Modeling Dependence in the Design of Whole Farm Insurance Contract

—A Copula-Based Model Approach

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Abstract:

The objective of this study is to evaluate and model the risks of corn and soybean production. This study focuses on the risk of revenue variability that arises from changes in prices, yields shortfalls or both. There are several models for price and yield risk factors for corn and soybeans. For instance, yield risks can be modeled by a family of Beta distributions, whereas price shocks can be modeled by log-normal distributions. In order to develop a multivariate model that preserves a given set of marginals, a copula approach can be used to characterize the joint yield and price risk of corn and soybeans, which are usually highly correlated. The copula approach has been spurred by the recent developments in the whole farm insurance (WFI), resulting in an increasing need for the modeling of multivariate risk factors and their interaction. As a part of the study, various copula models are investigated for their suitability in modeling yield and price risks. Finally, the proposed copula approach is illustrated with simulated data to calculate the premium rate of the whole farm insurance. Results show that WFI is superior to crop-specific insurance with premia 36% cheaper than the latter.

Key Words: Copula, Crop Insurance, Loss Distribution

1 Introduction

Federally regulated crop insurance programs have become a major source of subsidies to U.S. farmers. This program offers protection against risks in agricultural production such as yield shortfalls, price collapses and revenue losses. Agricultural risk is typically assumed to originate from the unanticipated movements in prices, yields and revenues. In designing and rating crop insurance contracts, it is important to understand the distributions of several risk variables interacting simultaneously, not in isolation of one another. Ignoring dependencies among risk factors can lead to biased estimates of the risk. For example, in the case of the natural hedge in which revenue is stabilized because of the negative relationship between crop yields and prices, if the negative relationship between price and yield is ignored, it will overestimate the risk of the revenue insurance. Thus, it is important to be able to adequately model dependence and multivariate outcomes.

The federal crop insurance program currently insures in excess of $70 billion in crops and livestock commodities, resulting in an increasing need for the modeling of multivariate risk factors and their interaction. A risk management tool known as crop revenue insurance protects crop producers from declines in both crop prices and yields. Revenue insurance products currently presented approximately 50% of U.S. corn and soybean insurance acres (USDA/RMA). There exists three revenue insurance (RI) programs: Crop Revenue Coverage (CRC), Revenue Assurance (RA) and Group Risk Income Protection (GRIP). These programs guarantee a certain level of crop revenue for a given crop or for all insurable crops grown on a farm, rather than just production, and pay an indemnity if revenues fall beneath the guarantee. This indemnity payment scheme deals with both price and yield risk and should be better correlated with farm's need.

One of the problems with pricing revenue insurance contracts is accounting for the dependence structure between the price and yield risks. It is difficult to calculate the premium rates for the revenue insurance when the joint distribution of two or more random variables need to be assessed. Many studies have evaluated the revenue risk by considering the degree of correlation between yields and prices. The modeling approach ranges from non-parametric (Goodwin and Ker, 1998) to parametric methods. Cobel, Heifner, and Zuniga (2000) have investigated these correlations in revenue insurance and found that there is strong correlation between price and farm yield and between farm and national yiled in certain crops and regions. The procedures utilized in the literature are to generate pairs of random variables from a given pair of marginals with a known degree of correlation. Spearman's rank correlation coefficient and Kendall's tau coefficient are two common measures of correlation which are invariant to monotone transformations. However, these procedures are limited in that only the first and second moments are matched (Goodwin and Ker, 2001). Also, these correlation structure measures characterize the dependence over the entire support of the risk variables, while many issues in risk management focus on the tail areas behavior. Besides the above limitations, theoretical models of risk management imply that crop-specific revenue insurance and whole-farm insurance should be derived from the joint probability distribution of yields and prices of one crop or multiple crops. To know the joint loss probability, the joint distribution of crop yields and prices must be known. This multidimensional composition calls for flexible models that can describe the major data properties, such as higher-order moments, fat tails, co-extreme movement and tail dependence.

