Reconsidering the early marginal productivity theory of distribution and interest[♦]

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INTRODUCTION

The 1960s' critique of capital theory argued that one cannot explain income distribution on the basis of 'social' marginal productivities of given (and fully employed) endowments of labour, land and capital. In the words of Pasinetti: 'Very far from embodying the relevant features of the general case, and from being a simplified way of expressing it, the one-commodity infinite-techniques construction is (…) revealed to be an entirely isolated case. As such, it can have no theoretical or practical relevance whatever. At the same time the whole traditional idea that lower and lower rates of profit are the natural and necessary consequence of further and further additions to "capital" is revealed to be false' (Pasinetti, 1969, pp. 522-3). The 'traditional idea' referred to by Pasinetti was developed in the XXth century and may be exemplified in Hicks's *Theory of Wages*[1] . This theory, in turn, is generally thought to have been the natural development of the early marginal productivity theory, put forward in the 1890s by a group of mathematically oriented economists such as Wicksteed (1894), Walras (1896), Barone (1896) and Wicksell (1893 and 1901) [2]. Yet the early theory -which was much less ambitious than its XXth century versions and, with the exception of Wicksell, is immune to capital theoretic criticism- is interesting in its own and it is open to different

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♦ I thank Ian Steedman and Carlo Panico for his useful comments and participants at a meeting on 'Growth and distribution' in Catania, 23-4 January 2004, for further stimulus.

[1] Hicks (1932b). The following passage represents exactly the object of Pasinetti's criticism: "as capital continues to grow, it is certain that that the more advantageous applications will be used up. (…) But of course this process involves a fall in the marginal product of capital and therefore of the rate of interest. Eventually the fall in interest will check saving, and the community whose technique does not progress will approach the "stationary state" of the classical economists" (Hicks, 1932b, p. 127-8).

[2] See, e.g., Stigler (1941). Also the 1960s criticism of capital theory seemed to assume some sort of homogeneity and of continuity between early and the XXth century marginal productivity theories of distribution.

developments. This Chapter is concerned with such a theory, with special reference to Wicksteed, Walras and Barone (WWB hereon).

Three main differences between the WWB theory and the 'Hicksian' version will be stressed in what follows: the former (i) concerns the *individual* price taking firm or industry, (ii) considers the distribution of the product between the holders of an *indefinite* number of *physical* inputs and (iii) considers the equality between the marginal productivity of an input and its real reward as a property of the *point* of equilibrium, whereas the latter concerns the economic system 'at once', considers the distribution of the 'Social Dividend' between the classical 'grand' factors of production and theorises marginal productivity as a principle of 'input demand'.

Such differences have recently been put to the fore by I. Steedman's textual analysis of Wicksteed's *Coordination*, (Steedman, 1992) which casts some doubts on the idea that the founder of marginal productivity theory was aimed at a "grandiose" explanation of distribution of the 'Social Dividend'. Quite the contrary, according to Steedman, "one should accept the work for what it is: a modest attempt to discuss certain aspects of the partial equilibrium theory of factor use in the face of given factor prices and a given rate of interest" (p. 35). Within these limits, marginal productivity theory is by no means committed to a Hicks-type theory of distribution: in particular, it would be quite unfortunate to say, in respect to the early authors, that marginal productivity "determines" the distribution of the product (either at firm or at social level) between the factors of production.

It will be of some interest, then, to reconsider as well some other early contributions to marginal productivity theory which can be more properly understood and interpreted on the same footing as Wicksteed's "modest attempt"[3]. Since these theories are perfectly sound within their own limits, it

[3] The position of Alfred Marshall should be singled out. In fact only in note XIV of the Mathematical Appendix of his *Principles* (Marshall, 1920) can we find a formal account of the equality between the reward of a factor service and the marginal product (marginal efficiency, or net product, in his terminology) multiplied by the product (demand) price. He seems, however, to be somewhat ambiguous on the relevance of this equality for the theory of distribution. On the one hand, in fact, he claims that 'this proposition is very important and contains within itself the kernel of the demand side of the theory of distribution' (Note XIV, p. 697); on the other hand, however, he says that 'the doctrine that the earnings

will also avail to ask what difficulties are encountered in the attempt of building a theory of factor demand on their basis and if the same basis can bear different constructions. It is to this that we seek to contribute in the three parts of the chapter.

In the first part it will be argued that Walras's treatment of marginal productivity served exactly the same purposes as Wicksteed's, and that his equations –as referred to the individual price taking firm- can in fact be considered "duals" to Wicksteed's equations. We shall argue that also the Italian scholar Enrico Barone pursued the same logic, placing a special emphasis on the problems of capital and interest.

The second part of the paper draws on a fundamental point that Hicks himself was to note in his *Value and Capital*: that under the assumptions which can be ascribed to WWB, it is impossible to change just *one* factor price without upsetting equilibrium altogether. This makes the usual kind of comparative static analysis and the usual notion of "factor demand" –on which the "grandiose" theory of distribution attacked by Pasinetti is based- quite problematic. It should be stressed, however, that this has *nothing to do* with capital theoretic criticism and is a much more fundamental obstacle to an interpretation of marginal theory as a theory of "factor demand". As we shall see, this important methodological point has been developed "positively" -in the sense of proposing a more complex comparative static analysis of factor use in relation to factor price- in a series of articles of the 1970s by Silberberg (1974a and 1974b) and others. Along similar lines, some further and much less "conventional" results have been reached in a series of articles by Steedman (1985, 1988, 1998), who worked out some analytical consequences deriving from the presence of *produced* means of production.

