A New Family of Hybrid 4-dof Parallel Mechanisms with Two Platforms and its Application to a Footpad Device

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ABSTRACT

This paper proposes a new family of four degrees-of-freedom (dof) parallel mechanisms with two platforms and its application to a footpad device that can simulate the spatial motions of the human foot. The new mechanism consists of front and rear platforms, and three limbs. Two limbs with 6-dof serial joints ($P$-$S$-$P$-$P$) are attached to each platform and are perpendicular to the base plate, while the middle limb is attached to the revolute joint that connects the front and rear platforms. The middle limb is driven by the 2-dof driving mechanism that is equivalent active serial prismatic and revolute joints ($P_c$-$R_c$), or prismatic and prismatic joints ($P_c$-$P_c$) with two base-fixed prismatic actuators. Since the middle limb perpendicular to the base plate has 3-dof serial joints ($P_c$-$R_c$-$R$ or $P_c$-$P_c$-$R$), two new 4-dof parallel mechanisms with two platforms can generate pitch motion of each platform, and roll and heave motions (1T-3R) or pitch motion of each platform and two translational motions (2T-2R) at both platforms, according to the type of the 2-dof driving mechanism. Kinematic
analyses of the 1T-3R mechanism were performed, including inverse and forward kinematics, and velocity analysis. Based on the 1T-3R mechanism, a footpad device was designed to generate foot trajectories for natural walking.

KEYWORDS: 4-dof Parallel Mechanism, Footpad, Walking Trajectory

1. INTRODUCTION

In the last fifteen years, there has been extensive research on the use of parallel robots for machine tools, such as for 5-axis milling and drilling machines, because of their desirable characteristics, including low inertia, high rigidity, compactness, and precise resolution as compared to serial mechanisms. In addition, non-machine-tool applications using parallel robots are becoming widespread in haptic interfaces, medical operations, teleoperations, welfare, etc.\(^1\) Until now, most designs were 6-dof (degrees of freedom) or 3-dof mechanisms. A few designs have been proposed for 4-dof parallel mechanisms: Koeverman’s flight simulator,\(^2\) Reboulet’s 4-dof wrist,\(^3\) and 4-dof parallel manipulator with RRRRR limbs\(^4\) have been proposed to provide three rotations (spherical motions) and one translation (z-axis). Moreover, hybrid parallel mechanisms,\(^5\)-\(^7\) 3T-R,\(^8\)-\(^10\) and 2T-2R\(^11\) parallel robots have been suggested, where the letters T and R represent translational and rotational motions of the platform. In addition, Gao et al.\(^12\) suggested several types of composite pairs and new kinds of sub-chains with 2-, 3-, 4-, and 5-dof parallel mechanisms. Similarly, Li and Huang\(^13\) suggested 3-, 4-, and 5-dof parallel mechanisms, where each limb consists of five revolute pairs. Fang and Tsai\(^14\) synthesized 4- and 5-dof parallel mechanisms with identical serial limbs.

Most parallel robots with 4-dof mechanisms consist of a single rigid platform. In
areas such as clinical medicine, however, robots with more than two platforms are necessary to support multi-body objects such as the human body. For example, an intelligent bed system\textsuperscript{15} should have multiple platforms to move the articulations of the human body. This mechanism uses a four-bar mechanism for the upper part and a slider-crank mechanism for the lower part. Unfortunately, the bed robot cannot generate roll motions, which are necessary for changing the position of a patient to prevent skin diseases such as bedsores. Moreover, this system does not need to generate rapid motions, since its purpose is to keep the patient positioned comfortably in bed.

For lower limb rehabilitation, Schmidt \textit{et al.}\textsuperscript{16} pointed out that the platform needs to generate relative rotation between the fore and rear foot in order to train patients for natural walking trajectories, although their mechanism could not generate this motion. Relative rotation at the metatarsal joint of the foot increases the naturalness of human walking and helps reduce the load on the knee joints, where high force and speed are required to achieve human locomotion.\textsuperscript{17} Therefore, any device to be used as a footpad should consist of at least two platforms to allow relative rotation between the fore and rear foot. Therefore, a parallel mechanism with two platforms is the solution that satisfies the desired performance of footpads in rehabilitation.