The need to model the multivariate distribution of yields and prices and the multivariate dependence structure associated with the copula method motivated this study to use copula approach. A joint probability distribution uniquely determines all lower dimensional marginal and conditional distributions. As a result, association between any set of random variables can be fully described by knowing the multivariate joint distribution. A promising tool used to study multivariate outcomes is the copula function, which was originated by Sklar (1959). Most of the related work in copulas was written after 1990s. The basic idea of a copula is to link the marginal distributions together to form the joint distribution. It is a parametrically-specified, joint distribution generated from given marginals. The attractive feature of parametrically specified copulas is that estimation and inference is based on standard maximum likelihood procedures, which make it feasible to efficiently estimate the assumed copula model.

The objective of this study is to model and evaluate multivariate risk factors and their interaction in corn and soybean production with implications for evaluating the potential for establishing a combination insurance or whole-farm revenue insurance that would address the risks of corn and soybean production. The multivariate modeling is based on the copula approach to reconstruct a multivariate joint distribution of the risks based on a given set of yields and prices marginal distributions as well as to separate the dependence structure and the marginal distributions in a multivariate distribution. As a part of this study, various parametric copula models such as Gaussian copula and t copula are investigated for their suitability in modeling yield and price risks.

This study focuses on the risk of whole-farm revenue variability that arises from changes in multi-crop random prices, random yields shortfalls or both. With whole farm coverage, all acres of farms crops insured in a county rather than an individual crop are covered under one insurance unit. Since most crop risks are not perfectly correlated ($|\rho| < 1$), whole farm insurance (WFI) provides more efficient coverage than insuring each crop with a specific policy. The rational of WFI is thus to pool all of a farm's insurable risks into a single policy that provides coverage at lower rates against the farm's revenue losses. If a farm grows two crops, corn and soybeans, an insurance policy based on the farm's total revenue will be cheaper than the sum of the premia of crops corn and soybeans for the same expected revenue (Hennessy et al. 1997). The effects of an increase in dependence between multiplicative risks on the actuarially fair premium value of an insurance contract are analyzed in this study. The gains of moving from a combination revenue insurance delivered by crop-specific policies to WFI are also evaluated in this study.

Based on the copula WFI model approach, the rest of the paper is organized as follows. Section 2 explores the copula approach for WFI insurance modeling by using a two-stage MLE method. Section 3 provides the empirical analysis of the copula model by using corn and soybean data. A simulation study is conducted in Section 4 to compute the premium rate of the WFI and crop-specific insurance based on the copula estimates from Section 4. The conclusion is that premia of WFI are 36% cheaper at a 75% coverage level.

2 Copula Based Models for Whole Farm Revenue Insurance

This section discusses the copula approach for whole-farm insurance modeling. In the WFI model, gross revenue is exposed to multiple risks, such as random yields and random prices of multiple crops, which may affect each other. It is important to model the distributions of these multivariate risk variables simultaneously by using a joint distribution model, which can capture the dependence structure between these risk factors.

2.1 Copula Functions and the Two-stage MLE Estimation Method

The copula is a tool for understanding relationships among multivariate random variables. The usefulness of copulas in modeling dependence stems from a famous theorem of Sklar (1959). Sklar's theorem states that any continuous multivariate distribution can be uniquely described by two parts: the univariate marginals and the multivariate dependence structure. The latter is represented by a copula. A m-dimensional copula $C(F_1(x_1), F_2(x_2), ..., F_m(x_m))$ is defined as any multivariate distribution function in the unit hypercube $[0,1]^m$, with uniform $U[0,1]$ marginal distributions. It can be shown (see Sklar, 1959) that every joint distribution $F(x_1, x_2, ..., x_m)$, whose marginal distribution functions are $F_1, F_2, ..., F_m$, can be written as:

$$F(x_1, ..., x_m) = C(F_1(x_1), ..., F_m(x_m); \theta), \tag{1}$$

where θ is a vector of parameters of the copula which are called the *dependence parameters*, which measure dependence between the marginals.

Sklar's theorem implies that copulas can be used to express a multivariate distribution in terms of its marginal distributions. The copula C is unique for a given distribution F if each marginal $F_j(x_j)$ is continuous. That is, the joint distribution of $x_1, \dots, x_m$ can be described by the marginal distribution $F_j(x_j)$ and the copula C. This result allows us to estimate joint distributions with a two-step process. The first step is to estimate appropriate marginal distributions of each random variable (not necessarily from the same family). The copula construction does not constrain the choice of marginal distributions. Second, estimate a copula by using one of the copula models and use it to capture the joint distribution, given the underlying marginal distributions. Alternatively, one can jointly estimate the marginals and the copula parameters.

