



On Determining Dipole Moments of a Magnetic Torquer Rod — Experiments and Discussions

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ABSTRACT

This paper describes the tests undertaken to determine the dipole moments of a magnetic torquer rod. Test methodology, experimental setups, and test results are presented in detail. The dipole moments of the torquer rod are determined by using different experimental setups and are also compared with those provided by the manufacturer of the devices. Certain setups would yield satisfactory results while others would not. The factors that have major effects on the magnetic field of the torquer rod are discussed based on our experiments.

RÉSUMÉ

Ce mémoire décrit les essais entrepris pour déterminer les moments dipolaires du coupleur magnétique. Le mémoire présente en détail la méthode employée pour réaliser les essais, les configurations expérimentales utilisées ainsi que les résultats obtenus. Les moments dipolaires du coupleur magnétique sont calculés grâce à diverses configurations expérimentales et sont même comparées aux configurations fournies par le fabricant du dispositif. Dans certains cas, quelques configurations pourraient donner des résultats satisfaisants. Les facteurs qui peuvent avoir une influence majeure sur le champs magnétique sont abordés dans le cadre des expériences réalisées.

1. INTRODUCTION

Magnetic torquer rods (also known as torquers, torque rods, torque bars, torqrods, or magnetorquers) are widely used as attitude control system (ACS) actuators for geostationary satellites, small satellites, and microsattellites. A magnetic torquer rod is essentially a long copper wire wound around a core. The core material could be special alloys or simply air. With a metal core, the magnetic torquer rod can generate a larger magnetic dipole moment at the expense of a larger residual moment as compared with the air-core type. Magnetic torquer rods are designed to generate controllable magnetic dipole moments that interact with the Earth's magnetic field and generate torques for active attitude control purposes for spacecraft (Sidi, 1997; Wertz, 1978). Torque T_{mag} of a magnetic torquer is given by the cross product of its magnetic dipole moment \mathbf{M} and the Earth's magnetic-field vector \mathbf{B} , i.e., $T_{\text{mag}} = \mathbf{M} \times \mathbf{B}$. Magnetic control systems have the characteristics of relatively light weight and require no moving parts, expendables, or complex hardware.

A necessary step before applying magnetic torquer rods in a satellite is to verify experimentally their properties, such as the linear range of the dipole moments, saturation moments, residual moments, power consumptions, time constants, etc. We have recently procured a set of magnetic torquer rods to be used in a microsattellite project from ZARM at the University of Bremen, Germany (Matthews, 1999). In the process of experimental verification of the torquers, we found that it was difficult to find information in the open literature on detailed experimental techniques of this kind. In addition, when measuring dipole moments of the magnetic torquer rods, we found that it was not trivial to obtain correct and accurate measurements. Special attention has to be paid to such factors as the configuration and the environment of the test setups. This paper presents our experience with the experiments undertaken to determine the dipole moments of the magnetic torquer rods. Test methodology, experimental setup, and the test results are included in this paper. The factors that would have significant effects on the test results are also discussed here. For the sake of simplicity, this paper is only concerned with one of the torquer rods mentioned above, and only Coil-A, between Coils-A and -B in the torquer rod, is considered.

In the rest of the paper, Section 2 presents a mathematical description of the magnetic field of a torquer rod. Section 3 describes our experiments and the results. Section 4 comments on some factors that have significant effects on the magnetic-

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field measurement of a torquer rod. Finally, Section 5 concludes the paper.

2. MAGNETIC FIELD OF A MAGNETIC TORQUER

The definition of variables for the magnetic torquer is as shown in **Figure 1**, where M represents the dipole moment of the torquer, θ is the angle with respect to the torquer axis, R is the distance from the center of the coil, and L is the effective coil length. Also, B is the magnetic-flux density, B_r and B_t are the radial and tangential components of B , respectively.