This paper presents a new family of spatial 4-dof parallel mechanisms with two platforms. One of the suggested mechanisms (1T-3R) can generate independent pitch motion at each platform, and roll and heave motions at both platforms, while the other (2T-2R) can generate independent pitch motion at each platform and two translational motions at both platforms. The new mechanism consists of front and rear platforms, and three limbs. Two limbs with 6-dof serial joints are attached to each platform and are perpendicular to the base plate, while the middle limb is attached to the revolute joint that connects the front and rear platforms. A parallel mechanism can be defined as a closed-loop mechanism in which the
mobile platform is connected to the base by at least two independent kinematic limbs.\textsuperscript{18} Since each platform of the suggested 4-dof mechanisms is connected to the base by two independent limbs and the two platforms are connected with a revolute joint, the suggested 4-dof mechanisms with two platforms can be explained as a combination of two independent 3-dof hybrid parallel mechanisms, and these have high potential for rehabilitation applications. Therefore, this paper analyzed the 1T-3R-type 4-dof parallel mechanism with two platforms in detail to consider the natural walking trajectory, including relative rotation at the foot.

The following section gives a kinematic description of the suggested mechanisms. Section 3 presents kinematic analyses of the 1T-3R-type mechanism, including inverse and forward kinematics, and velocity analysis. Section 4 presents a footpad device based on a 1T-3R-type parallel mechanism. Conclusions and future research are summarized in Section 5.

2. KINEMATIC DESCRIPTION OF THE MECHANISM

The special Plücker coordinates of the final motions generated by the platform of a parallel mechanism equal the intersection of the special Plücker coordinates of all limbs, which describe the displacement of the output link of a limb in a parallel mechanism.\textsuperscript{12} Since it is difficult to obtain limbs with specific degrees of freedom using traditional pairs, Gao \textit{et al.}\textsuperscript{12} suggested several types of composite pairs. However, they considered only composite pairs for one actuation at one limb; hybrid parallel mechanism with an \( n \)-dof platform connected to the base by \( m \) (\( m < n \)) independent kinematic limbs has limbs with one or more actuated joints.\textsuperscript{18} The reduced number of limbs increases the orientation workspace of the mechanism, which is important for rehabilitation applications. Although McCloy\textsuperscript{19} and Gosselin\textsuperscript{20} presented planar 2-dof parallel mechanisms with five and seven links, respectively, the motions of their mechanisms at end-effectors cannot be confined to exact 2-dof motions.
Therefore, it is necessary to devise 2-dof-driving mechanisms that generate exact 2-dof motions at the end-effectors. These confined motions at the end-effectors should be helpful in designing specific degrees of freedom at the platform of a hybrid parallel mechanism. Therefore, we first present 2-dof driving mechanisms in which the end-effectors have 2-dof motions for two actuations at one limb. Using these 2-dof driving mechanisms, we propose novel 4-dof parallel mechanisms with two platforms.

### 2.1. Two-dof driving mechanisms with two base-fixed prismatic actuators

Figures 1 and 2 show T-R- and T-T-type 2-dof driving mechanisms that can generate 2-dof motions, respectively. In the figures, the letters $P$, $R$, and, $S$ represent prismatic, revolute, and spherical joints, respectively. An underlined letter represents an actuated joint. The T-R-type driving mechanism in Figure 1(a) consists of passive prismatic joints ($P_3$ and $P_4$) between revolute joints ($R_1$ and $R_3$) at the upper ends of the active prismatic joints ($P_1$ and $P_2$) fixed to the base and end-effector of the driving mechanism, and includes a passive prismatic joint ($P_5$) between the revolute joint $R_2$ of the end-effector and base plate. $P_5$ allows the end-effector to move in the $z$-direction only and $R_2$ allows the end-effector to rotate about the $y$-axis only. A CAD model of the T-R-type driving mechanism is shown in Figure 1(b). The T-R-type driving mechanism can generate translation along the $z$-axis and rotation about the $y$-axis independently using two base-fixed prismatic actuators. Therefore, the T-R-type driving mechanism is conceptually equivalent to active serial prismatic and revolute joints ($P_e$-$R_e$).

By contrast, the T-T-type driving mechanism in Figure 2(a) is composed of a five-bar mechanism with two base-fixed active prismatic joints ($P_1$ and $P_2$), two passive prismatic joints ($P_3$ and $P_4$), and two revolute joints ($R_2$ and $R_3$) attached to the end-effector of the driving mechanism. The passive prismatic joints ($P_3$ and $P_4$) allow the end-effector to move along the $x$- and $z$-directions only. A CAD model of the T-T-type driving mechanism is shown...
in Figure 2(b). The T-T-type driving mechanism can generate two independent translational motions in the x-z plane using the active serial prismatic and prismatic joints ($P_e-P_e$).

