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... HASN'T IT?

A commentary on Eric Scerri's paper "Has Quantum Mechanics Explained the Periodic Table?", now published under the title "Just How Ab Initio is Ab Initio Quantum Chemistry?"

INTRODUCTION

Exact sciences cherish approximations. More often than not, resorting to approximations is a matter of *necessity*: that is the case when a problem cannot in principle be solved exactly. For instance, many-body problems fall all in this category, whether they are classical or quantum (see, e.g., Meyer, 1999). We note that here *many* means more than two; hence there are very many *many*-body problems. Approximations are also introduced when seeking a *qualitative understanding* of a problem: approximations (called in this context *models* or *treatments*) reveal the structure of problems and aid in identifying analogies with other problems, thus adding to the sense that we can make of them.

An excellent example of both of the above are the approximations developed to treat and to make sense of the periodic properties of the elements – meaning the *periodic electronic properties of atoms*. Let's keep in mind that nuclear properties are also periodic, but their periodicity is qualitatively different and its understanding requires a different model (see, e.g., Blatt and Weisskopf, 1979).

The electronic properties of atoms determine both the *spectra* and (largely) the *chemistry* of atoms. While the explanation of chemical reactivity must invoke bonding between two or more atoms (which is an altogether different many-body problem), atomic *spectra* reflect the properties of individual atoms and thus reveal their electronic properties *directly*. This is why spectroscopy can be used as a stringent test of our *quantitative understanding* of atomic properties – such as the energies of the electronic states.



Foundations of Chemistry 6: 117–132, 2004.

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The correspondence between the spectral properties of atoms and their chemistry was used by Niels Bohr to “deduce” the periodic table (see, e.g., Pais, 1991). In what follows, we’ll concentrate on the electronic properties of atoms as revealed by their spectra.

In quantum mechanics, only *few* problems can be solved *exactly* (i.e., analytically). These include the harmonic oscillator, the rigid rotor, and the *nonrelativistic one-electron atom*. The analytic solutions to these problems represent all there is to know about the problems. This knowledge is encapsulated in mathematical functions which relate all possible values of the parameters that characterize the problem to the problem’s energy and wavefunction. For instance, for a nonrelativistic one-electron atom, these parameters are the charge of the nucleus and its mass.

Approximations are usually inspired by analytic solutions to similar problems. For instance, the approximations to the N -electron atom are inspired by the hydrogen atom. Because of the prevailing *qualitative* similarity with hydrogen, the N -electron atom can in fact be described in a language adapted from hydrogen.

Approximations in quantum mechanics are based either on the *perturbation method* or the *variational method* or on their *combination*. Apart from that, *mathematical approximations*, such as power-series expansions or expansions in orthogonal polynomials, are always around, with their helping hand outstretched . . .

A special category of approximations takes advantage of the *correspondence between quantum and classical mechanics*. Quantum problems have classical aspects, which, of course, can be treated with classical mechanics. In such cases it makes a perfect sense to take classical mechanics to task, since in classical mechanics problems can often be solved analytically.

Approximations to the solution of a certain problem form a *hierarchy*, according to the degree of their *accuracy*; for many problems, an *arbitrary accuracy* can be achieved in principle and often in practice. A good example is the N -electron atom problem. Achieving an arbitrary accuracy of a quantity (such as energy) that arises in a given problem means that we can generate a correct digit in any decimal place of the quantity’s value.

Let’s illustrate the concept of arbitrary accuracy by considering the motion of mass points acting on one another by the gravitational

force; this problem is the principal subject of celestial mechanics (Goldstein, 1980). The quantity of interest is the path of any of the mass points, as expressed by the path's coordinates. If only two mass points are present, their motion is described by a Kepler's orbit (which is the analytic solution to such a two-body problem) and all we have to do in order to generate an arbitrary accuracy of the orbit's coordinates is to substitute for the parameters of the orbit (such as mass, gravitational constant, etc.) and carry out the numerical operations up to the decimal place that we wish to obtain (we shouldn't overdo this though and stop as soon as we reach the accuracy with which we know the problem's parameters). However, if a third mass point is present, the problem no longer has an analytic solution and the paths can only be obtained approximately. Such an approximate path is correct up to a certain decimal place, no matter how accurately we know the parameters of the problem or how far we carry out the numerical operations. The accuracy achieved may be *sufficient* for a comparison with an equally accurate observation (measurement) but not for a more accurate one. In the latter case we have to *require* a higher accuracy from the theory and so need to develop a better (higher) approximation that meets this requirement. This can be later compared with a more accurate measurement. The ensuing interplay between theory and experiment forces each towards a higher accuracy, and is credited with most of the progress achieved in physical sciences.

