

## Performance optimization of a long rod penetrator penetrating into a semi-infinite target considering bending characteristics

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### Abstract

In this article a new parameter is introduced that optimizes penetration depth of a long rod penetrating into a semi-infinite target. This parameter helps to optimize penetration depth when the projectile is subjected to transverse loading. This parameter is defined using a simple assumption governing bending moment and deflection of the rod as well as the experimental observation of long rod penetrators having aspect ratios ( $L/D$ ) of greater than 30. In this article the length of the rod that sustains allowable bending stress but does not fail is calculated. Using the results of the bending analysis and solution of the Alekseevskii & Tate (AT) equation, an analytical method to optimize crater depth for long rod penetrators of  $L/D > 30$  is obtained. The result of the optimization shows a 14% increase in penetration depth. Also the results of the optimization are compared with the experimental results.

**Key Words:** Optimization, Penetration, Long rod penetrators, Target, Aspect ratios, Bending characteristics.

### Introduction

Investigation and analysis of long rod projectiles penetrating into semi-infinite targets are of prime importance to researchers as well as applied scientists. Due to the complexity of the process and the interdisciplinary aspect of penetration phenomena, most of the work in this area is experimental in nature. When a long rod penetrator collides with a target, the penetrator experiences an impact load of high amplitude. The situation may arise that the rod will be subjected to transverse loadings and therefore bending will occur in the rod. Since for a long rod the ratio  $L/D$  is usually  $>10$ , these projectiles are susceptible to bending. This will allow the designer of the projectiles to confront bending in the rods. As noted by W. Lanz bending stresses in long rod projectiles are usually of high interest for jacketed rods with aspect ratio greater than 30 (Lehr, 1996). Modern targets

are used to compensate for transverse loading and as a result Lehr et al. performed several experiments to investigate the bending effects of long rod penetrators when penetrating into a modern target. Figure 1 shows an example of their test (Lehr and Wollmann, 2001; Lehr et al., 2001).



**Figure 1.** X-ray radiography of the bending of a long rod penetrator before penetration into a modern target.

As shown in this figure, the penetrator, due to high impact loading, is under high bending stresses. These bending stresses are due to inside barrel loadings as well as penetration into high strength materials. Some other examples can be found in the references (Drysdale, 1981; Chhabildas et al., 2005).

Wollman and Lehr also used experimental methods for the design of long rod penetrators. In their investigation, the normalized penetration depth,  $P/L$ , was determined experimentally. This experimentally calculated crater depth was then combined with equations governing bending rods to determine the characteristics of a penetrator to maximize penetration depth.

In this article, a new parameter called bending characteristic is introduced. Using this bending characteristic the equations governing bending and deformation of a long rod projectiles penetrating into a modern target are developed. The bending behavior of a long rod penetrator is modeled using a simple method. The basis of this analysis is the shape of the bending rod during impact. The model used is based on a bending beam under transverse acceleration.

This article also shows that the bending characteristic plays a significant role in the bending behavior of a long rod penetrator. The result of the analysis is then combined with equations and the result of the analysis is used to optimize penetration depth into semi-infinite targets. This means that the penetrator will penetrate deeper into a target without failure due to bending.

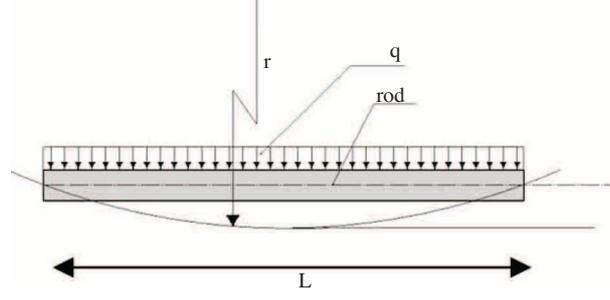
Here, the objective is to determine the behavior of a long rod penetrator under transverse loading. In order to reach this goal the effects of the relevant parameters like mass, length, diameter, and the ratio of  $L/D$  should be delineated properly. Referring to references, the validity of the assumption used in this model is established. The main advantage of the present method is its simplicity and direct applicability to the design process.