2.2. Four-dof mechanisms with two platforms

Based on the 2-dof driving mechanisms, 4-dof parallel mechanisms with two platforms have been developed. The proposed 4-dof mechanisms with two platforms consist of front and rear platforms and three limbs, as shown in Figure 3. Two outer limbs with 6-dof serial joints ($P-S-P-P$) are attached to both platforms and are perpendicular to the base plate, while the middle limb with 3-dof serial joints ($P_e-R_e-R$ or $P_e-P_e-R$) is attached to a revolute joint that connects the front and rear platforms. Since each platform has two limbs with one 6-dof serial joint ($P-S-P-P$) and one 3-dof serial joint ($P_e-R_e-R$ or $P_e-P_e-R$), the final output displacement of each platform is dependent only on that of the middle limb with its 3-dof serial joint ($P_e-R_e-R$ or $P_e-P_e-R$), which is the intersection of the special Plücker of two limbs and equals each platform. In addition, the two platforms are connected with a revolute joint. Therefore, the suggested mechanisms have four degrees of freedom in total. For the proposed 1T-3R mechanism shown in Figure 3(a), each platform has 3-dof, which are described by two rotations about the x- and y-directions and one translation along the z-direction. Therefore, the 1T-3R mechanism with two platforms can generate pitch motions at both platforms, and roll and heave motions with the T-R-type driving mechanism at the middle limb. For the 2T-2R-type mechanism with two platforms shown in Figure 3(b), the mechanism can generate pitch motions at both platforms, and x- and z-translational motions with the T-T-type driving mechanism at the middle limb.

In a real joint implementation using the 1T-3R mechanism with two platforms, the spherical joints can be replaced by revolute and universal joints, and the $P-P$ joints can be replaced by x-y stages with a linear motion (LM) guide, as shown in Figure 4. Note that
inserting x-y stages at the P-P joints causes unavoidable vertical offset between the platforms and the limbs with 6-dof serial joints (P-S-P-P). As a result, the revolute joint between the front and rear platforms cannot be located in exactly the same position as the revolute joint at the upper end of the middle limb. Therefore, the middle limb with a 3-dof (P-e-R-e-R) joint is attached to the rear platform. The positioning of the two revolute joints at different locations causes the pitching motions of the two platforms to be dependent on each other. Conversely, in a real joint implementation using the 2T-2R mechanism with two platforms, as shown in Figure 5, although there are vertical offsets between the platforms and the limbs with 6-dof serial joints (P-S-P-P), the revolute joint between the front and rear platforms can be located in exactly the same position as the revolute joint at the upper end of the middle limb. Therefore, this identical position at the two revolute joints allows the pitch motions of the two platforms to be independent.

2.3. Link-pair relationship with the suggested mechanisms

Figure 6 shows the link-pair relationship diagram for the 2-dof driving mechanism. The white boxes represent passive joints and the hatched boxes represent active joints. Lines between letters represent links. It is possible to consider each driving mechanism as equivalent to the two actuated joints (P-e-R-e or P-e-P-e) in terms of the number and type of degrees of motion. Figure 7 shows the link-pair relationships of the 1T-3R- and 2T-2R-type mechanisms with two platforms. The motions of the given mechanism can be verified using Grübler’s formula as:\textsuperscript{21}

\[ M = d(n - g - 1) + \sum_{i=1}^{g} f_i \]  \hspace{1cm} (1)

where \( d, n, g, \) and \( f_i \) represent the dimension of the feasible motion space of all the joints of a
given mechanism, the number of links including the ground, the number of constraints of the joint, and the degree of freedom at the \(i\)-th joint, respectively.

For both T-T- and T-R-type driving mechanisms, the mobility is as follows:

\[
M_{2\text{dof}} = 3(7 - 8 - 1) + 1 \times 8 = 2
\]

Moreover, if the driving mechanisms are considered equivalent to serial \((P_e^-R_e)\) or \((P_e^-P_e)\) joints, the mobility of the new 4-dof mechanisms with two platforms is:

\[
M_{4\text{dof}} = 6(11 - 12 - 1) + 1 \times 10 + 2 \times 0 + 3 \times 2 = 4
\]

If one limb with 6 joints \((P-S-P-P)\) is added to the 2-dof driving mechanism with a revolute joint, as shown in Figure 8, the mechanism will have an additional 1-dof motion. Therefore, the mobility of the mechanism can be generalized as follows:

\[
M = 2 + n
\]

where \(M\) is the mobility of the suggested mechanism and \(n\) is the number of platforms with a 6-joint \((P-S-P-P)\) limb and a revolute joint. Currently, for the suggested 4-dof mechanism, \(n = 2\). If \(n = 3\), the mechanism will have three platforms and be able to generate pitch motions at each platform, which could be applied to an intelligent bed system\(^15\) with additional roll motions at each platform.

