Network Formation

Preliminary and Incomplete

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Abstract

I study a network based mechanism of norm enforcement in a community where agents play a Prisoner's Dilemma with changing partners. Agents may choose a number of close friends. Communication to close friends within the network enforces cooperation. In a world with noiseless communication, it is optimal that each agent has close contacts to all other network members. Moreover, it is optimal to have a huge network size. If communication is noisy a lower number of close contacts is optimal. As the number of network members gets large and network members receive noisy information from all network members, norm enforcement fails. *JEL Classification Numbers: Keywords:*

1 Introduction

Network based mechanisms are important institutions to enforce trust and cooperative behavior in communities. The mechanisms work through communication and fear of punishment for misbehavior or anticipation of rewards in case of good behavior. For example, ethnic communities in the US like the Dominicans in New York, the Cubans

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in Miami use such systems to sustain informal credit channels (see Portes and Sensenbrenner [22]). The well known rotating credit associations among Asian immigrants, for example the Chinese on Java, rely on trust enforcement through network mechanisms as well (see Granovetter [13]).

The number of close friends network members have among each other is an important input to foster cooperation. Among close friends, information, judgements, gossip and so on are exchanged. A dense network, that is, a network where each network member has many close ties seems beneficial since each network member may use these ties as an effective threat to enforce cooperative behavior. It is less clear what the costs of a dense network are. In fact, an exacerbation of the obligations within a network can conspire exactly against the network.[1] Boissevain [6] shows with an example of two Maltesian inhabitants that people generally do not maintain close ties to all members in their network. The question is hence what an optimal number of contacts or close friends for each network member is and what might constrain the number of close friends in one's network.

Another important dimension with direct consequences for economic behavior is the network size, that is, the overall number of participants in a network. Granovetter [13] notes the success of the overseas Chinese on Java based on rotating credit associations while the Javanese themselfes failed in establishing such a mechanism. Being immigrants, the Chinese are a small community in relation to the Javanese: in a Javanese town dubbed Modjokuto they numbered 1.800 out of a total of approximately 18.000. Granovetter offers the following explanation. Successful Javanese face demands for a piece of the cake achieved from an unlimited number of other Javanese (relatives, kins, etc.). The Chinese immigrants did not suffer from such excessive claims, since their immigrant status simplified the process of "decoupling" from relatives and kins. I shall offer an alternative hypothesis: on the one hand, cooperating network members benefit from a huge number of network members since it is more likely that they interact with other - cooperating - network network members. On the other hand, large communities are plagued by gossip which might make monitoring more difficult. In

[1] see Portes and Sensenbrenner for interesting examples like faulty assaults or constraints on freedom.

particular, the number of close friends which is required to sustain cooperation might then be prohibitively high.

The present paper adresses these two issues in the setting of a repeated Prisoner's Dilemma with changing partners. I show that in a world with noisy communication it is in general not optimal to have close contacts to all other network members. Moreover, as the size of the network gets large, cooperation fails; if network formation is costly, no network sustaining cooperation might form at all.

I start out with a framework of a repeated game with changing partners à la Kandori [18]. Before repeated interaction starts, a subset of all agents form a network and each network member chooses a number of other network members - close contacts - to whom he communicates the events in each of his per-period interactions. If communication is frictionless harshest punishments are achieved by choosing as many contacts as possible. Moreover, it is optimal to have as many network members as possible. However, if communication is noisy, it is no longer optimal to have a maximum number of contacts possible. The reason is that noisy communication might lead to sanctions with positive probability although there was no misbehavior. This lowers the overall benefit from cooperation if the number of contacts is too large. The message of this is that in an optimal network there should not be communication and information flow among all members. However, private incentives to maintain close contacts might differ significantly from social incentives, so that an overinvestment in relations may occur.

Moreover, in a community there is gossip and individuals receive noisy messages from all network members. Then, as the number of network members gets large, cooperation cannot be sustained anymore. Intuitively, if the number of network members gets large, agents cannot distinguish between gossip information and truthful information; cooperation fails. This implies that if network formation were costly no network would form! The finding suggests also why in many communities we do *not* decentralized cooperation networks. If the number of community members gets large, other institutions such as courts are needed (see Milgrom, North and Weingast [20]).