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of a worker tend to be equal to the net product of his work, has by itself no real meaning; since in order to estimate net product, we have to take for granted all the expenses of production on the commodity on which he works, other than his own wages' (Book VI, Ch. II, pp. 429-30) . According to Stigler, Marshall 'hesitates at the final tenet of the theory, that the distributive share is *equal to* or *determined by* its marginal product (…) The fundamental reason Marshall rejects an outright marginal productivity theory seems to be the difficulty of measuring the marginal productivity of a productive service' (Stigler, 1941, pp. 347-8).

The final part of this paper concerns the marginal productivity theory of interest in relation to growth. It will be argued that the equality, in equilibrium, between the interest rate and the "marginal productivity of capital" *at the level of the price-taking productive unit* does not necessarily lead to a marginal productivity theory of interest, as in the Hicksian version. Nor does it necessarily lead to a Solowian theory of steady growth, in which the equality between the rate of interest and the rate of return on social capital plays a central role in the adaptation to full employment saving. That simple idea, instead, is in a sense theoretically 'neutral'[4] and is therefore compatible with other theories of interest. We shall argue, in particular, that it fits quite well with a *surplus* theory of interest. In fact, if all firms *can* pay a positive rate of interest (which each of them takes as given), then there logically follows that there should exist a positive difference between the production of commodities and the consumption of them for productive use. As we shall see, some of the "new" authors contemporary to Wicksteed and Walras, such as Wieser, Pantaleoni, and for certain aspects Barone himself, entertained some kind of just such a surplus conception of interest.

PART I) <u>The WWB marginal productivity theory of the competitive firm</u>

2. Even though the above mentioned early authors always speak of "factors of production" in relation to the problem of distribution, it is not arbitrary, from a modern standpoint, to replace their original phrase with the more neutral term "inputs"[5]. In fact, the phrase "factor of production" is evocative of an aggregative view of the productive services. Differently from such authors as Clark and Wicksell, this by no means was the idea pursued by Wicksteed, Walras and Barone: rather, they

[4] A similar point has been raised by Pasinetti in his discussion of the Fisherian concept the 'rate of return'. He argues that the equality between the rate of profit and the rate of return, if properly understood, is an *accounting identity* which is 'compatible with any explanation, *i.e.*, any theory, of the rate of profit' (Pasinetti, 1969, p. 514).

[5] According to Schultz (1929) the phrase "input per unit of output" had been introduced by some agricultural economists and "is to be commended (provided it is confined to physical relations) for its mnemonic properties" (p. 510, n.)

thought of physical production as the result of the application of an indefinite number of *physically specified items*. The fact that these items were usually listed, for concreteness, under the general *headings* of labour, land or capital services does not alter in the least their attitude towards a physical, analytical conception of the production process. As Wicksteed puts it, "The crude division of the factors of production into land, capital and labour must indeed be abandoned (…). We must regard every kind and quality of labour that can be distinguished from other kinds and qualities as a separate factor; and in the same way every kind of land will be taken as a separate factor. Still more important is it to insist that instead of speaking of so many £ worth of capital we shall speak of so many ploughs, so many tons of manure, and so many hourses" (Wicksteed, 1992, pp. 83-4). Similarly, Walras's coefficients of production are *ordered* under the headings of labour services, land services and capital goods services, but mathematically they represent an indefinite number of separate variables[6]. The same is true of Barone[7] and of Pareto[8]. From a modern standpoint, it is therefore convenient to list all of them under the general heading of "inputs" or of "inputs per unit of output" as the case may be.

3. Let us start with the basic analysis of Wicksteed's *Essay*. A perfectly competitive unit facing given input and output prices -represented by vector **w** and scalar *p*, respectively- is free to choose its own vector of input use, **x**. The amount produced, *y*, is a function of **x**, $y = f(\mathbf{x})$. Wicksteed explicitly assumed this function to be everywhere differentiable and homogeneous of degree 1.

He regarded as a truism the fact that 'if a man "is not worth his salt" he is discharged' (p. 59), and that, in competitive conditions, no "factor" need accept less (Steedman, 1992, p. 17-18). In

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[6] See Lesson 36 of Walras (1896).

[7] "L'imprenditore per la fabbricazione di un certo prodotto si vale di lavori, di terre, di capitali tecnici. I lavori saranno di diverse specie, come pure le terre, come pure i capitali tecnici: *per semplicità di esposizione*, supporremo che si tratti di *due* sole specie di lavoro, due sole specie di terre, due sole specie di capitali tecnici: la qual cosa (…) nulla toglie alla generalità dei ragionamenti che faremo" (Barone, 1936 [1896], p. 171).

[8] See Pareto (1971), pp. 720-29.

equilibrium, then, input use involves the equality between each input price and the value of its marginal product:

$$\mathbf{w} = pf_{\mathbf{x}}(\mathbf{x}). \qquad (1)$$

On the other hand, it is shown that[9]

$$py = pf_{\mathbf{x}}(\mathbf{x}) \cdot \mathbf{x} \qquad (2)$$

In equilibrium, when each input is paid the value of its marginal product, the value of the product is exactly exhausted.