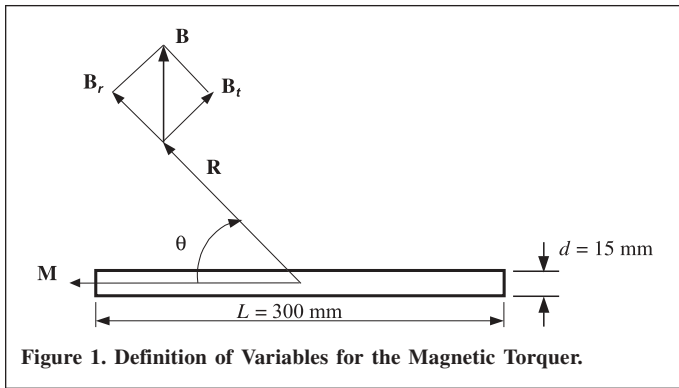


Figure 1. Definition of Variables for the Magnetic Torquer.

$$\mathbf{B} = \mathbf{B}_r + \mathbf{B}_t \quad (1)$$

The expressions to describe the magnetic field around a uniformly magnetized core of length L are found to be (Wiegand, 1999)

$$B_r = \frac{\mu_0}{4\pi} M \times \left[\frac{\frac{R}{L} - \frac{\cos \theta}{2}}{\left(R^2 - RL \cos \theta + \frac{L^2}{4} \right)^{3/2}} - \frac{\frac{R}{L} + \frac{\cos \theta}{2}}{\left(R^2 + RL \cos \theta + \frac{L^2}{4} \right)^{3/2}} \right] \quad (2)$$

$$B_t = \frac{\mu_0}{4\pi} M \times \left[\frac{\frac{\sin \theta}{2}}{\left(R^2 - RL \cos \theta + \frac{L^2}{4} \right)^{3/2}} + \frac{\frac{\sin \theta}{2}}{\left(R^2 + RL \cos \theta + \frac{L^2}{4} \right)^{3/2}} \right] \quad (3)$$

where $B_r = \|\mathbf{B}_r\|$, $B_t = \|\mathbf{B}_t\|$, $M = \|\mathbf{M}\|$, $R = \|\mathbf{R}\|$, and μ_0 is the vacuum permeability, which has the value $4\pi \times 10^{-7}$ H/m. The above equations are valid for the condition

$$R \geq L/2 \quad (4)$$

There are two special orientations along which the mathematical expressions of the magnetic field have relatively simple forms. One is in the axial direction of the torquer, where $\theta = 0^\circ$. The other is in the perpendicular direction along the mid-line of the torquer, where $\theta = 90^\circ$. The mathematical expressions of the magnetic field in these two directions are given as follows:

For $\theta = 90^\circ$:

$$\begin{cases} B_r = 0, \\ B_t = \frac{\mu_0}{4\pi} M \frac{1}{\left(R^2 + \frac{L^2}{4} \right)^{3/2}} \end{cases} \quad (5)$$

For $\theta = 0^\circ$:

$$\begin{cases} B_r = \frac{\mu_0}{4\pi} M \left[\frac{\frac{R}{L} - \frac{1}{2}}{\left(R^2 - RL + \frac{L^2}{4} \right)^{3/2}} - \frac{\frac{R}{L} + \frac{1}{2}}{\left(R^2 + RL + \frac{L^2}{4} \right)^{3/2}} \right] \\ B_t = 0 \end{cases} \quad (6)$$

3. DIPOLE MOMENT OF A MAGNETIC TORQUER

3.1. Introduction

We rewrite Equations (5) and (6) as follows:

If $\theta = 90^\circ$,

$$M = \frac{4\pi}{\mu_0} \cdot \left(R^2 + \frac{L^2}{4} \right)^{3/2} \cdot B_t \quad (7)$$



If $\theta = 0^\circ$,

$$M = \frac{4\pi}{\mu_0} \cdot \frac{1}{\frac{\frac{R}{L} - \frac{1}{2}}{\left(R^2 - RL + \frac{L^2}{4}\right)^{3/2}} - \frac{\frac{R}{L} + \frac{1}{2}}{\left(R^2 + RL + \frac{L^2}{4}\right)^{3/2}}} \cdot B_r \quad (8)$$

It is clear that the dipole moment of a magnetic torquer could be obtained if the magnetic-flux density is known. Now, the problem of obtaining the dipole moment of a magnetic torquer turns into the problem of obtaining the magnetic-flux density at a known location and orientation, which is discussed in the rest of this section.

3.2. Experimental Setup to Measure the Magnetic-Flux Density

The magnetic-flux density of the magnetic torquer can be measured using a magnetometer. Our experimental setup to measure the magnetic-flux density is shown schematically in **Figure 2**, where the magnetometer is a Mag-03MS made by Bartington Instruments Ltd. (Bartington Instruments Ltd., 1999), the magnetic torquer and its driver board is a MT6-2-CSA-001 and a DRV-5SN5, respectively, made by ZARM (Matthews, 1999). The labels X, Y, and Z represent the axes of the Bartington magnetometer. The positive Y-direction points into the paper and is not labeled. A photograph of part of the setup is shown in **Figure 3**.