The approximations near the top of a hierarchy usually lack *Anschaulichkeit*. Apart from technical aspects, this is one of the key differences with respect to an exact, analytic solution: an analytic solution offers not only an arbitrary accuracy but also provides the best insight into a problem – it reveals the problem's structure.

Quantum chemistry deals with formidable problems. A sufficient accuracy to treat, for instance, room-temperature kinetics of molecules containing several atoms each, is not yet available (see, e.g., Miller, 1998).

The ability to achieve the required accuracy depends on algorithms, computer technology, and on *smart shortcuts* that exploit the structure of problems. Which brings us back to models . . .

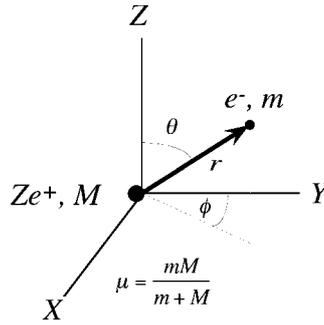


Figure 1. A coordinate system XYZ whose origin coincides with the center of mass of the hydrogenic atom with a nucleus of mass M and charge Ze^+ , and an electron of mass m and charge e^- . The separation between the nucleus and the electron is given by r ; it equals the separation between the center of mass and the particle with reduced mass μ .

QUANTUM MECHANICS OF THE ATOM

Hydrogenic atom. We begin with a brief look at a key atomic prototype, the hydrogen atom or any one-electron ion. It's a system that consists of a heavy nucleus, with mass M , and a light electron, with mass m , see Figure 1. The mass ratio is about 1836 for hydrogen. All the interesting physics that has to do with atomic structure happens in a coordinate system whose origin sits at the center of mass, which is just slightly shifted off the nucleus (in the absence of external fields, the center of mass itself moves uniformly). In the center-of-mass coordinates, the two-body system is equivalent to a one-body system with a reduced mass $\mu \equiv mM/(m + M)$ at a distance r from the center of mass (r is also equal to the separation of the nucleus and the electron).

The energy operator (i.e., the Hamiltonian) is (see, e.g., Weissbluth, 1978)

$$(1) \quad H = H_0 + H'$$

with

$$(2) \quad H_0 = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$$

which consists of the kinetic energy of the particle with the reduced mass μ (p is the momentum of the particle) and a potential energy

of the Coulomb attraction between the nucleus and the electron. The charge of the nucleus is an integer multiple, Z , of the positive elementary charge; hence Z coincides with the number of protons in the nucleus and is called the atomic number. The term H' describes additional interactions which we'll ignore for a moment.

The Schrödinger equation

$$(3) \quad H_0\psi = E\psi$$

for H_0 can be solved analytically and so yields the exact energies E and wavefunctions of the system. The wavefunctions

$$(4) \quad \psi = R_{n\ell}(r)Y_{\ell,m}(\theta, \phi) = \psi_{n\ell m}(r, \theta, \phi)$$

have radial, $R_{n\ell}(r)$, and angular, $Y_{\ell,m}(\theta, \phi)$, parts and are referred to as *orbitals*, in honor and memory of the early planetary model of the atom.

Apart from other effects, the H' term gives rise to an internal angular momentum of the electron, called spin, described by a wavefunction $\zeta(m_S)$. The total hydrogenic (i.e., one-electron) wavefunction

$$(5) \quad \psi = \psi_{n\ell m}(r, \theta, \phi)\zeta(m_S) = \psi_{n\ell m_\ell m_S}(\lambda)$$

is then called a *spin-orbital*. It depends on the spatial as well as spin coordinates, labeled summarily as λ . The bound states of the hydrogenic atom are quantized. The quantum numbers are integers that are related to one another by the inequalities:

$$(6) \quad 0 \leq \ell < n = 1, 2, 3, \dots; -\ell \leq m_\ell \leq \ell; m_S = \pm 1/2.$$

