# Digital Compensation of Nonlinear Distortion in Loudspeakers

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*Practical loudspeakers are nonlinear transducers, i.e. the acoustic output is not a linear representation of the electrical input signal. In this paper two loudspeakers are considered; a loudspeaker for reproduction of low frequencies (woofer) and a horn loudspeaker. Instead of improving the behavior of the two transducers by constructional means we apply a feedforward nonlinear digital inverse circuit. Results of a Volterra compensator for the woofer show a significant reduction of the second order harmonics at low frequencies, leaving higher order distortions unchanged. To the horn loudspeaker a more direct method is applied. This loudspeaker suffers from distortions caused by the adiabatic behavior of the air volumes in the compression driver. With this compensator we obtain a reduction of harmonic distortion, in the frequency span from 600 to 1100 Hz.*

## 1 Introduction

With the audio reproduction chain becoming more and more digital, the loudspeaker has become the weakest link between recording and reproduction. A method to improve the transfer behavior of electro-acoustical transducers is by means of magnetic or mechanical design changes. Disadvantage of this kind of changes is a more complex production and thus an increasing prize of the transducer. In this paper we improve the behavior of the loudspeaker by means of an algorithm on a Digital Signal Processor (DSP). With the decreasing prizes of processing hardware this may be a realistic solution to the mentioned design problem. The compensators are realized as feedforward controllers. Therefore an additional sensor like a microphone or an accelerometer is superfluous. Such sensors make the system more expensive, and their quality determines the effectiveness of the controller.

In this paper two types of loudspeakers are considered, an electrodynamic loudspeaker for low frequencies (woofer) and a midrange loudspeaker, a horn loudspeaker. First transducer suffers from distortions caused by the displacement dependent parameters. In order to have a good bass reproduction a large cone excursion is needed, this increases the already inherent nonlinear distortion. Horn loudspeakers are widely used in the area where high sound pressure levels and good directivity of sound is needed. They have a much higher efficiency than direct radiator loudspeakers. Disadvantages are its higher prize, greater size and higher nonlinear distortions. One part of the horn loudspeaker nonlinearity is caused by adiabatic compression of air in the driver.

## 2 Modeling

Starting point for design of a controller to compensate for nonlinearity in a system, is a good model. From the techniques available we choose to model the transducers by lumped parameter electrical equivalent circuits. Such circuits have a full connection towards physical properties of the transducer from which we can use as much expert knowledge as possible. This is the contrary with other modeling techniques like black-box modeling (e.g. NARMAX), although gray-box modeling (a mixture of deterministic and statistical modeling) is a promising technique for the future. In Figure 1 (a) and (b) both transducers are schematically depicted. First we will consider

