

PREDICTION OF FATIGUE FAILURES OF ALUMINUM DISC WHEELS USING THE FAILURE PROBABILITY CONTOUR BASED ON HISTORICAL TEST DATA

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ABSTRACT

Disc wheels intended for normal use on passenger cars have to pass three tests before going into production: the dynamic cornering fatigue test, the dynamic radial fatigue test, and the impact test. This paper describes a probability model for prediction of fatigue failures of aluminum disc wheels, which intends to better link the prediction using simulation results with historical test data. Finite element models of 54 aluminum wheels, which are already physically tested, are constructed to simulate the dynamic cornering fatigue test. Their mean stresses and stress amplitudes during the fatigue loading cycle are calculated and plotted on a two-dimensional plane. Matching with historical test data, the failure probability contour can be drawn. For a new wheel, the failure probability of dynamic cornering fatigue test can be read directly from this probability contour. The test result of the new wheel can be added into the set of historical test data and the failure probability contour is updated. Same procedure is directly applied to the fatigue prediction of dynamical radial fatigue test. At this point we only have 20 historical test data to construct the failure contour. The prediction will become more and more reliable as the number of historical test data increases.

Keywords: Fatigue test, Aluminum disc wheel, Failure probability

1. INTRODUCTION

Disc wheels intended for normal use on passenger cars have to pass three tests before going into production: the dynamic cornering fatigue test, the dynamic radial fatigue test, and the impact test. Fatigue prediction has been an important issue to the design of aluminum disc wheels. Karandikar and Fuchs [5] developed a computer-based system, including a CAD package, a finite element analysis program, and a fatigue life computation program, for predicting the fatigue life of wheels. Kaumle and Schnell [6] also developed a technique for fatigue testing using a rapid-prototyping system. On the rapid-prototyping wheel the stresses will be measured, areas with high stress can be detected early, and the fatigue behavior can be tested with the rapid-prototyping wheel.

The randomness of fatigue prediction due to the inherent uncertainties in loading, manufacturing

variability, and material properties has been commonly recognized. Probabilistic approaches are proposed to account for the uncertainties in fatigue prediction models for various industrial applications. For example, Shen and Nicholas [9] described the probabilistic analysis for high cycle fatigue design of gas turbine; De Lorenzo and Hull [3] used a fully instrumented test bicycle to quantify the loads input to an off-road bicycle as a result of surface-induced loads to provide data for fatigue prediction; Sheikh et al. [8] proposed a method for calculating the reliability of an assembly of rotating parts subjected to fatigue failure; Adrov [1] presented a probabilistic approach to predict airframe fatigue damage using a load spectrum.

In such models, probability distributions of the factors with uncertainties are often obtained from measurements in actual operating conditions. Given the probability distributions of these factors, the distribution of fatigue life is derived analytically or

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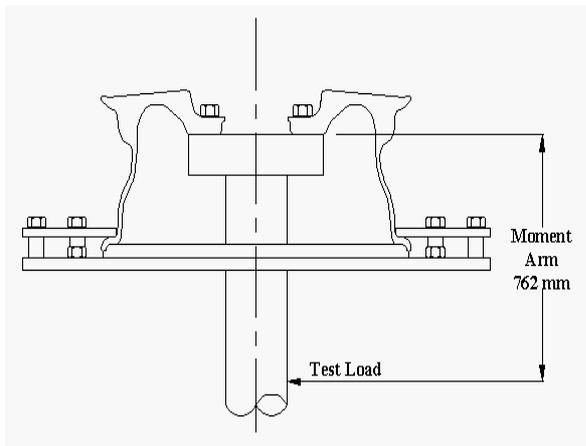


Figure 1. Typical setup of 90 degree loading method of cornering fatigue [7]

fitting with experimental data. Tallian [10] presented the results of a project aimed at improving the fatigue prediction of rolling bearings. A large experimental bearing life database is assembled, a mathematical fatigue prediction model is formulated, and the model is fitted to the historical experimental database. We will use similar approach in this research.