The paper is organized as follows. Section 2 sets up the model. Conditions for a network equilibrium without noise are derived in section 3. Section 4 contains the result on the optimal number of contacts with noise, while section 5 presents the result on cooperation failure when the number of network members gets large. Section 6 discusses this paper's relation to the literature.

2 The Model

There are $i = 1, ..., I > 2$ agents, where I is even. Time is discrete, $t = 1, 2, ...$ In each period, all agents get matched pairwise. If a match forms, the two agents play the following stage game.

	C	D
C	a, a	b, c
D	c, b	d, d

with $c > a > d > b$.

Each agents only observes his own private history but does not observe the per-period events in matches he is not involved in. Moreover, each agent does not observe the identities of his trading partners. For each agent, a strategies is a function from his private history to his action set. Each agents maximizes his average discounted payoff. Let $\delta \in (0, 1)$ denote the discount factor for all agents. This setting is equivalent to the ones studied by Kandori [18] or Ellison [10]. However, whereas Kandori and Ellison have punishments relying on contagion effects coming back to hit a noncooperator, I shall consider a more direct punishment mechanism. My direct punishments work for discount factors even smaller than the discount factor required to sustain Kandori's or Ellison's mechanism. I shall discuss this aspect in more detail below.

Network and Equilibrium Definition

Suppose that $N \leq I$ agents form a **network**. The number N is exogenous. With network formation the following assumptions hold.

(N1) in period 0 - before repeated interaction starts - all network members choose simultaneously $L_i \leq N - 1$ contacts or close friends. Agents who are not network members do not have this choice. Let $\mathcal{L}_i$ denote the set of contacts agent i has and let $\mathcal{N}$ denote the set of network members.

(N2) Relations among agents are not symmetric, that is $j \in \mathcal{L}_i \nRightarrow i \in \mathcal{L}_j$ for all $i, j \in \mathcal{N}$. Hence, agents $i \in \mathcal{N}$ and $j \in \mathcal{N}$ have mutual contacts to each other if and only if $j \in \mathcal{L}_i$ and $i \in \mathcal{L}_j$. Moreover, relations among agents are not transitiv, that is, $i \in \mathcal{L}_j \wedge z \in \mathcal{L}_i \nRightarrow z \in \mathcal{L}_j$ for all $i, j, z \in \mathcal{N}$.

(N3) repeated interaction is now as follows: in any period $t = 1, 2, ...,$ agents get matched pairwise. If two network members i and j meet they observe the number of contacts L_i and L_j of their partner and then choose their action, C or D.

(N4) after network members i and j have played the stage game, both i and j inform their respective contacts (L_i contacts for i and L_j contacts for j) of their partners' actions in the match. This information includes not only the action of, say, i but also his name.

(N5) all agents can get matched both with agents in the network and with agents outside the network.

(N6) all agents inside the network know and recognize the identities of other network members.

I do not model the process of network formation. I take a network as given and ask wether network members are willing to participate. A network of size N is beneficial if and only if all network members are willing to participate. This participation constrained is required to hold for all histories of play.

I consider equilibria involving the following strategies:

(E1) agents outside the network choose D in each period

(E2) agents inside the network choose C if they have no special information.

(E3) if a network member i deviated in a match with network member j, then j informs all his L_j contacts about this. From the next period on those L_j contacts punish i forever after.

(E4) Once a punishment phase has started (which implies that action D is played in all matches where punishment takes place), noncooperators cannot trigger punishments on their punishers.

To understand (E4), suppose that network member i did not cooperate when playing with network member j and that j informs all his L_j contacts about this. Assumption (E4) says that noncooperator i cannot trigger punishments on his punishers by informing his L_i contacts.

Let σ_n denote the strategy profile for network member described above while σ_{-n} denotes the strategy profile for all agents not in the network. The strategy profile for all agents is denoted by σ. The payoff $V_i^n(\sigma)$ for a network member, net of opportunity cost d, is given by

$$V_i^n(\sigma) = \frac{N-1}{I-1}a + \frac{I-N}{I-1}d - d. \tag{1}$$

The average expected payoff $V_i(\sigma)$ for an agent not in the network is $V_i(\sigma) = d$.

