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CONSERVATION LAWS IN A BEHAVIOR OF A COMPLETE MONOPOLIST II

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1. Introduction

Noether theorem [8] concerning with symmetries of the action integral or its generalization (Bessel-Hagen [2]) with those up to divergence plays an effective role for discovering conservation laws from the Lagrangian or the Hamiltonian structures of considering problem. In our previous paper [9], the theorem was carried into a consideration of a complete monopolist who behaves himself to maximize the profit over a period of time $[0, T]$:

$$\int_0^T (xp - C(x))\,dt,$$

where $C(x)$ is a cost function of his demand $x = D(\dot{p}(t), p(t))$ ($\dot{p} = dp/dt$) for producing and selling a single good of price $p(t)$ at time t. However, via the application of a suitable version of Noether theorem to the composite variational principle, the new operative procedure for the laws has been given by Caviglia [3, 5] and after then analyzed with various viewpoints by Mimura and Nôno [6, 7].

In this paper, in contrast with Noether theorem, the procedure can be applied effectively for the maximizing problem of a profit with constant rate ρ:

$$\int_0^T e^{-\rho t}(xp - C(x))\,dt,$$

in which the original Lagrangian $L(\dot{p}, p) = xp - C(x)$ is generalized as

$$\tilde{L}(\dot{p}, p, t) = e^{-\rho t} L(\dot{p}, p). \tag{1}$$

Similarly as in [9], the Lagrangian $L(\dot{p}, p)$ is assumed to be quadratic with respect to $\dot{p}$. It can be reduced to that obtained from the couples of demand and cost functions:

$$\begin{cases} D(\dot{p}, p) = ap + b\dot{p} + c \ \text{ or } \ D(\dot{p}, p) = \dfrac{1 - a\alpha}{\alpha} p - b\dot{p} - \dfrac{\beta + c\alpha}{\alpha}, \\ \\ C(x) = \alpha x^2 + \beta x + \gamma; \end{cases} \tag{2}$$

or

$$(3)\qquad \begin{cases} D(\dot{p}, p) = \dfrac{p}{2\alpha} \pm \sqrt{\omega\dot{p}^2 + \tau\dot{p}p + \kappa}\,, \\ \\ C(x) = \alpha x^2 + \gamma, \end{cases}$$

where $a, b, c, \alpha, \beta, \gamma, \omega, \tau, \kappa$ are all constants and $ab\alpha\omega \neq 0$; in which the couple of the former of D's and C in (2) was given by Allen [1] for the maximizing problem.

2. New derivation of conservation laws

Our discussion for discovering conserved quantities (first integrals) of the Euler-Lagrange equation with given Lagrangian $\tilde{L}(\dot{p}, p, t)$:

$$(4)\qquad \frac{d}{dt}\left(\frac{\partial \tilde{L}}{\partial \dot{p}}\right) - \frac{\partial \tilde{L}}{\partial p} = \ddot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}^2} + \dot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial p} + \frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial t} - \frac{\partial \tilde{L}}{\partial p} = 0$$

will begin with the theorem 6 in [6] (cf. [4], Theorem 3.2; [5], Theorem), which is reduced for the Lagrangian $\tilde{L}(\dot{p}, p, t)$ as follows:

THEOREM 1. *For given Lagrangian $\tilde{L}(\dot{p}, p, t)$, let $\xi_1(\dot{p}, p, t)$ and $\xi_2(\dot{p}, p, t)$ satisfy the equation*

$$(5)\qquad \frac{\partial^2 \tilde{L}}{\partial \dot{p}^2}\frac{d^2\xi}{dt^2} + \frac{d}{dt}\left(\frac{\partial^2 \tilde{L}}{\partial \dot{p}^2}\right)\frac{d\xi}{dt} + \left(\frac{d}{dt}\left(\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial p}\right) - \frac{\partial^2 \tilde{L}}{\partial p^2}\right)\xi = 0$$

on solution to (4). *Then the following conserved quantity Ω of* (4) *is constructed*:

$$(6)\qquad \Omega = \frac{\partial^2 \tilde{L}}{\partial \dot{p}^2}\left(\xi_1\frac{d\xi_2}{dt} - \xi_2\frac{d\xi_1}{dt}\right).$$