This paper describes a probability model for prediction of fatigue failures of aluminum disc wheels, which intends to better link the prediction using simulation results with historical test data. In various fatigue criterions, the mean stress and the stress amplitude are the two critical variables for fatigue prediction. In this paper, finite element models of aluminum wheels are constructed to simulate the dynamic cornering fatigue test. Fifty-four wheels, which are already physically tested, are analyzed and their mean stresses and stress amplitudes during the fatigue loading cycle are calculated and plotted on a two-dimensional plane. Instead of using the fatigue failure criterions commonly seen in literature, such as Goodman and Gerber's criterions [2], we match the analysis results with historical test data, and a mathematical model is used to fit the historical data to construct the "failure probability contour." For a new wheel, the failure probability of dynamic cornering fatigue test can be read directly from this probability contour. The test result of the new wheel can be added into the set of historical test data and the failure probability contour is updated.

Same procedure can be directly applied to the fatigue prediction of dynamical radial fatigue test. At this point we only have 20 historical test data to construct the failure contour. The prediction will become more and more reliable as the number of historical test data increases.

2. PREDICTING THE DYNAMIC CORNERING FATIGUE TEST

The dynamic cornering fatigue test simulates the loading condition of the wheels in normal driving. Figure 1 shows a typical setup of the 90-degree loading method of cornering fatigue, according to SAE J32 [7]. In the Figure, the downside outboard flange of rim of the wheel is clamped securely to the test device, and a rigid load arm shaft is attached to the mounting surface of the wheel. A test load applies on the arm shaft to provide a constant cyclical rotation bending moment. After being subjected to the required number of test cycles, there shall be no evidence of failure of the wheel, as indicated by propagation of a crack existing prior to test, new visible cracks penetrating through a section, or the inability of the wheel sustain load.

The finite element model used to simulate the dynamic cornering fatigue test has been described in great details in the authors' previous work [4]. Figure 2 shows the finite element model of an aluminum wheel. All degrees of freedom of the nodes on the downside outboard flange of the rim are fixed. The dynamic load is represented by 24 discrete loads 15 degrees apart. For each node, the maximum and minimum Von Mises stresses during the load cycle are extracted to obtain the mean stress σ_m and the stress amplitude σ_a of the node. The well-known Goodman and Gerber's criterions were first used for fatigue prediction in this study.

$$\text{Goodman's criterion: } \frac{\sigma_a}{S_e/n} + \frac{\sigma_m}{S_u/n} = 1 \quad (1)$$

$$\text{Gerber's criterion: } \frac{\sigma_a}{S_e/n} + \left(\frac{\sigma_m}{S_u/n} \right)^2 = 1 \quad (2)$$

where S_e is the endurance limit, S_u is the ultimate strength of the material, and n is the safety factor [2].

Figure 3 shows the two curves with safety factor $n = 1$. Each node of the finite element model is represented by its (σ_m, σ_a) . The top 1% nodes that are closest to the Goodman's line, which have the greatest possibility to fail in the fatigue test, are plotted in Figure 3. The average (σ_m, σ_a) of these 1% top stress nodes are calculated, as shown by the solid circle $(\sigma_m, \sigma_a)_{1\%}$ in Figure 3. This point is used to represent the wheel when checking with Goodman and Gerber's lines to predict whether the wheel will pass the cornering fatigue test.

Finite element models of the first 28 aluminum wheels, which were already physically tested by a local manufacturer, were constructed to simulate the cornering fatigue tests. The $(\sigma_m, \sigma_a)_{1\%}$ for each wheel was calculated and plotted in Figure 4, where "o" and



