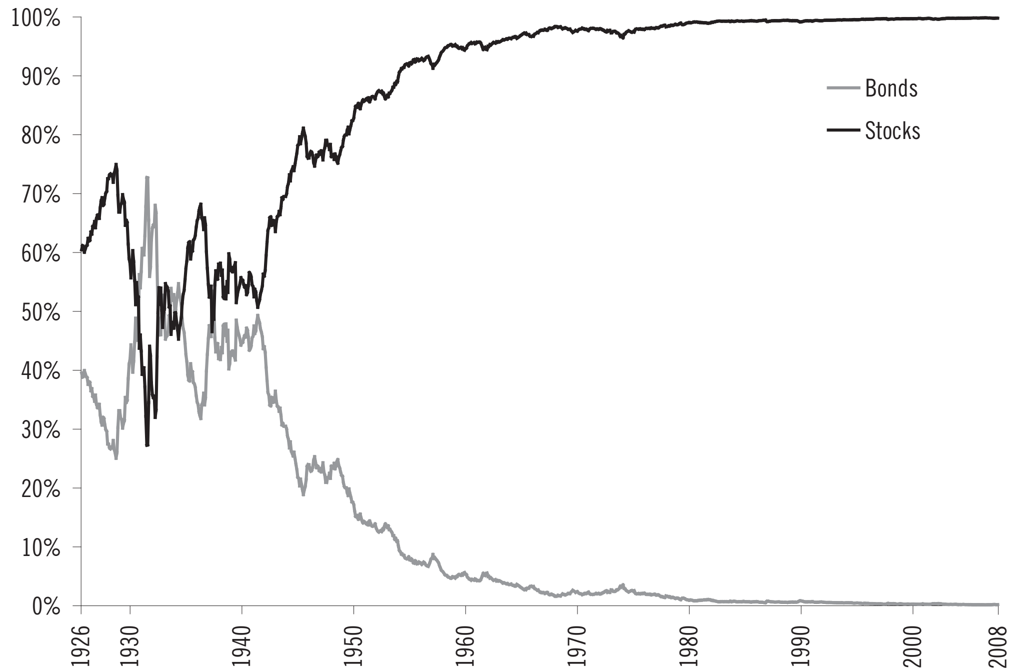
9. US bills, notes, and long-term bonds © *Stocks, Bonds, Bills, and Inflation Yearbook*™, Ibbotson Associates, Chicago (annually updated work by Roger G. Ibbotson and Rex A. Sinquefield).