Equation (1) is a frequent starting point of empirical applications of copulas. In this equation, the joint distribution is expressed in terms of its respective marginal distributions and a function C that binds them together. A substantial advantage of copula functions is that the marginal distributions may come from different families. This construction allows researchers to consider marginal distributions and dependence as two separate but related issues. For the empirical applications, the dependence parameter θ is the main focus of estimation. This equation will be used in fitting copula-based models to agricultural data in the design of whole farm insurance contract.

Using the relation, a large number of parametric families of copulas are presented in the literature (Nelson, 1999). The most common copulas in risk management are the t copula and the Gaussian (normal) copula.

The Gaussian copula takes the form

$$C(u_1, u_2; \theta) = \Phi_2(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \theta))$$

where Φ is the distribution function of the univariate standard normal distribution, Φ^{-1} is the inverse distribution function of the standard normal distribution and $\Phi_2(x_1, x_2; \theta)$ is the standard bivariate normal distribution with correlation parameter $\theta \in (-1, 1)$.

The t copula with γ degrees of freedom and correlation θ is

$$C(u_1, u_2; \theta_1, \theta_2) = T_{2\gamma}(T_{1\gamma}^{-1}(u_1), T_{1\gamma}^{-1}(u_2); \theta)$$

where $T_{1\gamma}$ is the distribution function of an univariate t-distribution with γ degrees of freedom and $T_{2\gamma}(x_1, x_2; \theta)$ denotes the distribution function of a bivariate t-distribution with γ degrees of freedom. The two dependence parameters are (γ, θ). The parameter γ controls the heaviness of the tails. As $\gamma \to \infty$, the t copula approaches Gaussian copula.

Given a collection of marginal densities, the multivariate distribution can be defined by applying a copula to the prescribed marginals by using equation (1). There are several parametric models for price and yield risk factors for corn and soybeans. For instance, yield risks are usually modeled by a family of Beta distributions, whereas price shocks are usually modeled by log-normal distributions. Copulas allow researchers to piece together joint distributions when only parametric form of the marginal distributions are known explicitly.

The density function of a multivariate distribution whose dependence structures is defined by a copula function can be obtained by differentiation of equation (1) and is given by

$$f(X_1, \ldots, X_m, \theta) = c(F_1(X_1), \ldots, F_m(X_m)) \prod_{j=1}^{m} f_j(X_j)$$

where f_j represents the marginal density function of x_j and c is the density function of the copula function as shown in equation (1). In other words,

$$c(u_1, u_2) = \frac{\partial^2 C(u_1, u_2)}{\partial u_1 \partial u_2} = \frac{f(F_1^{-1}(u_1), \ldots, F_m^{-1}(u_1)}{\prod_{j=1}^{m} f_j(F_j^{-1}(u_j))}$$

The log-likelihood function is

$$l(\theta) = \sum_{i=1}^{n} log(c(F_1(X_{i1}; \beta), \ldots, F_m(X_{im}; \beta); \alpha)) + \sum_{i=1}^{n} \sum_{j=1}^{m} log(f_i(X_{ij}; \beta))$$

where $u_1 = F_1(X_{i1}), u_2 = F_2(X_{i2})$.

The usual methodology used in the literature, as proposed by Joe and Xu (1996), requires one to estimate the MLE of the appropriate marginal in a two-stage approach. This method is called the inference function for margins (IFM) method. By using this method, the set of parameters of the model are estimated through a system of estimating equations, with each estimating equation being a score function (partial derivative of a log-likelihood) from some marginal distribution of the multivariate copula model. In the first step of the IFM method, the marginal parameter β_i is estimated by maximizing the loglikelihood of the m univariate margins separately

$$\hat{\beta}_{iIFM} = \arg\max_{\beta} \sum_{t=1}^{T} log f_i(X_{ti};\beta),$$

where $i = 1, 2, \ldots, m$ stand for parameters from each marginal distribution F_i. In the second step, the copula form is identified and the dependence parameters are estimated given the marginal estimates $\hat{\beta}_{IFM}$:

$$\hat{\theta} = \arg\max_{\alpha} \sum_{t=1}^{T} \text{log} c(F_1(X_{t1};\hat{\beta}_{IFM}), \ldots, F_m(X_{tm};\hat{\beta}_{IFM});\alpha)$$