### **Bending of long rod penetrator under uniform transverse loading**

An armor target designed to confront long rod penetrators often integrates a target plate motion in a manner that a strong interaction between the colliding materials leads to rod failure Figure 2.

Experiments performed by H. F. Lehr and E. Wollmann showed that long rod penetrators impacting a modern target undergo bending collapse. Considering the shape of the bending rod, the governing equations for the bending and deformation of long rods are developed. Based on the geometry of the long rods a prismatic rod with circular cross section is considered. This configuration can easily be extended to non-circular cross sections.

The beam is assumed to have 2 simple supports and a uniform load is assumed to be exerted on it. The loading and support conditions of this beam are considered at an arbitrary cross section and generating a torque,  $M$ . Parameter  $\varepsilon$  is considered to be the maximum deformation under this bending. For the loading condition depicted in Figure 2 the following equation can be written:



**Figure 2.** Length to diameter ratios for long rod penetrators having the same bending characteristics.

$$M = \frac{qx}{2}(L - x) \quad (1)$$

$$M_{\max} = \frac{qL^2}{8} \quad (2)$$

The internal load between the penetrator and the target is not considered. It is assumed that transverse loading is due to inertial force, and this is the only load that causes uniform lateral acceleration. This of course is considered to be a valid assumption. Based on Newton's second law we can write

$$F = \frac{\pi D^2}{4} L \rho a_q \quad (3)$$

In this equation lateral acceleration is defined as  $a_q = n.g$  where  $g$  is the acceleration due to gravity and  $n$  is a real and positive number. This equation can be used to relate the penetration depth to the transverse acceleration of the penetrator.

In fact, if the mass distribution does not change along the length of the projectile then, according to Lehr and Wollmann, the assumption is considered to be valid.

Substituting  $q = F/L$ , load per unit length we can write

$$q = \frac{\pi}{4} D^2 \rho a_q \quad (4)$$

Using Eq. (2), the bending stress sustained by the long rod under transverse loading is seen to be

$$\sigma_{\max} = \frac{qL^2}{8w} \quad (5)$$

where  $w$  is the section modulus.

Using the above equations, the maximum stress is found to be

$$\sigma_{\max} = \rho a_q \frac{L^2}{D} \quad (6)$$

Now a new parameter defined as  $(L/D) \times L$  and denoted by  $F_L$  is introduced. Calling this new parameter as bending characteristic, the maximum stress can be written as

$$\sigma_{\max} = \rho a_q F_L \quad (7)$$

On the other hand, the maximum deformation of the rod is calculated to be

$$\varepsilon_{\max} = \frac{5}{384} \frac{qL^4}{EI} \quad (8)$$

Using Eq. (4) it is noted that

$$\varepsilon_{\max} = \frac{5}{24} \frac{\rho a_q}{E} F_L^2 \quad (9)$$

Finally the curvature induced in the rod is found to be

$$k = \frac{M}{EI} = 2 \frac{\rho a_q}{E} \left(\frac{L}{D}\right)^2 \quad (10)$$

Equations (7), (9), and (10) are bending equations of the long rod in terms of  $F_L$  and  $L/D$ .

### Long rods under the same transverse acceleration

Here long rods with uniform bending and transverse acceleration are considered. Referring to Eq. (7) it is clear that if the dimensions of the rods are identical, parameter  $\rho a_q$  will remain unchanged.