Definition 1. *In an* **equilibrium with network formation***, (N1)-(N6) and (E1)-(E4) hold. Moreover, for all $i \in \mathcal{N}$ and for all histories of play,*

$$V_i^n(N^*) \geq 0. \tag{2}$$

3 Equilibrium

Lemma 1. *Truthful information transmission is always a best response.*

Proof. Pick agent $i \in \mathcal{N}$. Suppose that j in a match with i plays C. If agent i informs his L_i contacts that j played D, they punish C forever after, which does not increase i's payoff. Suppose that j played D. If agent i does not inform his contacts about this, his payoff does not increase either. □

W.l.o.g., let $d = 0$. To have network members cooperate, the following incentive constraint has to hold for all $i \in \mathcal{N}$:

$$\frac{N-1}{I-1}a \geq \frac{N-1}{I-1}c + \frac{\delta}{1-\delta}\left(\frac{N-L_j-1}{I-1}a\right) \tag{3}$$

for all $j \in \mathcal{L}_j$.

The benefit of cooperation is given by the probability of meeting a network member, $(N-1))/(I-1)$ times the payoff of cooperation, a. The benefit from defection is the defection payoff c and the continuation payoff. The continuation payoff from defection is zero with probability $L_j/(I-1)$ and with probability $(I-N)/(I-1)$ and a with probability $(N-L_j-1)/(I-1)$. Due to Lemma 1 it is always a best response for j's L_j contacts to punish i.

Solving inequality (3) yields

$$L_j \geq L^\circ = \frac{(N-1)(1-\delta)(c-a)}{\delta a}$$

as the minimum number of contacts each network members needs to have in order to sustain cooperation.[2]

To determine the number of contacts agents choose in period 0, I focus on network members' equilibrium choices which induce extremal equilibria in the sense of Abreu, [1] or [2]. That is, I consider a choice of close contacts such that any deviation by network member i is punished by play switching to the perfect equilibrium in which that player's payoff is lowest. Hence, it is optimal for each agent during the stage of contact formation that every agent chooses $L_i = N-1$. This number of links provides the harshest punishment for a noncooperator.

Considering network size, note that equilibrium payoff are increasing in N so agents prefer large networks. If $c > a/(1-\delta)$, the number of contacts required to sustain cooperation would strictly exceed $N-1$; cooperation based on a network mechanism fails in that case. The following Proposition summarizes.

[2]This number is actually $[L^\circ]+1$, where $[x]$ denotes the next largest integer to x.

Proposition 1. *(i) Cooperation is sustainable in the network if and only if*

$$L_i \geq \frac{(N-1)(1-\delta)(c-a)}{\delta a}$$

for all $i \in \mathcal{N}$.

(ii) Suppose that contact choices induce extremal equilibria. For all $N^* \geq 2$, it is then optimal to form a network in which each network member chooses $L_i = N-1$ contacts. Then punishment is maximal and cooperation in the network can be sustained for $\delta \geq (c-a)/c$.

(iii) Since equilibrium payoffs are increasing in N, it is optimal to have all agents in the network.

(iv) If $c > a/(1-\delta)$ it is not possible to sustain cooperation.

In Kandori [18] and Ellison [10] cooperation is sustained by punishments relying on contagion effects coming back to hit a noncooperator: if an agent defects, he plays action D from then onwards and so do all his partners on whom he has defected.

Proposition 2. *The discount factor necessary to sustain cooperation through network formation is smaller than the discount factor necessary to sustain cooperation through contagion.*

Proof. Cooperation through contagion requires a larger discount factor than cooperation through network formation if a noncooperator's continuation payoff after defection is larger.

With network formation, a noncooperator is punished immediately forever after a deviation. Hence, a noncooperator's continuation payoff after a deviation is zero.

In punishments relying on a contagion effects and it lasts at least $(I-2)/2$ periods until a noncooperator is punished with probability 1 in every period. Hence, in the first $(I-2)/2$ periods after a deviation there is always a strictly positive probability that a noncooperator is not punished. This implies that the infimum of a noncooperator's continuation payoff is strictly bounded away from zero. □

4 Noisy Communication

In the previous section it was optimal for each agent to chose as many contacts as possible. This choice was optimal since contacts never became active in equilibrium. A huge number of contacts or contacts is a very effective threat! However, in many networks, the same mechanism which supports cooperative actions may also be detrimental to the agents in the network. If my partner has many contacts, this is beneficial since it helps in sustaining cooperation. On the other hand, if communication is noisy, those contacts might not cooperate with me although I myself did cooperate. It may then be optimal if my partner has fewer than $N-1$ contacts. I shall derive the optimal number of contacts when there is noise in the transmission of information.