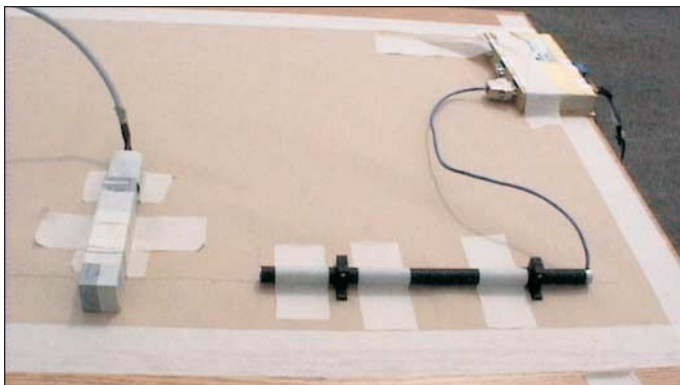
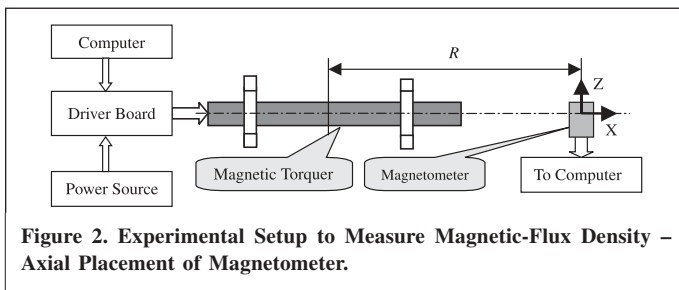


Figure 3. ZARM Magnetic Torquer, its driver, and Bartington Magnetometer.

However, it is not trivial to obtain the correct magnetic-field readings of the torquer using a magnetometer. It was found in our experiments that several factors have significant effects on the magnetic field of the torquer.

It was noticed that any metallic objects in the vicinity of a few metres of the magnetic torquer had a significant effect on the magnetic field of the torquer. These metal objects can be the metal structure of the building, the metallic components of the test equipment, the metal parts of the supporting table, and so on. It was also found that the position and the orientation of the magnetic-field sensor had significant effects on the magnetic field of the torquer or on the correctness of the measurement, which will be examined in more detail later in this paper.

To minimize the interference of the aforementioned factors on the magnetic field of the torquer, all tests were conducted in an open field. Further, special attention was paid to ensure there was no magnetic metal within 5 m of the torquer being tested. The only exception was the PC computer for gathering data and the DC power source of the magnetic torquer. The computer was 3.5 m away and the power source was 1.5 m away from the torquer. To minimize the interference of the magnetic-field sensor on the magnetic field of the torquer, the magnetometer was placed in the axial direction of the magnetic torquer, i.e., $\theta = 0^\circ$, and was 600 mm away from the center of the torquer, i.e., $R = 600$ mm. Satisfactory results were obtained with this experimental setup.

3.3. Test Procedure and Record

To obtain the magnetic-flux density, command voltage in the allowable operating range of $(-7, +7)$ V was applied to the magnetic torquer. The readings in the X-direction of the magnetometer are listed in **Table 1**. The sign of the measured data in **Table 1** follows the definition of the X-direction of the magnetometer in **Figure 2**. The test procedure was as follows:

- (1) The command voltage was applied starting from 0 V and gradually increased to 7 V, then subsequently decreased to 0 V again. Because of the hysteresis effect, the magnetic field at the second zero voltage was expected to be different from that at the starting zero voltage, which was observed in the measurement.
- (2) After reaching the second zero voltage mentioned above, there was a “power-off” period, which allowed the residual moment of the magnetic torquer to diminish. By trial and error, it was found that a period of 25 s would be enough to achieve this.
- (3) Following the above power-off period, the command voltage was gradually decreased from 0 to -7 V, and subsequently increased to 0 V again.