Exhibit 1

Asset Allocation without Rebalancing

July 1926-June 2008

Stocks and Bonds



Asset Classes

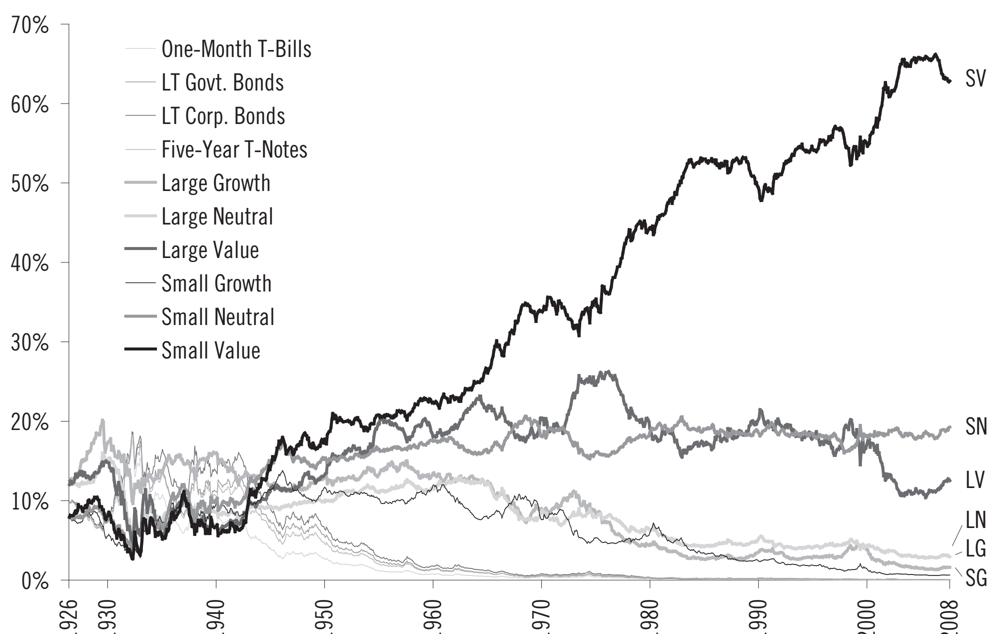
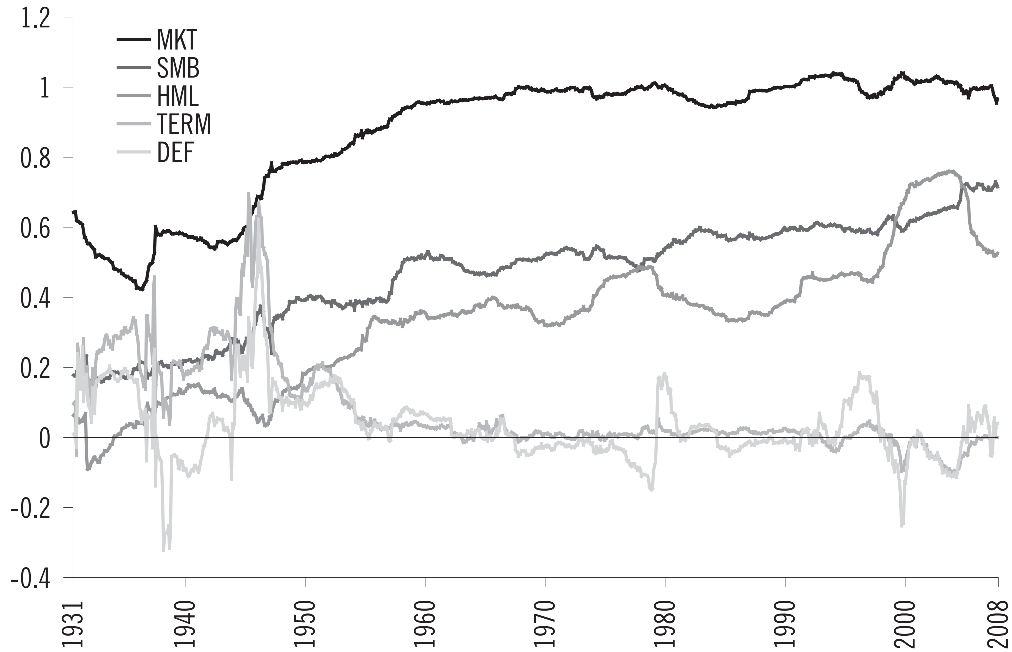


Exhibit 2

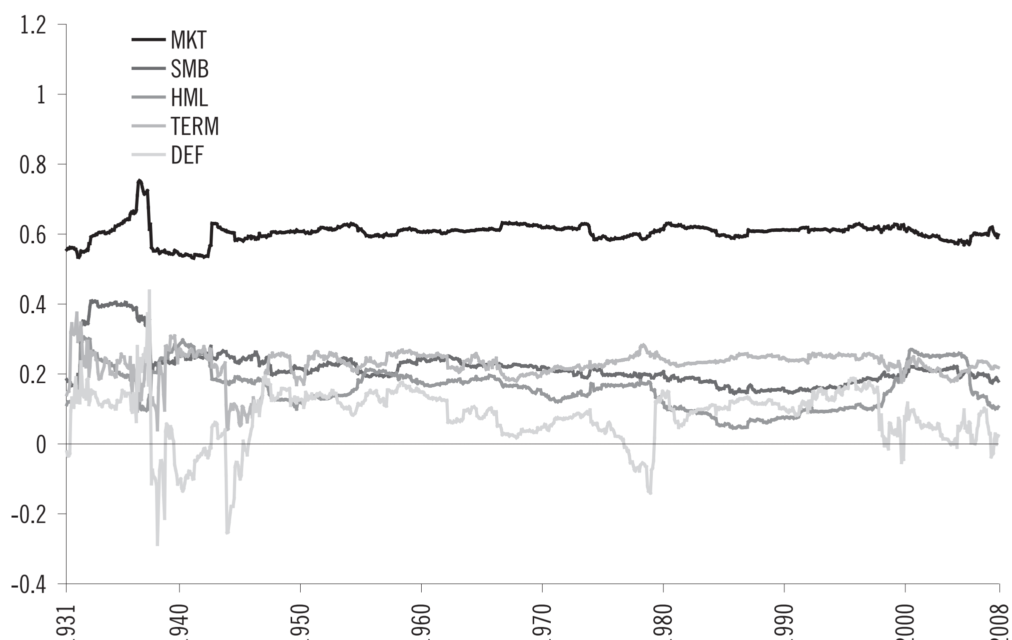
Coefficients from Rolling Five-Factor Regressions