4. We know that Walras claimed that his equations of production contained a marginal productivity theory of distribution which was more general than this[10]. His analysis differs from Wicksteed's in the fact that he *started* with the zero profit condition, which *automatically* involves that the output is exactly distributed between the inputs. Even though Walras's general equilibrium theory is carried on in terms of input *coefficients*[11], he departed from this practice in the treatment of marginal productivity, in order to make the connection with Wicksteed's argument more transparent. Formally, he considers each competitive industry, facing given input and output prices. The zero profit condition

$$py = \mathbf{wx} \qquad (3)$$

is assumed to be satisfied in each productive unit at "a predetermined quantity to be manufactured" and under "the condition that the cost of production be a minimum" (p. 384). Differentiating eq.(3)

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[9] Wicksteed presented a cumbersome original demonstration, even though Euler's theorem was immediately applicable, as Flux (1894) noted. The question of whether Wicksteed knew about Euler's theorem is discussed by Steedman (1992), p. 13.

[10] He also claimed to have been piqued by Wicksteed: "Mr. Wicksteed (…) would have been better inspired if he had not made such efforts to appear ignorant of the work of his predecessors" (Walras, 1969 [1900[4]], p. 495). These rather crude words have been considered unfair by the subsequent literature.

[11] At the turn of the XIXth century, the practice of Walras and Pareto was to use coefficients, whereas that of such authors as Wicksteed, Barone, Wieser and Pantaleoni was to use "absolute" amounts. But, as Pantaleoni remarks, "it is the same thing" (Pantaleoni, 1904, p. 326).

with respect to each input use, under the constraint of the production function, we can see that this condition is in fact fulfilled when eq. (1) holds[12].

Since Walras's eq. (3) presumes cost minimisation, it is quite natural to express his argument in terms of *cost functions*. Let $C(\mathbf{w}, y)$ be the cost function of a productive unit producing a certain commodity (suffix omitted), and let $c(\mathbf{w}, y) \equiv \frac{1}{y} C(\mathbf{w}, y)$. In equilibrium (when *maximum* profit is zero), we have

$$py = C(\mathbf{w}, y), \tag{4}$$

at any y such that

$$c(\mathbf{w}, y) = \frac{\partial C(\mathbf{w}, y)}{\partial y}. \tag{5}$$

Since $C(\mathbf{w}, y)$ is homogeneous of the first degree in **w**, eq. (4) may be re-written as

$$py = C_{\mathbf{w}} \cdot \mathbf{w} \tag{6}$$

Moreover, by Shephard's lemma, we have

$$\mathbf{x} = C_{\mathbf{w}}(\mathbf{w}, y) \tag{7}$$

We see at once that eq.s (6) and (7) are "duals" to eq.s (1) and (2): the cost function "replaces" the production function, and **w** "replaces" **x** (and conversely).

5. In this formalisation, Walras's assumption of "a predetermined quantity to be manufactured" can be given two distinct interpretations. The prevailing interpretation is that he assumed in fact constant returns to scale, so that the "predetermined quantity" y can be completely arbitrary and eq. (5) is *always* satisfied. In this interpretation, it can hardly be maintained that Walras's formulation of marginal productivity theory was more "general" than Wicksteed's: they refer to precisely the same theory, from different points of view. There is, however, a second possible interpretation. In

[12] See Walras (1969), p. 385.

the words of Hicks: "Wicksteed thought he had proved that it was a necessary condition for the truth of the marginal productivity theory that this curve [of average cost in relation to output] should be a horizontal straight line. Walras and Wicksell showed that it was only necessary that the curve should have a minimum point, and that in equilibrium output must be at that point" (Hicks, 1932b, p. 238). In this case, the "predetermined quantity" is uniquely determined by the condition of equality between average and marginal cost. We do not examine here the question whether there is any textual evidence in favour of this second interpretation, nor do we examine the even more difficult question of its coherence with Walras's overall theory of general equilibrium. We only notice that both interpretations are possible (as rational reconstructions, at least) when we limit ourselves to the equilibrium of the price-taking firm.

6. The formulation of marginal productivity theory in terms of U-shaped cost curves had some advantages, which were appreciated by later literature. The first advantage is that to some extent it meets the early objections raised by Edgeworth and by Pareto, concerning the *realism* of constant returns. We know that Edgeworth's sarcasm on Wicksteed's use of linearly homogeneous functions[13] has not been taken too seriously by the literature[14]. Pareto's criticism, however, has been quite influential. In the words of his often-quoted, and ironic, example: "If (…) one were to engage in the transportation business in Paris, it would be necessary to assume another business and another Paris" (p. 721; translation taken from Stigler). According to Pareto, one should recognise that, in most cases, the (cost minimising) coefficients of production do vary with the amount produced, and a reasonable assumption is that they are U-shaped.

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[13] As he says, "there is a magnificence in this generalisation which recalls the youth of philosophy. Justice is a perfect cube, said the ancient sage; and rational conduct is a homogeneous function, adds the modern savant". Edgeworth (1925) [1904], p. 31.

