

# From Coordination to Double-Crossing: Experiments on the Strategic Behavior of Groups

By Marco Castillo and Greg C. Leo\*

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**Abstract:** We explore the empirical relevance of coalitional deviations in strategic games. To do this, we implement a series of experiments with and without unstructured, nonbinding, preplay communication. The study is based on three-player games. One of the games has a unique strong Nash equilibrium and two coalition-proof Nash equilibria. Two of the games have a unique strong and a coalition-proof Nash equilibrium. We find clear evidence that communication affects play, but not in the way theory predicts. While in games with no communication the strong Nash equilibrium is played most often, in games with communication players move away from either strong or coalition-proof Nash equilibria. The failure of the concepts does not seem to be due to some strategy profile being more attractive *a priori*. Players agree to play a coalitionally dominated equilibrium even when egalitarian Pareto efficient *credible* deviations are available. This does not seem to be the result of a lack of understanding of the games either. Subjects learn to coordinate on self-enforcing equilibria and coalitional deviations. Indeed, we find evidence of double-crossing. Players propose deviations from which they attempt to deviate again with the help of a player not involved in the original deviation. Our results show the importance of coalitional reasoning in strategic games, but also the need to adjust theories to incentive-compatible information transmission.

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<sup>0</sup>\* Castillo: School of Public Policy, Georgia Institute of Technology, 685 Cherry Street, Atlanta, GA 30332, USA, mcastillo@gatech.edu; Leo: School of Economics, Georgia Institute of Technology, 781 Marietta Street, Atlanta, GA 30318, USA, gleo@gatech.edu. We thank, without implication, Ragan Petrie, Lise Vesterlund, Utku Unver and participants at the experimental seminar at Pittsburgh university for helpful comments.

# 1 Introduction

We explore the empirical relevance of coalition-proof type concepts in strategic games with communication. We do this by studying behavior in a series of experiments based on three-player games with and without nonbinding, preplay communication. While there is a long tradition, starting with the work of Aumann (1959) and more recently with the work of Bernheim, Peleg and Whinston (1988), on the strategic importance of coalitional deviations when communication is possible, there is very little field or experimental evidence on the effect that coalitional deviations has in strategic games. Our experiments are an attempt to close that gap.<sup>1,2</sup>

The experiments in this paper are based on three-player games with multiple Nash equilibria. We use three-player games because they allow us to test the importance of coalitional deviations by exploiting conflicts of interests among subgroups of players. A natural starting point to analyze multi-player games with unstructured communication is coalition-proof extensions of Nash equilibrium. These theories investigate what equilibria must be considered self-enforcing if coalitions can form.<sup>3</sup> Because these theories leave the details of communication unmodeled, they tend to have sharper predictions than the theories that model communication itself. The downside of not modelling communication explicitly is that some assumptions must be made on what coalitions can and cannot do. A common assumption is that only subcoalitions of a currently deviating coalition can deviate further. The rationale for this assumption is that information asymmetries would prevent these deviations from occurring.<sup>4</sup>

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<sup>1</sup>Moreno and Wooders (1998) seems to be the only experimental study on coalition-proofness in noncooperative games. They implemented a three-player version of the Matching Pennies game in which two players could gain by correlating their strategies. Because in their experiment the coalitional deviation was self-enforcing, the experiment tested players' capacity to implement correlated strategies through cheap talk.

<sup>2</sup>Early examples of experiments with communication are Cummings's and Harnett's (1969) study the effect of communication and information on multilateral bargaining games and Murnighan's and Roth's (1977) study of the predicted power of cooperative game theory concepts in three-player characteristic form games. More recently, Bolton, Chatterjee and McGuinn (2003) show that communication possibilities can have a large impact in multilateral bargaining.

<sup>3</sup>Different coalition concepts differ on what should be considered a credible deviation. Some examples are Berheim, Peleg and Whinston (1987), Charkravorty and Kahn (1993), Chakravorty and Sharkey (1994), Greenberg (1990, 1992), Moreno and Wooders (1996).

<sup>4</sup>Berheim, Peleg and Whinston (1987) warn of this issue while motivating their

As previously shown in the experimental literature, we find that communication has a large impact on the way games are played. However, our results show that behavior does not always match theoretical predictions. We find that while play converges to the unique strong Nash equilibrium in games without communication, play diverges from it when communication is allowed. This is true even in games where the strong Nash equilibrium coincides with the coalition-proof Nash equilibrium. That is, failure to play the coalition-proof Nash equilibrium is not due to the existence of an equilibrium that is naturally attractive. More importantly, we find that players agree to play an equilibrium that is dominated by an egalitarian and credible coalitional deviation. The evolution of play shows that subjects first learn to play individually self-enforcing agreements and then coalitionally self-enforcing agreements. In the process they learn to implement coalitional deviations that are not predicted by theory: players engage in double-crossing. Since double-crossing requires incentive-compatible information transmission, we provide evidence that players are more sophisticated than assumed by theory.

There are many situations where agreements by groups of players are likely to be important in designing institutions. Small investors face the problem that trading companies may have a stake in the performance of the firms they recommend for stock purchases. Countries imposing commercial sanctions against a particular country face the problem that a third country could weaken the sanctions through trade. Congressional leaders face the problem that legislators might endanger their agenda by engaging in vote trading. Teachers giving take-home exams face the problem that students might cheat, and students considering cheating face the problem that their leaked answers might benefit an unintended person. Public administrators face the problem of collusion among public officials and the public, and legislators worry about designing rules of the division of power that are robust to capture.<sup>5</sup>

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Coalition-proof Nash equilibrium.

<sup>5</sup>The theoretical literature provides several examples of the importance of coalitions on strategic behavior. For instance, Konishi, Breton and Weber (1999) discuss common agency games and Delgado and Moreno (2004) discuss equilibrium selection in oligopoly games. Genicot and Ray (2005) show that the extension of risk-sharing and reciprocity can be severely limited by the possibility of coalitional deviations. Dutta and Mutuswani (1997) discuss the efficiency of networks when groups can coordinate their actions and Konishi and Unver (2005) show relationships between coalition-proof equilibrium and group stability

Previous experimental research shows that preplay communication in two-player games can increase coordination (Crawford, 1998). For instance, preplay communication might increase efficiency in simple coordination games, but it may fail to do so in games where preferences between players are not aligned, as in the Battle of Sexes game (Cooper, DeJong, Forsythe and Ross, 1989). In other situations, risks associated with coordination failure might prevent successful coordination (Clark, Kay and Sefton, 1997; Burton and Sefton, 2000). But, overall, research shows that players try, and many times succeed, in coordinating on efficient equilibria (Charness, 2000; Burton, Loomes and Sefton, 1999). For instance, Blume, DeJong, Kim and Sprinkle (2001) show that coordination is possible even when messages do not have an *a priori* meaning, and Blume and Ortmann (2007) show preplay communication increases efficiency in median and minimum games. Finally, Weber (2006) shows that groups can solve coordination problems by growing slowly. Our experiments give further evidence of players' remarkable ability to coordinate their actions in sophisticated ways.