Figure 2. Simulating the dynamic cornering fatigue test

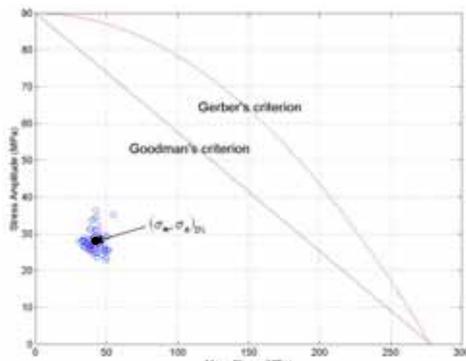


Figure 3. The Goodman and Gerber's lines for safety factor $n = 1$

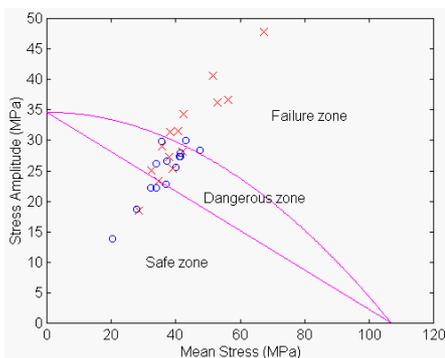


Figure 4. The Goodman and Gerber's lines for safety factor $n = 2.6$

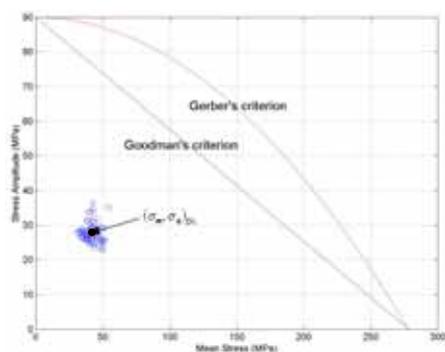


Figure 5. Prediction results of 26 new aluminum wheels

“x” represent whether the wheel actually passed or failed the cornering fatigue test.

Finite element models of the first 28 aluminum wheels, which were already physically tested by a

local manufacturer, were constructed to simulate the cornering fatigue tests. The $(\sigma_m, \sigma_a)_{1\%}$ for each wheel was calculated and plotted in Figure 4, where “o” and “x” represent whether the wheel actually passed or failed the cornering fatigue test.

These “historical data points” provide good references for choosing the safety factor n for this specific case. As shown in Figure 4, we found that for $n = 2.6$, the Goodman and Gerber lines best fit the 28 historical data points. Using this safety factor, we can then define the “safe zone,” “dangerous zone,” and “failure zone” for predicting whether a new aluminum wheel can pass the dynamic cornering fatigue test.

Following this study, another batch of 26 new aluminum wheels were predicted using this procedure. The prediction results were then matched with the physical test results. As shown in Figure 5, all 10 wheels that are in the safe zone did pass the cornering fatigue test. Three wheels fall in the dangerous zone and 1 of them did not pass the cornering fatigue test. Thirteen wheels fall in the failure zone, and 12 of them did not pass the cornering fatigue test. In this sample of 26 wheels, this procedure has a 96% success rate in correctly predicting whether the wheels will pass the dynamic cornering fatigue test.

3. PREDICTING THE DYNAMIC CORNERING FATIGUE TEST USING THE PROBABILITY MODEL

Though the procedure described above works well for practical purposes, there are still several fundamental problems. First, this procedure only provides three qualitative predictions: safe, dangerous, and fail. Referring to Figure 5, in the neighborhood of the lines between safe, dangerous, and failure zones, a small deviation may result in completely different predictions. Quantitative information of “how likely the wheel is going to fail” is desired, especially for the data points in the dangerous zone. Secondly, in this procedure, the prediction of a new aluminum wheel is made based on historical test data, and the safety factor n is the only “mathematical relation” used to fit the prediction with historical test data. A better mathematical model should be constructed to fit the historical data. As the number of historical data points increases, there should be a mechanism for updating this “relation” to improve the prediction.

Based on the experience of predicting the fatigue failure of aluminum disc wheel, this paper presents a probability model that intends to better link the prediction using simulation results with historical test data. Instead of the three qualitative predictions,

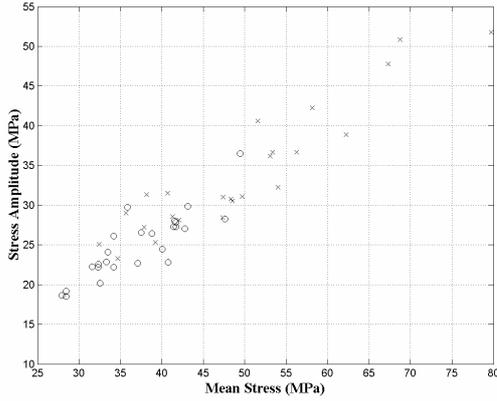


Figure 6. 54 historical data points of the cornering fatigue test

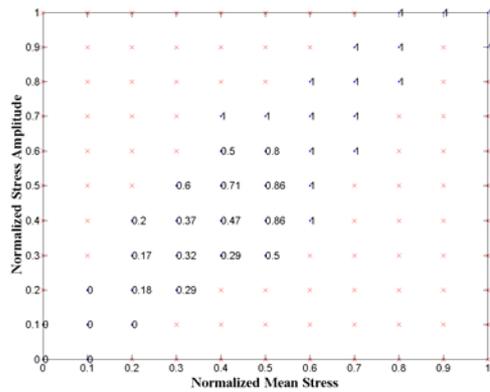


Figure 7. Failure probability on the grid points for $m=10$

this probability model generates the probability of failure based on historical test data.

