Exponential Controller for Robot Manipulators *

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Abstract: In this article, we propose a new control algorithm with exponential structure for the position control problem of robot manipulators. We have demonstrated by taking into account the full nonlinear and multivariable nature of the robot dynamics, that the overall closed–loop system is globally asymptotically stable through the Lyapunov's direct method together with LaSalle invariance principle. Experimental results of the proposed controller are presented on a three–degrees–of–freedom direct drive robot manipulator.

Keywords. Direct–drive Robot, Position Controllers, Lyapunov Function, Asymptotic Stability.

1 Introduction

Robot manipulators offer interesting challenges to control researchers owing to the non–linear and multivariable nature of their dynamical behavior. The position control of robot manipulators is the simplest aim in robot control. The goal of position control is to move the manipulator to a fixed desired configuration regardless of the initial joint position. It is well known that most of the present day industrial robots are equipped with simple controllers such as (proportional plus derivative) PD or (proportional plus integral plus derivative) PID types which are effective to achieve the positioning goal [1].

However, these controllers assume implicitly that the robot actuators are able to generate the requested torques. In practice it is not possible, because the actuators are constrained to supply limited torques. That is, if the output amplitude of the controller is out of the linear range of the actuator, then the torque supplied by the actuator to the manipulator joint will be different from that demanded by the controller.

Motivated by the practical interest of relying on control algorithms leading to global stability of the closed–loop system, we propose a control algorithm with exponential structure for the position control problem of robot manipulators.

In this paper, based on the energy shaping methodology [2] [3] [4], we have demonstrated by taking into account the full nonlinear and multivariable nature of the robot dynamics, that the overall closed–loop system is globally asymptotically stable through the Lyapunov's direct method together with LaSalle invariance principle.

This paper is organized as follows. In the Section 2, we recall the robot dynamics and its useful properties. In the Section 3, we present the new controller and its analysis

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global asymptotic stability for position control. Section 4 summarizes the main components of the experimental set–up. Section 5 contains the experimental results of the proposed controller on a direct–drive arm. Finally, we offer some conclusions in Section 6.

2 Robot Dynamics

The dynamics of a serial n-link rigid robot can be written as [5]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \qquad (1)$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n \times 1$ vector of input torques, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques, $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy due to gravity.

We assume that the robot links are joined together with revolute joints. Although the equation of motion (1) is complex, it has several fundamental properties which can be exploited to facilitate control system design. We use the following important property [5]:

Property 1. The matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ (defined using the Christoffel symbols) and the time derivative $\dot{M}(\boldsymbol{q})$ of the inertia matrix satisfy:

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} = \boldsymbol{0} \;\; \forall\, \boldsymbol{q},\dot{\boldsymbol{q}} \in \mathbb{R}^n. \qquad (2)$$

3 Exponential Controller for Robot Manipulators

This section presents the new controller with exponential structures and its stability analysis. The controller is integrated for three members such as: non–lineal function of the position error, saturated–derivative term plus gravitational compensation. Consider the following control scheme with gravity compensation given by

$$\boldsymbol{\tau} \;=\; K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v \tanh(\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) \qquad (3)$$

$$K_p(\tilde{\boldsymbol{q}}) \;=\; \mathrm{Diag}\,\{k_{pi}\,[\,1 - \alpha_i \exp^{-\alpha_i \tilde{q}_i^2}\,]\} \qquad (4)$$

where $i = 1 \cdots n$, $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ is the $n \times 1$ vector of position errors, $\boldsymbol{q}_d \in \mathbb{R}^n$ is the vector of desired positions $K_p(\tilde{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$ is the proportional gain which is a diagonal matrix, $k_{pi} > 0$, $\alpha_i \in \mathbb{R}_+$, $K_v \in \mathbb{R}^{n\times n}$ is a positive definite matrix, the so called derivative gain.

The control problem can be stated by selecting the design matrices K_p and K_v such that, the position error $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ vanishes asymptotically, i.e., $\lim_{t\to\infty} \tilde{\boldsymbol{q}}(t) = \boldsymbol{0} \in \mathbb{R}^n$ and keeping the applied torques constrained by the prescribed limits on actuators of the robot. To solve the control problem, we propose the following proposition:

Proposition. Considering the robot dynamic model (1) together with the control law (3), then the closed–loop system is globally asymptotically stable and the positioning aim $\lim_{t\to\infty} \boldsymbol{q}(t) = \boldsymbol{q}_d$ is achieved.

Proof: The closed–loop system equation obtained by combining the robot model (1) and control scheme (3) can be written as

$$\frac{d}{dt}\begin{bmatrix}\tilde{\mathbf{q}}\\ \dot{\mathbf{q}}\end{bmatrix} = \begin{bmatrix} -\dot{\mathbf{q}} \\ M^{-1}(\mathbf{q})\left[K_p(\tilde{\mathbf{q}})\tilde{\mathbf{q}} - K_v\tanh(\dot{\mathbf{q}}) - C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}\right]\end{bmatrix} \qquad (5)$$

which is an autonomous differential equation, and the origin of the state space is its unique equilibrium point.