The next section presents the experiments used in this study and discusses theoretical predictions. The section is followed with a description of the experimental procedure. We then report results from the experiment and conclude.

## 2 Theory and Hypotheses

This section presents the games used in the experiment and discusses the theoretical predictions of coalition-proof type models. All games are kept as similar as possible in an attempt to minimize idiosyncratic treatment effects. The games are presented in Table 1. All games have 3 players: row, column and matrix, and each player has two choices: A or B. Payoffs are arranged in the same order, first number for row, second number for column and third number for matrix.

We use three-player games because this is the minimum environment where coalitional deviations produce results different from coordination on self-enforcing Pareto efficient outcomes.

We use games that are slight variations of each other. For instance, all the games share the same Nash equilibria in pure strategies: (A, A, A), (B, A, B) and (A, B, B). Also, conditional on row playing A, column and

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in many-to-many matching problems.

matrix face an asymmetric coordination game.<sup>6</sup> The payoffs are such that if players were to play a mixed-strategy equilibrium, the predicted frequency of play of equilibrium (A, A, A) is between 23% and 27% and the predicted frequency of play of (A, B, B) is between 20% and 26%. Experimental research on the effects of nonbinding preplay communication in coordination games with asymmetric payoffs (see Camerer (2003), section 7.3) shows that people strive to break the asymmetry created by opposite incentives. One-sided communication tends to ameliorate the coordination problem while two-sided communication leaves it unchanged. There is no *a priori* reason to believe that pre-play communication will favor a particular equilibria over the other in our experiments. However, we note that (A, A, A) is self-enforcing and produces symmetric payoffs while neither (A, B, B) nor the mixed-strategy equilibria do (see below). We will refer to (A, A, A) as the symmetric Nash equilibrium.

A first hypothesis is that communication will produce more equilibrium play. In a sense, this requirement is minimal since players can use more complicated strategies once communication is allowed. In games with non-binding pre-play communication players can coordinate or even correlate their strategies. For instance, players can agree to play in the following manner: each player announces either A or B with equal probability, whatever the announcement, row always plays A. If column's and matrix's announcements coincide they both play A, and if their announcement disagree they both play B. The expected payoff of this agreement, in Game 1, is 2.75 for player row, 1.75 for player column and 1.25 for player matrix. These payoffs are higher than the payoffs resulting from the equilibrium in mixed strategies. Moreover, this agreement is implementable as a Nash equilibrium of the game with pre-play communication. Other agreements would be possible if mediated communication is possible (see Myerson, 1991).

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<sup>6</sup>The probability of playing A for player column in the mixed-strategy equilibrium is 2/5 for Game 1, 4/7 for Game 2, and 2/3 for Game 3. The probability of playing A for player matrix in the mixed-strategy equilibrium is 4/7 for Game 1, 2/5 for Game 2, and 2/5 for Game 3.

**Table 1 - Experimental Treatments**

Game 1					
A			B		
	A	B		A	B
A	$\frac{3}{2}, \frac{3}{2}, \frac{3}{2}$	0,0,0	A	0,0,0	4,2,1
B	0,0,0	0,0,0	B	0,4,0	2,2,2

Game 2					
A			B		
	A	B		A	B
A	$\frac{3}{2}, \frac{3}{2}, \frac{3}{2}$	0,0,0	A	0,0,0	4,1,2
B	0,0,0	2,2,2	B	0,0,0	0,0,4

Game 3					
A			B		
	A	B		A	B
A	$\frac{3}{2}, \frac{3}{2}, \frac{3}{2}$	0,0,0	A	0,0,0	4,1,3
B	0,0,0	2,2,2	B	0,0,0	0,0,4

A second hypothesis is that players will coordinate on equilibria that are also robust to deviations by coalitions (Aumann, 1959). For instance, while (B, A, B) is a Nash equilibrium of all the games, it can be deviated from by coalitions of players. In Game 1, row and matrix can switch to A instead and secure  $\frac{3}{2}$  for each. In Game 2 and Game 3, this equilibrium can also be challenged by the coalition of all players. The same argument can be used to argue that the mixed-strategy equilibria are not robust to deviations of coalitions. Indeed, in all games, the mixed strategy equilibria give strictly less to column and matrix than either the symmetric Nash equilibrium or (A, B, B). These players do better by compromising. Finally, note that the symmetric Nash equilibrium is not robust to deviations by coalitions either. In game 1, all players can agree to play (B, B, B) instead, and in Game 2 and Game 3, they can agree to play (B, B, A). These deviations strictly improve the payoffs of all players. We will refer to this alternative as the Pareto efficient allocation.<sup>7</sup> Also, since all games have a unique strong Nash

<sup>7</sup>The strong Nash equilibrium is also an efficient allocation. The abuse of terminology is done to ease notation.

equilibrium, we will refer to (A, B, B) as such.

The process of elimination of equilibria just described defines Aumann (1959) strong Nash equilibrium (SNE). SNE extends Nash equilibrium to the case where coalitions can implement deviations. SNE is similar to the core concept of cooperative game theory (Greenberg, 1992) and fails to exist for many games of economic interest (e.g., games with a Prisoner's Dilemma structure).

The main criticism of SNE is that it considers deviations that are not immune to further deviations. Take the case of a deviation from the symmetric Nash equilibrium to the Pareto efficient allocation in Game 1. It is clear that either row or column will deviate to (B, A, B) or (A, B, B) making player matrix worse-off. Therefore, the original deviation from the symmetric Nash equilibrium is not credible in this game and player matrix will be unwise to follow it.