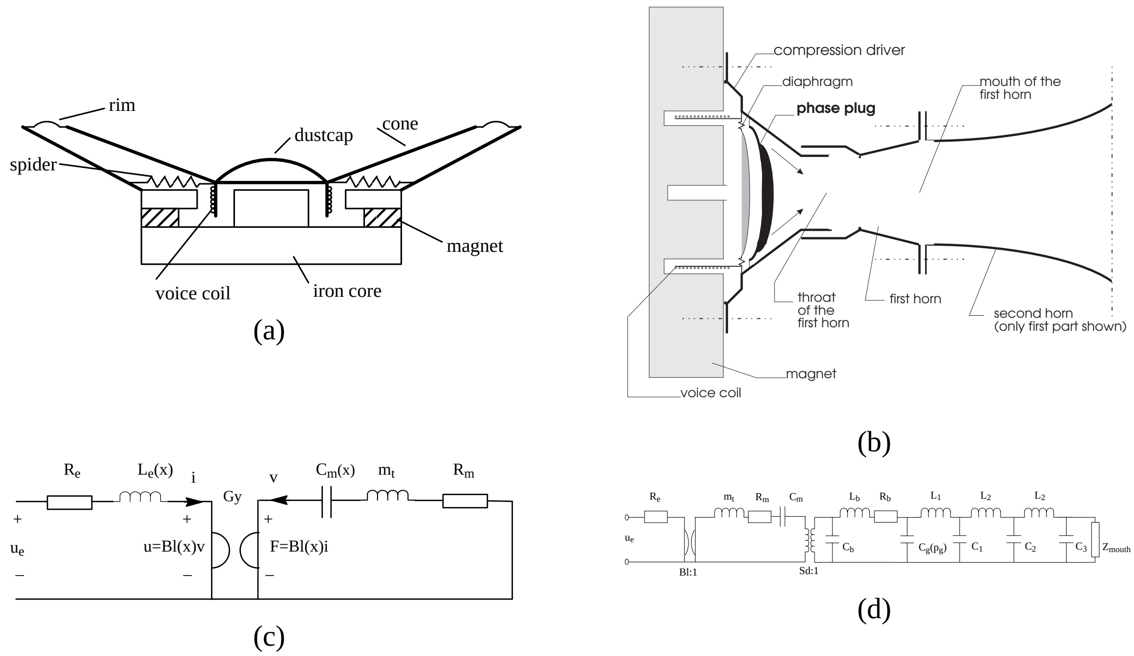


Figure 1: *The transducers under consideration; electrodynamical direct radiator (a) and horn loudspeaker (b) with their equivalent circuits in respectively (c) and (d).*

the woofer. Nonlinearities of this transducer are all caused by the displacement $x$ of the cone. Three nonlinearities are found to be of major influence [1]:

- the transduction between electric and mechanic domain
- the stiffness of the spider suspension
- the self inductance of the coil

The electrical equivalent circuit of a woofer inside a closed box is depicted in Figure 1 (c). It consists in the electrical domain of the series circuit of the voice coil resistance $R_e$ and displacement dependent self inductance $L_e(x)$. Coupling towards the mechanical domain is formed by a gyrator with displacement dependent coupling factor $Bl(x)$, also known as the force factor. In the mechanical domain we have the electrical analogue of a damped mass-spring system, formed by the resistance of the suspension $R_m$, the displacement dependent compliance of suspension and cabinet compliance $C_m(x)$, and the total mass of the moving cone and moving air in front of it $m_t$. From straightforward network analysis of the equivalent network for the woofer we find two coupled nonlinear differential equations [1]:

$$u_e = R_e i + \frac{dL_e(x)i}{dt} + Bl(x)\dot{x} \tag{1}$$

$$Bl(x)i = m_t\ddot{x} + R_m\dot{x} + \frac{1}{C_m(x)}x \tag{2}$$

The displacement dependent parameters $L_e(x)$, $Bl(x)$ and $C_m(x)$ are described by a truncated Taylor series expansion. The total nonlinear differential equation is obtained from substitution of Eq.(2) into Eq.(1).

The electrical equivalent circuit of the horn loudspeaker is more complex compared to the woofer. Although the electric and mechanic domain are the same as with the woofer, the acoustical domain contains more parameters. We will not discuss all of them but will focus in one parameter in particular: the nonlinear compliance $C_g(p_g)$. This compliance represents the thin air film between diaphragm and phase correction plug inside the driver, see Figure 1 (b). The phase correction plug is provided to prevent dips in the frequency response due to canceling sound waves traveling different paths towards the throat of the first horn. Disadvantage of the use of such a plug is the introduction of a thin air film which behaves nonlinear. When we assume adiabatic behavior of the air volumes in the driver we are able to determine a relation for the nonlinear compliance, which is approximated by a second order polynomial [2]. Like with the woofer we obtain a nonlinear differential equation from the circuit of Figure 1 (d) for the input voltage $u_e$:

$$u_e = \mathcal{L}^{-1}\left\{\frac{1}{H_{tot}(s)}\right\} * i_g + \mathcal{L}^{-1}\left\{H_1(s)\right\} * L_g(i_g)i_g + \mathcal{L}^{-1}\left\{H_2(s)\right\} * \frac{dL_g(i_g)i_g}{dt} \qquad (3)$$

where $H_{tot}(s)$, $H_1(s)$ and $H_2(s)$ are linear transfer functions determined by the pertinent parameters. They are transformed back into the time domain by the inverse Laplace transform: $\mathcal{L}^{-1}\{\}$ where * denotes convolution. The signal $i_g$ is the electrical equivalent of the acoustical pressure between diaphragm and phase plug, while $L_g(i_g)$ is the electrical equivalent of the nonlinear compliance $C_g(p_g)$.
Parameters of both transducers, linear and nonlinear, are determined from input impedance and sound pressure response measurements using least squares optimization.