In our case, we now have $28+26=54$ historical data points, as shown in Figure 6. Note that historical data points towards the upper right corner are more likely to fail. Given a new data point $(\sigma_m, \sigma_a)_{1\%}$, we can define a circle of radius r , which is centered on this new data point. We can then use the historical data points in the circle to predict the failure probability of the new data point. The failure probability can be easily calculate as $P_{fail} = \frac{N_{fail}}{N_{total}}$,

where N_{fail} is the number of historical data points in the circle that actually failed, and N_{total} is the total number of historical data points in the circle.

While this is a straightforward way to calculate the failure probability of a new data point using historical test data, there is a practical problem of how to define the radius r . If r is large, the circle may be too big to adequately represent the new data point. If the radius r is small (or the historical data points are sparse in the neighborhood of the new data point), the number of historical data points in the circle will be too few to generate a meaningful probability value.

Therefore, instead of calculating failure probability directly from the historical data points that fall in the circle, we try to draw a “probability contour” on the two-dimensional domain σ_m - σ_a first. To do this, the $x(\sigma_m)$ and $y(\sigma_a)$ axes of the 54 historical data points are normalized between 0 and 1, then we divided this domain into $m \times m$ rectangular grids. On each grid point, we can draw a circle of radius $r=1/m$, which is the length of the grid, then the failure probability of this grid point can be calculated from the historical data points that fall in this circle. Figure 7 shows the failure probabilities of the grid points for $m=10$.

The historical data points scatter along the diagonal of the σ_m - σ_a domain in this case, and only the 38 grid points along the diagonal have failure probability figures. These probability figures are extrapolated to the whole domain in order to draw the failure probability contour. The extrapolation is done in an iterative manner, and the 38 original probability figures in Figure 7 remain fixed during the extrapolation.

Several assumptions are considered during the extrapolation. Grid points toward the upper right corner should have a higher failure probability. Therefore we have the following assumption:

Therefore we have the following assumption:

$$P_{fail}(\sigma_m^i, \sigma_a^i) \leq P_{fail}(\sigma_m^j, \sigma_a^j), \tag{3}$$

if $\sigma_m^i \leq \sigma_m^j$, and $\sigma_a^i \leq \sigma_a^j$

All probability figures generated during the process have to satisfy Eq. (3). Moreover, since the probability value lies between 0 and 1, Eq. (3) also implies

$$\forall \sigma_m^i \leq \sigma_m^j, \text{ and } \sigma_a^i \leq \sigma_a^j, P_{fail}(\sigma_m^i, \sigma_a^i) = 0 \tag{4}$$

if $P_{fail}(\sigma_m^j, \sigma_a^j) = 0$

$$\forall \sigma_m^i \geq \sigma_m^j, \text{ and } \sigma_a^i \geq \sigma_a^j, P_{fail}(\sigma_m^i, \sigma_a^i) = 1 \tag{5}$$

if $P_{fail}(\sigma_m^j, \sigma_a^j) = 1$

These two equations are also necessary when extrapolating the probability figures to the boundary of the domain.

Figure 8 shows the results of the extrapolation, and finally Figure 9 shows the failure probability contour based on the 54 historical data points. Note that the Goodman and Gerber criterion are also plotted on the Figure for comparison. The failure probability of a new data point can be read directly from this Figure, even for data points in the “dangerous zone” between the Goodman and Gerber’s lines.

Table 1. Failure probability prediction of 8 new wheels

| Wheel No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $(\sigma_m, \sigma_a)_{1\%}$ | (40.0, 32.2) | (32.3, 26.3) | (43.6, 31.5) | (41.8, 30.8) | (33.3, 28.5) | (70.1, 52.7) | (40.2, 28.2) | (45.9, 38.3) |
| P_{fail} | 67.1% | 23.9% | 71.2% | 66.4% | 27.2% | 100% | 39.5% | 73.5% |
| Updated P_{fail} | 73.3% | 20.2% | 73.9% | 59.1% | 23.0% | 100% | 37.3% | 75.7% |
| Test result | Fail | Pass | Fail | Pass | Pass | Fail | Pass | Fail |

expected value of the number of trials of making and revising the die to determine whether the designer should accept this new design or redesigns the wheel. If the failure probability of a new wheel is P_{fail} , the expected value of trials is approximately $(1-P_{fail})+2P_{fail}$, neglecting the higher order terms. Therefore, for example, if a manufacturer has a policy that average number of trials of making and revising the die for all wheels should be lower than, say, 1.3, then $P_{fail}=0.3$. That is, designers should only accept the wheels whose predicted failure probabilities are lower than 0.3.