This criticism is summarized in the coalition-proof Nash equilibrium concept (Bernheim et al., 1987). An agreement is coalition-proof Nash equilibrium (CPNE) if it is immune to *credible* deviations by coalitions. The concept is nested. Taking the actions of others as given, a player's best response is always credible. Two players' deviations are credible if they are Nash equilibria of the game resulting from keeping the choices of the non-deviating players fixed. So, in three-player games, an agreement is CPNE if it is immune to credible deviations and there is no other immune agreement that makes all three players better off.

In Game 1, there are two CPNE: the symmetric Nash equilibrium and the strong Nash equilibrium.

In Game 2 and Game 3, the unique CPNE is the strong Nash equilibrium.<sup>8</sup>

The symmetric Nash equilibrium is no longer CPNE in these games because a deviation to the Pareto efficient allocation is now credible. This is so because row and column do not need matrix to implement it. If matrix does not know that the deviation has taken place he cannot deviate to (B, B, B). More importantly, neither row nor column have an incentive to further deviate from the Pareto efficient allocation once they reach it.<sup>9</sup> A third hypothesis is then that subjects will play according to CPNE: the symmetric Nash equilibrium should not be played in Game 2 and Game 3.

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<sup>8</sup>All strong Nash equilibria are coalition proof Nash equilibria.

<sup>9</sup>Coalition proof equilibrium (CPE:Moreno and Wooders, 1996) extends CPNE to the case in which correlated strategies are available. CPE also predicts that only (A,B,B) will survive in Game 2 and Game 3.

The nestedness of coalitional deviations insures that all deviators share knowledge of the deviations that have already taken place. However, this implicitly rules out the possibility that players might transmit truthful information about other players' deviations. I.e., CPNE rules out double-crossing.

Game 2 and game 3 are examples of the conflict between SNE and CPNE and the literature on games with cheap talk (Crawford, 2003). That is, there are coalitional deviations that require that some players (those not involved in the deviation) believe a message that is untruthful. For instance, in Game 2 and Game 3, a deviation from the symmetric Nash equilibrium to the Pareto efficient allocation requires that matrix implicitly agree to not play his best-response to this deviation. In other words, these theories seem to assume too little rationality.<sup>10</sup>

There is a second reason why the recommendation of CPNE for Game 2 and Game 3 is not entirely convincing. It depends critically on the assumption that only the members of a deviating coalition can deviate further. However, row does have an incentive to deviate further with matrix's help. That is, row and matrix have an incentive to double-cross column.

Why would row try to deviate from the Pareto efficient allocation? And, why would matrix cooperate if he cannot verify that a previous deviation has occurred? Row would like to reveal that the deviation has occurred because, once column has committed to play B, the remaining game between row and matrix has a unique Nash equilibrium, the strong Nash equilibrium. And this equilibrium gives row his highest possible payoff in Game 2 and Game 3.

Row doesn't have an incentive to lie about the occurrence of this deviation. Suppose, on the contrary, that row convinces matrix that column is playing B when he is not. As mentioned above, in the remaining game between row and matrix, matrix has a dominant strategy, B. That is, matrix will likely play B. But, if row was lying about column playing B, and matrix believes this lie and acts accordingly, his lie will cause everyone to earn nothing. If matrix understands row's incentive, he will likely take row's announcement seriously.

Given the possibility of double-crossing, column should consider that a deviation from the symmetric Nash equilibrium to the Pareto efficient allo-

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<sup>10</sup>To be clear, we are not criticizing equilibrium theories, which define stability in terms of absence of profitable deviations, we are pointing out that such a deviation is not a Nash equilibrium of the game with an added round of costless preplay communication.



cation is not profitable.<sup>11</sup> We hypothesize that double-crossing takes place making (A, A, A) robust to deviations by coalitions.

In summary, strong Nash equilibrium predicts that in all games only strategy profile (A, B, B) will be selected. Coalition-proof equilibrium, on the other hand, only predicts that strategy profile (A, B, B) will be selected in Game 2 and Game 3. In Game 1, Coalition-proof Nash equilibrium does not rule out the selection of strategy profile (A, A, A). Finally, strategy profile (A, A, A) is not ruled out in either Game 2 or Game 3 if subjects are able to coordinate on double-crossing.

### 3 Experimental Design

Experimental sessions of the three games described in the previous section with and without communication were implemented at the University of Wisconsin-Madison and at the Georgia Institute of Technology. A total of 90 students participated in four sessions consisting of 21 or 24 participants in Wisconsin-Madison (Game 1 and Game 2 with communication). Except for session 1 of Game 1, which consisted of 10 rounds, all the sessions consisted of 13 rounds of play. All the participants were told that they were going to play 10 to 15 rounds of the game as time permitted.<sup>12</sup> A total of 180 students participated in 10 sessions consisting of 18 participants at Georgia Tech. All sessions consisted of 15 rounds of play and all participants were told that they were going to play 10 to 20 round of the game as time permitted.<sup>13</sup>

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<sup>11</sup>To our knowledge, the only concepts that extend CPNE to the case in which these deviations are possible are Chakravorty and Sharkey's (1993) Consistent coalition proof equilibrium (CCPE) and Chakravorty and Kahn's (1994) Universal coalition proof equilibrium (UCPE). For our purposes, only UCPE captures the logic of our argument.

Universal coalition proof suggests that after a deviation has occurred, some of the deviators could invite some new players to deviate further. The new players would play along provided that any move that could harm them would harm at least one of the original deviators. This condition would give new players peace of mind that they are not been deceived. However, (A, A, A) is not UCPE for Game 2. The reason being that at (B, B, A) there is not a deviation that makes both player row and player column strictly better off. (A, A, A) is a UCPE in Game 3.

<sup>12</sup>Instructions can be found at [www.prism.gatech.edu/~mc338/game3no.pdf](http://www.prism.gatech.edu/~mc338/game3no.pdf), [www.prism.gatech.edu/~mc338/game3comm.pdf](http://www.prism.gatech.edu/~mc338/game3comm.pdf).

<sup>13</sup>The choice of the last round was made to prevent last round effects. A random stopping rule could have been used, but our method insure comparable session sizes. We did not find any evidence that subjects anticipated the end of a session.

So, in total there are 270 subjects. Each experimental session lasted about an hour and a half, and each subject received an average of \$20.35 (std = \$13.29), plus a \$6 show-up fee.