Particularly for $\xi = \dot{p}$, the left hand side of (5) is written as

$$\begin{aligned}
&\dddot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}^2} + \ddot{p}\frac{d}{dt}\left(\frac{\partial^2 \tilde{L}}{\partial \dot{p}^2}\right) + \dot{p}\left(\frac{d}{dt}\left(\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial p}\right) - \frac{\partial^2 \tilde{L}}{\partial p^2}\right) \\
&= \frac{d}{dt}\left(\ddot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}^2} + \dot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial p} + \frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial t} - \frac{\partial \tilde{L}}{\partial p}\right) - \frac{d}{dt}\left(\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial t}\right) + \frac{d}{dt}\left(\frac{\partial \tilde{L}}{\partial p}\right) - \ddot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial p} - \dot{p}\frac{\partial^2 \tilde{L}}{\partial p^2} \\
&= \frac{d}{dt}\left(\frac{d}{dt}\left(\frac{\partial \tilde{L}}{\partial \dot{p}}\right) - \frac{\partial \tilde{L}}{\partial p}\right) - \frac{d}{dt}\left(\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial t}\right) + \frac{\partial^2 \tilde{L}}{\partial p\partial t},
\end{aligned}$$

in which, for the Lagrangian of the form (1), the identity follows

$$-\frac{d}{dt}\left(\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial t}\right) + \frac{\partial^2 \tilde{L}}{\partial p\partial t} = \rho\left(\frac{d}{dt}\left(e^{-\rho t}\frac{\partial L}{\partial \dot{p}}\right) - e^{-\rho t}\frac{\partial L}{\partial p}\right)$$

$$= \rho\left(\frac{d}{dt}\left(\frac{\partial \tilde{L}}{\partial \dot{p}}\right) - \frac{\partial \tilde{L}}{\partial p}\right).$$

Therefore $\xi = \dot{p}$ satisfies the equation (5) on solution to (4). So, $\dot{p}$ is substituted for ξ_1 in (6), while ξ_2 is left in (6) as $\xi_2 = \xi$, and then the resulting term $(\partial^2 L/\partial \dot{p}^2) \cdot (d\xi_1/dt) = \ddot{p}\partial^2 L/\partial \dot{p}^2$ is rewritten by (4) to conclude:

THEOREM 2. *For given Lagrangian* $\tilde{L}(\dot{p}, p, t) = e^{-\rho t}L(\dot{p}, p)$, *let* $\xi(\dot{p}, p, t)$ *satisfy the equation* (5) *on solution to* (4). *Then the following conserved quantity of* (4) *is constructed*:

$$\Omega = \dot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}^2}\frac{d\xi}{dt} + \left(\dot{p}\frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial p} + \frac{\partial^2 \tilde{L}}{\partial \dot{p}\partial t} - \frac{\partial \tilde{L}}{\partial p}\right)\xi. \tag{7}$$

Now, in the Lagrangian $\tilde{L}(\dot{p}, p, t) = e^{-\rho t}L(\dot{p}, p)$, assume that $L(\dot{p}, p)$ takes the form

$$L(\dot{p}, p) = k\dot{p}^2 + (rp + s)\dot{p} + lp^2 + mp + n \quad (k, r, s, l, m, n\text{: const.}, \ k \neq 0). \tag{8}$$

Then, in the theorem 2, the equation (5) is reduced to

$$2k\frac{d^2\xi}{dt^2} - 2k\rho\frac{d\xi}{dt} - (2l + r\rho)\xi = 0, \tag{9}$$

and the conserved quantity (7) is also to

$$\Omega = \left(2k\dot{p}\frac{d\xi}{dt} - (2k\rho\dot{p} + (2l + r\rho)p + m + s\rho)\xi\right)e^{-\rho t}. \tag{10}$$