I model the presence of noise as follows. Suppose that there is noise in the stage of the game where each agent informs his L_i contacts about the behavior of his partner j in a given period. Two things can happen: the partner j of agent i did deviate, but nobody of i's contacts received the message and all of i's contacts continue to believe that j did cooperate. Or, j did not deviate, but the L_i contacts of agent i did receive the message that agent j did deviate. Each agent reports still truthfully what happenend in any period to his L_i contacts but information transmission is noisy.

Formally, each match between two agents generates one of two signals at each period. The signal space is given by $x \in X = \{\underline{x}, \overline{x}\}$ and is the same for all matches. Let, for all $i, j \in \mathcal{N}$,

$$\begin{aligned}
\alpha &= \Pr(L_i \text{ contacts receive signal } \underline{x} \mid j \text{ played } C) \\
1-\alpha &= \Pr(L_i \text{ contacts receive signal } \overline{x} \mid j \text{ played } C) \\
\beta &= \Pr(L_i \text{ contacts receive signal } \underline{x} \mid j \text{ played } D) \\
1-\beta &= \Pr(L_i \text{ contacts receive signal } \overline{x} \mid j \text{ played } D).
\end{aligned}$$

Let $\beta < \alpha$. All contacts of a network members receive the same signal.

The noise in communication may be interpreted as noise in the technology, for example email, which is used to transmit information. Another interpretation is that each agent, with positive probability, reports mistakenly as compared what actually happened. Although there is no direct incentive for doing so in the present framework, such behavior is observed: Portes and Sensenbrenner [22] cite studies of the rise of commercial enterprises in Bali. There, successful enterprises were assaulted by job- and loanseeking kinsmen.

The timing is now as follows: in period 0 all agents in the network choose their contacts. Then agents get matched. Each agent receives a signal which is generated by the previous match his partner was involved in. I assume that each agent obtains such a signal in all periods $t = 1, 2, ...$ Then, each agents observes the number of contacts his partner has and after they play the stage game. Actions in the stage game generate signals which again get transmitted to the agents' contacts. All this continues forever.

I consider equilibria with the following properties. Each network member cooperates if he receives the signal $\underline{x}$ ("good" signal) about the previous behavior of his new partner. If an agents receives a signal $\overline{x}$ ("bad" signal)the referring agent is punished forever after.

Proposition 3. (*i*) *There exists a symmetric equilibrium with every agent having L contacts, where*

$$L \geq \frac{(N-1)(1-\delta\alpha)(c-a)}{\delta(\alpha-\beta)a}.$$

Punishment is induced by bad signals, $\overline{x}$, providing incentives for cooperation which occurs on good signals, $\underline{x}$.

(*ii*) *Equilibrium payoffs are equal to*

$$V_i^+ = \frac{[(1-\delta)(N-1)+\delta(1-\alpha)(N-L-1)]a}{(I-1)(1-\delta\alpha)} \tag{4}$$

for all $i \in \mathcal{N}$ and are monotonically decreasing in L.

(*iii*) *The optimal number of contacts for each network member L^* is given by*

$$L^* = \frac{(N-1)(1-\delta\alpha)(c-a)}{\delta(\alpha-\beta)a}. \tag{5}$$

(*iv*) *It is beneficial to have a network if and only if*

$$a > \frac{(1-\alpha)}{(1-\beta)}c. \tag{6}$$

If (6) holds, it is optimal to choose N as large as possible

Proof. Denote by V_i^+ the payoff from the proposed equilibrium strategy profile. It is given by

$$V_i^+ = (1-\delta)\frac{N-1}{I-1}a + \delta\alpha V^+ + \delta(1-\alpha)\frac{N-L_j-1}{I-1}, \tag{7}$$

where L_j are the contacts each partner $j \in \mathcal{N}$ of i has. Using symmetry, $L_j = L$ for all $j \in \mathcal{N}\backslash i$, this can be rewritten as

$$V_i^+ = \frac{[(1-\delta)(N-1) + \delta(1-\alpha)(N-L-1)]a}{(I-1)(1-\delta\alpha)}. \tag{8}$$

This proves part (ii) of the Proposition. Moreover, the following incentive constraint has to hold.