We'll take a particular notice of the principal and orbital quantum numbers n and ℓ . Let's note that the values of 0, 1, 2, 3, 4, 5 are designated with the letters *s, p, d, f, g, h, i*

Figure 2 shows the hydrogenic energies. In the simple, non-relativistic Coulomb case, the energy of the hydrogenic states depends solely on the nuclear charge and the principal quantum number. There are *infinitely many bound states*, with a negative energy; with increasing n they approach the ionization limit located at zero energy. The effect of the nuclear charge is quite dramatic and

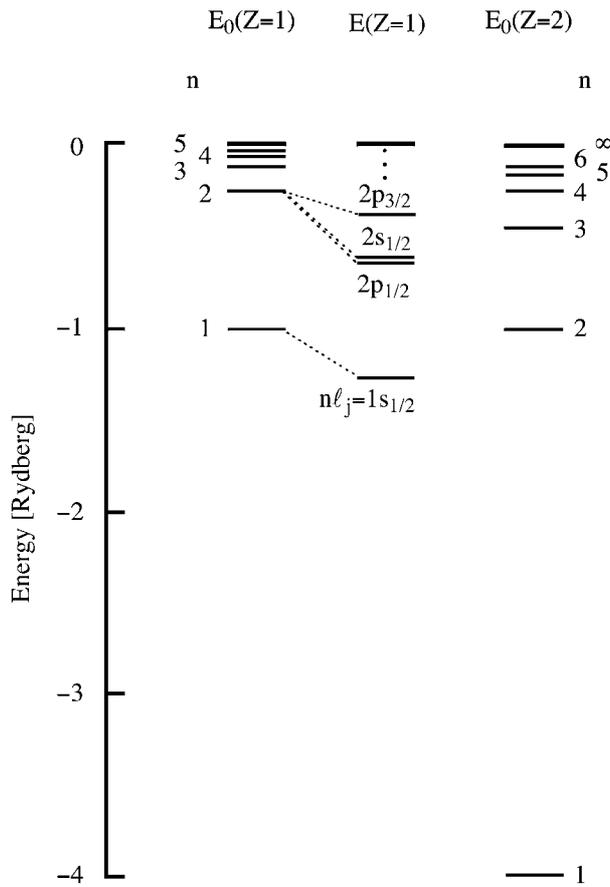


Figure 2. The hydrogenic energies. In the simple, nonrelativistic Coulomb case (left and right portion), the energy of the hydrogenic states depends solely on the nuclear charge and the principal quantum number. There are *infinitely many bound states* with a negative energy; with increasing n they approach the ionization limit located at zero energy. The central portion of the figure shows schematically what happens when we take into account the additional interactions, due to H' . See text.

is shown in the right portion of the figure. For instance, a change of Z from 1 to 2 increases the binding energy of the system 4-times.

The central portion of the figure shows schematically what happens when we take into account some of the additional interactions, due to H' . These are mainly relativistic, and can be approximated by (Weissbluth, 1979)

$$(7) \quad E' = -\frac{Z^4\alpha^2}{n^3} \left(\frac{1}{j + 1/2} - \frac{3}{4n} \right)$$

with $j = \ell \pm 1/2$ and $\alpha \equiv e^2/\hbar c$ (the fine-structure constant, $\alpha \approx 1/137$). Figure 2 illustrates that even a hydrogen atom in its full glory is fairly complicated, and cannot be treated analytically. This is the case even without hyperfine interactions (i.e., interactions between the multipole moments of the nucleus and the electrons) – which I left out of E' altogether. And will leave it that way throughout this account, except for mentioning that hyperfine transitions in atomic hydrogen are at the core of as big and as substantial a field of research as radioastronomy . . .

Let's make here one and only historical detour (Friedrich and Herschbach, 1998). The hydrogenic energy formula

$$(8) \quad E_n = -\frac{me^4}{\hbar^2} \left(\frac{Z}{n}\right)^2$$

rewritten here in CGS units, was discovered by Bohr in 1913. Applied to the Rydberg-Ritz combination rule, it yields the spectral energies of transitions between different hydrogenic states characterized by quantum numbers $n_1 < n_2$

$$(9) \quad \Delta E = E_2 - E_1 = \frac{me^4 Z^2}{\hbar^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