[14] According to Stigler, for instance, "some of his arguments are nothing more than ridicule; the remainder are based upon rather obvious misapprehensions" (p. 341).

To be sure, Pareto had a peculiar notion of the "coefficients of production". By way of a digression, let's briefly consider this notion. Pareto's coefficients are "measured by finding the change in the service corresponding to an infinitesimal change in the quantity produced" (Schultz, 1929, p. 520): Walras coefficients are average concepts (inputs per unit of output), whereas Pareto's coefficients are marginal concepts (variation in inputs per unit of variation of output). In terms of the cost function, Pareto's coefficients are the second order derivatives:

$$\mathbf{b}' = \left[\frac{\partial^2 C(\mathbf{w}, y)}{\partial w_i \partial y} \right], \; i = 1, 2, \cdots, n \, .$$

By homogeneity of degree one in $\mathbf{w}$ of $\frac{\partial C(\mathbf{w}, y)}{\partial y}$, we have $\mathbf{wb}' = \frac{\partial \mathbf{C}(\mathbf{w}, y)}{\partial y}$. At an output such that $c(\mathbf{w}, y) = \frac{\partial C(\mathbf{w}, y)}{\partial y}$, this involves $p = \mathbf{wb}'$: Pareto's coefficients, therefore, have the required "adding up" property, in equilibrium[15].

A second obvious advantage of U-shaped curves is that they avoid the problems of indeterminacy which notoriously affects the case of constant returns, even though, of course, in the latter case, one obtains determinate input levels *per unit of output*.

[15] A second criticism made by Pareto was that there are many instances of fixed proportions between some inputs and the output, like the case of iron ore and pig iron (Cf. Pareto, 1971, p. 721). In such cases there is no ground for the idea that one may change one input–and the output as well- *keeping all the others constant*. As he says, the equalities between each input price and the marginal product in value "require that the variations of [the quantities of input use] be independent of each other. This is not the case when one coefficient of production is constant" (p. 274, n. *a*). He insisted, therefore, that "one cannot use the theory of marginal product without taking into account these corrections" (p. 724). In effect, many economists of the time (and of the previous generation) stressed the role of complementarity, as they called it, quite a lot. Complementary goods and factors featured prominently in Gossen's and Menger's theory; Wieser thought that Menger's "loss principle" (that rudimentary anticipation of the concept of marginal productivity) was incoherent precisely because of complementarity. Pantaleoni expressed the same idea borrowing from chemistry the property of "definite proportions". Also Irving Fisher stressed input complementarity, as opposed to substitutability, on the basis of the Marshallian classification of joint and composite demand for (and supply of) goods and services.

7. The basic marginal productivity theory presented so far ignores that production takes time, that some payments for securing input services are made in *advance*, and that normally a *positive* interest is charged on such advances. For this reason, Barone objected that it was a "rudimentary" theory, valid only as a first approximation[16]. The formal integration of interest into the marginal productivity theory of distribution was the major concern of many authors at the end of the XIXth century, and of Boehm-Bawerk, Wicksell and Barone in particular. We need not review all the complex capital theoretic literature of the time, however, and shall limit ourselves only to the very fundamental and simple aspect concerning the equilibrium of the individual, price-taking agent. In this regard, Barone's treatment is quite explicit and clear. To consider the most simple case, let *all* input services require an anticipation for the whole amount at the beginning of the period, and let all known processes require the same length of time. If the *given* rate of interest is z, we have just to replace eq. (1) with[17]

$$(1+z)\mathbf{w} = pf_{\mathbf{x}}(\mathbf{x}) \qquad (1A)$$

Rearranging eq. (1A) slightly, we have $\frac{\mathbf{w}}{p} = \frac{1}{(1+z)} f_{\mathbf{x}}(\mathbf{x})$, which expresses formally Barone's main conclusion that "in terms of a product, each firm remunerates each factor (different kinds of labour and of land, and different technical capitals) at a rate equal to the marginal productivity of the same factor, diminished by interest on the corresponding portion of the capital of anticipation" (pp. 190-91). As Steedman (1992) noted, also Wicksteed "only ever considers the position of the individual agent, faced with given factor prices and a given rate of interest" (p. 29), and, in this context, eq.s (1A), (2) can be shown to involve the equality between the "marginal productivity of capital" and the interest factor. In fact, let $k = \mathbf{wx}$ be the capital advanced, and define $dk = \mathbf{w}(\mathbf{dx})$. Assuming all amounts to be positive, at a profit maximising choice in $\mathbf{x}$ we have $pf_{\mathbf{x}}(\mathbf{dx}) - (1+z)dk = 0$; but

[16] Cf. Barone (1936), p. 221.

[17] Curiously enough, the "Paretian" Barone did not carry out his argument in terms of coefficients: rather, he used "absolute" amounts of input use, like Pantaleoni.

$pf_{\mathbf{x}}(\mathbf{dx}) = d(py)$, so that $d(py)/dk = (1+z)$. "Needless to say, this is a purely partial equilibrium statement, based entirely on quantities being adjusted to given 'prices'" (p. 29).