In order to carry out the stability analysis of equation (5), we propose the following Lyapunov function candidate:

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \quad \mathcal{K}(\dot{\boldsymbol{q}},\tilde{\boldsymbol{q}}) + \mathcal{U}(\tilde{\boldsymbol{q}})$$

$$\mathcal{U}(\tilde{\boldsymbol{q}}) = \begin{bmatrix} k_{p1}\sqrt{\tilde{q}_1^2 + \exp_1^{-\alpha_1 \tilde{q}^2} - 1} \\ k_{p2}\sqrt{\tilde{q}_1^2 + \exp_1^{-\alpha_1 \tilde{q}^2} - 1} \\ \vdots \\ k_{pn}\sqrt{\tilde{q}_1^2 + \exp_1^{-\alpha_1 \tilde{q}^2} - 1} \end{bmatrix}^T \begin{bmatrix} k_{p1}\sqrt{\tilde{q}_1^2 + \exp_1^{-\alpha_1 \tilde{q}^2} - 1} \\ k_{p2}\sqrt{\tilde{q}_1^2 + \exp_1^{-\alpha_1 \tilde{q}^2} - 1} \\ \vdots \\ k_{pn}\sqrt{\tilde{q}_1^2 + \exp_1^{-\alpha_1 \tilde{q}^2} - 1} \end{bmatrix}$$

$$\mathcal{K}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}$$

The first term of $V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}})$ is a positive definite function with respect to $\dot{\boldsymbol{q}}$ because $M(\boldsymbol{q})$ is a positive definite matrix. The second one of Lyapunov function candidate (6) is also a positive definite function with respect to position error $\tilde{\boldsymbol{q}}$ because K_p is a diagonal matrix.

The time derivative of Lyapunov function candidate (6) along the trajectories of the closed–loop equation (5) and after some algebra by using the property 1, it can be written as follows:

$$\dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T K_v \tanh(\dot{\boldsymbol{q}}) \leq 0, \quad (7)$$

which is a globally negative semidefinite function and therefore, we conclude stability of the equilibrium point. In order to prove asymptotic stability, we exploit the autonomous nature of the closed–loop equation (5) to apply the LaSalle's theorem [6]. In the region

$$\begin{aligned} \Omega &= \left\{ \begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} \in \mathbb{R}^{2n} : \dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) = 0 \right\} \\ &= \left\{ \tilde{\boldsymbol{q}} \in \mathbb{R}^n, \dot{\boldsymbol{q}} = \boldsymbol{0} \in \mathbb{R}^n \right\}, \end{aligned}$$

the unique invariant is $\tilde{\boldsymbol{q}} = \boldsymbol{0}$ and $\dot{\boldsymbol{q}} = \boldsymbol{0}$, all solutions of (5) will globally asymptotically converge to Ω as $t \to \infty$.

4 Experimental Set-Up

We have been designed and built at The Benemérita Universidad Autónoma de Puebla an experimental system for research of robot control algorithms. It is a direct–drive robot manipulator with three degrees of freedom moving in three–dimensional space (see Figure 1). The experimental robot consists of links made of 6061 aluminum actuated by brushless direct drive servo actuator from Parker Compumotor to drive the joints without gear reduction. Advantages of this type of direct-drive actuator includes freedom from backslash and significantly lower joint friction compared to actuators with gear drives. The motors used in the robot manipulator are listed in Table I. Position information is obtained from incremental encoders located on the motors. The standard backwards difference algorithm applied to the joint position measurements was used to generate the velocity signals. In addition to position sensors and motor drivers, it also includes a motion control board manufactured by Precision MicroDynamic Inc., which is used to obtain the joint positions. The control algorithm runs on a Pentium–II (333 Mhz) host computer. With reference

Table I: Servos of the experimental robot.

Link	Model	Torque [Nm]	p/rev
Base	DM1050A	50	1024000
Shoulder	DM1150A	150	1024000
Elbow	DM1015B	15	655360

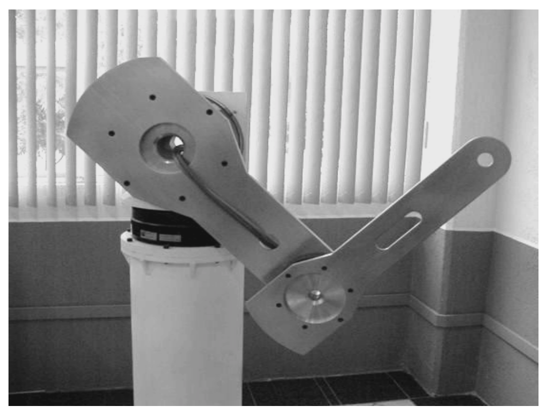

Figure 1: Experimental robot.

to our direct–drive robot, only the gravitational vector is required to implement the new controller(3) (see [7]).