Each round of play in the games with communication consisted of three stages, a communication stage that lasted at most three minutes, a decision-making stage and a payoff-action-review stage without time constraints.<sup>14</sup> At the beginning of the experiment, each subject was randomly assigned a color (Red, Blue or Green), corresponding to the role of row, column, and matrix. Each player kept the same color/role for the entire experiment. In each round of play, groups consisting of one player of each color were randomly formed. We made sure that no exact group of three players was ever in sequence. The same procedure was followed in the sessions without communication with the obvious exception that the communication stage was removed.

In sessions with communication, experimental instructions were read out loud. After all participants finished reading the instructions, they faced a screen as in Figure 1 in which they were informed which color/role they had been assigned. The panels in the screen changed to signal the change of the stage of the game. For instance, the upper left window changes only to show which round of play was being played. The payoff matrix and subject's role were always visible. Payoffs were presented in the corresponding color to make the reading of the matrix easier. Subjects were informed of their color only when the game started. Indeed, the experimenter did not know ahead of time which role each person was going to be assigned. The upper right window changed with each stage of the game. In Figure 1 the window corresponding to the communication-stage is shown. This window changed after three minutes to a menu in which subjects were asked to make their decisions. After all subjects had submitted their decisions in a round, the upper right window changed again to show the outcome of that round, including the decisions and payoffs for each player. After all players were done reviewing the results of the rounds and payoffs, subjects were regrouped and a new communication stage started a new round of play. The bottom panel in Figure 1 showed the messages sent by other players. Messages could either be sent publicly to the other two players or be sent privately to only one of them.

In the no-communication sessions, all the procedures were the same except

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<sup>14</sup>The sessions in Madison had a time constraint for the review of payoff stage that lasted 20 seconds. The constraint was never binding.

that subjects did not have three minutes to send messages. Also, in Figure 1, the right-hand panel was only a decision-making screen.<sup>15</sup>

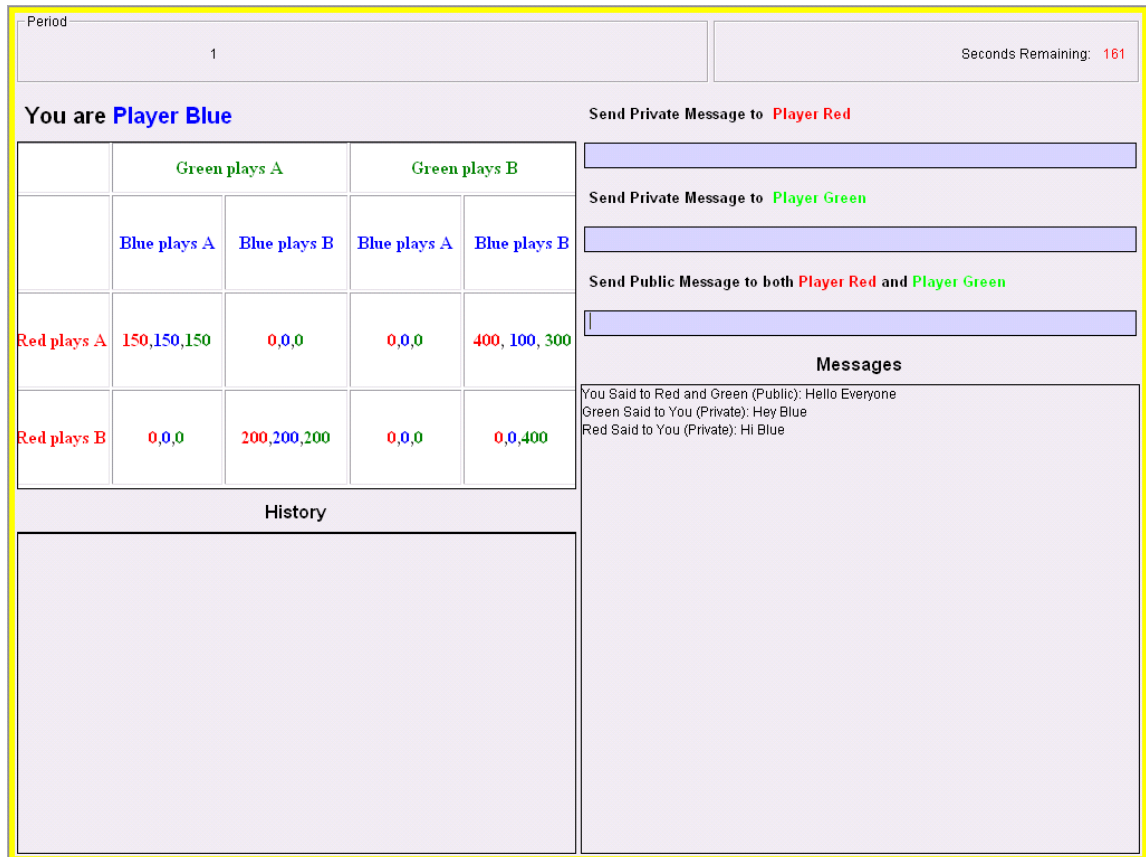


Figure 1. Experiment Screen for the Communication Stage

## 4 Results

### 4.1 Basic Results - No Communication

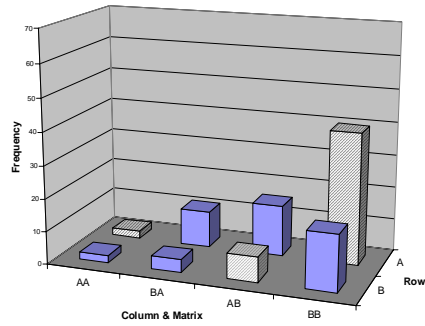
This section presents a general overview of play in the games without communication. Extra information and statistical tests are presented in the appendix. The graphs in Figure 2 present the frequency with which each strategy profile was chosen. The bars in the back correspond to row choice

<sup>15</sup>All the experiments used z-tree (Fischbacher, 2007).

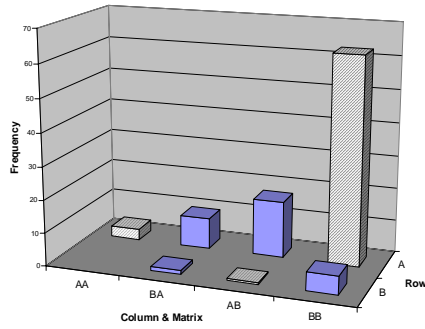
of A and the front bars correspond to row choice of B. The leftward 4 bars correspond to matrix choice of A and the remaining 4 rightward bars correspond to matrix choice of B. The bars named AA and AB correspond to column choice of A and the bars named BA and BB correspond to column choice of B. The graphs reproduce the presentation of the game matrix in Table 1 to facilitate the presentation.