More complex cases might of course be considered: the argument would not change substantially if we were to introduce inputs *not* requiring any anticipation and others needing anticipations distributed continuously through time[18]. The consideration of a *variable* length of time is more problematic. Barone seems to be quite simplistic, but formally correct, in this respect: he just assumes that production is an increasing function of the length of time through which given input services are distributed; in equilibrium, the value-marginal productivity of "time" is then equal to the interest payment (on all anticipations)[19].

Reverting briefly to our simple case, it will be useful for later reference to formulate the dual counterpart of eq.s (1A), (2). With constant returns to scale, we have

$$py = y(1+z)c_{\mathbf{w}}\mathbf{w} \qquad (6A)$$

$$\mathbf{x} = yc_{\mathbf{w}}(\mathbf{w}) \qquad (7A)$$

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[18] This is in fact the case assumed by Wicksell (p. 187) and Barone (p. 182-3).

[19] Like Boehm-Bawerk, he used the formula of *simple* interest, so that the value-marginal productivity of "time" is equal in equilibrium to the capital of anticipation *times* the rate of interest: see Barone (1936) [1896], p. 222, eq. (10). In our notation, Barone's equations read:

$$y = f(\mathbf{x}, t) \qquad (A)$$

$$k = \mathbf{wx} \qquad (B)$$

$$py = \mathbf{wx}(1+zt) \qquad (C)$$

with

$$pf_{\mathbf{x}} = (1+zt)\mathbf{w}\ ;\quad pf_t = zk \qquad (D)$$

It is quite clear that multiplying both sides of the first condition (D) by any vector $(\mathbf{dx})$ we obtain $(1+zt) = d(py)/dk$, and rearranging slightly the second condition, we obtain $z = \frac{\partial(py)}{\partial t}\frac{1}{k}$: at the level of the individual price-taking agent, and using the formula of simple interest, one may indifferently interpret $(1+zt)$ as the value-marginal productivity of value-capital and z as the value-marginal productivity of "time", per unit of value-capital.

Dividing through by *y* one could, of course, reformulate the system in terms of coefficients per unit of output. One may also assume U-shaped (average and marginal) cost curves, on the basis of eq.s (4)-(6).

PART II) Can we force "input demand" into the WWB theory ?

8. In the foregoing sections it has always been assumed that the individual agent faces given prices of the product and of the input services, and a given rate of interest. It should now be stressed that these given "prices" cannot be *arbitrary*. This depends, of course, on the fact that the early authors considered firms as being always in *long-period* equilibrium, earning a maximum profit equal to zero. Apart from the necessity of obtaining an "adding-up" property of distribution, they had very good reasons for adopting this long-period view in relation to marginal productivity. In fact, the longer the time considered, the more plausible is the possibility of substituting (marginally or completely) one input with another along a production function. This is a point which Hicks, for instance, stressed very much when defending marginal productivity theory against the attacks of Pareto, who advocated the "definite proportions" case, especially when produced inputs are involved[20]. But if competition is assumed to lead firms, in the long-run, to exploit many possibilities of substitution, the same competitive processes should coherently be assumed to eliminate any possibility of positive profits, as well as any situation of loss.

Now the condition that maximum profits are zero, *automatically* establishes a relation between input prices and the product price. So, with constant-returns-to-scale, the Walrasian "production equations" can be expressed as

$$\mathbf{p} = \mathbf{c}(\mathbf{w}) \qquad (8)$$

[20] "If we do not allow the entrepreneur time to replace his equipment, the old difficulty of fixed proportions is absolutely unescapable". Hicks (1932a), p. 86.

$$\mathbf{B} = \mathbf{c}_{\mathbf{w}}(\mathbf{w}). \qquad (9)$$

where **B** denotes the matrix of coefficients per unit of output. Now **B** is determined in accordance with marginal productivity theory at any arbitrary **w**, but this automatically involves that each productive unit faces a certain, determinate, product price. Or, differently, one may take one product as the numéraire, and find that there is a certain relationship between "real" input prices and between them and the (relative) product prices. Replacing constant with U-shaped cost curves would not change matters in this respect, as should be clear from equations (4)-(7). Nor would a positive interest rate eliminate the need to limit marginal productivity theory to *specific* price configurations, as can be seen in eq.s (6A), (7A). On the other hand, it is intuitive that even *further* sources of price interrelatedness should be taken into account if we consider that a great many firms produce commodities which are used as inputs by other firms.

When we limit our attention to the characteristics of a *point* of equilibrium, as Wicksteed, Walras and Barone did, this poses no problem: one has just to choose a consistent system of prices at which input use and marginal productivities are evaluated. Long-period price interrelatedness, on the contrary, poses some serious problems for the conception of marginal productivity theory as a theory of "factor demand". This conception, in fact, is typically based on a partial equilibrium comparative static analysis in which *one* price is changed *keeping all other prices constant*.

9. The fundamental problem with this kind of comparative static analysis can be seen very simply by differentiating totally any of eq.s (8) (commodity suffix omitted):

$$dp = \sum_i b_i dw_i \,. \qquad (10)$$

If we consider only positively used inputs, it is simply impossible to change one price in isolation without destroying equilibrium: either we set $dp = 0$ and allow at least two input prices to change, or we set $dw_i = 0$ for all i but one, and allow the output price to change. The point was plainly recognised by the most accurate neoclassical authors, such as Hicks and Allen. The former argued

that "it is not possible for the price of one factor (or product) to change, there being no change in the prices of all other factors and products, without upsetting equilibrium altogether"(J.R. Hicks (1946), p. 322), and Allen concluded that "in the long-run, if all elements are variable and returns are constant, the analysis tends to break down or to become inappropriate"(R.G.D. Allen (1957), p. 617).