$$V_i^+ \geq (1-\delta)\frac{N-1}{I-1}c + \delta\beta V_i^+ + \delta(1-\beta)\frac{N-1}{I-1}a. \tag{9}$$

Plugging in the expression for V_i^+ one obtains for the incentive constraint

$$\frac{[(1-\delta)(N-1) + \delta(1-\alpha)(N-1)]a}{(I-1)(1-\delta\alpha)}(1-\delta\beta) \geq$$
$$(1-\delta)\frac{N-1}{I-1}c + \delta(1-\beta)\frac{N-1}{I-1}a. \tag{10}$$

The problem of the network is to maximize each network member's utility, that is,

$$\max_{L} \quad V_i^+$$

for all $i \in \mathcal{N}$ subject to (10). As can be seen, it is in the interest of each agent i that L is chosen as low as possible. The solution is hence given by equating the incentive constraint, solving for L and denoting the solution L^*. This proves parts (i) and (iii) of the Proposition.

Equilibrium payoffs are

$$V_i^+ = \frac{N-1}{I-1}\left(\frac{(1-\beta)a - (1-\alpha)c}{\alpha-\beta}\right). \tag{11}$$

for all $i \in \mathcal{N}$. Hence, there is network formation if and only if

$$a > \frac{(1-\alpha)}{(1-\beta)}c. \tag{12}$$

If this condition holds, it is still optimal to choose N as large as possible. This proves part (iv). □

The proposition states that it may well be optimal to restrict the number of contacts and close contacts each agent has. In particular,

$$L^* < N - 1 \quad \Leftrightarrow \quad a > \frac{(1-\delta\alpha)}{(1-\delta\beta)}c.$$

If the benefit from cooperation, a, is large, it is optimal to have less than $N-1$ contacts in the network. Hence, the optimal network might not be connected. [3] Connectedness is a standard assumption in much of recent work on social learning and local interaction; see e.g. Anderlini and Ianni [3], Ellison [9] or Ellison and Fudenberg [11].

The *private incentives* for network formation differ significantly from the social incentives. It is easy to see that, for each network member, a choice of $N-1$ contacts is again an equilibrium of the network formation game in period 0 if the contacts choice is supposed to induce extremal equilibria.

As one easily checks, the following comparative statics results hold.

$$\frac{\partial L^*}{\partial a} < 0, \quad \frac{\partial L^*}{\partial \delta} < 0, \quad \frac{\partial L^*}{\partial c} > 0, \quad \frac{\partial L^*}{\partial N} > 0, \quad \frac{\partial L^*}{\partial \alpha} < 0, \quad \frac{\partial L^*}{\partial \beta} > 0 \qquad (13)$$

The optimal number of contacts decreases as α increases (β decreases). Less noise in communication reduces the need for powerful punishments so less close contacts are necessary to sustain cooperation. On the other hand, if either the network size N or incentives for deviation, c, is large many contacts are needed.

Ellison [10] constructs an equilibrium based on contagion effects that approaches the cooperative level as the amount of noise tends to zero and as the discount factor tends to one. It remains an open question, which institution - network or contagion - is more beneficial for the agents if the amount of noise remains positive for a *given* discount factor.

5 Gossip

In the previous section, the distribution of the signal depended only on the outcome of any given match. In this section I add "more" noise to the situation and model

[3] In the language of graph theory, a network is connected if there is a path between every pair of agents.

network gossip. I assume that each agent is informed about the behavior of a friend's partner not only through a more or less informative signal which is generated from any given match. Rather, each agent receives a noisy message from each other network member. The interpretation of this is that even network member who did not observe an agent's behavior gossip about what that agent did.

Formally, for any given agent i, the new partner of i in any given period receives now

- one signal which is informative according to the above defined probabilities α and β. It still holds that $\alpha > \beta$. Suppose for simplicity that $\alpha = 1$ und $\beta = 0$.
- and also signals from all the other $N-2$ agents in the network. Those agents did not observe what i did in the previous period. So, they gossip and transmit their gossip to the new partner of i: each of the $N-2$ signals can be $\underline{x}$ with probability γ and $\overline{x}$ with probability $1-\gamma$. The probability γ is not conditioned on the action agent i and his partner took in the previous period. An agent cannot observe who sent which signal, the signal technology is hence anonymous in the sense of Green [14].