The first observation is that, in the absence of communication, subjects tend to play the strong Nash equilibrium of the games. This fact is stronger in Game 2 and Game 3. We find that in all the games matrix rarely chooses to play A and that row strongly prefers to play A also. This results therefore shows that subjects do find these games similar. Interestingly, strong Nash equilibrium is less attractive when an alternative coalition-proof Nash equilibrium is available (Game 1). We should remark that some rows play B in Game 1. In this game, row has a weakly dominant strategy, A. Statistical tests show that the play in Game 2 and Game 3 without communication is not different, but that play in Game 1 is different to Game 2 and Game 3 combined.

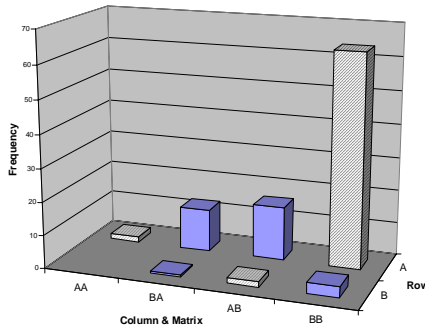
The received wisdom on coordination games with asymmetric payoffs, like the Battle of the Sexes game (Camerer, 2003), is that subjects strive to break the asymmetry inherent in these games. Without communication, subjects are relatively successful in coordinating play on an asymmetric outcome.



a. Game 1 - All Rounds



b. Game 2 - All Rounds



c. Game 3 - All Rounds

Figure 2. Distribution of Play - No Communication

## 4.2 Basic Results - Communication

This section presents a general overview of play in the games with communication. Extra information and statistical tests are presented in the appendix.

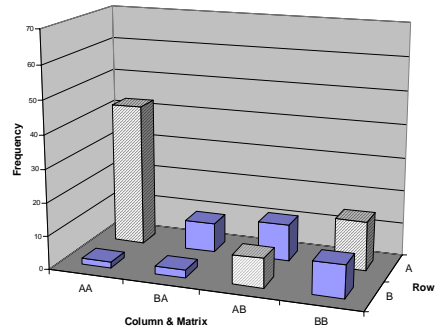
The graphs in Figure 3 present the frequency with which each strategy profile was chosen. The graphs follows the description given in the previous section.

The first observation is that communication affects play significantly. The distribution of play is different in the sessions with and without communication for every one of the games. This result gives us confidence that preplay communication has an important effect on the way games are played. That communication affects play is to be expected given previous experimental evidence.<sup>16</sup>

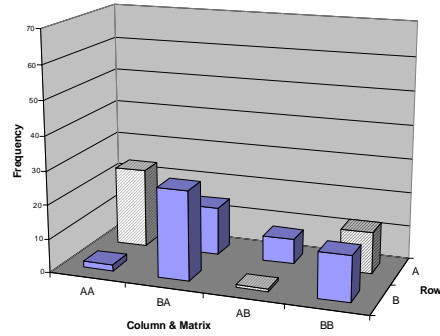
In the absence of communication, the symmetric Nash equilibrium is rarely played. But with communication, it is selected frequently. Analysis of the evolution of play also shows that the symmetric Nash equilibrium is played more frequently as Game 1 progresses. In Game 2 and Game 3, its frequency remains above 30% across all rounds of play.

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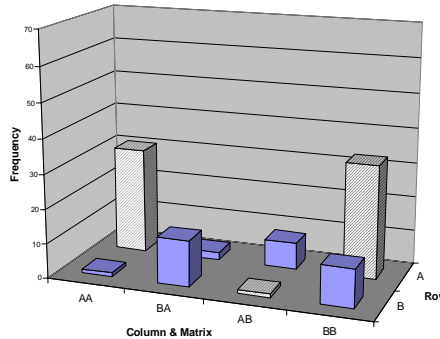
<sup>16</sup>See Kagel and Roth (1995, chapter 3.2), Camerer (2003, chapter 7.2), Crawford (1998), Blume, DeJong, Kim and Sprinkle (2001), Clark, Kay and Sefton (2001), Charness (2000), and Blume and Ortmann (2007) among many.



a. Game 1 - All Rounds



b. Game 2 - All Rounds



c. Game 3 - All Rounds

Figure 3. Distribution of Play - Communication

The increased frequency of the symmetric Nash equilibrium in Game 1 can be understood by the fact that it is a CPNE of the game. However, this is not true in Game 2 and Game 3. Row and column have a credible deviation to the Pareto efficient allocation. If matrix plays A, row and column face a

simple coordination game. Most experimental evidence suggests that subjects have little difficulty in coordinating on payoff dominant Nash equilibria that are also risk dominant when communication is available. The recurrence of the symmetric Nash equilibrium is even more puzzling because a deviation to the Pareto efficient allocation benefits all players equally. Fairness considerations cannot therefore explain the recurrence of the symmetric Nash equilibrium. Moreover, Figure 3 shows that an increase on the frequency of play of the strong Nash equilibrium *does not* diminish the frequency of play of the symmetric Nash equilibrium.

Figure 2 and Figure 3 together present the puzzling result that subjects have an easier time coordinating on a strong Nash equilibrium when communication is not available than when it is. This result cannot be attributed to saliency of the symmetric Nash equilibrium alone.

The next sections analyze the evolution of messages as a way to assess whether this result is due to lack of coalitional reasoning or due to the lack of credibility and/or sustainability of coalitional deviations.

### 4.3 The Distribution of Messages

Table 2 shows the distribution of messages sent by a player to the other two partners of play. Messages have been coded from the actual dialogue preceding play. We maintain a very conservative classification of messages, in the sense that a message was classified as A only if it was evident that a player was promising to play strategy A. Non-committal messages or silence were coded as “other.” The convention to construct this table is as follows: for row, message AA means that he promised to column and matrix to play A. Message AB would mean that he promised to play A to column and promised to play B to matrix. For column, the first letter denotes the promise made to row and the second letter denotes the promise made to matrix. For matrix, the first letter denotes the promise made to row and the second letter denotes the promise made to column. For example, Table 4 says that in the last rounds of Game 2 row, column and matrix sent messages BA 24%, 23%, and 7% of the time.

