

Optimizing IP Networks for Uncertain Demands Using Outbound Traffic Constraints

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Abstract

Conventional routing management approaches, which are based on *given* and often "worst case" traffic matrices, are not sufficient anymore to deal with the increasing growth of the number of endpoints and diverse applications in IP networks. Traffic variations are becoming a more and more important issue, especially when making long-term network planning decisions as well as for medium-term network provisioning policies. In this paper, we address an offline metric-based traffic engineering (TE) problem in IP networks for uncertain demands, subject to several simple outbound traffic constraints. The resulting models belong to the *polyhedral* traffic model. They are intuitively tractable and particularly appropriate for TE approach based on heuristic or metaheuristic frameworks. Impacts of the proposed uncertainty models on link utilization are compared. We also provide some computational results in terms of network utilization and other performance measures.

keywords : routing, traffic engineering, IP networks, demand uncertainty

1 Introduction

Traffic routing consists of carrying traffic from sources to destinations, using the available network resources [3]. Using ordinary approaches, routes are selected in a way to optimize overall network cost and performance based on a *given* traffic matrix. Thus, quality of the resulting routing pattern is very dependent on the precision of the traffic matrix. Shift in traffic may result in undesirable network performance and for a long-term time scale it may affect proper operation of the network. In the context of IP networks, traffic *uncertainty* is becoming a more and more important issue due to considerable growth rates in terms of both size and number of different services, which in turn make precise forecast of traffic demands very difficult. From network operators' point of view, it may be desirable having a routing configuration which is sufficiently flexible to capture certain traffic variations, while keeping resource utilization as efficient as possible. Considering traffic uncertainty for design and planning of IP networks has recently attracted much attention [3, 5, 7, 9, 10, 12]. From the authors' point of view, there are three main approaches: (i) based on some probabilistic traffic assumptions as in [9, 10]; (ii) based on the *polyhedral* model [3], where vectors of traffic demands are bounded and satisfy some linear inequalities [3, 5, 12]; and (iii) based on multiple demand matrices [7]. In this paper we address traffic uncertainty as simple forms of the second model, where only a few constraints for *outbound* traffic from each node need to be specified. The main benefits of such simple constraints are among other things : (i) it needs only little information of the traffic to provide bounds in performance; and (ii) the impact of traffic uncertainty is intuitively tractable and it can be used in conjunction with solving approaches based on metaheuristic frameworks. The second benefit is particularly important, since so far to the best of our knowledge, the polyhedral traffic model is always solved using mathematical programming [3, 12]. We use these traffic models in the context of offline metric-based traffic engineering (TE) for IP networks running an IGP (Interior Gateway Protocol) like OSPF (Open Shortest Path First) or IS-IS (Intermediate System to Intermediate System). In these networks, TE can be deployed by optimizing the parameters used for routing decisions [2, 4, 6, 8, 11, 14]. These parameters (also known as weights or metrics) are administratively assigned to each link in the network and used by routers to compute shortest paths to each destination for routing of the demands. Though here we consider a TE problem in classical IP domains, with appropriate modifications, a similar approach can also be applied for other routing schemes (e.g. Multi-Protocol Label Switching (MPLS) [1, 16]) and for long-term network provisioning purposes such as capacity and topology planning [10], since controlling traffic (TE) is an integral part of them. The remainder of this paper is organized as follows. The following section introduces some notations and mathematically describes the problem of metric-based TE for uncertain demands using outbound traffic constraints. In Section 3 we present some results for the network show in Figure 2. Finally, Section 4 gives some concluding remarks.