Things would not be different if we were to allow for a positive rate of interest. In fact, differentiating (6A) totally, one gets:

$$dp = (1+z)\sum_i b_i dw_i + \sum_i b_i w_i d(1+z). \qquad (11)$$

As we can see, a change in the rate of interest must be accompanied by a change in one input price and/or by a change in the product price.

Nor would it help to assume U-shaped, rather than constant, cost curves. In fact, substituting $c(\mathbf{w}, y) \equiv \frac{1}{y} C(\mathbf{w}, y)$ into (4), and differentiating (4) and (5) totally, we have

$$\begin{aligned} dp &= \sum_i b_i dw_i \\ dy &= \left(\frac{\partial^2 C}{\partial y^2}\right)^{-1} \sum_i \left(b_i - \frac{\partial^2 C}{\partial w_i \partial y}\right) dw_i \end{aligned} \qquad (12)$$

In this case a change in one input price is accompanied by a change in *both* the output price and the output quantity.

10. It is incoherent, then, to insist on a comparative static analysis based on ceteris paribus. In fact 'metaphors such as "short run" have been used to gloss over the inconsistency of holding output price constant in the face of changing costs through changing factor prices' (Silberberg, 1974b, p. 734): but we have seen that marginal productivity theory would be deprived of any interest in the "short run". A *different* and more complex kind of comparative static analysis is required, which takes some "concomitant" price changes into account. An interesting example, concerning U-

shaped cost curves, consists in a series of articles[21] by E. Silberberg and others, who 'have investigated the comparative statics of the "neoclassical" firm in a competitive environment with the specific proviso that output price adjusts in response to changes in factor prices'[22]. Silberberg works out the consequences of the fact that in a long-run equilibrium "the value of the marginal product (…)[is] equal to the wage rate, for each factor, *with the added stipulation* that the output price be set at the level of minimum average cost". We need not enter into the details of his analysis. His *conclusion* is that, with this added stipulation, the use of an input need not be negatively related to its price, whereas the use of an input *per unit of output* is related to the input price in the usual way.

11. Further steps are to be made –and far less conventional results are obtained- when we explicitly allow for *produced* inputs. In this case, we cannot ignore the fact that a combination of price changes which keeps one firm in equilibrium may automatically drive other firms –those which use its product as an input- out of equilibrium. This point has been developed by Steedman in a series of articles (Steedman, 1985, 1988, 1998) and in Opocher (2002).

Without entering into the details of what Steedman calls "full industry equilibrium" analysis (Steedman, 1998, p. 196), a fundamental methodological point is worth discussing here.

Let us first consider the very simple case of a firm which uses its own product as an input. Denoting now by **w** the price vector of *primary* inputs and by **b** their use per unit of output, we have in equilibrium

$$p = c(\mathbf{w},\, p), \qquad (13)$$

$$\mathbf{b} = c_{\mathbf{w}}(\mathbf{w},\, p);\; a = c_p(\mathbf{w},\, p), \qquad (14)$$

[21] See Silberberg (1974a), Silberberg (1974b), and the bibliography quoted therein.

[22] Silberberg (1974b), p. 734.

where *a* denotes the cost minimising use of the "own product". Differentiating (13) totally, and making some substitutions, one gets:

$$p\hat{p} = \sum_i b_i w_i \hat{w}_i + ap\hat{p},$$

where $\hat{x} = dx/x$. But $ap = p - \sum_i b_i w_i$. Thence

$$0 = \sum_i b_i w_i (\hat{w}_i - \hat{p}). \qquad (15)$$

Whenever *p* rises with respect to one primary input price, at the same time it falls with respect to at least one other. *Any* conception of an input "demand curve" –not to speak of the simple conception based on ceteris paribus- seems to be simply meaningless, in the long-run context, when referred to a *produced* input.

The more complex, and realistic, case of a firm which uses the products of *other* firms leads to similar results. One should coherently consider now the equilibrium of the firm at prices which keep *all* the firms involved in long-period equilibrium, that is

$$\mathbf{p} = \mathbf{c}(\mathbf{w}, \mathbf{p}) \qquad (16)$$

$$\mathbf{B} = \mathbf{c}_{\mathbf{w}}(\mathbf{w}, \mathbf{p}); \ \mathbf{A} = \mathbf{c}_{\mathbf{p}}(\mathbf{w}, \mathbf{p}), \qquad (17)$$

where **p** and **c** are vectors, and **B**, **A** are matrices.

In the neighbourhood of any vector $(\mathbf{w}, \mathbf{p})$ satisfying (16) there is a linear relationship between **p** and **w**:

$\mathbf{p} = \mathbf{H}\mathbf{w}$ with $\mathbf{H} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$.

In particular, we have

$p_j = \mathbf{h}_j \mathbf{w}$, where $\mathbf{h}_j$ is the jth row of **H** and commodity j is assumed to be an input to some firms/industries.