2.2 Copula-Based Whole Farm Insurance Model and Price-Yield Dependence and Cross Price-Yield Dependence

To model the risk of whole-farm revenue (WFI) variability that arises from changes in multi-crop prices, yields shortfalls or both, the copula method can be applied. Suppose a farm grows two crops, corn and soybean, an insurance based on the farm's total revenue and the sum of the premia of corn and soybean for the same expected revenue can be compared based on a four-dimensional copula and two two-dimensional copulas, respectively.

The WFI contract has the standard indemnity scheme of the form

$$max[(\lambda R^e - R), 0]$$

where $R = Y_cP_c + Y_sP_s$ is the revenue, $R^e = E(R)$ is the expected revenue and $\lambda \in (0, 1])$is the coverage level. P_i and $Y_i, i = c, s$ are the non-negative random variables representing price and yield risks of corn and soybean, respectively, with the joint probability distribution F in a certain copula function form $F(P_c, P_s, Y_c, Y_s) = C(F_1(P_c), F_2(P_s), F_3(Y_c), F_4(Y_s); \theta)$. If $R < \lambda R^e$, the insurer will pay $(\lambda R^e - R)$ as an indemnity. An actuarially fair premium for the WFI is equal to the expected loss of this contract. To calculate the premium of a WFI contract based on the copula we estimated, suppose an insurance contract will insure some proportion λ of the mean crop revenue (R^e), the expected loss (in bushels) for this insurance contract that guarantees $\lambda \times 100\%$ of the predicted revenue (R^e) takes the form of

$$\mathrm{EL}(R) = E[(\lambda R^e - R)I(R \leq \lambda R^e)]$$

where R denotes the observed annual WFI revenue and R^e represents the predicted revenue. The expected value of the revenue risk R^e is with respected to the marginal risk measures of price and yield risk. The marginal distribution of yield and price can be inferred from the historical yield and price data. To construct the full joint distribution of yield and price risk, we therefore need a copula.

3 Empirical Framework

This section illustrates the copula modeling of WFI revenue risk by using a four-dimensional copula model. The guarantee of WFI revenue insurance is based on market price and the actual yield. The data used in this analysis are the National Agricultural Statistics Service (NASS) county-level corn and soybean yields of Iowa from 1960 to 2006 and the difference between the futures prices at harvest and at planting for corn and soybeans from the Chicago Board of Trade (CBOT) for the same time period.

3.1 Futures Price Data

Current revenue insurance programs use Chicago Board of Trade futures market prices and the historic average of the actual yields to compute the revenue coverage and guarantee. The Chicago Board of Trade's December corn futures contract and November soybean futures contract data from 1960 to 2007 are considered.

A base market price is determined during February (planting time) by averaging new-crop futures prices for December corn futures contract and November soybeans futures contract. A delivery price is determined by averaging the new crop futures prices during delivery time for both corn (in December) and soybeans (in November). The data used in this study are February corn futures price data (defined as $P_{c,t}^{2,12}$) and December (harvest time) corn futures price data (defined as $P_{c,t}^{12,12}$) of each year from 1960 to 2007; February soybean futures price data (defined as $P_{s,t}^{2,11}$) and November (harvest time) soybean futures price data (defined as $P_{s,t}^{11,11}$) of each year from 1960 to 2007.