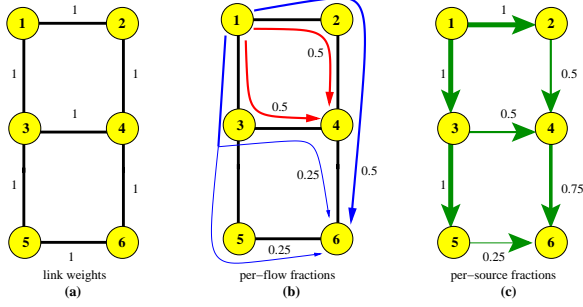


Figure 1: An example of routing demands using the ECMP rule for calculating link loads

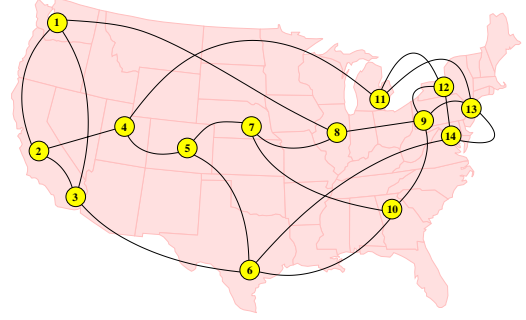


Figure 2: An example ISP network (14 nodes, 22 bidirectional-links)

2 Problem Description

Metric-based TE Using Outbound Traffic Constraints. In a classical IP network running an IGP, demands are routed along shortest paths with respect to metric values (weights), which are assigned to each link in the network. In the existence of multiple shortest paths, traffic will be split over those paths roughly evenly. This is known as the ECMP (Equal Cost Multi-Path) rule. For a *given* traffic matrix, metric-based TE approaches as discussed in [2, 4, 6, 8, 11, 14], will try to find a set of metric values (a weight-system) that optimizes performance e.g. with respect to network utilization. There are basically two types of weight-systems i.e. (i) that for multi-path routing strategy by taking the advantage of ECMP capability of routers; and (ii) that for *unique* single-path routing strategy. In this paper we consider the problem of metric-based TE for uncertain demands, which satisfy several outbound traffic constraints, for both multi-path and unique single-path routing strategies. The term "outbound" is used for traffic originating from a node. The approach discussed in this paper, can surely be applied for "inbound" (i.e. terminating) traffic by a small modification in the formulation below, but not for both, outbound and inbound, simultaneously. However, as will be discussed at the end of this section, it is possible to indirectly limit inbound traffic by specifying more information for outbound traffic. Figure 1(a) shows a small network and the corresponding metric value for each link, Figure 1(b) several routing paths for traffic originating from node 1, and Figure 1(c) the resulting maximum load fraction on the links affected by uncertain traffic from node 1: Setting the metric values homogeneously causes split of traffic destined to node 4 and 6 as shown in Figure 1(b), while traffic to the rest of the nodes is not split and follows the paths (1 – 2), (1 – 3) and (1 – 3 – 5), respectively. Thus the link (1, 3) for instance, is occupied by four different flows i.e. those destined to nodes 3, 4, 5 and 6 with the *per-flow* traffic portion of 100%, 50%, 100% and 50%, respectively. Since traffic is considered as uncertain and we are given only the maximum aggregate traffic values (as expressed by inequality (1) below), a single flow in the *worst* case can occupy the whole resources allocated for traffic aggregate. In the above example, it means: (i) 100% of the total traffic originated from node 1 could occupy the link (1, 3) for for the cases where the traffic aggregate is assigned entirely to the flow terminating at nodes 3 or 5; (ii) 50% of the total traffic could occupy the link (3, 4) for the case where the traffic aggregate is assigned entirely to the flow terminating at node 4; etc. We will now formulate the problem. Given is a directed network $G = (N, A)$, where N is the set of nodes representing the network's routers and A is the set of arcs representing the network's links. Each link $(i, j) \in A$ has a capacity $c_{i,j}$. A demand $f^{u,v}$ denotes the demand to be carried from source u to destination v , $u \neq v \in N$. $f^{u,v}$ can vary over time but it still has to satisfy the following outbound traffic constraints:

$$\sum_{v \in N \setminus \{u\}} f^{u,v} \leq f_{\text{out}}^u \quad (1)$$

A real variable $l_{i,j}^u$ is associated with the load on link (i, j) resulting from flow aggregate f^u originating from node u . Let $A^{u,v} = \{A_1^{u,v}, \dots, A_k^{u,v}, \dots, A_K^{u,v}\}$ be defined as the set of shortest paths for the flow $f^{u,v}$, $A_k^{u,v} = \{(n_1^k = u, n_2^k), \dots, (n_{s-1}^k, n_s^k = v)\}$ as the set of links that belong to the shortest path k for the flow $f^{u,v}$, and $\xi_k^{u,v}$ as a (normalized) fraction of $f^{u,v}$ that is routed through $A_k^{u,v}$ (calculated using the ECMP rule). The total load fraction of the flow $f^{u,v}$ that is routed through the link (i, j) is expressed by