Now,  $F_L$  is used to express the ratio of the length to diameter as follows:

$$F_L = \frac{L}{D} \quad L = \left(\frac{L}{D}\right)^2 D \quad (11)$$

As seen, the length of the projectile is to the power 2, which signifies the importance of the rod length. Also  $F_L$  can be related to bending stress in the following manner:

$$\sigma \propto \left(\frac{L}{D}L\right) = \left(\frac{L}{D}\right)^2 D = F_L, \quad (12)$$

$$\sigma \propto \left(\frac{L}{D}L\right)^2 = \left(\frac{L}{D}\right)^4 D^2 = F_L^2, \quad (13)$$

and

$$k \propto \left(\frac{L}{D}\right)^2 \quad (14)$$

Now, if the dimensions of the rod are chosen in such a way that  $F_L$  remains constant, that is

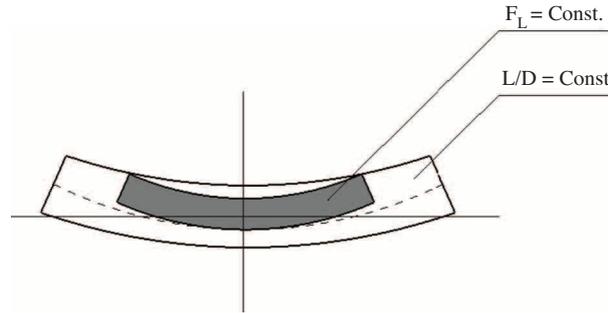
$$F_L = \frac{L^2}{D} = \left(\frac{L}{D}\right)^2 D = \text{const.} \quad (15)$$

then, for uniform long rod penetrators having the same mass and accelerations, the following results are deduced:

1. For a long rod penetrator undergoing transverse acceleration, the resulting stress due to the bending is a function of penetrator length and is inversely proportional to the diameter of the projectile. In this case, deflection of the rod is proportional to the fourth power of length and inversely proportional to the square of the projectile diameter.

2. For long rod penetrators having the same length to diameter ratio, bending stress is directly proportional to the diameter of the long rod, but deflection is a function of diameter squared.
3. Curvature and bending stress are both proportional to the square of length to diameter ratio.
4. The new parameter,  $F_L$ , is an appropriate criterion to reflect bending in a long rod penetrator subject to transverse loadings.

In order to properly define the role of  $F_L$  in the variation of  $L$  and  $D$  2 rods of identical  $F_L$  but with different diameter and length are considered as shown in Figure 3.



**Figure 3.** Comparison of optimized penetration depth.

Here, it is seen that these penetrators sustain different loads even though they are subjected to the same transverse accelerations. Figure 3 shows the bending behavior of 2 long rod penetrators having the same  $F_L$ . For these penetrators, the diameter ratio is  $\sqrt{2}\sqrt{2}$ , while deflection of the 2 rods is equal.

### The effects of the $L/D$ and $L^2/D$ on the variation in $L$ and $D$

Here, in order to determine the importance of the parameter  $F_L$ , a comparison is made with the  $L/D$  ratio of a long rod.

For the comparison a reference penetrator with the following properties is considered:

Diameter = 0.025 m

Length = 0.85 m

Density=17,500 kg/m<sup>3</sup>

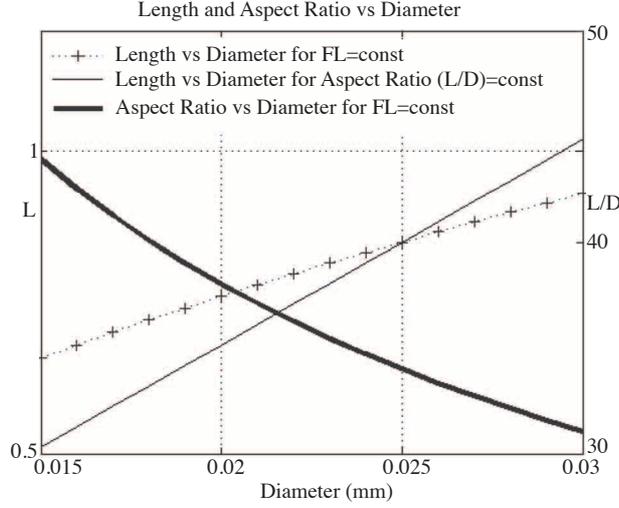
Modules of elasticity = 350 MPa, Transverse acceleration = 1000 m/s<sup>2</sup>, allowable bending stress = 484 MPa.