The first thing to observe in Table 2 is that the conversation among players was different across games. In the earlier rounds of play, and with the exception of Game 3, most subjects used messages that did not distinguish the recipient. For instance, column and row most frequently used message BB and matrix used message BB in Game 1 and AA in Game 2 and Game 3.



This suggests that subjects were striving to coordinate on a strategy profile that was acceptable to all. The second observation is that dialogue changes across games as the experiment progresses. In Game 1 players attempt more frequently to coordinate on the symmetric Nash equilibrium. In Game 2 and Game 3, subjects start using messages that are customized to different recipients. This is clear evidence that subjects in Game 2 and Game 3 understood the existence of coalitional deviations.

**Table 2**  
**Distribution of Messages Sent**  
**Game 1**

	Rounds 1-7				Rounds 8-10(13)		
	Row	Column	Matrix		Row	Column	Matrix
AA	25.71	15.24	32.38	AA	71.21	54.55	71.21
AB	0.95	0.00	0.00	AB	0.00	0.00	0.00
BA	0.00	0.00	0.00	BA	0.00	0.00	0.00
BB	60.00	73.33	57.14	BB	21.21	40.91	15.15
Other <sup>a</sup>	13.33	11.43	10.48	Other <sup>a</sup>	7.58	4.55	13.64

Game 2

	Rounds 1-7				Rounds 8-13(15)		
	Row	Column	Matrix		Row	Column	Matrix
AA	20.63	20.11	84.13	AA	40.32	39.78	81.18
AB	1.06	1.06	0.00	AB	1.08	0.00	0.00
BA	6.35	1.59	3.70	BA	24.19	22.58	6.99
BB	59.79	67.20	4.76	BB	24.19	30.65	9.14
Other <sup>a</sup>	12.17	10.05	7.41	Other <sup>a</sup>	10.22	6.99	2.69

Game 3

	Rounds 1-7				Rounds 8-15		
	Row	Column	Matrix		Row	Column	Matrix
AA	29.76	25.00	63.10	AA	61.46	47.92	50.00
AB	1.19	1.19	0.00	AB	1.04	0.00	0.00
BA	17.86	0.00	19.05	BA	26.04	19.79	12.50
BB	42.86	67.86	10.71	BB	3.13	31.25	33.33
Other <sup>a</sup>	8.33	5.95	7.14	Other <sup>a</sup>	8.33	1.04	4.17

<sup>a</sup> Includes no messages or messages to one player only.

The difference in play across games reported in Figure 2 and Figure 3 might be an expression of differences in initial attempts to coordinate on an equilibrium. For instance, the relatively more frequent play of the symmetric Nash equilibrium in Game 1 might be due to early failure to coordinate on the Pareto efficient allocation and not due to coalitional reasoning. The evidence suggests otherwise. There is no evidence that subjects were relatively more adept at coordinating on the symmetric Nash equilibrium or failing to coordinate on the Pareto efficient allocation in Game 1 than in other games (more on this later).

Before looking at the joint distribution of messages and play, we present two examples of the dialogues that took place during the games. Both examples are clear evidence that subjects learn to use the incentives of the game to their advantage. The examples belong to Game 3, but similar examples are also found in Game 2. Table 4 shows an example of a successful coordination by row and column on the play of the Pareto efficient allocation. As the dialogue shows, row and column make an effort to convince matrix that they will play A. Importantly, matrix recognizes that row and column have an incentive to coordinate on the Pareto efficient allocation once the symmetric Nash equilibrium has been agreed. He also recognizes that enough confidence on a deviation to the Pareto efficient allocation will prompt further deviations.

Table 4. Transcribed Dialogue (*extract*) - Example 1

Round 9, Game 3, played (B, B, A), payoff (200, 200, 200)

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*Row (To All)*: what should we do?

*Column (To All)*: a a a

*Row (To All)*: sounds good to me

*Column (To All)*: so matrix doesn't screw us over

*Matrix (To All)*: alright, straight up, im picking A, because otherwise somebody goes crappy and i end up with 0 somehow

*Row (To All)*: I hear that I have already been screwed over by matrix

*Row (To Column)*: hey why don't we pick b so that we all end up making 200each... as well as the matrix player

*Column (To Row)*: are you sure you trust matrix?

*Row (To Column)*: what do you want to do

*Column (To Row)*: I've been screwed over that way before... I think we should stick to a

*Row (To Column)*: I mean if he thinks that we are picking a he doesn't get paid unless he pick a...see so don't let him know that he COULD screw us over

*Row (To All)*: im going with a as well

*Row (To Column)*: im really picking B

*Matrix (To All)*: have you guys decided? don't tell me if its 200, so then I can't screw you over

---

Table 5 presents an example of double-crossing. Row and column attempt to deviate from the symmetric Nash equilibrium to the Pareto efficient allocation by convincing matrix that they both will play A. Row realizes that he can convince matrix that such a deviation has taken place and the both can further deviate from the Pareto efficient allocation to the strong Nash equilibrium. This particular group succeed in implementing the unique coalition-proof Nash equilibrium by means of deception.

Table 5. Transcribed Dialogue (*extract*) - Example 2

Round 14, Game 3, play (A, B, B), payoffs (400, 100, 300)

---

*Row (To All)*:150s?

*Column (To All)*: sure

*Column (To Row)*: Row, if we make green think we're going for 150, we can both choose B and get 200

*Row (To Column)*: I was going to tell u that too

*Row (To All)*: 150s

*Matrix (To All)*: Fine, 150s

*Row (To Column)*: so we are going for 200s? Rite

*Row (To Matrix)*: hey matrix

*Matrix (To Row)*: Yeah, whats up

*Column (To Row)*: make matrix think it's 150 though

*Row (To Matrix)*: column is actually going for 200 200 200... so play 413...but don't tell him

*Matrix (To Row)*: Gotcha

*Column (To All)*: 150 ensures no cheating

*Row (To Column)*: 222 nice...