$$\beta_{i,j}^{u,v} = \sum_k \sum_{l \in A_k^{u,v}} \delta_{i,j}^l \cdot \xi_k^{u,v} \quad (2)$$

where

$$\sum_k \xi_k^{u,v} = 1 \quad \text{and} \quad \delta_{i,j}^l = \begin{cases} 1 & \text{if } l = (i, j) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Note that for the special case of *unique* shortest path routing i.e. $K = 1$, (2) becomes $\beta_{i,j}^{u,v} = \sum_{l \in A_1^{u,v}} \delta_{i,j}^l$, which has the value of 1 if $(i,j) \in A_1^{u,v}$ and 0 otherwise. A real variable $l_{i,j}^u$ is associated with the load on link (i,j) resulting from flow aggregate originating from u . The total load on the link (i,j) , denoted by $l_{i,j}$, can be computed as follows:

$$l_{i,j} = \sum_u l_{i,j}^u \quad (4)$$

where

$$l_{i,j}^u = f_{\text{out}}^u \cdot \max_{v \in N \setminus \{u\}} \beta_{i,j}^{u,v} \quad (5)$$

For a given set of maximum outbound traffic $F = (f_{\text{out}}^u)$ from each node $u \in N$, the problem is to find a set of metric values $W = (w_{i,j}), \forall (i,j) \in A$, which optimizes network performance. This can be formulated as :

$$P1 \begin{cases} \min \{ \rho_{\max} \} \\ \rho_{i,j} \leq \rho_{\max}, \forall (i,j) \in A \end{cases} \quad (6)$$

where $\rho_{i,j} = \frac{l_{i,j}}{c_{i,j}}$ is the utilization of the link (i,j) . With (6) we prefer solutions with a low ρ_{\max} , which implies that the network is better utilized. Furthermore, for comparison it might be of interest to maximize the *uniform* outbound traffic i.e. $f_{\text{out}}^u = f^c, \forall u \in N$. Given the maximum allowable link utilization ρ_{\max}^c , this problem variant can be expressed as follows:

$$P2 \begin{cases} \max \{ f^c \} \\ \rho_{i,j} \leq \rho_{\max}^c, \forall (i,j) \in A \end{cases} \quad (7)$$

For solving the problem we are using the heuristic approach based on simulated-annealing as shortly discussed in [11]. During the search process, a temporary solution representing a certain W is chosen. Having this weight system and the given demand parameters, we can compute load distribution over the network and thereafter associate the corresponding solution with a quality measure according to the objective in (6) or (7). Although a solution is feasible if all link utilization values are less or equal than 100%, we do not explicitly apply this constraint i.e. a utilization value during the search process may exceed the value of 100%. Therefore, validity of final solutions needs always to be checked at the end of the optimization.

Varying Traffic Constraints. Looking at (1) it is obvious that providing minimal information f_{out}^u will result in a highly asymmetric situation since the terminating traffic at each node v is upperbounded by $\sum_{t \in N \setminus \{v\}} f_{\text{out}}^t$. This bound, as will be discussed in the next section, will be reached only in rare cases. To indirectly limit the terminating traffic to each node, we can additionally provide a second parameter $\varphi_{\text{out}}^u \leq f_{\text{out}}^u$ as the maximum capacity that can be occupied by a single flow (corresponding to a certain node pair). That is

$$f^{u,v} \leq \varphi_{\text{out}}^u; \forall v \in N \setminus \{u\} \quad (8)$$