June 1931-June 2008

Non-Rebalanced



Annually Rebalanced



explained by their exposures to five risk factors: the market return in excess of the one-month Treasury bill rate (MKT), the return on small capitalization stocks in excess of the return on large capitalization stocks (SMB), the return on value stocks in excess of the return on growth stocks (HML), the return on long-term government bonds in excess of the one-month Treasury bill rate (TERM), and the return of long-term corporate bonds in excess of the return on long-term government bonds (DEF). Any return that cannot be explained by these five risk factors appears in the alpha term of a five-factor regression.

The factor loadings on the five Fama/French risk factors are plotted for both the non-rebalanced and the annually rebalanced portfolios in Exhibit 2. Because the coefficients in the five-factor regression may not be constant over the entire 82-year sample, I estimate rolling five-factor regressions using the previous 60 months of data. For example, coefficients reported for December 1990 are estimated using monthly data from January 1986 to December 1990. The risk factor exposures display a tendency to trend in a particular direction in the non-rebalanced portfolio, as shown in the top panel of Exhibit 2. The TERM and DEF risk factors mainly explain bond returns, and thus it is no surprise that the exposures to these two factors tend to zero as the share of bonds in the non-rebalanced portfolio gets close to zero. The coefficient on the market factor increases from about 0.6 to 1, echoing the increasing overall share of stocks in the non-rebalanced portfolio. The loadings on SMB and HML also trend upward, reflecting the increased exposures to small and value stocks. In contrast, maintaining the targeted risk exposures, which should be the primary goal of rebalancing, is successfully achieved in the annually rebalanced portfolio. The factor loadings of the rebalanced portfolio, shown in the lower panel of Exhibit 2, are fairly stable over the entire sample.

If rebalancing produces returns in excess of the risks captured by these five factors, the five-factor alphas of the rebalanced portfolio should be larger than the alpha of the portfolios that are seldom or never rebalanced. The differences in alpha, plotted in the upper panel of Exhibit 3, should be positive on average. Aside from some large positive differences in the 1930s, the alphas of the rebalanced portfolio do not appear to be reliably larger than the portfolios rebalanced seldom or never. In fact, looking at the t-statistics for the differences in the lower panel of Exhibit 3, these differences are significantly positive about the same number of times they are significantly negative. A few significant instances, both negative and positive, are exactly what one should expect from random noise given so many observations.

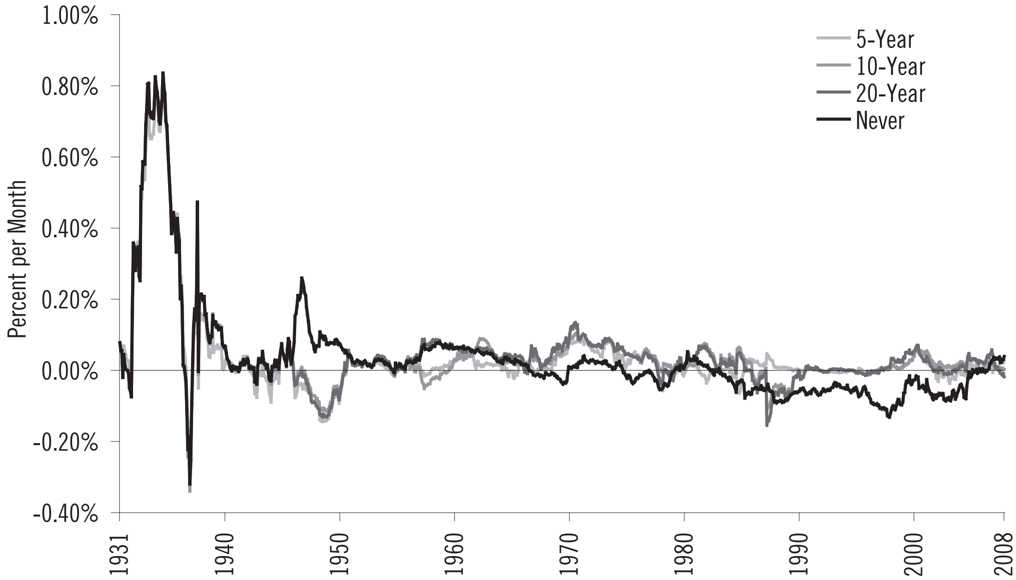
The results from the analysis of 82 years of data highlight the danger when interpreting return differences over short samples. Over short samples, it is possible that rebalanced portfolios can have higher risk-adjusted returns than portfolios rebalanced infrequently or not at all. However, the historical data shows that the reverse also occurs. Over the entire 82-year sample, it is clear that the rebalanced portfolio had no significant advantage over the portfolios rebalanced less frequently. It would be premature to declare the existence of positive rebalancing return benefits using 10 to 20 years of data because the period is too short and the data is too volatile to distinguish higher expected returns from luck.