3. KINEMATICS OF THE 1T-3R TYPE

The kinematic relationships of the 1T-3R-type mechanism are derived to design an optimal footpad device. In order to analyze the proposed mechanism, the kinematic parameters are shown in Figure 9. The fixed base reference frame, \(O_b (X_b, Y_b, Z_b)\), is located at the bottom center of the 2-dof driving mechanism on the base plate. A local reference frame for the 2-dof driving mechanism, \(O_m (X_m, Y_m, Z_m)\), is located at the center of the upper end of the middle limb. A local reference frame for the revolute joint, \(O_p (X_p, Y_p, Z_p)\), is centered
between the front and rear platforms. The front, \( \mathbf{O}_f (X_f, Y_f, Z_f) \), mobile reference frames is located on the y-axes of reference frame \( \mathbf{O}_p \) of its platform and the rear, \( \mathbf{O}_r (X_r, Y_r, Z_r) \), mobile reference frame is located on the intersection line of the upper plane of the rear platform with the \( Y_m-Z_m \) plane of the reference frame \( \mathbf{O}_m \).

Points \( S_f \) and \( S_r \) denote the base center positions of passive spherical joints and points \( B_f \) and \( B_r \) denote the base positions of active prismatic actuators at each limb. \( Z_{ost} \) denotes the vertical offset between the spherical joint and platform.

If \( Z_{ost} = 0 \), the pitch motions of the rear and front platforms are independent and the local reference frame \( \mathbf{O}_m (X_m, Y_m, Z_m) \) of the 2-dof driving mechanism is located at the center of the revolute joint between the front and rear platforms, since the position of the revolute joint at the upper end of the middle limb is identical to that of the revolute joint between the front and rear platforms. Since an x-y stage should be inserted between the spherical joint and the rear platform in the physical realization stage, \( Z_{ost} > 0 \).

### 3.1 Inverse kinematics

The inverse kinematics computes the prismatic actuator displacements \( (L_f, L_r, L_{m1}, L_{m2}) \) of the mechanism given the position and orientation \( (Z_f, Z_r, \theta_f, \theta_r, \phi) \) of the platforms, where \( Z_f \) and \( Z_r \) are the respective heights of the front and rear platforms from the base, \( \theta_f \) and \( \theta_r \) are the pitch motions at each platform, and \( \phi \) is the common roll motion of both platforms. The relative rotation angle \( \theta_m \) between the front and rear platforms can be obtained as:

\[
\theta_m = \theta_f - \theta_r
\]

The height of the platform is represented by

\[
Z_i = \mathbf{O}_i(z) \quad (i = f, r)
\]

where \( \mathbf{O}_i(z) \) is the z-axis coordinate of the front or rear mobile reference frame with respect to the base reference frame. Position \( \mathbf{O}_i \) can be represented in the base reference frame \( (X_b, Y_b, \)
as:

\[ \mathbf{O}_r = \mathbf{O}_m + R_y(\phi)R_x(\theta_r)T^r_m (0,-L_{cr}, Z_{ost}) \]  \quad (4)

where \( \mathbf{R} \) is the rotation matrix, \( T^r_m(x,y,z) \) is the translation matrix from local reference frame \( \mathbf{O}_m (X_m, Y_m, Z_m) \) of the 2-dof driving mechanism to the rear mobile reference frame \( \mathbf{O}_r (X_r, Y_r, Z_r) \), the position \( \mathbf{O}_m \) from the base reference frame is \([0,0,L_m]^T\), and \( L_{cr} \) is the distance from the center position \( \mathbf{O}_p \) of the revolute joint to position \( \mathbf{O}_r \) of the rear mobile reference frames.