Each reading of the magnetic field in **Table 1** is composed of two components. One is generated by the magnetic torquer. The other is contributed by the Earth’s magnetic field. What is of interesting to us only is the component of the torquer. This component is obtained by subtracting the Earth’s magnetic field from the measurement. In this work, the component of the

**Table 1. Test record and dipole moments of the magnetic torquer.**CSA-001 (Coil A) for $\theta = 0^\circ$ and $R = 600$ mm

No.	Voltage (V)	Test 1		Test 2	
		Magnetic flux density in X-direction (G)	Dipole moment (Am^2)	Magnetic flux density in X-direction (G)	Dipole moment (Am^2)
1	0	-0.0571	0.0014	-0.0572	-0.0087
2	1	-0.0434	1.3050	-0.0432	1.3122
3	2	-0.0295	2.6259	-0.0295	2.6158
4	3	-0.0155	3.9526	-0.0156	3.9367
5	4.2	-0.0001	5.4155	-0.0001	5.4054
6	5	0.0072	6.1078	0.0070	6.0862
7	6	0.0140	6.7567	0.0142	6.7610
8	7	0.0199	7.3100	0.0199	7.3086
9	6	0.0143	6.7828	0.0143	6.7756
10	5	0.0074	6.1282	0.0074	6.1151
11	4.2	0.0004	5.4590	0.0004	5.4575
12	3	-0.0152	3.9787	-0.0152	3.9743
13	2	-0.0291	2.6665	-0.0289	2.6737
14	1	-0.0429	1.3543	-0.0431	1.3267
15	0	-0.0567	0.0420	-0.0567	0.0376
Power off (25 s)					
16	0	-0.0572	-0.0014	-0.0570	0.0087
17	-1	-0.0707	-1.2848	-0.0705	-1.2747
18	-2	-0.0846	-2.6085	-0.0844	-2.5984
19	-3	-0.0984	-3.9151	-0.0983	-3.9136
20	-4.2	-0.1138	-5.3750	-0.1137	-5.3764
21	-5	-0.1211	-6.0673	-0.1208	-6.0486
22	-6	-0.1280	-6.7220	-0.1277	-6.7090
23	-7	-0.1336	-7.2579	-0.1334	-7.2449
24	-6	-0.1282	-6.7423	-0.1279	-6.7206
25	-5	-0.1214	-6.0963	-0.1212	-6.0862
26	-4.2	-0.1143	-5.4214	-0.1142	-5.4199
27	-3	-0.0987	-3.9469	-0.0987	-3.9541
28	-2	-0.0849	-2.6375	-0.0849	-2.6448
29	-1	-0.0710	-1.3137	-0.0710	-1.3268
30	0	-0.0572	-0.0014	-0.0571	-0.0058

Earth's magnetic field was approximated by the measured field before any command voltage was applied to the torquer, i.e., the field at zero voltage without a hysteresis effect. This was obtained by using the mathematical average of the two magnetic-field measurements: the initial field before any command voltage is applied and the field measurement at zero voltage immediately after the power-off period, which corresponds to the lines Nos. 1 and 16 in **Table 1**.

3.4. Dipole Moment

In calculating the dipole moment M using Equation (8), the positive direction of the magnetic field follows the definition in **Figure 1**. Comparing **Figures 1** and **2**, it is clear that the positive B_r direction is in the positive X-direction of the magnetometer. The calculated dipole moments of the torquer are also listed in **Table 1**, where the effect of the Earth's magnetic field has been subtracted from the measured data in calculating the dipole moments.

The differences in the dipole moments between those measured and those specified by the manufacturer are

presented in **Table 2**, where the percentage error is calculated from

$$\text{error(\%)} = \frac{\text{measured} - \text{factory specified}}{|\text{factory specified}|} \times 100\% \quad (9)$$

However, it was noted that the manufacturer specified dipole moment at $V = 0$ V is zero, where the above equation is not applicable.

The dipole moments and the errors in percentage are graphically illustrated in **Figure 4**.

From **Figure 4** and **Table 2**, it is found that the measured dipole moments of the magnetic torquer coincide very well with those provided by the manufacturer. The maximum deviation is only 3%.