Hence, each agent receives $N-1$ signals about the behavior of his new partner. Each agent then uses a simple rule to evaluate the $N-1$ signals and to condition his future actions on the signals.

- If a sufficiently large share $K \in (0,1)$ of the signals is $\underline{x}$, an agent cooperates with his new partner. Otherwise, he chooses action D.

Let $|\underline{x}|$ denote the number of signals with the value $\underline{x}$ an agent received about his new partner and define, for all $i, j \in \mathcal{N}$,

$$\begin{aligned}
\widehat{\alpha} &= \Pr(L_i \text{ contacts receive at least } (N-1)\cdot K \text{ signals } \underline{x} \mid j \text{ played } C) \\
\widehat{\beta} &= \Pr(L_i \text{ contacts receive at least } (N-1)\cdot K \text{ signals } \underline{x} \mid j \text{ played } D).
\end{aligned}$$

I assume $\widehat{\alpha} > \widehat{\beta}$. All network members receive the same signals. I shall for further use rewrite these probabilities as

$$\widehat{\alpha} = \Pr(|\underline{x}|/(N-1) \geq K|C)$$

and

$$\widehat{\beta} = \Pr(|\underline{x}|/(N-1) \geq K|D).$$

Since these probabilities are the same for $i, j \in \mathcal{N}, j \neq i$, any subscripts are omitted. Basically, we have now a situation as in section 4, but with "more" noise.

Given the probabilities $\widehat{\alpha}$ and $\widehat{\beta}$, one can repeate the exercise from the previous section and solve for the number of L contacts each agent receives. It is obvious from the previous section that this number L is given by

$$\widehat{L}^* = \frac{(N-1)(1-\delta\widehat{\alpha})(c-a)}{\delta(\widehat{\alpha}-\widehat{\beta})a}. \tag{14}$$

I now analyse the possibilities for network cooperation as N gets large. Recall that the incentive constraint is given by

$$\begin{aligned}(1-\delta)\frac{N-1}{I-1}a + \delta\widehat{\alpha}V^+ + \delta(1-\widehat{\alpha})\frac{N-L_j-1}{I-1}a \geq \\ (1-\delta)\frac{N-1}{I-1}c + \delta\widehat{\beta}V^+ + \delta(1-\widehat{\beta})\frac{N-L_j-1}{I-1}a,\end{aligned} \tag{15}$$

for all $i, j \in \mathcal{N}, j \neq i$. This can be rewritten as

$$\widehat{\alpha} - \widehat{\beta} \geq \frac{(N-1)(1-\delta\alpha)(c-a)}{\delta L_j a}. \tag{16}$$

Proposition 4. *There exists $\overline{N}$ such that for all $N \geq \overline{N}$ cooperation cannot be sustained.*

Proof. Note that

$$\widehat{\alpha} = \Pr(|\underline{x}|/(N-1) \geq K|C) = \Pr(|\underline{x}|/ \geq K(N-1)-1|C).$$

This follows from the fact that in case of the partner's action being C, any new partner of any of the agents involved in the match obtains one signal $\underline{x}$ for sure (Recall that

$\alpha = 1$). This lowers the critical bound of other signals an agent has to obtain to continue with cooperation.

On the other hand, since $\beta = 0$, we have

$$\widehat{\beta} = \Pr(|\underline{x}| \geq K(N-1)|D) = \Pr(|\underline{x}| \geq K(N-1)|D).$$

The argument is now that $\lim_{N\to\infty} |\widehat{\alpha} - \widehat{\beta}| = 0$ while the right hand side of (16) tends to infinity as $N \to \infty$. To see this, note that

$$\widehat{\beta} = 1 - \sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k}$$

and

$$\widehat{\alpha} = 1 - \sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k}.$$

Then, $\widehat{\alpha} - \widehat{\beta}$ is given by

$$\sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k} -$$

$$\sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k}.$$

Note that

$$\sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k} =$$

$$\sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k} + \gamma^{K(N-1)}.$$

Hence, $|\widehat{\alpha} - \widehat{\beta}| = \gamma^{K(N-1)}$ and $\lim_{N\to\infty} |\widehat{\alpha} - \widehat{\beta}| = 0$. So there exists $\varepsilon > 0$ such that $|\widehat{\alpha} - \widehat{\beta}| < \varepsilon$ for all $N \geq \overline{N}$. Moreover, as $N \to \infty$, the right hand side of (16), tends to infinity. But since the left hand side of the equation cannot be larger than ε, there exists $\overline{N}$ such that for all $N \geq \overline{N}$ equation (16) does not hold.