Differentiating totally, we get, after some manipulation

$$0 = \sum_i \frac{h_{ji} w_i \left(\hat{p}_j - \hat{w}_i \right)}{p_j}.$$

If, in the neighbourhood of a point satisfying (16), we have

$\hat{p}_j - \hat{w}_i > 0$ for primary input i

there necessarily exists a primary input s such that

$\hat{p}_j - \hat{w}_s < 0.$

The price of *any* produced input increases relative to one primary input price *and* decreases relative to another or others. Once again the phrase "price increase (or decrease)", so fundamental in any notion of an input demand curve, would be deprived of any meaning in the case of produced inputs.

PART III) The WWB marginal productivity theory and the surplus approach

12. The marginal productivity theory of distribution is frequently presented as an alternative (if not a reaction) to the Classical "surplus" approach to distribution, interest and growth. Yet it would be a mistake to think that marginal productivity theory at the level of the individual, price taking unit – which was the main concern of the early authors- is incompatible with a surplus theory of interest. It is one thing to say that the *price taking* firm equates the value-marginal productivity of value-capital of anticipations to the *given* interest factor –and a completely different thing to ask under what conditions all firms have *the possibility* of paying a positive interest. An answer to the second question, which is closely related to the wider question of what determines the rate of interest to be paid by all firms, involves some "surplus" considerations, as we shall see. On the other hand it would also be incorrect to think of the "new" economists at the end of the XIXth century as completely extraneous to any surplus theory of interest.

13. Considering the late XIXth century's conditions of life of workers –especially those of the commonest kind - it was perfectly natural to many economists of the time to assume that production required an *anticipation* of necessaries. This anticipation was considered by some of them as an amount of *commodities*, rather than as a sum of value. For instance, according to Pantaleoni, the anticipated wage "consists of direct [consumption] *commodities* that are absolutely consumed by industrial processes" (p. 307; emphasis added). Barone shared a similar view: "production can take place only if besides workers, lands and technical capitals there is also a fund of *consumption commodities* by means of which, during the production process, not only the needs of the workers, but also those of the other participants to production (…) can be met. A clear understanding of this point is essential for the theory of capital and interest". (Barone, 1936, pp. 180-81; my translation; emphasis added).

Neither Barone nor Pantaleoni formalised this idea in the framework of marginal productivity theory. By way of a rational reconstruction, however, it is not difficult to see that such a formalisation is possible. We basically need that the services rendered by the various inputs be reduced to a "productive consumption" of commodities. To this end, let us reconsider the "dual" version of Barone's equations, as presented in our eq.s (6A) and (7A). Adopting Pantaleoni's and Barone's point of view, the vector of prices for input services, **w**, expresses the *values* of the amounts of "productive consumption" per unit of the various services. Let us assume that a unit of input service j receives amounts of commodities $(\chi_1, \chi_2, \cdots \chi_n)$ which satisfy the equation

$$0 = \varphi_j(\chi_1, \chi_2, \cdots \chi_n), \quad j = 1, 2, \cdots, m .$$

We may assume any degree of "substitutability" between commodities, of course. The equation may indifferently be interpreted either in the strict sense dictated by the biological "maintenance" of workers, or in the wider sense of securing some other kind of negotiated "needs".

Now, if each firm behaves rationally, we have

$$w_j = \min_{\chi}\left(\chi\mathbf{p} \middle| 0 = \varphi_j(\chi_1, \chi_2, \cdots \chi_n)\right)$$

Let us denote the minimum value function by $w_j(\mathbf{p})$. By Shephard's lemma, we have, of course,

$$\frac{\partial w_j}{\partial p_i} = \chi_{ij} .$$

In vector/matrix notation, we have

$\mathbf{w} = \mathbf{w}(\mathbf{p})$, with

$\mathbf{w_p} = \mathbf{X}$, and, of course, $\mathbf{w} = \mathbf{w_p p}$.

The unit cost function of the individual firm can now be expressed as

$$c((1+z)\mathbf{w}(\mathbf{p})) = (1+z)c(\mathbf{w}(\mathbf{p}))$$

Eq. (6A) can now be reformulated as

$$p = (1+z)c_{\mathbf{w}}\mathbf{w_p p}$$

or $p = (1+z)\mathbf{bXp}$

Now $\mathbf{b} \cdot \mathbf{X}$ gives the vector of amounts of commodities anticipated *per unit of product*. Setting $\mathbf{b} \cdot \mathbf{X} \equiv \mathbf{a}$, we have then

$$p = (1+z)\mathbf{ap} . \qquad (18)$$

It should be stressed that (18) refers to a *point* of equilibrium, in which the given rate of interest and the given price vector are consistent with zero (maximum) profits in *all* firms.

14. In Pantaleoni's and Barone's terminology, $\mathbf{ap}$ is the value of the "capital of anticipations" per unit of output in an individual firm. Pantaleoni is very accurate in stressing that this capital of anticipation

"which remunerates labour *is a flow and not a fund*. In fact, if the entrepreneur's capital were not continually made up again by the proceeds of production, it could only serve once for the payment of wages" (p. 307; emphasis in original).

In order that this circular flow may continue over time, therefore, it is necessary that, in all firms *taken together*, "productive consumption" be compensated, or more than compensated, by the

"proceeds of production". Denoting by **A** the *matrix* formed by vectors **a,** as referred to each *industry*, and by **y** the vector of the amounts produced in each industry, we must have

$\mathbf{y} \geq \mathbf{yA}$.