Since the characteristic polynomial of (9) has the roots

$$\frac{\rho}{2} \pm \sqrt{\frac{\rho^2}{4} + \frac{r\rho}{2k} + \frac{l}{k}},$$

by putting

$$\lambda = \sqrt{|\sigma|} \quad \text{where} \quad \sigma = \frac{\rho^2}{4} + \frac{r\rho}{2k} + \frac{l}{k}, \tag{11}$$

the independent solutions of (9) can be determined as

$$\begin{aligned} &\xi_{11} = e^{(\lambda + \rho/2)t}, && \xi_{12} = e^{-(\lambda - \rho/2)t}, && \text{if } \sigma > 0; \\ &\xi_{21} = e^{\rho t/2}\sin \lambda t, && \xi_{22} = e^{\rho t/2}\cos \lambda t, && \text{if } \sigma < 0; \\ &\xi_{31} = e^{\rho t/2}, && \xi_{32} = te^{\rho t/2}, && \text{if } \sigma = 0. \end{aligned}$$

These solutions are carried into (10), according to the case of σ, to have the final conclusion:

THEOREM 3. *For given Lagrangian* $\tilde{L}(\dot{p}, p, t) = e^{-\rho t}L(\dot{p}, p)$ *with* $L(\dot{p}, p)$ *of* (8), *there exist the following pairs of conserved quantities*

$$\begin{cases} \Omega_{11} = (2k\lambda\dot{p} - f)e^{(\lambda-\rho/2)t}, \\ \Omega_{12} = (2k\lambda\dot{p} + f)e^{-(\lambda+\rho/2)t}, \end{cases} \quad \text{if } \sigma > 0;$$

$$\begin{cases} \Omega_{21} = (2k\lambda\dot{p}\cos\lambda t - f\sin\lambda t)e^{-\rho t/2}, \\ \Omega_{22} = (2k\lambda\dot{p}\sin\lambda t + f\cos\lambda t)e^{-\rho t/2}, \end{cases} \quad \text{if } \sigma < 0;$$

$$\begin{cases} \Omega_{31} = fe^{-\rho t/2}, \\ \Omega_{32} = (2k\dot{p} - tf)e^{-\rho t/2}, \end{cases} \quad \text{if } \sigma = 0;$$

where $f = k\rho\dot{p} + (2l + r\rho)p + m + s\rho$. *Moreover, assuming that* $2l + r\rho \neq 0$, $\dot{p}$ *and be eliminated respectively in each pair of the quantities* Ω_{i1} *and* Ω_{i2} $(i = 1, 2, 3)$ *to obtain the trajectories* $p = p(t)$ *of the Euler-Lagrange equation* (4):

$$p = A_1e^{(\lambda+\rho/2)t} + A_2e^{-(\lambda-\rho/2)t} - p_0, \qquad \text{if } \sigma > 0;$$

$$p = \left(A_1\left(\cos\lambda t - \frac{\rho}{2\lambda}\sin\lambda t\right) + A_2\left(\sin\lambda t + \frac{\rho}{2\lambda}\cos\lambda t\right)\right)e^{\rho t/2} - p_0, \qquad \text{if } \sigma < 0;$$

$$p = (A_1t + A_2)e^{\rho t/2} - p_0, \qquad \text{if } \sigma = 0;$$

where A_1 *and* A_2 *are arbitrary constants and* $p_0 = (m + s\rho)/(2l + r\rho)$.

REMARK 1. The first pair in the theorem 3 is rewritten as

$$\begin{cases} k(2\lambda - \rho)\dot{p} - (2l - r\rho)p = \Omega_{11}e^{-(\lambda-\rho/2)t} + m + s\rho, \\ k(2\lambda + \rho)\dot{p} + (2l + r\rho)p = \Omega_{12}e^{(\lambda+\rho/2)t} - m - s\rho, \end{cases}$$

whose coefficients of $\dot{p}$ and p have the determinant

$$\begin{vmatrix} k(2\lambda - \rho) & -2l - r\rho \\ k(2\lambda + \rho) & 2l + r\rho \end{vmatrix} = 4k\lambda(2l + r\rho),$$

and similarly $2k\lambda(2l + r\rho)$ and $-2k(2l + r\rho)$ according respectively to the second and the third ones. So that, since $k \neq 0$ and $\lambda = \sqrt{|\sigma|}$, $\dot{p}$ can be eliminated to determine p in each pair whenever $2l + r\rho \neq 0$.