Similarly, the coordinate position \( \mathbf{O}_f \) of the front mobile reference frame can be represented in the base reference frame \((X_b, Y_b, Z_b)\) as (see Figure 1(a)):

\[ \mathbf{O}_f = \mathbf{O}_m + R_y(\phi)R_x(\theta_r)T^p_m (0,0, Z_{ost}) + R_y(\phi)R_x(\theta_f)T^f_p (0,L_{cf},0) \]  \quad (5)

where \( L_{cf} \) is the distance from center position \( \mathbf{O}_p \) of the revolute joint to coordinate position \( \mathbf{O}_f \) in the front mobile reference frame.

The height \( L_m \) from the base reference frame to the local reference frame of the 2-dof driving mechanism can be computed using equations (3), (4), and (5) as:

\[ L_m = Z_i - \cos(\phi) \cos(\theta_r)Z_{ost} + \delta \cos(\phi) \sin(\theta_i)L_{ci} \quad (i=f \text{ or } r) \]  \quad (6)

where \( \delta \) is the selection index among the two platforms. If \( i=f \), \( \delta=-1 \), while if \( i=r \), \( \delta=1 \). The lengths of the actuators of the 2-dof driving mechanism are then obtained as:

\[ L_{m1} = L_m + \frac{L_{base} \tan \phi}{2}, \quad L_{m2} = L_m - \frac{L_{base} \tan \phi}{2} \]  \quad (7)

where \( L_{m1}, L_{m2}, \) and \( L_{base} \) are the active lengths of the left and right actuators, and the distance between the two active prismatic joints of the 2-dof mechanism, respectively.

Note that the spherical joint positions \( \mathbf{S}_f \) can be obtained geometrically from the intersection of the prismatic actuator axis with the \( F \)-plane of the front platform, which is represented by
\[ P(X) = \tilde{f}^T \cdot (X - F_p) = 0 \]  

(8) 

where \( \tilde{f} \) is the vector normal to the \( F \)-plane, \( X \) is any point on the plane, and \( F_p \) is a determined point on the plane. The point \( F_p \) can be represented in the base reference frame as 

\[ F_p = O_m + R_y(\phi)R_x(\theta_r)T_m^p (0,0,Z_{ost}) + R_y(\phi)R_x(\theta_f)T_p^f (0,L_{cf},-Z_{ost}) \]  

(9) 

Inserting \( S_f \) on the \( F \)-plane into \( X \) in Equation (8) gives an equation with the unknown prismatic joint length \( L_f \) in a closed form. Position \( S_f \) of the spherical joint attached to the front platform can be represented by 

\[ S_f = [0,L_a,L_f]^T \]  

(10) 

where \( L_a \) is the distance from the origin in the base reference frame to the point of the front prismatic actuator. Using two rotation matrixes, vector \( \tilde{f} \) normal to the \( F \)-plane is easily found as follows: 

\[ \tilde{f} = R_y(\phi)R_x(\theta_f)[0,0,1]^T \]  

(11) 

Inserting equations (9), (10), and (11) into Equation (8), the front length \( L_f \) of the active prismatic joint is then given by: 

\[
L_f = (\cos(\theta_f)\cos(\theta_r)Z_{ost} + \sin(\theta_f)\sin(\theta_r)Z_{ost} - Z_{ost} + \cos(\theta_f)\cos(\phi)L_m\cos(\phi) / \cos(\theta_f)
\]

(12) 

Similarly, the rear length \( L_r \) of the active prismatic joint is computed by inserting position \( S_r \) on the \( R \)-plane into the equation: 

\[ P(X) = \tilde{r}^T \cdot (X - O_m) = 0 \]  

(13) 

where \( \tilde{r} \) is the vector normal to the \( R \)-plane. Position \( S_r \) of the spherical joint attached to the rear platform is represented by: 

\[ S_r = [0,-L_h,L_r]^T \]  

(14)
where \( L_b \) is the distance from the origin in the base reference frame to the point of the rear prismatic actuator. In addition, using two rotation matrixes, the vector \( \vec{r} \) normal to the \( R \)-plane can be found as follows:

\[
\vec{r} = R_y(\phi)R_x(\theta_r)\begin{bmatrix} 0,0,1 \end{bmatrix}^T
\]  

(15)

Inserting equations (14) and (15) into Equation (13), the unknown active prismatic joint \( L_r \) can be solved in a closed form

\[
L_r = (-\sin(\theta_r)L_m + \cos(\theta_r)\cos(\phi)L_m) / \cos(\phi) / \cos(\theta_r)
\]  

(16)

Note that if \( Z_{ost} = 0 \), \( L_f \) in Equation (12) has no \( \theta_r \) term, which means that the pitch motions between the front and rear platforms are independent.