5 Experimental Results

To support our theoretical developments, extensive experimental tests were carried out with the controllers family (3) on a direct–drive robot showed in Figure 1. In the experimental results the friction phenomena for compensation purposes were not modeled, that is, the controller (3) did not show any type of friction compensation.

$$\begin{aligned} \tau_1 &= k_{p1}\left[1-\alpha_1 \exp^{-\alpha_1 \tilde{q}_1^2}\right]\tilde{q}_1 \\ &\quad -k_{v1}\tanh(\dot{q}_1) \end{aligned} \qquad (8)$$

$$\begin{aligned} \tau_2 &= k_{p2}\left[1-\alpha_2 \exp^{-\alpha_2 \tilde{q}_2^2}\right]\tilde{q}_2 \\ &\quad -k_{v2}\tanh(\dot{q}_2) + 38.45\ \sin(q_2) \\ &\quad +1.82\sin(q_2+q_3) \end{aligned} \qquad (9)$$

$$\begin{aligned} \tau_3 &= k_{p3}\left[1-\alpha_3 \exp^{-\alpha_3 \tilde{q}_3^2}\right]\tilde{q}_3 \\ &\quad -k_{v3}\tanh(\dot{q}_3) + 38.45\ \sin(q_2) \\ &\quad +1.82\sin(q_2+q_3) \end{aligned} \qquad (10)$$

It is assumed that the supply torques of the servomotors are bounded, that is, $\tau_i(\cdot)$ $i = 1\cdots 3$ have the constraint $|\tau| \leq \tau_{max}$ as is showed in the Table I, so that let the control value at the upper or lower bound whenever the bound is exceeded.

We have selected identical desired joint positions for the controller such as: $q_{d_1} = 45$ [degrees], $q_{d_2} = 45$ [degrees] and $q_{d_3} = 90$ [degrees] for the base, shoulder and elbow joints, respectively. The initial positions and velocities were set to zero. The algorithm (8–10) has been written in C language with a sampling time for control of 2.5 msec., and it was implemented at 50 micro–seconds. The controller gains were tuned with following values. Proportional gains are: $k_{p1} = 1.12$ Nm/degree, $k_{p2} = 1.8$ Nm/degree, $k_{p3} = 0.168$ Nm/degree, and Derivative gains k_{v1}=0.13 Nm–sec/degree, k_{v2}=0.5 Nm–sec/degree, k_{v3}=0.013 Nm–sec/degree.

The applied torques of the controllers (8–10) as well as their experimental position errors corresponding to the base, shoulder and elbow joints, respectively are depicted in Figure 2. Note that all torque profiles are within the actuators' torque saturation limits. In these figures can be noted that the position errors after a smooth transient, all components tend asymptotically to a small neighborhood of zero.

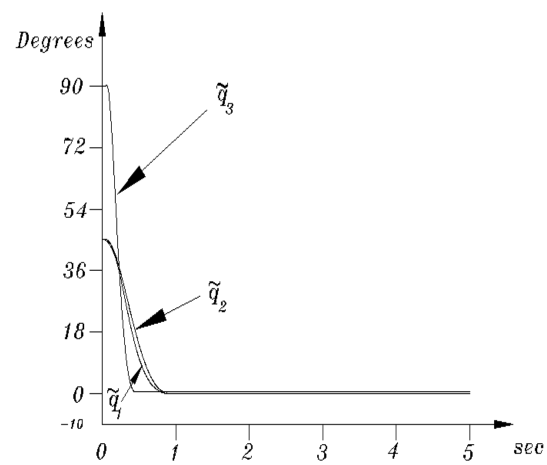

Figure 2: Position errors.

In fact, a thorough analysis of the obtained from experimental data it has not only brief transient, but also the error tends faster to zero without going into the saturation zone of the actuator's torques.

6 Conclusions

In this paper, we have proposed a new controller with gravity compensation for position control of robot manipulators, supported by a rigorous stability analysis. For stability

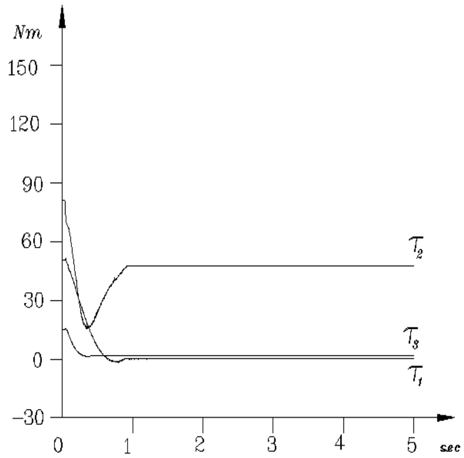

Figure 3: Applied torques.

purpose, the tuning procedure for the new controller is suffice to select proportional gain as diagonal matrix and derivative gain as symmetric positive definite matrix in order to ensure global asymptotic stability.

Experiments results in real–time on a three degree–of–freedom direct drive robot system have been carried out to show the stability and performance. From the experimental results the usefulness of the family of controllers can be concluded.

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