Seventeen years earlier, Charles Pickering, a Harvard astronomer, had discovered in starlight a remarkably regular series of spectral lines, one not seen in the laboratory spectra of hydrogen. Bohr noted that these unassigned lines could be ascribed to the helium atomic ion: like hydrogen, it has only one electron, but the helium nucleus contains two protons and so has $Z = 2$. Accordingly, Bohr's formula predicted that, for given n , the separation of the spectral lines of the He-ion should be increased by a factor of 4 with respect to hydrogen,

$$(10) \quad \frac{\Delta E(He^+)}{\Delta E(H)} = \frac{Z(He^+)}{Z(H)} = 4.$$

This nicely accounted for the spectral pattern, but a spectroscopist then pointed out that the measured factor would need to be 4.0016 rather than 4 to fit the lines accurately. Bohr responded that, for simplicity, he had previously approximated the mass of the nucleus as infinitely heavier than that of the electron. The correction, implemented by replacing the electron mass with the corresponding

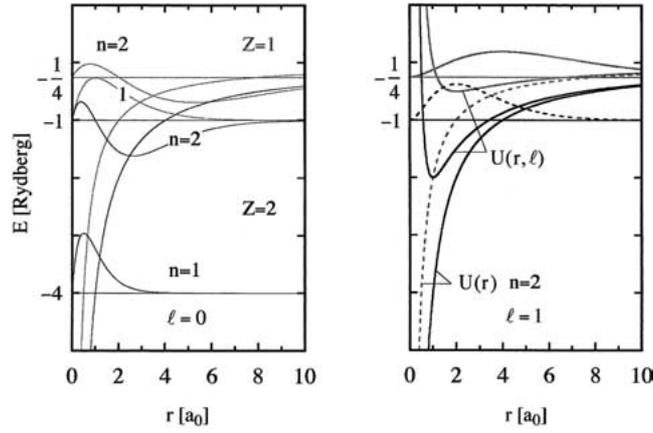


Figure 3. Coulomb potential, effective potential, energy levels and wavefunctions for a hydrogenic atom. The grey curves correspond to $Z = 1$ and the black ones to $Z = 2$. In the left panel $\ell = 0$; for both $n = 1$ and $n = 2$, the electron is much closer to the nucleus for $Z = 2$ than for $Z = 1$. In the right panel $\ell = 1$; the effective potential throws the electron further away from the nucleus than for the $\ell = 0$ case. This effect is more pronounced for $Z = 1$ than $Z = 2$, due to the weaker bonding which renders the effective potential rather flat. The atom's energy is expressed in Rydberg units (1 Rydberg is equal to 2.17991×10^{-18} J) and the separation between the nucleus and the electron in Bohr radii, a_0 (a_0 is equal to 5.29177×10^{-11} m). See text.

reduced mass of the atom or ion, $m \rightarrow \mu \equiv mM/(m + M)$, could be easily and precisely evaluated since it depended only on accurately known mass ratios. Bohr thus obtained

$$(11) \quad \frac{\Delta E(\text{He}^+)}{\Delta E(\text{H})} = \frac{Z(\text{He}^+)\mu(\text{He}^+)}{Z(\text{H})\mu(\text{H})} = 4.00163$$

in gratifying and unprecedented quantitative agreement with the spectroscopic data. Einstein commented that this was “an enormous achievement.”

Figure 3 summarizes the hydrogenic properties that are most relevant for multi-electron atoms. The *radial wavefunctions*, which determine how far from the nucleus the electron is likely to be found, can be viewed as pertaining to a *one-dimensional motion in an effective potential* (see, e.g., Landau and Lifshitz, 1977)

$$(12) \quad \left(-\frac{d^2}{dr^2} - \frac{2Z}{r} + \frac{\ell(\ell + 1)}{r^2} \right) r R_{n\ell} = E_n r R_{n\ell}.$$

The effective potential

$$(13) \quad U(r, \ell) = U(r) + \frac{\ell(\ell + 1)}{r^2}$$

consists of the Coulomb attraction

$$(14) \quad U(r) = -\frac{2Z}{r}$$

and the centrifugal repulsion between the nucleus and the electron, $\ell(\ell + 1)/r^2$. Since we know the effective potential and the eigenenergies, we can glean the form of the radial wavefunction - even without solving the radial equation. This is because the wavefunction can only have significant values within the range of electron separations demarcated by the effective potential.