If $2l + r\rho = 0$ (this turns into the case of $\sigma = (\rho/2)^2 > 0$), since $\lambda = \sqrt{|\sigma|} = |\rho|/2$, according to the signs of $\rho \neq 0$, the quantity Ω_{11} or Ω_{12} in the couple Ω_{1i} $(i = 1, 2)$ is reduced to

$$\Omega_{12} = (2k\rho\dot{p} + m + s\rho)e^{-\rho t}, \qquad \text{if } \rho > 0,$$

$$\Omega_{11} = -(2k\rho\dot{p} + m + s\rho)e^{-\rho t}, \qquad \text{if } \rho < 0,$$

while the partners are $\mp(m+s\rho)$ for $\rho \gtrless 0$. Consequently, whenever $\rho \neq 0$, $\dot{p}$ is written as

$$\dot{p} = Ae^{\rho t} - \frac{m+s\rho}{2k\rho} \qquad (A: \text{const.}),$$

which is integrated to obtain the trajectory

$$p = -\frac{m+s\rho}{2k\rho}t + A_1 e^{\rho t} + A_2 \qquad (A_1, A_2: \text{const.}).$$

Moreover, in addition, if $\rho = 0$, i.e., $l = 0$, the quantity Ω_{32} is reduced to

$$\Omega_{32} = 2k\dot{p} - mt, \text{ i.e., } \dot{p} = \frac{m}{2k}t + \frac{\Omega_{32}}{2k},$$

(while $\Omega_{31} = m$), which determines the trajectory

$$p = \frac{m}{4k}t^2 + A_1 t + A_2 \qquad (A_1, A_2: \text{const.}).$$

REMARK 2. Particularly let $\rho = 0$. Then the pairs of the conserved quantities for $\sigma = l/k > 0$ and $\sigma = l/k < 0$ in the theorem 3 are reduced respectively to

$$\begin{cases} \Omega_{11}^0 = (2k\lambda\dot{p} - 2lp - m)e^{\lambda t}, \\ \Omega_{12}^0 = (2k\lambda\dot{p} + 2lp + m)e^{-\lambda t}, \end{cases} \qquad \text{if } \sigma > 0;$$

$$\begin{cases} \Omega_{21}^0 = 2k\lambda\dot{p}\cos\lambda t - (2lp + m)\sin\lambda t, \\ \Omega_{22}^0 = 2k\lambda\dot{p}\sin\lambda t + (2lp + m)\cos\lambda t, \end{cases} \qquad \text{if } \sigma < 0;$$

which, in view of $l/\lambda = \pm k\lambda (\sigma \gtrless 0)$, lead to the conserved quantities obtained in ([9], Theorem 1) by multiplying $1/(2k\lambda)$.

3. Conservation laws in the maximizing problem

Theorem 3 is now applied to the Lagrangian $\tilde{L}(\dot{p}, p, t) = e^{-\rho t}L(\dot{p}, p)$, in which, by the couple of demand and cost functions of (2) or (3), the Lagrangian $L(\dot{p}, p) = xp - C(x)$ is given as (8) with the respective coefficients (refer to [9], p. 11):

$$\text{for (2):} \begin{cases} k = -b^2\alpha, \ r = b(1 - 2a\alpha), \ s = -b(2c\alpha + \beta), \\ l = a(1 - a\alpha), \ m = c - (2c\alpha + \beta)a, \ n = -c^2\alpha - c\beta - \gamma; \end{cases} \qquad \text{or}$$

$$\text{for (3):} \begin{cases} k = \alpha\omega, \ r = -\alpha\tau, \ s = 0, \\ l = \dfrac{4\alpha^2\kappa - 1}{4\alpha}, \ m = 0, \ n = -\gamma. \end{cases}$$

Under the circumstances, σ of (11) is reduced respectively to

(12) for (2): $$\sigma=\frac{a(a\alpha-1)}{b^2\alpha}+\frac{b\alpha\rho^2+2\rho(2a\alpha-1)}{4b\alpha},\ \text{or}$$

(13) for (3): $$\sigma=\frac{4\alpha^2\kappa-1}{4\alpha^2\omega}+\frac{\rho^2\omega-2\rho\tau}{4\omega}.$$