Exhibit 3

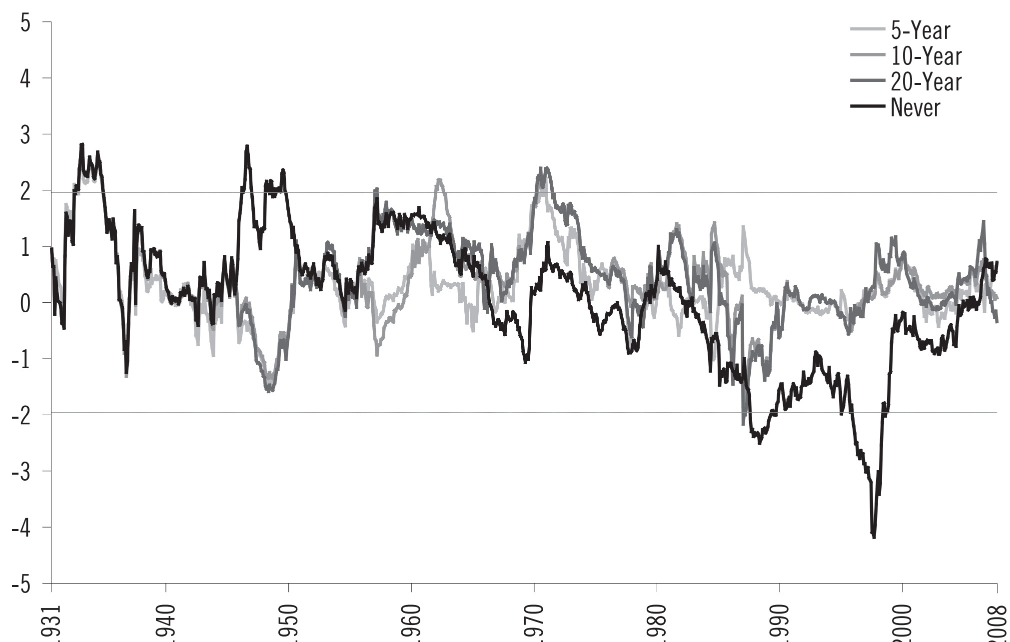
Difference in Alphas from Rolling Five-Factor Regressions

June 1931-June 2008

Annually Rebalanced Alpha – Infrequent or Never Rebalanced Alpha



T-Statistic



Conclusions

While it is true that the details of a rebalancing strategy will affect portfolio returns, trying to predict which strategy will have the highest returns going forward will likely lead one down a path of unproductive data mining. It is fairly easy to search over a number of strategies to find the one that works the best in a given sample and for a particular portfolio. However, finding one that will continue to provide better returns out of sample is a much more difficult if not impossible task.

Aside from avoiding excessive trading, there are no optimal rebalancing rules that will yield the highest returns on all portfolios and in every period. The good news for investors is that without an optimal way to rebalance, the burden of producing returns through optimal rebalancing is lifted. Return generation is again the responsibility of the market, which sets prices to compensate investors for the risks they bear. The primary motivation for rebalancing should not be the pursuit of higher returns, as returns are determined through the asset allocation, not through rebalancing.

The bad news, of course, is that there is no easy one-size-fits-all rebalancing solution. Rebalancing decisions should be driven by the need to maintain an allocation with a risk and return profile appropriate for each investor. The optimal rebalancing strategy will differ for each investor, depending on their unique sensitivities to deviations from the target allocation, transaction frequency, and tax costs.

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