By regressing the actual futures price on the February predicted futures price for both corn and soybeans, the estimates of the intercept and slope parameter can be obtained. $P_{c,t}^{12,12} = \alpha_c + \beta_c * P_{c,t}^{2,12} + e_{c,t}$, $P_{s,t}^{11,11} = \alpha_s + \beta_s * P_{s,t}^{2,11} + e_{s,t}$. As showed in Table 1, the estimated intercept α is not significantly different from 0 and the estimated slope $\beta_i, i = c, s$ is not significantly different from 1 for both regressions. Thus, the logarithmic price shocks for corn (defined as $\tilde{P}_{c,t}$) and soybeans (defined as $\tilde{P}_{s,t}$ are approximately equal to the difference between the logarithm prices in harvest time and in planting time. That is, $\tilde{P}_{c,t} = \log(P_{c,t}^{12,12}) - \log(P_{c,t}^{2,12})$, $\tilde{P}_{s,t} = \log(P_{s,t}^{11,11}) - \log(P_{s,t}^{2,11})$. It is typical assumption that the price is distributed as log-normal. Therefore, the logarithmic price shocks ($\tilde{P}_{c,t}$ and $\tilde{P}_{s,t}$) are normally distributed. The summary statistics for the price shocks are shown in Table 2.

3.2 Yield Data

The yield data used here are the county-level Iowa annual yields data for corn and soybeans from 1960 to 2006 in Adair county. By assuming that errors are proportional to the predicted mean, a proportional adjustment can be applied to obtain the detrended yield shocks. This detrending method is ad hoc but it is very common in practice. To do so, regress the yields on a quadratic time trend and output the residuals. $y_t = \alpha + \beta_1 t + \beta_2 t^2 + e_t$, where y_t is the observed crop yield data in year t. Then, calculate the residual $\hat{e}_t$ and the predicted yield for 2006 as $\hat{y}_{2006}$, the normalized and detrended data will be given by the following

$$\tilde{y}_t = \hat{y}_{2006}(1 + \frac{\hat{e}_t}{\hat{y}_t}).$$

where $\hat{e}_t$ is the residual for time t by regressing the yields on a quadratic time trend, and $\hat{y}_{2006}$ is the predicted yield for 2006.

3.3 Fitting 4-dimensional Copula for WFI Model

By using the data of price and yield shock of corn and soybeans ($\tilde{p}_c$,$\tilde{p}_s$,$\tilde{y}_c$,$\tilde{y}_s$), the model of prices and yields for corn and soybeans can be estimated by using a 4-dimensional Gaussian or t copula with the correlation matrix

$$\rho = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_4 \\ \rho_1 & 1 & \rho_3 & \rho_5 \\ \rho_2 & \rho_3 & 1 & \rho_6 \\ \rho_4 & \rho_5 & \rho_6 & 1 \end{bmatrix}$$

where the matrix of ρ is known as the unstructured dispersion matrix and ρ_js are dispersion parameters. Since the one of the good property of copula is invariance to monotonic transformations of the marginals, the dispersion matrix determines the dependence structure of the distribution functions $F_j(x_j)$ as well as multivariate variables x_js. In the case of WFI copula model, x_j stands for price and yields of corn and soybeans.

To use the two-step copula model fitting method, the initial step is to determine the appropriate marginals. It is popular to fit univariate marginals of the logarithmic price shocks as a normal distribution with parameters μ and σ of the distribution function and to fit univariate marginals of yields as a general Beta distribution. The Chi-square goodness-of-fit statistics also suggest that the yield marginal is Beta and the logarithmic price shock marginal is Normal. The Maximum Likelihood Estimate of μ and σ for corn logarithmic price shocks and soybean logarithmic price shocks are $(-0.03, 0.20)$ and $(0.02, 0.16)$ respectively. The marginal MLEs of yields shocks are estimated with corn yields $y_c \sim Beta(7.01, 2.09, 0, 203.55)$, soybean yield $y_s \sim Beta(17.60, 7.66, 0, 65.60)$. In the second step, the dependence parameters for corn and soybeans are estimated given the marginal estimates from the first step. The estimated results of the 4 dimensional Gaussian copula ($LLF = 34.43631$) and t copula ($LLF = 36.5453$) are shown in Tables 4 and 6. The corresponding correlation matrices for (p_c, p_s, y_c, y_s) implied by the t and Gaussian copula with Normal and Beta marginals are shown as Tables 5 and 7.

These results show that Gaussian copula and t copula exhibit similar dependence structures among price and yield shocks of corn and soybeans. The own-crop and cross-crop correlations of price and yield are negative, while the cross-crop price correlation (0.73 in Gaussian copula and 0.74 in t copula) and cross-crop yield correlation (0.68 in Gaussian copula and 0.71 in t copula) are positive. The magnitude of the negative correlations of prices and yields in t copula is higher, which means that the t copula implies higher overall dependence than Gaussian copula.