Under the above conditions,  $L/D$  ratio and  $F_L$  are 34 and 28.9 respectively (Lehr and Wollmann, 2001). Figure 4 shows the plot of the length versus diameter, for the constant aspect ratio.

This figure also shows the plot of aspect ratio versus diameter for a constant  $F_L$ .

As seen from Figure 4, the lines with constant slope drawn as solid line is for the projectiles having constant diameters. The dotted line is for the penetrators with the same bending characteristic,  $F_L$ . The thick solid line shows the variation in aspect ratio for the long penetrators whose bending characteristics are the same.

The intersection of constant  $L/D$  and  $F_L = \text{constant}$  expresses the specifications of the reference projectile. As shown in Figure 4 increasing the diameter and keeping  $F_L$  constant, the length of the penetrator increases much less than the case when the  $L/D$  ratio is constant.



**Figure 4.** Length to diameter ratios for long penetrators having the same bending characteristics.

If  $L/D$  remains constant, then increasing the diameter from 0.015 to 0.03 m doubles the length, while holding  $F_L$  constant the length of the penetrator is increased by 40%. In other words, while there is an approximately 60% reduction in penetrator length,  $L/D$  ratio is reduced only by 27%. This means that the variation in length and diameter has a positive effect on the  $L/D$  ratio.

As bending characteristic indicates, the relation between length and diameter is such that  $L$  and  $D$  can be reduced while a desirable aspect ratio can be achieved. This is clearly shown in Figure 5. and discussed in the next section.

### Long rod penetrators with the same bending stress and energy

The design of a long rod penetrator to optimize penetration depth is usually accomplished with special attention to kinetic energy and bending stresses of the penetrator. This is due to special conditions in colliding with a modern target, including limitations in available energy and bending stress due to the deflection of the rod.

Now consideration is given to a penetrator whose kinetic energy is constant. In this case quantities such as  $D$ ,  $L$ , and  $\nu$  do not change independently. These quantities should also satisfy the following relations:

$$E = \frac{1}{2}mv^2 = \text{Const.} \quad (16)$$

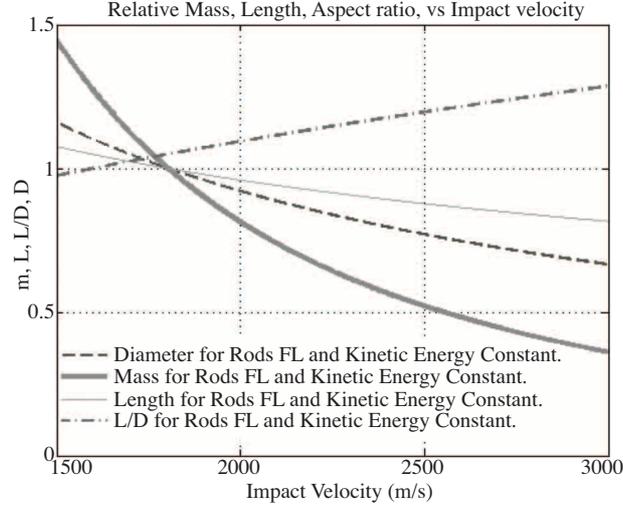
$$E = \frac{1}{8}\pi\rho D^2Lv^2 \quad (17)$$

Based on the above equations and the fact that  $F_L$  is a constant, the initial length of the penetrator is determined as follows:

$$L = f_E^{\frac{1}{5}} \left( \frac{L^2}{v} \right)^{\frac{2}{5}} \quad (18)$$

where  $F_E = 8E/\rho\pi$ .

As seen from Eq. (18), when kinetic energy and  $F_L$  are constant, the length of the projectile is a function of velocity only and obviously cannot change independently. On the other hand, this equation gives the initial length of the projectiles that have the same energy and bending stresses.