5. PREDICTING THE DYNAMIC RADIAL FATIGUE TEST USING THE FAILURE PROBABILITY CONTOUR

Figure 11 is a typical setup of the dynamic radial fatigue test, according to SAE J328 [7]. In the Figure, the test wheel is constrained by bolts through the PCD. The driven rotatable drum, whose axis is parallel to the axis of the test wheel, presents a smooth surface wider than the section width of the loaded test tire section width. The test wheel and tire provide loading normal to the surface of drum and in line radially with the center of test wheel and the drum. After being subjected to the required number of test cycles, there shall be no evidence of failure of the wheel, as indicated by propagation of a crack existing prior to test, new visible cracks penetrating through a section, or the inability of the wheel to sustain load.

Finite element analysis is used to simulate the dynamic radial fatigue test. As shown in Figure 12, the wheel is glued to a base through the PCD, and all degrees of freedom of the bottom of the base are constrained. The dynamic load is represented by 20 discrete loads 18 degrees apart. Similar to the simulation of dynamic cornering fatigue test, (σ_m, σ_a) during the load cycle are calculated for each node of the finite element model. The average of the top 1% nodes that are closest to the Goodman's line $(\sigma_m, \sigma_a)_{1\%}$ is used to represent the wheel when predicting whether the wheel will pass the radial fatigue test.

Finite element models of 20 aluminum wheels, which were already physically tested by a local

manufacturer, were constructed. The $(\sigma_m, \sigma_a)_{1\%}$ for each wheel was calculated and plotted in Figure 13, where "o" and "x" represent whether the wheel actually passed or failed the radial fatigue test. Using the same procedure presented in the previous sections, Figure 14 shows the failure probability contour for $m=10$, based on the 20 historical data points. The failure probability of dynamic radial fatigue test of a new wheel can be read directly from this figure. In this case, we used much fewer historical data points to construct the failure probability contour. The prediction will become more and more reliable as the number of historical data points increases.

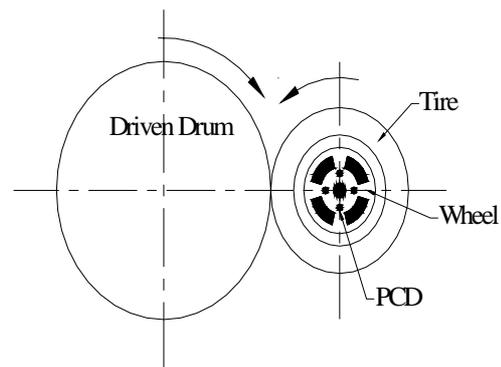


Figure 11. Typical setup of dynamic radial fatigue test

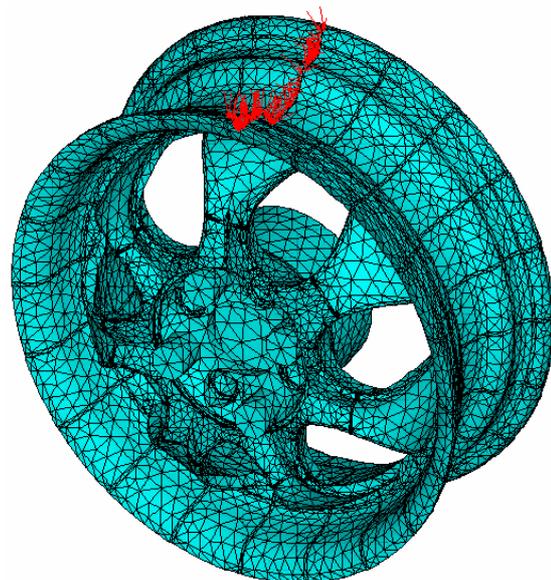


Figure 12. Simulating the dynamic radial fatigue test

6. DISCUSSIONS AND CONCLUSIONS

This paper presents a procedure that predicts the fatigue failure probability based on historical test data. Using this probability model, we can now provide quantitative information of “how likely the wheel is going to fail” using simulation results, and we also have a mechanism to update the model to improve the prediction as the number of historical data points increases.