And, in view of $(2l+r\rho)/(2k)=\sigma-\rho^2/4$ in (11), i.e.,

$$\frac{2l+r\rho}{2k\lambda}=\sqrt{\sigma}-\frac{\rho^2}{4\sqrt{\sigma}},\qquad \text{if } \sigma>0;$$

$$\frac{2l+r\rho}{2k\lambda}=-\sqrt{-\sigma}-\frac{\rho^2}{4\sqrt{-\sigma}},\qquad \text{if } \sigma<0;$$

$$2l+r\rho=-\frac{k\rho^2}{2},\qquad \text{if } \sigma=0;$$

the function f in the theorem 3 is rewritten as

$$\frac{f}{2k\lambda}=\frac{\rho}{2\sqrt{\sigma}}\dot{p}+\left(\sqrt{\sigma}-\frac{\rho^2}{4\sqrt{\sigma}}\right)p+\frac{m+s\rho}{2k\sqrt{\sigma}},\qquad \text{if } \sigma>0;$$

$$\frac{f}{2k\lambda}=\frac{\rho}{2\sqrt{-\sigma}}\dot{p}-\left(\sqrt{-\sigma}+\frac{\rho^2}{4\sqrt{-\sigma}}\right)p+\frac{m+s\rho}{2k\sqrt{-\sigma}},\qquad \text{if } \sigma<0;$$

$$\frac{f}{k}=\rho\dot{p}-\frac{\rho^2}{2}p+\frac{m+s\rho}{k},\qquad \text{if } \sigma=0.$$

Moreover, by putting

$$\Xi=\rho\dot{p}-\frac{\rho^2}{2}p+\frac{m+s\rho}{k},$$

which has the appearance

(14) for (2): $$\Xi=\rho\left(\dot{p}-\frac{\rho}{2}p+\frac{2c\alpha+\beta}{b\alpha}\right)+\frac{(2c\alpha+\beta)a-c}{b^2\alpha},\ \text{or}$$

(15) for (3): $$\Xi=\rho\left(\dot{p}-\frac{\rho}{2}p\right),$$

it follows that

$$\frac{f}{2k\lambda}=\sqrt{\sigma}p+\frac{\Xi}{2\sqrt{\sigma}},\qquad \text{if } \sigma>0;$$

$$\frac{f}{2k\lambda} = -\sqrt{-\sigma}\,p + \frac{\Xi}{2\sqrt{-\sigma}}, \qquad \text{if } \sigma < 0;$$

$$\frac{f}{k} = \Xi, \qquad \text{if } \sigma = 0.$$

These characters are used for the calculation of the conserved quantities $\Omega_{ij}/(2k\lambda)$ for $\sigma \gtrless 0$ and Ω_{3j}/k for $\sigma = 0$ $(i, j = 1, 2)$. Thus the theorem 3 can be reviewed as follows:

THEOREM 4. *Let a complete monopolist have the couple of demand and the cost functions of* (2) *or* (3). *Then, in his behavior of maximizing the profit over a period of time with the Lagrangian* $\tilde{L}(\dot{p}, p, t) = e^{-\rho t}(xp - C(x))$, *there exist the following pairs of conserved quantities*

$$\begin{cases} \Xi_{11} = \left(\dot{p} - \sqrt{\sigma}\,p - \dfrac{\Xi}{2\sqrt{\sigma}}\right)e^{(\sqrt{\sigma}-\rho/2)t}, \\ \Xi_{12} = \left(\dot{p} + \sqrt{\sigma}\,p + \dfrac{\Xi}{2\sqrt{\sigma}}\right)e^{-(\sqrt{\sigma}+\rho/2)t}, \end{cases} \qquad \text{if } \sigma > 0;$$

$$\begin{cases} \Xi_{21} = \left(\dot{p}\cos\sqrt{-\sigma}\,t + \left(\sqrt{-\sigma}\,p - \dfrac{\Xi}{2\sqrt{-\sigma}}\right)\sin\sqrt{-\sigma}\,t\right)e^{-\rho t/2}, \\ \Xi_{22} = \left(\dot{p}\sin\sqrt{-\sigma}\,t - \left(\sqrt{-\sigma}\,p + \dfrac{\Xi}{2\sqrt{-\sigma}}\right)\cos\sqrt{-\sigma}\,t\right)e^{-\rho t/2}, \end{cases} \qquad \text{if } \sigma < 0;$$