In the left panel, the Coulomb attraction and the effective potential coincide, since $\ell = 0$ there. The grey curves correspond to $Z = 1$ and the black ones to $Z = 2$. One can see that for both $n = 1$ and $n = 2$, the electron is much closer to the nucleus for $Z = 2$ than for $Z = 1$. This effect is due to the stronger binding (lower eigenenergy) at the larger Z -value.

In the right panel, the Coulomb attraction and the effective potential are different, since $\ell = 1$ there. One can see that the effective potential throws the electron further away from the nucleus than for the $\ell = 0$ case. This effect is more pronounced for $Z = 1$ than $Z = 2$, due to the weaker bonding which renders the effective potential rather flat. Note that the number of *nodes* of the radial wavefunction (i.e., the number of times it runs through zero, excluding $r = 0$) is $(n - \ell - 1)$.

These lessons are to be kept in mind when considering the effects of varying ℓ and of the shielding of the nuclear charge in multi-electron atoms.

N-electron atom. In treating electronic properties of N -electron atoms we start with what we know, namely the hydrogenic atom.

First, we construct the Hamiltonian. This we do by summing up, over all N electrons, the one-electron hydrogenic operators, consisting of the kinetic energy of each electron and of its Coulomb energy in the field of the nucleus with charge $Ze = Ne$. This yields

$$(15) \quad H_1 = \sum_{i=1}^N \frac{p_i^2}{2\mu} - \frac{Ze^2}{r_i}.$$

We also sum over all electron-electron interactions

$$(16) \quad H_2 = \sum_{i < j}^N \frac{e^2}{r_{ij}}$$

and all spin-orbit interactions of the N electrons

$$(17) \quad H_3 = \sum_{i=1}^N \xi_i \mathbf{l}_i \cdot \mathbf{s}_i.$$

Here \mathbf{l}_i and \mathbf{s}_i are orbital and spin angular momentum operators of the electrons and ξ_i is a constant. The resulting Hamiltonian is then the sum

$$(18) \quad H = H_1 + H_2 + H_3$$

of the three contributions (see, e.g., Weissbluth, 1979). The orbital and spin angular momenta \mathbf{L} and \mathbf{S} of the atom are given by the vector sum of the corresponding one-electron angular momenta:

$$(19) \quad \mathbf{L} = \sum_{i=1}^N \mathbf{l}_i \quad \& \quad \mathbf{S} = \sum_{i=1}^N \mathbf{s}_i.$$

This suggests constructing the N -electron wavefunctions according to the well-established *angular momentum coupling schemes* (see, e.g., Weissbluth, 1979). Also, the angular momenta, together with their sum, the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$, can be used to label the states of the atom,

$$(20) \quad {}^{2S+1}L_J.$$

It is a custom in atomic spectroscopy, caustically commented on by Pauling (1970), to designate the values of $L = 0, 1, 2, 3, 4, 5$ with the letters $S, P, D, F, G, H, I \dots$. An electron configuration, given by the quantum numbers n and ℓ , gives rise to spectral terms, described by quantum numbers L and S , which in turn consist of energy levels, characterized by the quantum number J . This is illustrated in Figure 4.

Second, since the wavefunction cannot be found analytically for an N -electron Hamiltonian, and since both the perturbation and variational treatments require a trial wavefunction, we have to construct

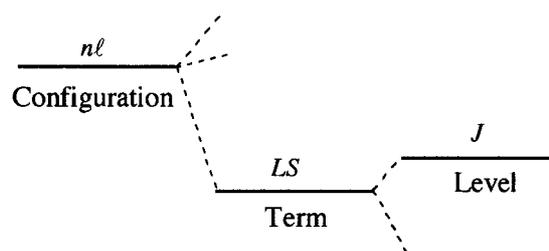


Figure 4. The relation between an electron configuration, given by the values of n and ℓ ; a term, given by the total quantum numbers L and S ; and a level with a total angular momentum quantum number J .

one. We do this by representing the electronic state of the N -electron atom in terms of a product of N spin-orbitals

$$(21) \quad \Psi(\lambda_1, \lambda_2, \dots, \lambda_N) = \sum_i \psi^{(j)}(\lambda_i)$$

where $\psi^{(j)}$ is a one-electron configuration (i.e., a particular combination of the quantum numbers n , ℓ , m_ℓ , m_s) of electron i whose coordinates (spatial and spin) are λ_i .