**Figure 5.** Variation in mass, length, diameter, and aspect ratio versus velocity for the projectile with kinetic energy of 11.822 MJ and  $F_L = 28.9$ .

Figure 5 shows the primary quantities of long rod penetrators as a function of impact velocity for the case when  $F_L$  and kinetic energy are constant. As shown in this figure, for all the long rod penetrators as velocity increases other quantities like mass, length, and diameter decrease while the aspect ratio is increased. On the other hand, when  $F_L$  is held constant, the length and diameter of the rod will vary in a manner that with decreases in parameters like mass, length, and diameter with respect to the reference projectile, the aspect ratio is increased. This is true even though the bending stress sustained by the projectile is not exceeding the allowable bending stress and deflection of the projectile. In this situation the penetrator does not fail when it collides with the target.

### Optimized penetration depth into semi-infinite targets for rods of constant bending stress and energy

During the past decades several engineering models have been developed to model penetration into semi-infinite targets. Allekseevski and Tate's model, known as the AT model, is one of the models that has been used extensively as a reference during the past 4 decades. Based on this model the following equations have been used for the determination of penetration depth into a semi-infinite target.

$$Y_p + \frac{1}{2}\rho_p(v-u)^2 = \frac{1}{2}\rho_T u^2 + R_T \quad (19)$$

$$Y_p = -\rho_P l \frac{dv}{dt} \quad (20)$$

$$\frac{dl}{dt} = -(v-u) \quad (21)$$

$$\frac{dP}{dt} = u \quad \text{or} \quad P = \int_0^T u dt \quad (22)$$

Equation (19) is known as the modified Bernoulli equation and it is based on the penetration of a fluid jet into an incompressible semi-infinite fluid target, considering dynamic strength of the target and projectile.

Equation (20) is the Newton's second law and Eq. (21) is the projectile erosion rate. Equation (22) is used to calculate penetration depth.

Here  $R_t$  and  $Y_p$  are the dynamic strength of the target and projectile, respectively.  $\rho_t$  is the density of the target and  $\rho_p$  is the density of the projectile. Also  $u$  and  $\nu$  are the instantaneous penetration and impact velocity, respectively. These equations are non-linear and their solutions can give crater depth, penetration velocity, and instantaneous length of the rod.

Tate presented a closed form solution of the above equations for the case when  $R_t/Y_p$  is an odd number. An analytical solution using Bessel functions was also developed by Segletes and Walters. Furthermore, Walter et al. presented an explicit solution for the AT equations without elimination of time variable.

In this article, these equations are solved numerically by considering the following pair of coupled differential equations as

$$\frac{dv}{dl} = \frac{Y_p}{\rho_p l (v - u)} \quad , \quad \frac{dv}{dP} = -\frac{Y_p}{\rho_p u l} \quad (23)$$

where  $Y_p$  is dynamic strength of the long-rod

The result obtained from these equations is the non-dimensionalized  $P/L$ , where  $P$  is the penetration depth and  $L$  is the initial penetrator length. Then the plot of  $P/L$  versus impact velocity for tungsten alloy rods penetrating into rolled homogeneous armor is presented as shown in Figure 6 (Zukas, 1991).

As seen in this figure when the velocity is increased beyond the hydrodynamic limit of colliding materials,  $P/L$  will approach the classical density law  $\sqrt{\rho_p/\rho_t}$ .