---

#### 4.4 Agreements, Deviations and Double-Crossing

This section analyzes the distribution of play given agreements (or lack thereof) and the evolution of coalitional deviations. Table 6 and Table 7 present the strategy profiles implemented by row, column and matrix (AAA, AAB, etc.) given messages sent. Table 6 presents data from early rounds and Table 7 presents data from round 8 on. The convention to read the tables is as follows: message "AB,AB,AA" means that row announced the play of A to column and the play of B to matrix, column announced the play of A to row and the play of B to matrix, and matrix announced the play of A to both row and column. The classifications are not exclusive. For instance, message "AB,AB,." includes "AB,AB,AA", "AB,AB,BA" and "AB,AB,BB".<sup>17</sup> Table 6 and Table 7 provide the total number of groups in parentheses to facilitate the analysis. Each separate panel within a table represents a game.

Table 6 shows that Nash equilibria can be self-enforcing agreements.

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<sup>17</sup>"O" denotes messages that are neither "A" nor "B".

Across all experiments, agreements to play A by all players is frequently implemented. For instance, in Game 1, 8 out of 9 agreements to play the symmetric Nash equilibrium were implemented in the first 7 rounds. This agreement was successfully implemented 17 out of 20 times in Game 2 and 14 out of 15 times in Game 3. While the agreement to play the strong Nash equilibrium was rarely observed, it was always followed when announced in Game 3.

Table 6 also shows that agreements on the Pareto efficient allocation are likely to fail. Only 11 out of 51 attempts are successful in Game 1, 42 out 99 in Game 2, and 10 out 33 in Game 3. The failures to implement the Pareto efficient allocation in Game 2 and Game 3 are due to deviations by matrix, but also by deviations by row (25 out of 99 times in Game 2 and 6 out of 33 in Game 3). This indicates that row anticipated a deviation by matrix. This also shows that row and matrix both have incentives to deviate from the Pareto efficient allocation. In early rounds of play subjects seem mainly to try to learn what agreements are individually self-enforcing.

**Table 6. Frequency of Play Given Agreement - Rounds 1 - 7**

<b>Game 1 (Groups = 105)</b>									
<b>Agreement/Play</b>	AAA	AAB	ABA	ABB	BAA	BAB	BBA	BBB	Total
AA,AA,AA	8	0	0	0	1	0	0	0	9
AA,BB,BB	0	1	2	0	0	0	0	0	3
BB,BB,BB	1	10	2	13	0	14	0	11	51
AA..;OO..	24	4	5	2	2	0	0	2	39
Other	20	5	7	5	1	0	2	2	42

<b>Game 2 (Groups = 189)</b>									
<b>Agreement/Play</b>	AAA	AAB	ABA	ABB	BAA	BAB	BBA	BBB	Total
AA,AA,AA	17	1	2	0	0	0	0	0	20
AA,BB,BB	0	1	0	1	0	0	0	0	2
BB,BB,AA	1	1	14	9	1	0	42	31	99
AA..;OO..	25	8	12	6	0	3	3	3	60
BA,BA,AA	0	0	0	0	0	0	2	0	2
BA,BA,.	0	0	0	0	0	0	2	0	2
Other	2	3	6	5	3	0	3	3	25

<b>Game 3 (Groups = 84)</b>									
<b>Agreement/Play</b>	AAA	AAB	ABA	ABB	BAA	BAB	BBA	BBB	Total
AA,AA,AA	14	1	0	0	0	0	0	0	15
AA,BB,BB	0	0	0	5	0	0	0	0	5
BB,BB,AA	0	0	3	3	1	1	10	15	33
AA,..;OO..	18	5	0	5	1	0	1	0	30
BA,BA,AA	0	0	0	0	0	0	0	0	0
BA,BA,.	0	0	0	0	0	0	0	0	0
Other	1	1	1	12	0	0	0	1	16

Table 7 presents the joint distribution of agreements and play for the last rounds of the experiment. There is an increase of Nash equilibrium play as games progress. In all games, agreements on either the symmetric Nash equilibrium or the strong Nash equilibrium are frequently followed. Proposals to implement the Pareto efficient allocation disappear almost completely in Game 1 and Game 3. The only exception is Game 2, where a significant

proportion of subjects agree to implement the Pareto efficient allocation. This result is almost completely explained by only 1 of the 4 sessions of Game 2.

We should note that the small change in payoffs introduced in Game 3 with respect to Game 2 facilitated coordination among players a great deal. As shown in Figure 3, Game 3 has fewer deviations from equilibrium play than Game 2. We will see that this small change had consequence on coalitional play as well.

As predicted by the theory, Table 7 shows repeated attempts to coordinate on coalitional deviations. Twenty-nine times in Game 2 and 18 times in Game 3 row and column attempted a deviation from the symmetric Nash equilibrium to the Pareto efficient allocation. These attempts were frequently successful anytime player matrix explicitly committed to play A. Table 7 also shows failures to implement this deviation. However, these deviations failed not because column did not follow them, but because either row or matrix changed their minds. Row “AB,AB,” shows that whenever matrix did not announce to both other players to play A, the likelihood of failing to implement a coalitional deviations increased. While players were able to figure out that coalitional deviations were profitable, we have provided evidence before that deviations from deviations happened.

Either matrix figured out a deviation from the symmetric Nash equilibrium was in the making or explicitly coordinated a double-crossing of column with row. Matrix sent message B to row and message A to column in 13 cases in Game 2 and in 7 cases in Game 3. In 17 of 21 cases this coordination resulted in the implementation of (A, B, B). More in depth analysis shows that the frequency of the symmetric Nash equilibrium and the Pareto efficient allocation under column “Other” for Game 2 and Game 3 are due to this kind of coalitional deviations. The fact that this type of double-crossing is relatively stronger in Game 3 seems to be due to the fact that a deviation from the Pareto efficient allocation to the strong Nash equilibrium by row and matrix is strict.

We conclude by observing that making double-crossing easier in Game 3 does not diminish the frequency of play of the symmetric Nash equilibrium. It does decrease the play of Pareto efficient allocation instead. That is, stronger coalitional incentives strengthen the frequency of strategy profiles that are not coalition-proof. Our results show that coalitional reasoning is important in our games. However, these deviations are more sophisticated than those predicted by CPNE. As suggested by Farrel and Rabin (1996),

we should expect communication everytime messages are self-signaling (the player reveals his intentions if and only if they are true). In these situations, double-crossing is possible and the restriction on coalitional deviations to strict subcoalitions seems to be too strict.