Third, we take into account that the electrons which are to occupy the spin-orbitals are *fermions*. Fermions are one of the two *fundamental varieties of indistinguishable particles*, the other variety being *bosons*. While bosons like each other's company and don't mind sharing the same state, fermions are individualistic, and would never share the same state with another fermion. Therefore, each spin-orbital in an atom can only be occupied by at most one electron. The last statement is also known as the *Pauli principle*. If electrons were bosons, they would all collapse into the ground state and all atoms would be chemically much the same.

An N -electron wavefunction can be conveniently represented in terms of a Slater determinant which takes automatically into account the fermionic nature of the electrons (see, e.g., Weissbluth, 1979)

$$(22) \quad \Psi_k = (N!)^{-1/2} \begin{vmatrix} \psi^{(1)}(\lambda_1) & \psi^{(2)}(\lambda_1) & \dots & \psi^{(N)}(\lambda_1) \\ \psi^{(1)}(\lambda_2) & \psi^{(2)}(\lambda_2) & \dots & \psi^{(N)}(\lambda_2) \\ \vdots & \vdots & \ddots & \vdots \\ \psi^{(1)}(\lambda_N) & \psi^{(2)}(\lambda_N) & \dots & \psi^{(N)}(\lambda_N) \end{vmatrix}.$$

A Slater determinant pertains to a certain set $k = \{L, S, J\}$ of the L , S , J values for a fixed J . Which Slater determinant to pick in

order to represent, say, the ground state of the N -electron atom, is a big question – unless we want to take them *all*, or if not all then *very many* of them . . . The method of *configuration interaction* (see, e.g., Weissbluth, 1979) indeed expands the N -electron wavefunction in terms of a complete set of Slater determinants

$$(23) \quad \Psi = \sum_{k=1}^{\infty} a_k \Psi_k.$$

The configuration interaction method has become practical with the advent of the computer. Let us note that J is a good quantum number, due to the spherical symmetry of the atom. The quantum numbers L and S are not, but can still be used, along with J , to label the electronic states.

The coefficients a_k can be computed by the variational principal with *arbitrary accuracy for ground state atoms*. This means that the *ground state energies* and other properties can be also evaluated with *arbitrary accuracy*.

The above treatment does not include corrections for the relativistic kinetic energies of the electrons, hyperfine structure, and other atomic idiosyncrasies . . .

The approximations to the N -electron atom form a hierarchy:

- (1) The *one-electron approximation* consists of a single Slater determinant

$$(24) \quad \Psi \approx \Psi_k$$

for a given $k = \{L, S, J\}$.

- (2) the *Hartree-Fock approximation* consists also of a single Slater determinant (and so is a one-electron approximation) but one that yields the most accurate energy

$$(25) \quad \Psi \approx (\Psi_k)_{\text{optimal}}$$

This is found by the method of self-consistent field (see, e.g., Weissbluth, 1979).

- (3) The *configuration interaction* method that expands the wavefunction into a large set of Slater determinants accounts for the *correlation energy* among the electrons. A clever trick, based

on dimensional scaling, makes it possible to evaluate about 90% of the correlation energy for ground state atoms essentially without any effort, just by solving a quartic equation (Kais and Herschbach, 1994). That's a huge simplification: Hartree-Fock combined with such an evaluation of the correlation energy then gives essentially the same accuracy as a large configuration interaction computation.

Are there any more such shortcuts? Do they reveal that the N -body Coulomb problem has a simpler structure than we think?

The *simplest model* of an N -electron atom, not included in the above hierarchy of "canonical" approximations, just seeks to *assign* the energy *levels* and to determine their relative *ordering*. This model, which Scerri seems to identify as *the* quantum mechanical solution to the N -electron atom (Scerri, 2004), assumes that each electron is in a hydrogenic spin-orbital and that electrons are independent of one another . . . except that electrons with a given principal quantum number are more hydrogen-like for large Z than for low Z because of less shielding by inner electrons:

- (a) for high Z , shells with same n and different ℓ have similar energies and are grouped together (like in hydrogen)
- (b) for small Z , shells with same n and different ℓ have less similar energies and often separate (unlike in hydrogen)

This is illustrated in the correlation diagram between states corresponding to $Z = 20$ and 90 , Figure 5.