$$\begin{cases} \Xi_{31} = \Xi e^{-\rho t/2}, \\ \Xi_{32} = 2\dot{p} - t\Xi e^{-\rho t/2}, \end{cases} \qquad \text{if } \sigma = 0;$$

where σ *and* Ξ *are of the forms* (12) *and* (14) *for* (2), *or of the forms* (13) *and* (15) *for* (3), *respectively. Moreover, assuming* $2a(a\alpha - 1) + b\rho(2a\alpha - 1) \neq 0$ *in* (2) *or* $2\alpha^2(2\kappa - \rho\tau) \neq 1$ *in* (3), *the trajectories* $p = p(t)$ *for the maximizing problem are determined completely as*

$$p = A_1 e^{(\sqrt{\sigma}+\rho/2)t} + A_2 e^{-(\sqrt{\sigma}-\rho/2)t} - p_0, \qquad \text{if } \sigma > 0,$$

$$p = \left(A_1\left(\cos\sqrt{-\sigma}\,t - \frac{\rho}{2\sqrt{-\sigma}}\sin\sqrt{-\sigma}\,t\right) + A_2\left(\sin\sqrt{\sigma}\,t + \frac{\rho}{2\sqrt{-\sigma}}\cos\sqrt{-\sigma}\,t\right)\right)e^{\rho t/2} - p_0, \qquad \text{if } \sigma < 0,$$

$$p = (A_1 t + A_2)e^{\rho t/2} - p_0, \qquad \text{if } \sigma = 0;$$

where p_0 *is the constant*

$$\text{for (2)}: \quad p_0 = \frac{(2c\alpha + \beta)(a + b\rho) - c}{2a(a\alpha - 1) + b\rho(2a\alpha - 1)}, \quad \text{or}$$

$$\text{for (3)}: \quad p_0 = 0.$$

REMARK 3. Consider the case of $2l + r\rho = 0$, i.e., $2a(a\alpha - 1) + b\rho(2a\alpha - 1) = 0$ in (2) or $2\alpha^2(2\kappa - \tau\rho) = 1$ in (3). Then, as seen in the remark 1, the trajectories are determined respectively as

$$\text{for (2)}: \begin{cases} p = -\dfrac{(2c\alpha - \beta)(a + b\rho) - c}{2b^2\alpha\rho} t + A_1 e^{\rho t} + A_2, & \text{if } \rho \neq 0, \\ p = \dfrac{(2c\alpha + \beta)a - c}{4b^2\alpha} t^2 + A_1 t + A_2, & \text{if } \rho = 0, a\alpha = 1; \end{cases} \quad \text{or}$$

$$\text{for (3)}: \begin{cases} p = A_1 e^{\rho t} + A_2, & \text{if } \rho \neq 0, \\ p = A_1 t + A_2, & \text{if } \rho = 0, 4\alpha^2\kappa = 1. \end{cases}$$

REMARK 4. Particularly let $\rho = 0$. Then σ of (12) or (13) is reduced respectively to

$$\sigma = \frac{a(a\alpha - 1)}{b^2\alpha} \quad \text{or} \quad \sigma = \frac{4\alpha^2\kappa - 1}{4\alpha^2\omega},$$

and Ξ of (14) or (15) is also respectively to

$$\Xi = \frac{(2c\alpha + \beta)a - c}{b^2\alpha} \quad \text{or} \quad \Xi = 0.$$

Therefore, in the reduction, the pairs of conserved quantities Ξ_{i1} and Ξ_{i2} $(i = 1, 2)$ in the theorem 3 lead to those obtained in ([9], Theorem 2, in which the denominators of the fractions in Ξ_{21} and Ξ_{22} for the case (i) with $\sigma > 0$ or $\sigma < 0$ should be multiplied by 2, and the minus sign in Ξ_{22} for the case (ii) with $\sigma > 0$ should be replaced with plus sign).

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