3.2 Forward kinematics

The forward kinematics problem of the suggested mechanism can be defined as follows: given the displacements \((L_f, L_r, L_{m1}, L_{m2})\) of the actuated joints, compute the positions \(Z_i\) \((i = f, r)\) and orientation \(\theta_f, \theta_r, \phi\) of the movable platforms with respect to the base fixed reference frame. First, using Equation (7), \(L_m\) and \(\phi\) can be computed as:

\[
L_m = (L_{m1} + L_{m2}) / 2, \ \phi = \tan^{-1}((L_{m1} - L_{m2}) / L_{base})
\]  

(17)

Then, arranging Equation (16) with respect to \(\theta_r\) and (12) with respect to \(\theta_f\) gives:

\[
a_i \sin(\theta_i) + b_i \cos(\theta_i) = c_i \ (i = f, r)
\]

\[
a_r = L_b
\]

\[
b_r = \cos(\phi)(L_r - L_m)
\]

\[
c_r = 0
\]

\[
a_f = L_a + \sin(\theta_f)Z_{ost}
\]

\[
b_f = \cos(\theta_f)Z_{ost} + \cos(\phi)(L_m - L_f)
\]

\[
c_f = Z_{ost}
\]  

(18)
Solving Equation (18), $\theta_i$ is represented by

$$\theta_i = \sin^{-1}\left(\frac{c_i}{\sqrt{a_i^2 + b_i^2}}\right) - \tan^{-1}\left(\frac{b_i}{a_i}\right) \quad (i = f, r)$$

Note that if $L_r = L_m$, the pitch rotation $\theta_r$ of the rear platform is always zero and if $L_f = L_m$ and $\theta_r = 0$, the pitch rotation $\theta_f$ of the front platform becomes zero, regardless of the roll angle. Finally, the height $Z_i$ of each mobile platform can be obtained by solving Equation (6) with the computed $L_m$, $\phi$, and $\theta_i$. Note that the forward kinematics for the suggested mechanism can be solved in simple closed forms.

3.3 Velocity analysis

Differentiating both sides of equations (7), (12), and (16) with respect to time and rearranging the equations, the velocity equations can be written in matrix form as:

$$\dot{\textbf{q}} = \textbf{J}_x \dot{\textbf{x}}, \quad \dot{\textbf{x}} = \textbf{J}_{\theta,i} \dot{\Theta}_i$$

$$\dot{\textbf{q}}_i = \textbf{J}_x \textbf{J}_{\theta,i} \dot{\Theta}_i$$

where $\dot{\textbf{q}}_i^T = [\dot{L}_f, \dot{L}_r, \dot{L}_{m1}, \dot{L}_{m2}]$ is the vector of the linear velocities of the actuators, $\dot{\textbf{x}}_i = [\dot{\Theta}_f, \dot{\Theta}_r, \dot{L}_m, \dot{\phi}]^T$ is the vector of the parameter velocities at the 2-dof driving mechanism reference frame, $\dot{\Theta}_i = [\dot{\Theta}_f, \dot{\Theta}_r, \dot{Z}_i, \dot{\phi}]^T$ is the vector of the parameter velocities of each mobile reference frame, and the Jacobian matrixes $\textbf{J}_x$ and $\textbf{J}_{\theta,i}$ are listed in the appendix.

Using the determinant of matrix $\textbf{J}_x$ in Equation (21), the singularities of the mechanism can be analyzed. The determinant of matrix $\textbf{J}_x$ is derived as:

$$\text{Det}(\textbf{J}) = (L_a + (\sin(\theta_r) - \sin(\theta_l))Z_{ost})L_{base}(1 + \tan^2(\phi)/(\cos^2(\phi)\cos^2(\theta)\cos^2(\theta)))$$

where $\text{Det}(\textbf{J})$ represents the determinant of matrix $\textbf{J}$. Note that regardless of the mechanism configurations, the determinants of Jacobians $\textbf{J}_{\theta,f}$ and $\textbf{J}_{\theta,r}$ are both unity. When $\text{Det}(\textbf{J}_x) = 0$,
the following singularity conditions can be found:

\[
\sin(\theta_f) - \sin(\theta_r) = L_a / Z_{ost}, \quad \theta_f = \theta_r = \phi = 90^\circ
\]  \hspace{1cm} (23)

Therefore, to avoid singularities of the mechanism, \( L_a/Z_{ost} \) should be selected to have a value larger than 2. Simultaneously, angles \( \theta_f, \theta_r, \) and \( \phi \) should be within \( \pm 90^\circ \), which can easily be achieved in the design by limiting the actuator lengths.