**Table 7. Frequency of Play Given Agreement - Rounds 8 - end**

<b>Game 1 (Groups = 66)</b>									
<b>Agreement/Play</b>	AAA	AAB	ABA	ABB	BAA	BAB	BBA	BBB	Total
AA,AA,AA	32	0	0	0	0	0	0	0	32
AA,BB,BB	0	0	0	0	0	0	0	0	0
BB,BB,BB	0	1	0	3	0	0	0	0	4
AA..;OO..	43	2	2	4	0	0	0	0	51
Other	1	0	2	0	1	1	0	2	7

<b>Game 2 (Groups = 186)</b>									
<b>Agreement/Play</b>	AAA	AAB	ABA	ABB	BAA	BAB	BBA	BBB	Total
AA,AA,AA	34	8	2	0	1	0	0	0	45
AA,BB,BB	0	1	0	3	0	0	0	0	4
BB,BB,AA	0	0	2	3	0	0	21	9	35
AA..;OO..	53	12	13	6	2	0	2	0	88
BA,BA,AA	0	0	3	2	0	0	17	4	26
BA,BA,.	0	0	3	4	0	0	18	4	29
Other	8	3	7	13	1	0	9	0	41

<b>Game 3 (Groups = 96)</b>									
<b>Agreement/Play</b>	AAA	AAB	ABA	ABB	BAA	BAB	BBA	BBB	Total
AA,AA,AA	30	1	0	0	0	0	0	0	31
AA,BB,BB	0	0	0	15	0	0	0	1	16
BB,BB,AA	0	0	0	0	0	0	1	1	2
AA,..;OO..	35	6	0	19	0	1	0	1	62
BA,BA,AA	0	0	0	1	0	0	12	0	13
BA,BA,.	0	0	0	5	0	0	12	1	18
Other	1	2	0	10	0	0	0	1	14



## 5 Conclusions

This research set out to investigate the relevance of coalitional deviation as a selection criteria in strategic games. Previous experiments have shown that communication in two-player games increases coordination and could help avoid Pareto inefficient equilibria (Charness, 2000; Burton et al., 1999). Talk might be cheap, but it is not worthless. We confirm that nonbinding pre-play communication affects the play of games. Subjects are able to navigate through incentives and find agreements that are self-enforcing. Moreover, we find that coalitional deviations are important in determining what agreements are self-enforcing.

Paradoxically, we find that without communication, play converges strongly to a coalition-proof equilibrium, but that with communication play moves away from it. Our results show that this cannot be completely explained by fairness considerations. Instead, we find that coordination on complicated coalitional deviations might play a role. Indeed, an important finding of our experiments is that once subjects understand the incentives of the games at hand, they can engage in sophisticated play. Either players openly plan on double-crossing each other or engage in incentive-compatible information transmission. We find that coalition-proofness might fail not only because of bounded rationality, but because equilibrium models assume too little rationality. The kind of deceptions observed in our games are consistent with the arguments in Crawford (2003) that boundedly rational agents might lie and get away with it sometimes. Interestingly, we find an evolution towards more and more involved forms of deception.

Our results show that the answer to the question of what constitutes a coalition-proof agreement is a complicated one. As previous experimental research has shown, subjects demonstrate an enormous ability to coordinate their actions. Our experiments encourage further experimentations to better understand the importance of self-signaling messages in multiplayer strategic games and the role of farsightedness in play.

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## 6 Appendix

**Table A.1**  
Frequency of Play - All Rounds (Percent)

No Communication				Communication							
Game 1											
A		B		A		B					
A	B	A	B	A	B	A	B				
A	2.22	11.11	A	15.56	40.00	A	42.69	8.77	A	11.11	14.62
B	2.22	3.89	B	7.78	17.22	B	1.75	2.34	B	8.77	9.94
Game 2											
A		B		A		B					
A	B	A	B	A	B	A	B				
A	3.33	9.44	A	17.22	62.78	A	23.47	14.40	A	7.47	12.27
B	0.00	1.11	B	0.56	5.56	B	1.87	26.40	B	0.80	13.33
Game 3											
A		B		A		B					
A	B	A	B	A	B	A	B				
A	1.67	12.78	A	16.11	63.89	A	30.56	2.22	A	7.78	32.78
B	0.00	0.56	B	1.67	3.33	B	1.11	13.33	B	1.11	11.11
$\chi^2$ tests: <ul style="list-style-type: none"> <li><math>H_o : Game_{comm}^1 = Game_{comm}^2</math>, p-value = 0.008</li> <li><math>H_o : Game_{comm}^1 = Game_{comm}^3</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{comm}^2 = Game_{comm}^3</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^1 = Game_{no\_comm}^{2,3}</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^2 = Game_{no\_comm}^3</math>, p-value = 0.634</li> <li><math>H_o : Game_{no\_comm}^1 = Game_{comm}^1</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^2 = Game_{comm}^2</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^3 = Game_{comm}^3</math>, p-value &lt; 0.000</li> </ul>											

**Table A.2**  
**Frequency of Play - Rounds 8 - end (Percent)**

No Communication				Communication							
Game 1											
A		B		A		B					
A	B	A	B	A	B	A	B				
A	0.00	10.42	A	18.75	41.67	A	66.67	6.06	A	4.55	10.61
B	2.08	5.21	B	5.21	16.67	B	1.52	3.03	B	1.52	6.06
Game 2											
A		B		A		B					
A	B	A	B	A	B	A	B				
A	1.04	4.17	A	16.67	72.92	A	32.26	11.83	A	8.06	12.90
B	0.00	1.04	B	0.00	4.17	B	1.61	26.34	B	0.00	6.99
Game 3											
A		B		A		B					
A	B	A	B	A	B	A	B				
A	0.00	12.50	A	15.63	69.79	A	37.50	0.00	A	8.33	35.42
B	0.00	0.00	B	1.04	1.04	B	0.00	13.54	B	1.04	4.17
$\chi^2$ tests: <ul style="list-style-type: none"> <li><math>H_o : Game_{comm}^1 = Game_{comm}^2</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{comm}^1 = Game_{comm}^3</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{comm}^2 = Game_{comm}^3</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^1 = Game_{no\_comm}^{2,3}</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^2 = Game_{no\_comm}^3</math>, p-value = 0.179</li> <li><math>H_o : Game_{no\_comm}^1 = Game_{comm}^1</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^2 = Game_{comm}^2</math>, p-value &lt; 0.000</li> <li><math>H_o : Game_{no\_comm}^3 = Game_{comm}^3</math>, p-value &lt; 0.000</li> </ul>											