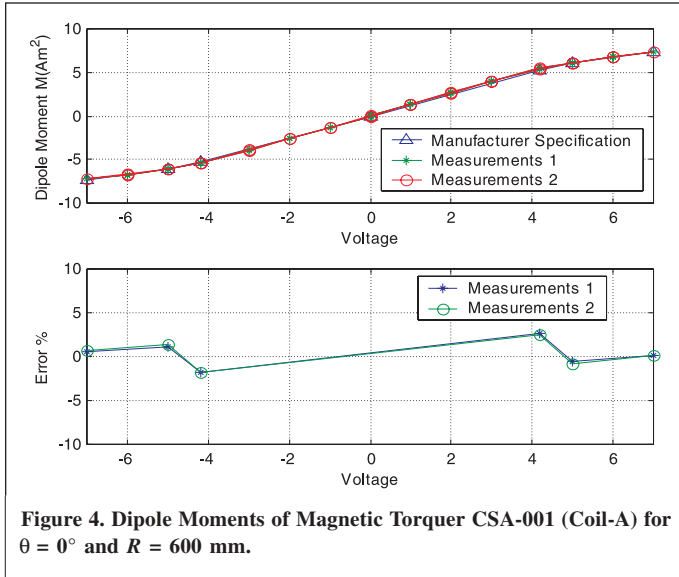


Figure 4. Dipole Moments of Magnetic Torquer CSA-001 (Coil-A) for $\theta = 0^\circ$ and $R = 600$ mm.

4. REMARKS ON THE TEST SETUP

As mentioned in Section 3.2, the location and orientation of the magnetometer, as well as the environment of the test setup all have significant effects on the magnetic-field measurement of the magnetic torquer, which will be looked at more closely in this section.

4.1. Location of the Magnetic-Field Sensor in the Axial Direction of the Torquer

We repeated the experiment described in Section 3.3 using the same test setup as shown in **Figures 2 and 3**, except that the magnetometer is placed closer to the magnetic torquer. In this experiment setup $R = 300$ mm. The magnetic-flux densities were measured when different command voltages were applied

to the torquer. Then the dipole moments of the torquer were calculated using Equation (8), and the errors in percentage were obtained using Equation (9). The results are graphically illustrated in **Figure 5**, and are also listed in **Table 3**. The maximum error in this case is about 11%, which is larger than the previous case where the maximum error was only 3%.

The relatively large error in this situation may be the result of misalignment and misplacement of the magnetic-field sensor, i.e., θ being not strictly 0° and R being not strictly 300 mm, which are often unavoidable in the experiments. Because the closer the field sensor is to the torquer, the greater the magnetic-field gradient is. Therefore, the effect of the same amount of misalignment and misplacement would be magnified in the measurement error.

On the other hand, if the location of the field sensor were very far from the torquer, the magnetic field would be very weak, where the sensor and the environment noises would have significant effects on the measurement.

In our experiments, it was found that relatively good results could be achieved by placing the magnetic-field sensor along the axial direction of the torquer at a distance of about 600 mm ($\sim 2L$).

4.2. Magnetic-Field Measurement with $\theta = 90^\circ$ and inside a Building

We repeated the same experiment as described in Section 3.3. However, in this experiment, the setup was placed inside a building and the magnetometer was placed in the perpendicular direction of the torquer as shown in **Figure 6**, where $\theta = 90^\circ$ and $R = 155$ mm. Labels X , Y , and Z are the axes of the Bartington magnetometer. The positive Y -direction points into the paper and is not labeled.

The sign of the measurement follows the definition of the X -direction of the magnetometer in **Figure 6**. In the calculation of the dipole moment M using Equation (7), the positive direction

Table 2. Dipole moments and errors of CSA-001 (Coil-A) for $\theta = 0^\circ$ and $R = 600$ mm.

Dipole moment		Test 1			Test 2		
Voltage (V)	Factory specified (Am ²)	(A m ²)	Error (%)	Error (%) (Average hysteresis)	(A m ²)	Error (%)	Error (%) (Average hysteresis)
0	0	0.0014	N/A	N/A	-0.0087	N/A	N/A
4.2	5.3	5.4155	2.18	2.59	5.4054	1.99	2.48
5	6.15	6.1078	-0.69	-0.53	6.0862	-1.04	-0.81
7	7.3	7.3100	0.14	0.14	7.3086	0.12	0.12
5	6.15	6.1282	-0.36	N/A	6.1151	-0.57	N/A
4.2	5.3	5.4590	3.00		5.4575	2.97	
0	0	0.0420	N/A		0.0376	N/A	
0	0	-0.0014	N/A	N/A	0.0087	N/A	N/A
-4.2	-5.3	-5.3750	-1.42	-1.86	-5.3764	-1.44	-1.85
-5	-6.15	-6.0673	1.34	1.11	-6.0486	1.65	1.35
-7	-7.3	-7.2579	0.58	0.58	-7.2449	0.76	0.76
-5	-6.15	-6.0963	0.87	N/A	-6.0862	1.04	N/A
-4.2	-5.3	-5.4214	-2.29		-5.4199	-2.26	
0	0	-0.0014	N/A		-0.0058	N/A	