Using this constraint we upperbound the terminating traffic at node v by the value of $\sum_{t \in N \setminus \{v\}} \varphi_{\text{out}}^t$ or $\varphi \cdot (|N| - 1)$ for the case of $\varphi_{\text{out}}^u = \varphi, \forall u$. Thus, (5) now can be expressed by:

$$l_{i,j}^u = \min(f_{\text{out}}^u \cdot \max_{v \in N \setminus \{u\}} \beta_{i,j}^{u,v}, \varphi_{\text{out}}^u \cdot \sum_{v \in N \setminus \{u\}} \beta_{i,j}^{u,v}) \quad (9)$$

The second possibility to indirectly limit the terminating traffic at each node is to generalize (1). Let Ω_r^u be defined as a set of destination nodes belonging to group r for traffic originating from u , where $\cup_r \Omega_r^u = N \setminus \{u\}$ and $\Omega_r^u \cap \Omega_s^u = \emptyset, s \neq r$. For each node u and Ω_r^u , we specify a maximum outbound traffic $f_{r,\text{out}}^u$. Thus, (1) and (5) can now be expressed by:

$$\sum_{v \in \Omega_r^u} f^{u,v} \leq f_{r,\text{out}}^u \quad (10)$$

$$l_{i,j}^u = \sum_r (f_{r,\text{out}}^u \cdot \max_{v \in \Omega_r^u} \beta_{i,j}^{u,v}) \quad (11)$$

In this case, the terminating traffic at v is upperbounded by $\sum_{t \in N \setminus \{v\}} \sum_r f_{r,\text{out}}^t \cdot \delta_r^{t,v}$, where $\delta_r^{t,v}$ having the value of 1 if Ω_r^t contains v , and 0 otherwise. Generalization can also be made for (8) by specifying the maximum flow to different groups of nodes

$\varphi_{r,\text{out}}^u \leq f_{r,\text{out}}^u$. In this case (8) and (5) are replaced by (12) and (13), respectively. The terminating traffic at v is now limited by $\sum_{t \in N \setminus \{v\}} \sum_r \varphi_{r,\text{out}}^t \cdot \delta_r^{t,v}$. Table 1 summarizes all possible uncertainty models together with outbound traffic constraints that have to be satisfied.

$$f^{u,v} \leq \varphi_{r,\text{out}}^u; \forall v \in \Omega_r^u \quad (12)$$

$$l_{i,j}^u = \sum_r \min(f_{r,\text{out}}^u \cdot \max_{v \in \Omega_r^u} \beta_{i,j}^{u,v}, \varphi_{r,\text{out}}^u \cdot \sum_{v \in \Omega_r^u} \beta_{i,j}^{u,v}) \quad (13)$$

| Model | Notation | Constraints |
|-----------------------------|----------|-------------|
| outbound | A1 | (1) |
| outbound + max-flow | A2 | (1) (8) |
| outbound + group | A3 | (10) |
| outbound + max-flow + group | A4 | (10) (12) |

Table 1: Several demand uncertainty models based on outbound traffic constraints.

| | | |
|-----------------------------|----------------------------------|---------------------------------|
| $\Omega_1^1 = \{2, 3, 8\}$ | $\Omega_1^6 = \{3, 5, 10, 14\}$ | $\Omega_1^{11} = \{4, 12, 13\}$ |
| $\Omega_1^2 = \{1, 3, 4\}$ | $\Omega_1^7 = \{5, 8, 10\}$ | $\Omega_1^{12} = \{9, 11, 14\}$ |
| $\Omega_1^3 = \{1, 2, 6\}$ | $\Omega_1^8 = \{1, 7, 9\}$ | $\Omega_1^{13} = \{9, 11, 14\}$ |
| $\Omega_1^4 = \{2, 5, 11\}$ | $\Omega_1^9 = \{8, 10, 12, 13\}$ | $\Omega_1^{14} = \{6, 12, 13\}$ |
| $\Omega_1^5 = \{4, 6, 7\}$ | $\Omega_1^{10} = \{6, 7, 9\}$ | |

Table 2: The parameter Ω_i^u for case study.