Now, penetration depth can be written as

$$P = \frac{P}{L} \times L \quad (24)$$

Moreover, the results of various experiments show that penetration depth is a function of impact velocity and  $L/D$  ratio (Anderson et al., 1994; Anderson et al., 1995). Therefore, normalized penetration depth can be defined as a function of  $\nu$  and  $L/D$ .

$$\frac{P}{L} = f\left(\nu, \frac{L}{D}\right) \quad (25)$$

As the  $L/D$  ratio becomes greater than 30,  $P/L$  will become independent of this ratio and therefore this ratio can be written as

$$\frac{P}{L} = f(\nu) \quad (26)$$

Now, combining Eqs. (18) and (24) the penetration depth in terms of impact velocity is written as

$$P = f_E^{\frac{1}{5}} \left( \frac{L^2}{v} \right)^{\frac{2}{5}} \left( \frac{P}{L} \right) \quad (27)$$

Differentiating the above equation with respect to  $\nu$  and equating it to zero gives the maximum penetration depth

$$\frac{dP}{d\nu} = 0 \quad (28)$$

or

$$\frac{2}{5} \cdot \frac{\left(\frac{P}{L}\right)}{v} = \frac{d\left(\frac{P}{L}\right)}{dv} \quad (29)$$

This means that the slope of the curve function value is divided by  $\nu$ .

A similar equation with somewhat different coefficients was obtained by Frank and Zook (Lehr and Wollmann, 2001).  $P/L$  as a function of  $\nu$  is 2/5 times experimentally. Their calculations resulted into the following equation, which is very similar to Eq. (21):

$$\frac{2}{3} \cdot \frac{\left(\frac{P}{L}\right)}{v} = \frac{d\left(\frac{P}{L}\right)}{dv} \quad (30)$$

By comparing Eqs. (29) and (30) it is seen that the difference between the equations is in the constant terms. However, one important point to note is that the bending stress has not been taken into account in the derivation of Eq. (30), while in the derivation of Eq. (29) attention has been paid to the failure of the projectile during impact.

## Results and Discussion

Equation (29) gives the points on the plot of normalized penetration depth  $P/L$  where the slope of the tangent line is equal to the slope of the lines drawn between the curve and point  $(\nu, 0.4 P/L)$ . At the extreme points of these lines a locus is constructed. The graph of this locus is very similar to the graph of  $P/L$  and the only difference is the existence of a multiplication factor of 0.4.

Figure 6 shows the locus of these lines along with the plot of  $P/L$ . Therefore the slope of all the lines which are drawn between the origin and all the points on the plot of  $0.4 P/L$  are the solution of the Eq. (29). Therefore, by characterizing a point on the  $P/L$  plot where the slope of the tangent line to the graph  $\frac{d(P/L)}{dv}$  at that point is equal to the slope of the line drawn between the origin and the end point of  $0.4 P/L$ , the maximum effectiveness of the projectile ( $P/L$ ) is obtained. Since AT equations are solved numerically, differentiation is also performed numerically and the point where the difference between this line and numerical differentiation is minimized is the desired solution.

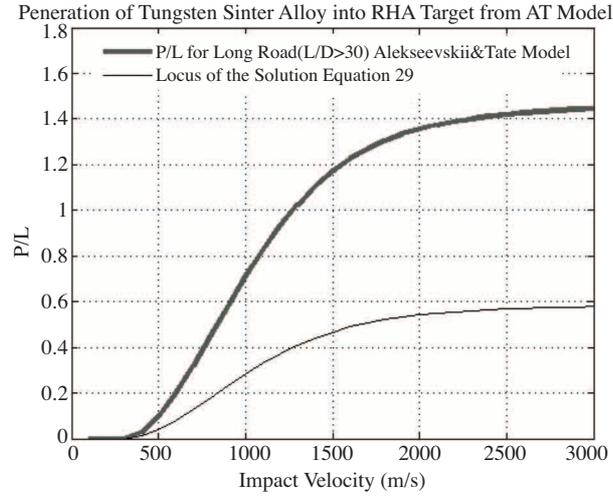
Referring to Figure 6 it is seen that the impact velocity at this point is equal to 2767 m/s. Of course this lies in the hypervelocity range. As it is known, in the hypervelocity range, the dynamic strengths of the colliding materials are neglected and the following equation is used to relate  $v$  and  $u$ :

$$\frac{1}{2}\rho_p(v-u)^2 = \frac{1}{2}\rho_T u^2 \quad (31)$$

Now, employing Eq. (21), the penetration depth is found to be

$$P = L\sqrt{\frac{\rho_p}{\rho_t}} \quad (32)$$

which is known as the classical density law.