The copula-based revenue risk model can be fit by different types of Copula functions. Some criterion need to be evaluated in order to select the best copula model. Some typical model selection methods can be used such as Akaike Information Criterion (AIC) and Bayesian Information Criterion of Schwarz (BIC). Various parametric copula models estimated in this sections such as Gaussian copula and t copula can be investigated for their suitability in modeling yield, price and revenue risks. From the estimation, the AIC of t copula ($AIC = -59.1$) is smaller than the AIC of the Gaussian copula ($AIC = -56.86$) and the log-likelihood value of t copula is greater than the log-likelihood value of the Gaussian copula, which imply better goodness-of-fit of t copula than the Gaussian copula.

4 Simulation Study and Policy Implication

In this section, a Monte Carlo simulation method is used to simulate the multi-dimensional variables of prices and yields of corn and soybeans from the copula function estimated above, which preserves the rank correlations of these variables. The joint revenue loss and the premium rates of the revenue insurance contract can be obtained at certain coverage levels. Several scenarios are conducted in which gross crop revenue is supported at the 75 and 85 percent level, respectively. The mean and variance of expected revenue and the expected loss and actuarially-fair premium rates for a whole-farm revenue insurance policy that guarantees a certain percent of the expected level of revenue are calculated.

Suppose that a representative Iowa farm in Adair county grows corn and soybeans with a equal proportion. The farm's guaranteed yields for crop insurance purposes for the two crops are equal to county trend yields. The dependence structure is imposed by the 4-dimensional copula as estimated in section 3. One million revenue series were drawn from the 4-dimensional copulas which preserved the dependence structure among prices and yields of corn and soybean as well as the Normal logarithm price shocks and Beta yields. Those draws $(\tilde{p}_{c,t}, \tilde{p}_{s,t}, \tilde{y}_{c,t}, \tilde{y}_{s,t})$ are used to calculate the expected loss and premium rates for the revenue insurance contract at different coverage levels at year 2006. The predicted harvest delivery price of corn is $\hat{P}^{c,12}_{2006} = exp(\tilde{P}_{c,2006} + log(P^2_{c,2006}))$, in which $1,000,000$ of realized predicted $\hat{P}^{12}_{c,2006}$ can be obtained. There are $1,000,000$ of predicted corn yield $\tilde{y}_{2006}$ of 2006 as well. The predicted revenue realization of corn is $\hat{R}_{c,2006} = \hat{P}^{12}_{c,2006} * \tilde{y}_{c,2006}$. Likewise, the predicted revenue realization of soybeans can be calculated as $\hat{R}_{s,2006} = \hat{P}^{12}_{s,2006} * \tilde{y}_{s,2006}$. Based on the simulated predicted revenue, the expected revenue loss and premium rate for crop-specific insurance contract and whole farm insurance contract can be calculated at 75% and 85 % level respectively as shown in Tables 8 through 12. The actuarially fair premium value of a yield insurance contract can also be obtained by calculating the dollar value of yields given a predetermined price. The yield series of corn and soybeans are simulated from the estimated Gaussian and t copula respectively and the expected loss of the yield of corn and soybean are calculated. The dollar value of the yield loss is then obtained by multiplying the expected yield loss and the predetermined prices.

Table 8 shows that the actuarially fair premium value of a WFI contract implied from a Gaussian copula at 75% coverage is \$4.44, while the premium implied from a t copula at the same coverage is \$3.63 as shown in Table 9. This indicates that higher magnitude of dependence structure implied by a t copla tends to result in lower premium than that from a Gaussian copula given the same level of revenue insurance coverage (\$511).