The onset of states with a given ℓ as a function of Z is beautifully explained by the *Thomas-Fermi model* in terms of the mean electron density. According to the model (see, e.g., Landau and Lifshitz, 1977)

$$(26) \quad Z \approx 0.17(2\ell + 1)^3$$

which yields $\ell = 1, 2, 3, 4$ at $Z = 5, 21, 58, 124$, respectively.

Thus the hydrogenic spin-orbitals, augmented by the Z -dependent energy-level hierarchy, also called the *Aufbau principle*, and combined with the *Pauli principle* and the *Hund rules* (see, e.g., Morgan and Kutzelnigg, 1993) are good enough to give correct electron configurations of most states for most atoms! This remarkable *prevalence* of the *hydrogenic structure* in N -electron atoms is

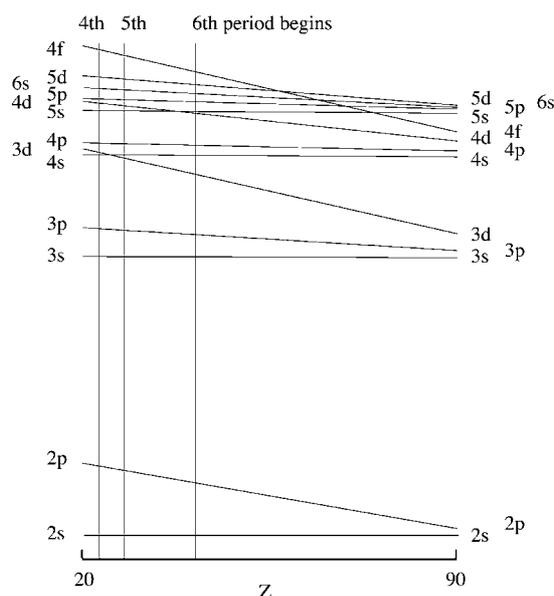


Figure 5. Correlation diagram between atomic energy levels at atomic numbers $Z \approx 20$ and $Z \approx 90$. The vertical lines mark the onset of the 4th, 5th, and 6th periods. Adapted from Herzberg (1944).

mainly a *consequence* of the *disparity between the large nuclear mass and the small mass of the electron*. The Z -dependence of the Aufbau principle accounts for the change of the relative roles of the Coulomb repulsion between the electrons (dominant at small Z) and the electronic spin-orbit coupling (dominant at large Z).

Despite its striking success in predicting electron configurations of most ground state atoms, this simplest model of an N -electron atom needs to be supplanted by one of the three canonical approximations whenever a quantitative information about the electronic structure of atoms is sought. This does not come as a surprise, given the qualitative nature of the model.

SUMMARY

Although the highly accurate methods of quantum mechanics allow us to compute *what* the energies and other properties of N -electron atoms are, the methods are rather reticent about *why* the properties are what they are. Therefore, models, whether simple or sophisti-

cated, add a crucial explanatory ingredient to our understanding of atoms. More than that, an understanding of the structure of problems suggests better and often more straightforward approximations, such as the dimensional scaling for computations of correlation energy.

The development of both computations and models is bound to continue as long as there is a need for a greater accuracy, justified by increasingly accurate measurements.

A push for augmenting the accuracy of atomic properties is coming, for instance, from experiments that use atoms (and molecules) in tests of fundamental symmetries in nature, such as the time-reversal symmetry (T-symmetry) or the symmetrization postulate (Pauli's principle). These experiments may take us into the realm of physics beyond the Standard Model (see, e.g., Hinds, 1997). T-symmetry violation is tantamount to a nonvanishing permanent electric dipole moment (EDM) of the electron and of other elementary particles. Since the contenders for physics beyond the Standard Model give definite predictions about the magnitude of EDM, one can eliminate those which predict a value exceeding a measured upper bound of the EDM. The EDM is measured via its interaction with an electric field. High- Z atoms or molecules containing high- Z atoms possess huge internal fields and so are especially well suited to reveal the EDM. What makes these experiments exceedingly attractive is that a nonvanishing EDM could help explain the *baryonic asymmetry* of the universe (i.e., why particles so dramatically outnumber antiparticles). In this way, atoms, that were once imagined as tiny planetary systems, may teach us about aspects of the universe that reach far beyond the solar system or the Milky Way.

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