**Figure 6.** Plot of normalized penetration depth  $P/L$  versus  $0.4 P/L$  versus impact velocity.

Appropriate numerical values for length and density of the projectile and target can now be employed in Eq. (32) to calculate penetration depth. This will result in a penetration depth of 1.083 m.

This shows only an error of 2.9%, which signifies the validity of Eq. (32) at the hypervelocity range.

Equation (32) delineates the fact that in the hypervelocity range penetration depth is independent of impact velocity. It is to be noted also that independence of velocity at the hypervelocity range is not true for certain materials like aluminum (Zukas, 1991) and thus the above equation should not be used indiscriminately for all the materials.

The plots of normalized penetration depth  $P/L$  and penetration depth for the projectiles whose aspect ratios are greater than 30 are presented in Lehr et al. (1996) and Anderson et al. (1995). The experimental plots are the basis of the comparison in this paper, and therefore the results of the calculations are compared with the experimental results in the references (Anderson et al., 1995; Lehr et al., 1996).

Table 1 shows the results of the comparison of the projectile proposed in this article, the reference projectile (projectile whose characteristics are given), and the projectile used in the experimental investigation by Lehr and Wollmann.

**Table 1.** Comparison of optimized penetration depth.

Name	Diameter m	Length m	Aspect ratio -	Mass g	Impact velocity m/s	Penetration depth m
Reference projectile (Lehr and Wollmann, 2001)	0.025	0.850	34	7300	1800	0.904
Optimized projectile	0.0178	0.716	40	3136	2767	1.0276
Experimental projectile (Lehr and Wollmann, 2001)	0.018	0.723	40	3250	2700	1.058
Fluid jet projectile	-	0.723	-	-	2726	1.082

Compared to the reference penetrator, it is seen that the optimization process resulted in a rod of smaller length and diameter but higher aspect ratio. This conclusion was obtained based on the analysis of the AT model considering bending characteristics and the test results obtained by Lehr and Wollmann. Compared to

the projectile used in experiment by Lehr and Wollmann it is seen that at velocity 2767 m/s the penetration depth for the proposed projectile in this article is 1.0276 m, which is less than the 1.058 m presented in the experimental results. This may be due to the negligence of the transient phase, which is of no significance in the hypervelocity regime.

The diameter and length of the designed projectile are less than the diameter and length of the reference projectile, while aspect ratio and penetration depth increased up to 18% and 14% respectively with respect to the reference projectile.

An important factor in long rod penetrator effectiveness is the ratio of the length to diameter. Increasing the aspect ratio of the rod will increase the effectiveness of the projectile. On the other hand, increasing the aspect ratio of the long rod penetrators weakens the ability of the penetrator to sustain transverse loadings as well as bending stresses.

Armor designers use this concept to design armors to defeat projectiles. The penetration depth of a projectile can be predicted by classical density law (fluid jet) in comparison with designed projectile as well as experimental results. It is clearly seen that the classical density law gives very good results when the velocity of colliding materials is high.

## Conclusions

In this article, a simple model considering the bending of a long rod penetrator during impact was developed. Combining the results of the bending analysis and AT penetration model an analytical method for depth optimization of long rod penetrators was developed.

Here a new parameter called bending characteristic ( $L^2/D$ ) was also introduced. This parameter showed that length is of prime importance in the penetration process. It was also noted that the length and diameter of the rod can be reduced simultaneously while the L/D ratio is increased and the penetrator does not fail due to bending when penetrating into a modern target.

The main advantage of this analytical method is its simplicity and direct applicability to design process. The required parameters are the dynamic strength of target and penetrator, which are easily obtained from Tate's model. It is emphasized that the velocity regime in this article is hypervelocity and the classical density law gives a very good result.

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