Tables 10 and 11 show that the actuarially fair premium value of crop-specific revenue insurance and the WFI implied by a Gaussian copula and a t copula at 85% coverage. The results are consistent with those at 75% coverage. Table 12 presents actuarially fair crop-specific and whole-farm revenue assurance premiums for different types of copulas and for different coverage levels. Let D be the difference between crop specific premiums and the whole-farm premium. The difference D ranges from \$2.46 in the Gaussian copula and \$2.06 in the t copula at 75% coverage, which implies the premium of WFI is 36% cheaper than the sum of the premiums of two crop-specific revenue insurance. At 85% coverage level, the premium of WFI is 39% cheaper than the combination of the two crop-specific revenue insurance, which shows that the portifolio effect increases as the coverage level increases. It indicates a big difference between crop-specific coverage and whole-farm coverage revenue insurance. These results suggest that there is a reduction in the actuarialy fair premium for whole-farm revenue insurance relative to single-crop insurance. The reduction is driven by portifolio effects of pooling across crops if a farmer compares single-crop and whole-farm crop insurance at the same coverage level for all crops. Therefore, the WFI on farm's gross revenue is less costly than the sum of crop-specific insurance policies at the same level of liability.

5 Conclusion

This study has taken a close look at the efficiency of whole-farm insurance when the crop producer faces joint yield and price risk and grows corn and soybeans by using a copula model approach. Systematic risk of price and yields has been studied as it is very important in crop revenue insurance. The empirical analysis in this study shows that the whole-farm contracts are more efficient as a risk management tools than the combination of the crop-specific contracts. A combination of crop-specific contracts is more expensive than a single whole-farm contract at the same protection level. Based on these results, the crop producers would switch from purchasing the crop-specific revenue insurance contracts to the WFI contract. The social planner should also take this into consideration when the crop insurance program is designed. For example, the subsidy plan in crop insurance program should favor crop producers who purchase whole-farm contract to improve the efficiency of crop insurance program.

This study contributes to the assessment of the new whole-farm revenue insurance which provides overall coverage to all farms' crops. The idea of whole-farm insurance is to pool all of a farm's insurable risks into a single insurance policy that provides cheaper premium rate at the same protection level against the gross farm revenue losses. The results in this analysis support this idea. Simulation results indicate that a WFI scheme that guarantees a certain expected revenue to producers could provide approximately the same level of coverage as the sum of crop-specific programs at as little as 64% ($3.63/$5.69) the cost at 75% coverage level, and at as little as 76% the cost at 85% coverage level. This shows that the accuracy and efficiency of modeling cross-crop yield and price correlations and rating of whole-farm insurance contracts are improved by using this copula approach. The result will have implications to pricing whole farm insurance products which cover crop revenues from corn and soybeans. These results are also crucial for conducting better risk management to producers from the whole-farm crop production risks.

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Table 1. Equation Estimates: Corn and Soybean Price shocks

Variable	Parameter Estimate	Standard Error	t Ratio[a]
............Regression with intercept............			
Intercept(Corn Futures Price)	0.84	0.42	2.01
$P_{c,t}^{2,12}$	0.84	0.07	10.68*
Intercept(Soybean Futures Price)	0.69	0.35	1.99
$P_{s,t}^{2,11}$	0.89	0.06	15.87*
............Regression without intercept............			
$P_{c,t}^{2,12}$	0.99	0.01	180.96*
$P_{s,t}^{2,11}$	1.003	0.003	262.17*

[a]An "*" indicates statistical significance at the $\alpha = .05$ or smaller level.

Table 2: Variable Descriptions and Summary Statistics[a] (1960-2006)

Variable	Description	Mean[b]	Standard Deviation
$P_{c,t}^{2,12}$	Predicted December Futures Price of Corn at February planting time of year t	221.58	71.20
$P_{c,t}^{12,12}$	Realized December Futures Price of Corn of year t	214.23	72.37
$P_{s,t}^{2,11}$	Predicted November Futures Price of Soybeans at February planting time of year t	510.67	178.07
$P_{s,t}^{11,11}$	Realized November Futures Price of Corn of year t	519.80	181.20
$\tilde{P}_{c,t}$	Logarithm Price shocks for corn	−0.03	0.20
$\tilde{P}_{s,t}$	Logarithm Price shocks for soybeans	0.02	0.16

[a]Data source: Chicago Board of Trade (CBOT). Number of observation is 48. Year $t = 1960, 1961, ...2007$.

[b]The units of the futures price are cents and quarter-cents/bushel (5000 bushels per contract).

Table 3. Equation Estimates: Detrending Corn Yield

Variable	Parameter Estimate	Standard Error	t Ratio[a]
..................Adair County, Iowa (Fips = 19001)			
Intercept	32.03	5.47	5.85*
t	0.47	0.31	1.52
t^2	0.012	0.004	3.43*

[a]An "*" indicates statistical significance at the $\alpha = .10$ or smaller level.