3 Results and Analysis

For the following discussion we use the network as shown in Figure 2 consisting of 14 nodes and 44 directed links (each of 2.5 Gbps capacity). The maximum outbound demand f_{out}^u for uncertainty model A1 and A2 is set as follows: 300 Mbps for nodes $\{10, 13\}$; 200 Mbps for nodes $\{2, 6, 7, 9, 11, 14\}$; and 100 Mbps for the rest of the nodes. For A3 and A4, destination nodes $N \setminus \{u\}$ are classified to two different groups ($\forall u$), where $f_{1,\text{out}}^u = 0.6f_{\text{out}}^u$ and $f_{2,\text{out}}^u = 0.4f_{\text{out}}^u$. Table 2 shows the parameter Ω_i^u for different nodes u and Ω_2^u is set as $N \setminus (\Omega_1^u \cup \{u\})$. The maximum flow φ_{out}^u for uncertainty model A2 as well as $\varphi_{1,\text{out}}^u$ and $\varphi_{2,\text{out}}^u$ for A4 is set homogeneously to the value of 40 Mbps.

Initial Network Utilization. Figure 3 shows the impact of each uncertainty model on link utilization resulting from inverse capacity metrics (denoted by InvCap), which in this case matches that resulting from unit metrics due to the homogeneity of link capacities. Figure 3(a) clearly indicates the benefit of inequality (8), by showing differences between link utilization calculated for A1 and for A2: the link utilization for A1 is much larger than that for A2. Figure 3(b) shows link utilization for uncertainty model A1 relative to that for A3 and signifies the benefit of (10). The maximum flow constraint (8) and the general outbound traffic constraint (10) are not dominating each other as displayed in Figure 3(c). These constraints can also be applied simultaneously to achieve better network efficiency as illustrated in Figure 3(d), which compares link utilization for A2 and A3 relative to that of A4. Figures 4(a) and 4(b) show link utilizations using InvCap metrics for uncertainty model A1 (the last histogram in both graphs) compared to randomly generated *traffic matrices*, that do not violate the constraints. Each histogram (except the last one) represents maximum utilization on each link for 10 different traffic matrices. Thus in each graph we compare the utilization computed by (5) with 100 randomly generated traffic matrices. In Figure 4(a) all of the aggregate demand f^u is carried by a single flow $f^{u,v}$, $u \neq v$, while in Figure 4(b) each element $f^{u,v}$ is randomly distributed in the interval $[0, f^u - \sum_{t=1}^{v-1} f^{u,t}]$; $u \neq t$, $u \neq v$. Using both demand generation strategies, the value of ρ_{max} found in all experiments is always below 40% and the number of links that have utilizations which exactly match that resulting from A1 is below the value of 25%. This fact supports the asymmetric property of (1) that has been addressed in Section 2.

Optimization Results Table 3 displays typical computation results with regard to some performance parameters. It basically shows the optimization results both for P1 and P2, compared to the performance obtained by the original routing pattern. The last three columns indicate: the number of different flows carried by a link (ω^{link}), the number of hops for a path (h^{path}) and the path delay (δ^{path}), which is modelled statically and mainly determined by propagation time. For the P1 case, using inverse capacity metrics, 56 flows are split and the maximum value of ρ_{max} is bounded by 57.6%. After optimization, it can be reduced to the value of 40.2% for multi shortest paths (MSP) case and correspondingly 44.2% for the unique shortest path (USP) case. Comparing the rows MSP and USP, the *probability* to obtain a better value of ρ_{max} is something that can be taken for granted, since the solution space for MSP is much larger than that for USP. But the smaller values for the number of flows that are split both for P1 and P2 compared to the InvCap case indicate that splitting traffic will not always bring better performance with respect to maximum utilization in the network. The average number of different flows carried by a link for the USP is lower than that for the MSP case. This can be seen as a logical impact of each routing strategy. The average values of the parameter h^{path} and δ^{path} do not differ very much implying that the network topology provides flexibility for routing. The maximum utilization before and after optimization for each uncertainty model is displayed in Figures 4(c) and 4(d). The first graph shows the optimization result based on uncertainty model A1, while the second graph illustrates that based on A2. With respect to the parameter ρ_{max} , a better routing pattern for a certain model is not necessarily better for the others. This can be seen in Figure 4(c), where the value of ρ_{max} : (a) for A2 after optimization is worse than that before optimization; and (b) for A3 after optimization in the USP case is

better than that in the MSP case. A similar situation can also be seen in 4(d), especially by comparing the results in both USP and MSP case for uncertainty model A3 with those for the other models.