It is straightforward to extend the Proposition for any $\alpha \in (0, 1)$ and any $\beta \in (0, 1), \alpha > \beta$.

□

This result might provide an explanation for the fact that immigrant groups are sometimes more successful in business than a country's inhabitants. Suppose that there is a small cost which each network member has to bear or forming the network. Proposition 4 implies that, for N large enough the network does not form at all since it does not yield any cooperation gains at all. Immigrant groups, at least in some cases, are small relative to the home population (recall the example from the Chinese experience on Java from the introduction).

The finding also suggests why in many communities we do *not* decentralized cooperation networks. If the number of community members gets large, other institutions such as courts are needed (see Milgrom, North and Weingast [20]).

6 Related Literature

This work is related to a number of literature which study networks in a social science context. There is an extensive literature on social networks from a sociological perspective (see Boissevain [6] or Wellmann and Berkowitz [24]). Moreover, there is a growing literature on strategic network formation which focusses on individual incentives to form links, e.g. Bala and Goyal [5], Dutta and Mutuswami [8]) or Jackson and Wolinsky [17]. In particular, Bala and Goyal mention the role of a network as a means of information flow. These works do not model explicit benefits and costs of network formation as the present paper does but rather use value functions to allocate the surplus among network members. Costs are exogenous and interpreted as cost for time and effort spent to maintain links. Papers which explicitly model network benefits from network formation in a more microeconomic context are Boorman [7]on the impact of networks on job search and Kranton and Minehart [19] who analyse buyer-supplier networks. There is also a formal cooperative-game theoretic literature which includes, for example, games with communication structure (Aumann and Myerson [4], van den Nouweland [21].)

Greif [16] and Milgrom, North and Weingast [20] model medieval institutions such as courts and trader coalitions supporting cooperation in trading games with changing partners. Milgrom, North and Weingast model the role of courts in enforcing cooperation in a basic setting similar to the one in my paper. As mentioned above, the existence of courts might be explained by the fact that large communities are not able to use decentralied networks to enforce cooperation (Proposition 4). Greif's remarkable work on the Maghribi trader's coalition studies an efficiency wage based mechanism to support cooperation in a repeated game and complete information framework. Both papers do not consider any cost of network formation.

On a more theoretical view, this work draws on the literature on repeated games with imperfect monitoring (Fudenberg, Levine and Maskin [12], Green [14], Green and Porter [15] and Sambourian [23]) and repeated games with random matching. Kandori [18] and Ellison [10] are the most influential works in repeated games with changing partners. They both propose a contagion mechanism to enforce cooperation. In contrast to Kandori, Ellison uses for most of his analysis a public randomization device to prevent some of the problems in Kandori's analysis. For example, Kandori's contagion equilibrium might only work for extreme stage game payoffs. While punishments in these papers work without any institutions, the present paper, as the works by Milgrom, North and Weingast and Greif, suggests that institutions might perform better since they provide more effective way of punishments. Kandori also proposes an exogenously given mechanism of information transmission to support cooperation.

Finally, several authors have considered the possibility that large population models may prevent cooperation in repeated games (see Ellison [10], Green [14] and Sambourian [23]). Green and Sambourian discuss models with a noisy observation of an aggregate statistic in a setting where agents do not change partners. My work is an extension of their results to the case of random matching. Ellison's analysis suggests that large population might not be all that bad, however, the amount of noise tends to zero in his analysis.

The main contribution of this paper lies in the explicit modelling of benefits *and* cost of network formation in a context where information flow provided by a network is significant: in repeated interaction with changing partners. In particular, while all of the literature mentioned above takes cost of network formation exogenous, I model costs endogenous. This comes from the view that the mechanism which supports cooperation through contacts in the first place might be detrimental to the community if applied to heavily. The paper also points out that networks might not function without frictions, an aspect which is missing in almost all of the network literature cited above[4]. Moreover, the paper analyzes benefits and costs of the overall network size and offers an alternative hypothesis to existing explanations in sociology (see the introduction of this paper and Granovetter [13]) .

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