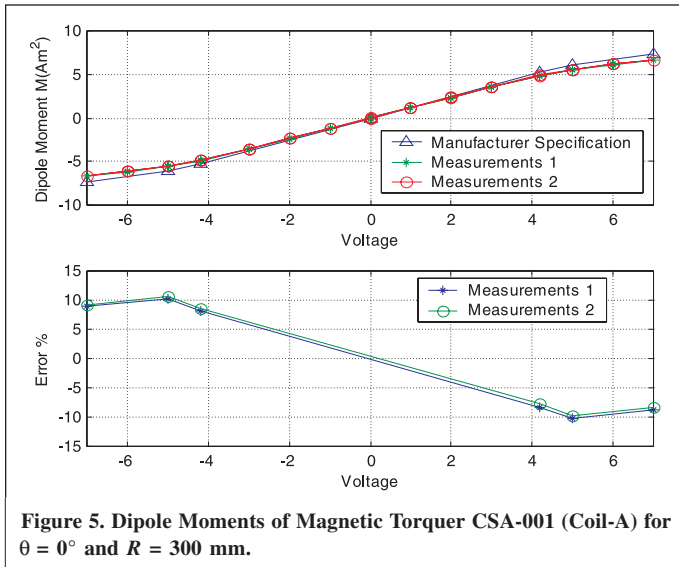


Figure 5. Dipole Moments of Magnetic Torquer CSA-001 (Coil-A) for $\theta = 0^\circ$ and $R = 300$ mm.

of the magnetic field follows the definition in **Figure 1**. Comparing **Figures 1** and **6**, it is clear that the positive B_t direction is in the negative X -direction of the magnetometer.

The dipole moments and the errors in percentage are graphically illustrated in **Figure 7**, and are also listed in **Table 4**, where the dipole moments are obtained using Equation (7) and the errors are obtained using Equation (9). The maximum error in this case is about 33%, which is much larger than that in both of the previous experiments, where the maximum errors were 3% and 11%, respectively.

Here, the measured magnetic field is much stronger than that expected. There is an apparent concentration of magnetic-field lines at the location of the magnetometer.

This experiment was conducted in a laboratory inside a building. Metallic objects in the laboratory and the metals in the building structure could distort the distribution of the magnetic-field lines, and consequently contribute to the errors of the

measurement. More factors affecting the magnetic field of the torquer could exist in this situation; these are left open for future study.

5. CONCLUSIONS

The magnetic-flux densities of a magnetic torquer were measured by a magnetometer and using different test setups. The dipole moments of the magnetic torquer were subsequently calculated from the magnetic-flux density measurements. It was found that such factors as metallic objects in the vicinity of the torquer as well as the location and orientation of the magnetic-field sensor have significant effects on the magnetic field of the torquer, and must be carefully considered in designing the experiments. Satisfactory results are obtained by conducting experiments in an open field without metallic objects in the vicinity of the torquer, and by placing the magnetic-field sensor in the axial direction of the torquer and at a distance about 600 mm ($\sim 2L$) from the center of the torquer.

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Table 3. Dipole moments and errors of CSA-001 (Coil-A) for $\theta = 0^\circ$ and $R = 300$ mm.

Voltage (V)	Dipole moment		Test 1		Test 2		Error (%) (Average hysteresis)
	Factory specified (Am^2)		(Am^2)	Error (%)	(Am^2)	Error (%)	
0	0		0.0155	N/A	N/A	-0.0066	N/A
4.2	5.3		4.8409	-8.66	-8.25	4.8642	-8.22
5	6.15		5.5060	-10.47	-10.18	5.5302	-10.08
7	7.3		6.6617	-8.74	-8.74	6.6894	-8.36
5	6.15		5.5421	-9.88	N/A	5.5659	-9.50
4.2	5.3		4.8849	-7.83		4.9117	-7.33
0	0		0.0148	N/A		0.0367	~ 0
0	0		-0.0155	N/A	N/A	0.0066	~ 0
-4.2	-5.3		-4.8437	8.61	8.11	-4.8262	8.94
-5	-6.15		-5.5034	10.51	10.24	-5.4843	10.82
-7	-7.3		-6.6508	8.89	8.89	-6.6284	9.20
-5	-6.15		-5.5377	9.96	N/A	-5.5091	10.42
-4.2	-5.3		-4.8972	7.60		-4.8746	8.03
0	0		-0.0317	N/A		-0.0078	~ 0