4 Conclusion

In this paper we have considered the problem of offline metric-based traffic engineering for uncertain demands. Traffic uncertainty is modeled using several simple outbound traffic constraints, that are intuitively trackable and particularly appropriate for TE approach based on heuristic or metaheuristic frameworks. Our results for the basic model A1 show that it is sufficiently flexible to capture a large set of traffic variations, including those with very high asymmetric properties, where all originating traffic is entirely concentrated in flows to a single destination node. Specifying more demand information as maximum flow or outbound traffic to groups of nodes instead of that to a single group, could significantly save network resources although this might also reduce the number of traffic variations being supported.

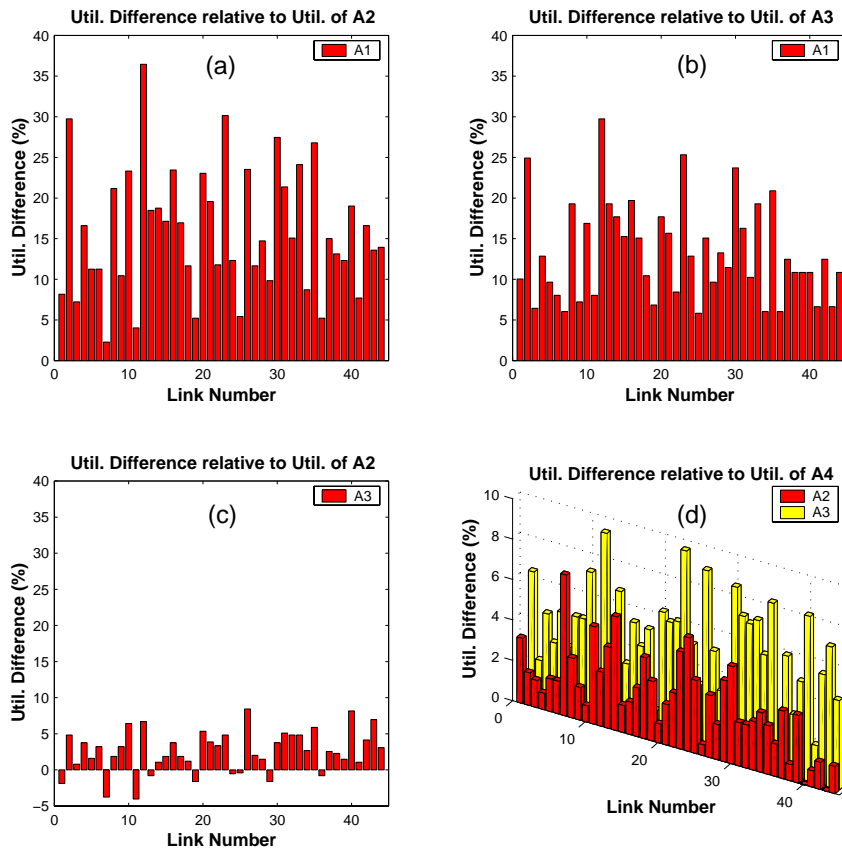


Figure 3: Comparison of link utilization for different models using inverse capacity (InvCap) metrics.