This point was fairly clear to Pantaleoni:

if we suppose that (…) the industrial production of a country were technically so ill-directed that the sum of utilities[23] produced were less than that of the utilities consumed, the wage fund would go on decreasing until it vanished altogether, whilst, on the opposite hypothesis, it would continuously increase (p. 307).

Another contemporary author, Wieser, whose works Pantaleoni appreciated very much, was even clearer in this respect:

Every bit of capital, rightly employed, produces directly a gross return of goods different from itself, and finally, after the necessary exchange between similar gross returns, reproduces itself and yields a return. In this sense machines, tools, raw materials, auxiliary materials, in short, all forms of concrete capital, the smallest and the most perishable, even those from which, materially speaking, nothing passes over into the product, replace themselves and yield a surplus. From this point of view every piece of coal which is burned for purposes of production creates, in the last resort, another similar piece of coal, and, beyond that, a perishable net return (Wieser, 1893, p. 133).

Formally, Wieser's "perishable net return" for the various commodities is the vector of semipositive differences between the terms of **y** and the corresponding terms of **yA** [24]. Now the fact that

[23] In this and in many other passages of Pantaleoni's work, "utility" is considered a mere unit of measure of physical commodities.

[24] "In the gross return must be found newly produced all the consumed capital, and beyond this there must be a certain surplus" (Wieser, 1893, p. 125). A more comprehensive presentation of Wieser's particular theory of interest is presented in Opocher (2004).

$\mathbf{y} - \mathbf{yA} \geq \mathbf{0}$ has intimate relations with the circumstance that firms face a positive interest rate. In fact, multiplying the terms of the above inequality by a (*any*) positive **p**, we obtain

$\mathbf{yp} - \mathbf{yAp} > 0$.

It follows that

the *value* of gross return and the *value* of capital can never be assimilated: there will always be a difference – viz. the *value* of the net return. (…) The subtrahend is somewhat less than the minuend, and the required residue of *interest* must be the result (Wieser, 1893, p. 142; emphasis added).

Using eq.(18), Wieser's conclusion can be formalised with the equation:

$\mathbf{yp} - \mathbf{yAp} = z\mathbf{yAp}$.

We have here a sketch of the surplus theory of interest which was to be developed some 35 years later by Sraffa and by von Neumann, and which, as we have argued, is by no means contradictory with a marginal productivity theory, under the condition that this theory is simply formulated at the level of the individual competitive productive unit.

CONCLUDING REMARKS

15. In the aggregate version of the marginal productivity theory of distribution the amounts of "social factors" are taken as given, and marginal productivity dictates real factor prices. If an external force causes the amount of one factor to increase, then its price falls, by diminishing marginal productivity, reducing the incentive to further growth.

It has been argued in this survey that the early marginal productivity theory of Wicksteed, Walras and Barone stopped far short of this kind of conclusion. Their formal analysis, in fact, concerns the individual productive unit which faces a given system of prices: they simply show that a) "factor prices" are equal, in equilibrium, to marginal productivities; b) competitive forces drive maximum profit to zero. To say that factor supply *determines* "factor prices" *via* marginal productivities would be quite inappropriate in this context. Moreover, as we have seen, these early authors never

forgot that individual firms minimise costs with respect to a number of physical inputs –many of which consist in the products of other firms- and not with respect to the three grand, aggregate "factors".

A second fundamental difference between the early authors and the aggregate version is that the former never ventured into a formal comparative static analysis of equilibrium: in particular, they did not extend their analysis in order to relate input *demand* by the firm to input price. Nor did they –to say the same from another point of view- develop a formal notion of marginal productivity as of a "demand price", which falls as factor use rises. We have argued that such an extension encounters serious logical problems, as the most accurate later literature has shown. In fact, one can hardly change one price in isolation without automatically pushing the firm and/or other related firms out of long-period equilibrium: hence, one should coherently consider a series of collateral price changes. Comparative static analysis based on marginal conditions is indeed possible and interesting, but one must be aware that it is much more complex than standard partial equilibrium comparative statics; moreover the very expression "price rise" turns to be completely empty when referred to produced inputs.

This sharp distinction between a marginal productivity theory concerned with the price-taking firm, with its individual produced and primary inputs, and with the properties of an equilibrium point, on one hand, and a marginal productivity theory concerned with the social product, with the grand factors of production, and with factor "demand" in the face of fixed supplies, on the other, has some interesting consequences for the theory of interest. The fact that the "interest-taking" firm equalises its "marginal productivity of capital" to the interest rate does not indeed say too much on the cause and nature of positive interest. A completely different question is to ask when *all* firms in all industries *can* pay positive interest. As we have argued in these pages, a marginal productivity theory of the "interest-taking" firm is quite consistent with a surplus theory of interest: in fact, it was a very common idea among the "new" economists of the late XIXth century that "factor services" require an anticipation of commodities which were "consumed" in production. This is the

main premise of a circular view of production in the economy as a whole, and some authors –like Wieser, as we have seen- drew, well before Sraffa and von Neumann, the logical consequence that a positive rate of interest depends on a permanent positive difference between flows of production and flows of productive consumption.

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