Computer simulations are often used to predict the performance of a new product, but the results from computer simulation still have to be confirmed by physical testing. Therefore historical test results are very valuable to manufacturers and should be systematically preserved and utilized. Based on the experience of predicting the fatigue failure of aluminum disc wheel, we can summarize a probability model that intends to better link the simulation results with historical test data. The whole idea is rather straightforward. There are three basic assumptions in this model:

- (1) The desired prediction using simulation results is a pass-fail type of prediction.
- (2) The essential indices that affect performance being predicted, evaluated from computer simulation, have been identified.
- (3) There are enough historical data points as references of the prediction.

Under these three assumptions, a design is expressed by a data point (\mathbf{I}, p) in this model, where \mathbf{I} is the vector of performance indices obtained from computer simulation, and p is the probability of whether this data point will fail in the physical test. Assume that there is a set of q historical data points (\mathbf{I}_i, p_i) , $i = 1, 2, \dots, q$, that are already physically tested. Note that for a historical data point, p_i is either zero (the corresponding design passes) or one (the corresponding design fails).

Now a new candidate design j is evaluated using computer simulation to obtain its vector of performance indices \mathbf{I}_j . From \mathbf{I}_j , we wish to predict p_j , the probability of whether this candidate design j will fail in the physical test, from the set of n historical data points. To do this, we define a hypersphere whose center is \mathbf{I}_j and radius is r . The n historical data points are checked to see if $\|\mathbf{I}_i - \mathbf{I}_j\| < r$, that is, whether the historical data point i falls in the hypersphere. Assume that there are m data points in the hypersphere, (\mathbf{I}_k, p_k) , $k = 1, 2, \dots, m$, we can

obtain $p_j = \frac{\sum_{k=1}^m p_k}{m}$. After this candidate design j is physically tested, it becomes a “historical data point,” and the set of historical data points is updated.

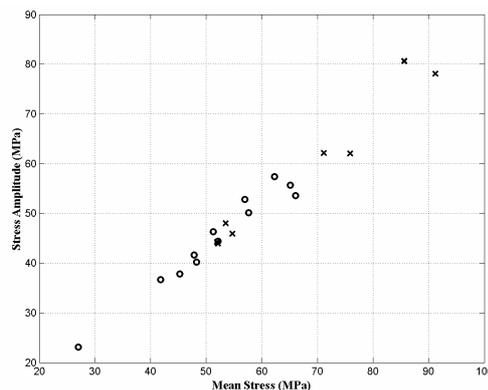


Figure 13. 20 history data points of the radial fatigue test

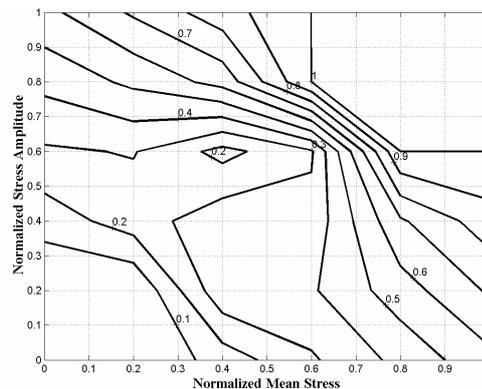


Figure 14. The failure probability contour based on 20 historical test data

A practical problem of using this model is how to define a proper radius r for the hypersphere. For the problems having only 2 or less components in \mathbf{I} , as in our aluminum wheel case, a probability contour can be drawn on the two-dimensional domain formed by the 2 components of \mathbf{I} , using the set of n historical data points (\mathbf{I}_i, p_i) , $i = 1, 2, \dots, q$. Then p_j can be read directly from the probability contour.

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運用歷史測試資料作鋁合金輪圈疲勞破壞機率預測

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摘要

汽車輪圈量產前必須通過三種測試：動態彎曲疲勞測試、動態徑向疲勞測試、與衝擊測試。本文描述一結合輪圈電腦模擬結果與輪圈歷史測試資料，以預測鋁合金輪圈動態彎曲疲勞測試及動態徑向疲勞測試之疲勞破壞機率的模式。本文首先以多個鋁合金輪圈有限元素分析與實際的測試結果之比對，建立破壞機率的等高線分布圖，對於一新設計輪圈，動態彎曲疲勞測試的破壞機率可以從此機率等高線分布圖中直接的讀出，此新輪圈的實際測試結果亦會被加入到歷史測試資料中，更新其破壞機率等高線圖，此預測方式將會因歷史測試資料的增加與更新而更加的可靠。

關鍵詞：疲勞測試，鋁合金輪圈，破壞機率

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