Table 4. The Estimates of the Dependence Parameters of the Gaussian Copula

Gussian	Estimate	Std. Error	P-value
ρ_1	0.73	0.055	< 0.001
ρ_2	−0.16	0.14	0.25
ρ_3	−0.27	0.13	0.04
ρ_4	−0.17	0.14	0.23
ρ_5	−0.29	0.13	0.03
ρ_6	0.68	0.065	< 0.001

Table 5. The Dependence Structure Implied by the Gaussian Copula

	Corn Price	Soybean Price	Corn Yield	Soybean Yield
Corn Price	1	0.73	−0.16	−0.17
Soybean Price		1	−0.27	−0.29
Corn Yield			1	0.68
Soybean Yield				1

Table 6. The Estimates of the Dependence Parameters of the t Copula

Gussian	Estimate	Std. Error	P-value
ρ_1	0.74	0.062	< 0.001
ρ_2	−0.31	0.15	0.042
ρ_3	−0.31	0.14	0.026
ρ_4	−0.29	0.15	0.06
ρ_5	−0.26	0.15	0.084
ρ_6	0.71	0.068	< 0.001
df	7	3.68	0.05

Table 7. The Dependence Structure Implied by the t Copula

	Corn Price	Soybean Price	Corn Yield	Soybean Yield
Corn Price	1	0.74	-0.31	-0.29
Soybean Price		1	-0.31	-0.26
Corn Yield			1	0.71
Soybean Yield				1

Table 8: The Revenue Insurance Implied from a Gaussian-copula at 75% Coverage

	Liability	Expected Loss	Premium Rate	Premium
Corn(Yield)	295.10	3.84	0.0130	3.84
Soybean(Yield)	216.65	0.57	0.0026	0.57
WFI (Yield)	511.25	2.82	0.0055	2.82
corn(Revenue)	294.65	5.83	0.0198	5.83
soybean(Revenue)	216.60	1.08	0.0050	1.08
WFI(Revenue)	511.25	4.44	0.0087	4.44

Table 9: The Revenue Insurance Implied from a t-copula at 75% Coverage

	Liability	Expected Loss	Premium Rate	Premium
Corn(Yield)	293.92	3.78	0.0130	3.78
Soybean(Yield)	216.65	0.58	0.0027	0.58
WFI (Yield)	511.01	2.96	0.0058	2.96
corn(Revenue)	294.46	4.43	0.0152	4.43
soybean(Revenue)	216.56	1.26	0.0059	1.26
WFI(Revenue)	511.01	3.63	0.0072	3.63

Table 10: **The Revenue Insurance Implied from a Gaussian-copula at 85% Coverage**

	Liability	E(loss)	Premium Rate	Premium
Corn(Yield)	334.45	9.21	0.0275	9.21
Soybean(Yield)	245.53	2.71	0.011	2.71
WFI (Yield)	579.42	9.42	0.0163	9.42
corn(Revenue)	333.90	13.62	0.041	13.62
soybean(Revenue)	245.52	4.53	0.018	4.53
WFI(Revenue)	579.42	14.30	0.025	14.30

Table 11: The Revenue Insurance Implied from a t-copula at 85% Coverage

	Liability	Expected Loss	Premium Rate	Premium
Corn(Yield)	334.46	9.24	0.0276	9.24
Soybean(Yield)	245.53	2.72	0.011	2.72
WFI (Yield)	579.32	9.66	0.0167	9.66
corn(Revenue)	333.83	11.18	0.033	11.18
soybean(Revenue)	245.49	4.60	0.0187	4.60
WFI(Revenue)	579.32	12.01	0.0207	12.01

Table 12: Premium Comparison

	Crop Type			
Copula	Corn	Soybeans	Total Premium for Crop Specific	Total Premium for Whole-Farm
Gaussian copula (75%)	5.83	1.08	6.91	4.44
t copula (75%)	4.43	1.26	5.69	3.63
Gaussian copula (85%)	13.62	4.53	18.15	14.30
t copula (85%)	11.18	4.60	15.78	12.01