4. DEVELOPING THE PROTOTYPE

To restore normal walking in patients through neurological rehabilitation, suitable task-specific training using robots might be very helpful to reduce the physical effort required of therapists. Therefore, a footpad device attached to the feet of patients should be designed to generate natural walking trajectories. In addition, the footpad device should have sufficient workspace to simulate natural walking on various surfaces, and should be able to support human weight. Based on normal gait requirements, a footpad device was developed based on the suggested 1T-3R-type mechanism.

Computer simulations suggested that the conditions to be satisfied for gait requirements are: \( L_{\text{base}} = 10 \text{ cm}, \ L_a = 5 \text{ cm}, \ L_b = 15 \text{ cm}, \) and actuator stroke = 20 cm. Figure 10 shows the workspace envelope of the designed footpad mechanism based on a discrete workspace analysis using the inverse kinematics of the mechanism and actuator length constraints. Figure 10(a) shows the orientation workspace about pitch \( \theta_f \) and roll \( \phi \) versus the front platform heave \( Z_f \); while Figure 10(b) is the orientation workspace of pitch \( \theta_r \) and roll \( \phi \) versus the rear platform heave \( Z_r \). With this design, the maximum heave motion is 20 cm, and the maximum pitch angles of the front and rear platforms exceed \( \pm 50^\circ \) and \( \pm 70^\circ \), respectively. The maximum roll angle of the platforms exceeds \( \pm 60^\circ \). Note that although the roll angles of the two platforms are the same, since they have a common 2-dof driving mechanism for heave
and roll motion, the achievable orientation workspace of pitch angles $\theta_f$ and $\theta_r$ of the two platforms differs owing to the different lengths ($L_a, L_b$) of the platforms. Figure 11 shows the maximum z-axis forces that the rear platform can handle with a single actuator with a maximum force of 1 kN with respect to pitch angle and the distance $L_{cr}$ from the center position $O_p$ of the revolute joint to position $O_r$ of the rear mobile reference frames. Since the trajectory of the gravity reaction force due to body weight moves from the heel to the toe during ground contact, the footpad device should support the reaction force at any contact point of the front and rear platforms of the footpad mechanism. In Figure 11, the maximum allowable force exceeds 3 kN at $L_{cr} = 5$ cm, while the minimum allowable force is 1 kN at $L_{cr} = 15$ cm. The force-bearing capability of the front platform showed the same trends as that of the rear platform. These motion ranges and force capabilities of the footpad device may simulate natural walking trajectories while supporting a normal human. Figure 12 shows the manufactured footpad device, which has x-y stages with two serial LM guides inserted between spherical joints and platforms.

5. CONCLUSIONS

This paper presents a new family of 4-dof parallel mechanisms, which consist of two platforms, two $P_S-P-P$ limbs, one ($P_S-R-P$- or $P_S-R$) middle limb, and a revolute joint that connects the front and rear platforms. The type of 2-dof driving mechanism at the middle limb determines the motions of the 4-dof mechanism with two platforms. We proposed T-R- and T-T-type 2-dof driving mechanisms, which can generate one translation and one rotation or two translations with two base-fixed prismatic joints, respectively. Therefore, the 4-dof parallel mechanism with the T-R-type driving mechanism (1T-3R) can generate pitch motion at each platform, roll and heave motions at both platforms, while that with the T-T type driving mechanism (2T-2R) can generate pitch motion of each platform and two translational motions.
at both platforms. For the 1T-3R-type mechanism, kinematic analyses involving inverse kinematics, forward kinematics, and velocity analysis were performed. In addition, a novel footpad device was developed based on the suggested mechanism with the 1T-3R type, and the natural trajectories of the human foot were simulated using this footpad device. The structure of the footpad device allows relative rotation between the fore and rear foot. Moreover, since the 2-dof driving mechanism has very simple kinematic solutions, the computed load of the proposed 4-dof footpad device is much smaller than that of general parallel mechanisms. In addition, the platform of the proposed device has only two serial chains, while the general parallel mechanism has four serial chains. This smaller number of limbs could increase the orientation workspace of a parallel mechanism. Therefore, the mechanism in the footpad device has characteristics that are intermediate between those of parallel and serial mechanisms, and provides an adequate solution for a foot rehabilitation device that needs relatively high stiffness to support human weight and a wide orientation workspace for foot motions. In the future, our footpad device will be combined with a planar device, which will generate planar motions for walking during the gait cycle. In addition, haptic effects for various terrains using position/force control will be developed to apply the device for lower limb rehabilitation.