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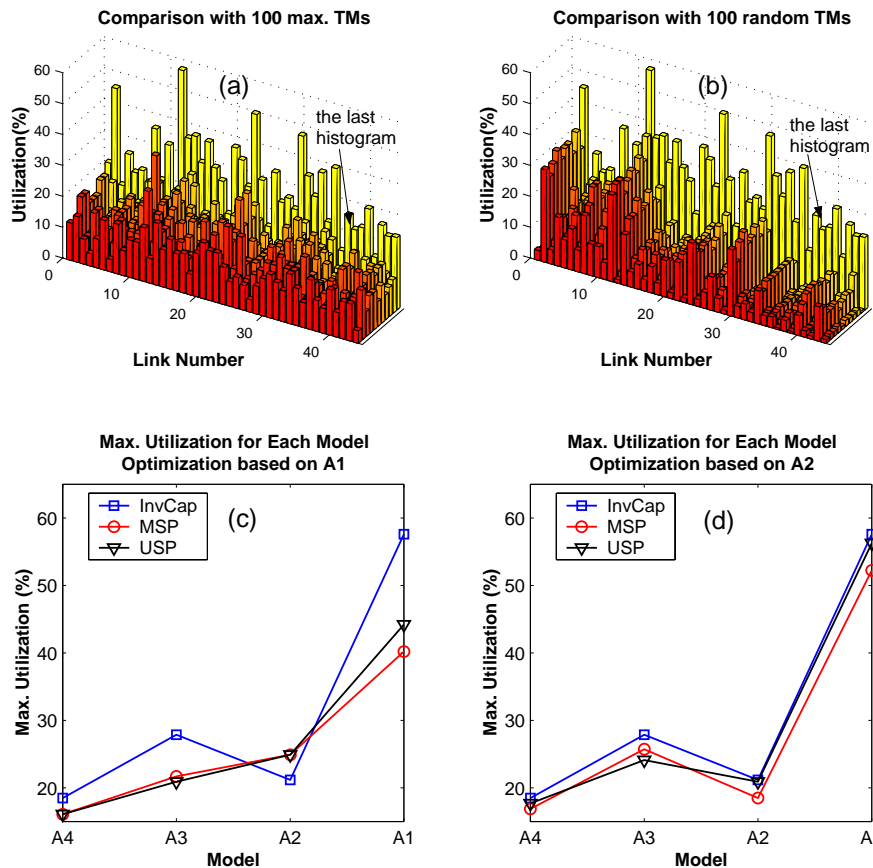


Figure 4: Comparison of link utilization resulting from A1 model versus that from randomly generated fixed traffic matrices using InvCap metrics (a)(b); Comparison of the maximum utilization before and after optimization based on A1 (c) and based on A2 (d).

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| | | f^c (Mbps) | ρ_{\max} (%) | $\bar{\rho}$ (%) | #flows (split) | ω^{link} | | h^{path} | | δ^{path} (ms) | | |
|--------|-----|-----------------|----------------------|---------------------|-------------------|------------------------|-------|-------------------|-------|-----------------------------|---------|-------|
| | | | | | | max | ave | max | ave | max | ave | |
| InvCap | A1 | — | 57.61 | 28.50 | 56 | 17 | 12.68 | 3 | 2.179 | 43.542 | 18.8868 | |
| | A2 | — | 21.17 | 12.69 | | | | | | | | |
| | A3 | — | 27.87 | 15.26 | | | | | | | | |
| | A4 | — | 18.49 | 10.18 | | | | | | | | |
| P1 | MSP | A1 | — | 40.19 | 2 | 17 | 9.18 | 4 | 2.196 | 38.6 | 18.2794 | |
| | | A2 | — | 24.92 | | | | | | | | 12.97 |
| | | A3 | — | 21.70 | | | | | | | | 15.49 |
| | | A4 | — | 16.08 | | | | | | | | 10.34 |
| | USP | A1 | — | 44.21 | 28.50 | 0 | 19 | 9.09 | 4 | 2.198 | 48.56 | 18.33 |
| | | A2 | — | 24.92 | 12.72 | | | | | | | |
| | | A3 | — | 20.90 | 15.34 | | | | | | | |
| | | A4 | — | 16.08 | 10.34 | | | | | | | |
| P2 | MSP | A1 | 180.9 | 40 | 30.1 | 8 | 17 | 9.6 | 4 | 2.2083 | 43.5 | 18.28 |
| | USP | A1 | 142.2 | 40 | 23.6 | 0 | 19 | 9.1 | 5 | 2.2088 | 53.5 | 19.35 |

Table 3: Some typical computation results for optimization based on A1.