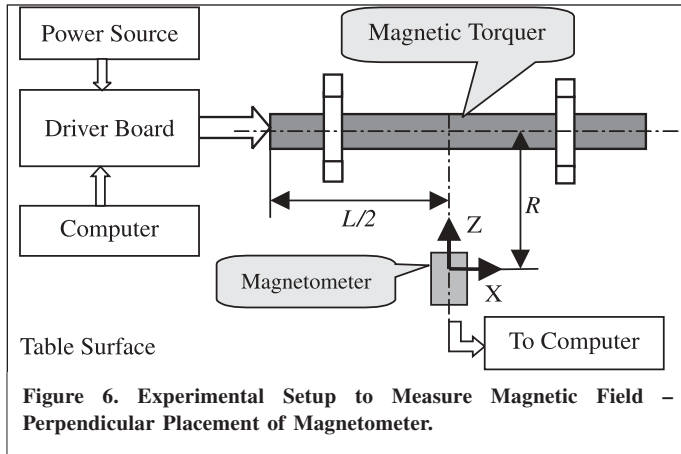


Figure 6. Experimental Setup to Measure Magnetic Field - Perpendicular Placement of Magnetometer.

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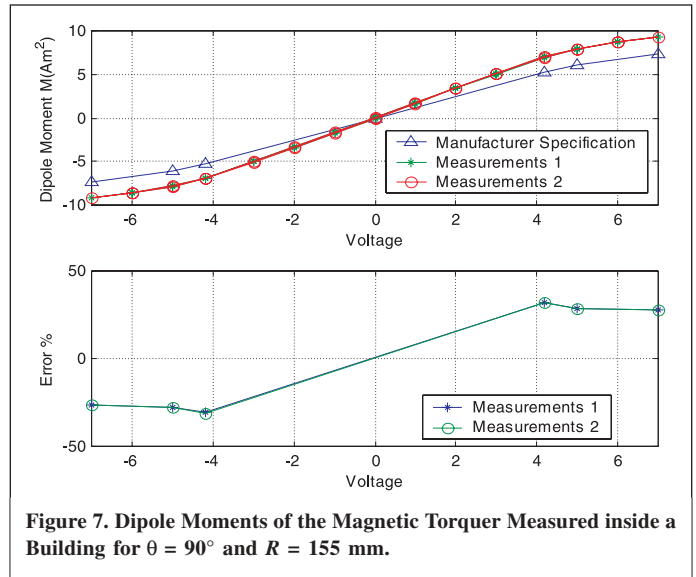


Figure 7. Dipole Moments of the Magnetic Torquer Measured inside a Building for $\theta = 90^\circ$ and $R = 155$ mm.

Table 4. Dipole moments for $\theta = 90^\circ$ and $R = 155$ mm and tested inside a building.

Voltage (V)	Factory specified (Am ²)	Dipole moment			Dipole moment		
		Test 1 (Am ²)	Error (%)	Error (%) (Average hysteresis)	Test 2 (Am ²)	Error (%)	Error (%) (Average hysteresis)
0	0	0.0003	N/A	N/A	-0.0027	N/A	N/A
4.2	5.3	6.9458	31.05	31.74	6.9558	31.24	31.98
5	6	7.8841	28.20	28.58	7.8891	28.28	28.66
7	7.3	9.3111	27.55	27.55	9.3161	27.62	27.62
5	6	7.9312	28.96	N/A	7.9362	29.04	N/A
4.2	5.3	7.0190	32.43		7.0341	32.71	
0	0	0.0465	N/A		0.0535	N/A	
0	0	0.0043	N/A	N/A	0.0074	N/A	N/A
-4.2	-5.3	-6.8969	-30.13	-30.80	-6.9019	-30.23	-30.92
-5	-6	-7.8131	-27.04	-27.44	-7.8182	-27.12	-27.48
-7	-7.3	-9.2291	-26.43	-26.43	-9.2291	-26.43	-26.43
-5	-6	-7.8623	-27.84	N/A	-7.8623	-27.84	N/A
-4.2	-5.3	-6.9682	-31.47		-6.9752	-31.61	
0	0	-0.0027	N/A		-0.0027	N/A	