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APPENDIX

\[ \dot{q}_i = J_x J_{\theta_i} \dot{\theta}_i \quad (i = f, r) \]

\[
J_x = \begin{bmatrix}
\frac{\partial L_f}{\partial \theta_f} & \frac{\partial L_f}{\partial \theta_r} & 1 & \frac{\partial L_f}{\partial \phi} \\
0 & \frac{\partial L_r}{\partial \theta_r} & 1 & \frac{\partial L_r}{\partial \phi} \\
0 & 0 & 1 & \frac{\partial L_{m1}}{\partial \phi} \\
0 & 0 & 1 & \frac{\partial L_{m2}}{\partial \phi}
\end{bmatrix}
\]

\[
\frac{\partial L_f}{\partial \theta_f} = (L_a + \sin(\theta_r)Z_{ost} - \sin(\theta_f)Z_{ost})/\cos(\phi)/\cos^2(\theta_f)
\]

\[
\frac{\partial L_f}{\partial \theta_r} = -Z_{ost}(\cos(\theta_f)\sin(\theta_r) - \sin(\theta_f)\cos(\theta_r))/\cos(\phi)/\cos(\theta_f)
\]

\[
\frac{\partial L_f}{\partial \phi} = \sin(\phi)(\cos(\theta_f)\cos(\theta_r)Z_{ost} + \sin(\theta_f)L_a + \sin(\theta_r)\sin(\theta_f)Z_{ost} - Z_{ost})/\cos^2(\phi)/\cos(\theta_f)
\]

\[
\frac{\partial L_r}{\partial \theta_r} = -L_a/\cos(\phi)/\cos^2(\theta_r), \quad \frac{\partial L_r}{\partial \phi} = -\sin(\phi)\sin(\theta_r)L_a/\cos^2(\phi)/\cos(\theta_r)
\]

\[
\frac{\partial L_{m1}}{\partial \phi} = 1/2L_{base}(1 + \tan^2(\phi)), \quad \frac{\partial L_{m2}}{\partial \phi} = -1/2L_{base}(1 + \tan^2(\phi))
\]

\[
J_{\theta_i} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\frac{\partial L}{\partial \theta_f} = -\cos(\phi)\cos(\theta_f)L_{cf} \quad , \quad \frac{\partial L}{\partial \theta_r} = \cos(\phi)\sin(\theta_r)Z_{ost}
\]

\[
\frac{\partial L_{m}}{\partial \phi} = \sin(\phi)\cos(\theta_r)Z_{ost} + \sin(\phi)\sin(\theta_f)L_{cf}
\]
\[
J_{\epsilon,r} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & \frac{\partial L_m}{\partial \theta_r} & 1 & \frac{\partial L_m}{\partial \phi} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\frac{\partial L_m}{\partial \theta_r} = -\cos(\phi) \cos(\theta_r)L_{cr} + \cos(\phi) \sin(\theta_r)Z_{ost}, \quad \frac{\partial L_m}{\partial \phi} = -\sin(\phi) \sin(\theta_r)L_{cr} + \sin(\phi) \cos(\theta_r)Z_{ost}
\]
Figure 1. T-R-type driving mechanism ($z$ and $\phi$) with two base-fixed active prismatic joints

Figure 2. T-T-type ($x$ and $z$) with two base-fixed active prismatic joints
Figure 3. New 4-dof mechanisms with two platforms

Figure 4. The real implementation of the 1T-3R-type mechanism with two platforms

Figure 5. The real implementation of the 2T-2R-type mechanism with two platforms
Figure 6. The joint structure of 2-dof driving mechanisms

Figure 7. The link-pair relationship diagram of the proposed 4-dof parallel mechanism with two platforms

Figure 8. The kinematic arrangement of the mechanism of multiple platforms with n+2-dof
Figure 9. The kinematic model of the footpad mechanism (1T-3R type)

(a) Front platform (b) Rear platform

Figure 10. Workspaces of the footpad mechanism

Figure 11. The maximum z-axis forces of the rear platform at a single actuator at 1 kN
Figure 12. The footpad device with the suggested mechanism (1T-3R)