# 3 Compensators

## 3.1 Volterra compensator

From the nonlinear differential equations of both transducers we derive a nonlinear inverse filter to eliminate the nonlinear behavior. First we will consider a compensator based on a Volterra series expansion of the transducer. The output $y(t)$ characterized by a continuous time Volterra series is given by:

$$y(t) = \int_{-\infty}^{\infty} h_1(\tau)x(t-\tau)d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1\tau_2 + \dots \qquad (4)$$

where x(t) is the system input and $h_n(\tau_1,\dots,\tau_n)$ are the generalized impulse responses, also called kernels. Similar to linear systems we can determine, using the multi-dimensional Laplace transform, the kernels in the frequency domain. The system response is now determined by a summation of all kernel responses, i.e. the Volterra series can be seen as a Taylor series with memory. From this nonlinear system representation we determine a compensator which eliminates the kernels $h_2(\tau_1,\tau_2)$ and higher. Truncation of the Volterra series after the second order kernel will yield a compensator which eliminates only second order distortion. From the nonlinear model of the woofer given by Eqs. (1) and (2) the second order Volterra compensator kernel in the frequency domain is found to be [1]:

$$\begin{aligned} K_2(s_1,s_2) &= H_x(s_1)H_x(s_2)\{a + b(s_1+s_2) + c(s_1+s_2)^2 + \\ &\quad d(s_1+s_2)^3 - s_1s_2\{e + f(s_1+s_2)\}\} \end{aligned} \qquad (5)$$

with $H_x(s)$ the linear transfer function from input voltage to displacement of the loudspeaker, and $a$ through $f$ constant parameters formed by linear and nonlinear parameters. Direct implementation of this compensator yields an inefficient implementation which needs five differentiators [1]. A more efficient form is found if we first rewrite Eq.(5) into:

$$\begin{aligned} K_2(s_1,s_2) &= \frac{1}{2}H_x(s_1)H_x(s_2)\{E(s_1^3+s_2^3) + G(s_1^2s_2+s_1s_2^2) + D(s_1^2+s_2^2) + \\ &\quad C(s_1+s_2) + 2Fs_1s_2 + 2B + A\left\{\frac{1}{H_x(s_1)} + \frac{1}{H_x(s_2)}\right\}\} \end{aligned} \qquad (6)$$

with $A$ through $F$ again constant parameters containing linear and nonlinear parameters of the loudspeaker. Normally a Volterra kernel, in general of the form $K_g(s_1, s_2) = K_a(s_1)K_b(s_2)K_c(s_1 + s_2)$, is synthesized using three linear filters and one multiplier [3]. However, from the form of Eq.(6) it is possible to obtain a more efficient implementation as for example for the part $\frac{1}{2}(s_2^2 s_1 + s_1 s_2^2)$ is given in Figure 2. Using this and other simplification techniques we obtain

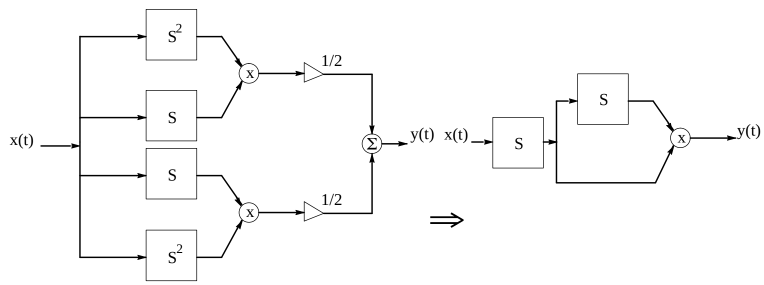


Figure 2: *Efficient kernel implementation.*

a second order compensator as depicted in Figure 3, where the factor M is the gain of the amplifier between filter and loudspeaker. In this figure the s-blocks are the differentiators, only three in this implementation. The differentiators were implemented as IIR filters based on the Simpson integration rule [5]. This is a very efficient implementation compared with its FIR (e.g. Parks-McClellan) equiripple counterparts. Single disadvantage is the fractional group-delay of 0.6 sample. Therefore we use linear interpolation to obtain an equal delay in the parallel paths of the algorithm. Linear filter $H_x(z)$ is obtained from its continuous frequency domain counterpart by means of the bilinear transformation.

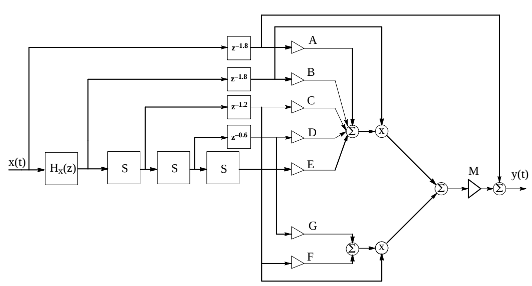


Figure 3: *Second order Volterra compensator.*

### 3.2 Horn compensator

For derivation of a compensator for the horn loudspeaker we did not used the Volterra series approach. Because of the much higher order of the model it is more attractive to use a more transducer related method. Such a method was first applied to a woofer by Klippel [4]. Based on the nonlinear differential equation, the nonlinear part is 'mirrored' to the electrical domain. When using this method towards Eq. (3) we find for the output of the correction filter:

$$u_{out} = u_{in} + \frac{1}{M}\mathcal{L}^{-1}\{H_1(s)\} * L_g(i_g)i_g + \frac{1}{M}\mathcal{L}^{-1}\{H_2(s)\} * \frac{dL_g(i_g)i_g}{dt} \qquad (7)$$

with $u_{in}$ the input of the filter and M a constant factor which is the gain of an amplifier between filter and loudspeaker. The signal $i_g$ is synthesized by means of an additional linear filter $H_{tot}(s)$ (see Eq.(3)). The compensator for the horn loudspeaker is schematically given in Figure 4. The synthesized $i_g$ is supplied to the static nonlinear $L_g(i_g)$ block whose output is, using inverse filtering formed by $H_1$, $H_2$ and differentiator $s$, summed to the input signal of the filter. Additional delay blocks are again needed to obtain equal delay in the different paths of the algorithm. To obtain stable implementations for $H_1$ and $H_2$, which are portions of the inverse filter of $H_{tot}$, it is necessary that $H_{tot}$ is minimum phase. In practice this is not the case, but fortunately its zeros are near fold-over and we therefore make a little error if we shift them inside the unity circle.

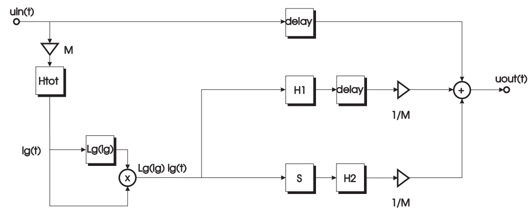


Figure 4: *Horn compensator.*

## 4 Results

Both compensation circuits are implemented in real-time on a TMS320C30 DSP which is mounted on a PC board together with AD- and DA-converters, anti-aliasing filters etc. Using the high level design and simulation package Alta-group SPW, C-Code is automatically generated from schematic entry of the algorithms. Results of the compensator for the woofer, measured with a microphone in the near field, are depicted in Figure 5. Clearly seen is the significant reduction of second order harmonic distortion in the frequency band of interest. Highest distortion occurs in the low frequency area because voice coil excursions are maximum at these frequencies. Performance of the reduction is driving level independent and third and higher order harmonics are not increased. Compared with previous results on reduction with Volterra compensators [6], we have obtained a substantial better performance in a wider frequency span.

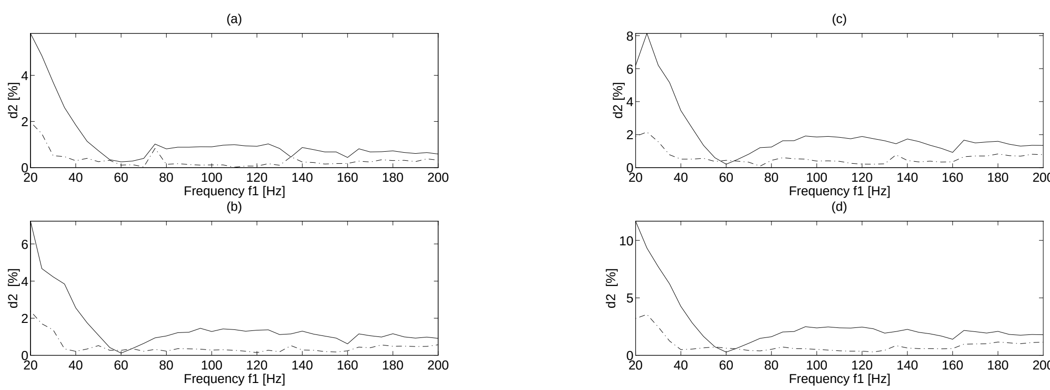


Figure 5: *Second order distortion ($d_2$) as function of the driving frequency $f_1$ at different driving levels without (solid) and with (dash-dotted) Volterra compensator: (a) 2.5V, (b) 3.75V, (c) 5V and (d) 6.25V.*

In the horn compensator the nonlinear inductance $L_g(i_g)$ is approximated by a second order polynomial from which we expect to reduce second and third order harmonics. Comparison of the model of the horn loudspeaker with real distortion measurements show that distortions produced by the nonlinear compliance in the driver are found in the frequency span from 600 to 1100 Hz. It is therefore not surprising that we obtain only reduction in this span as is clear from Figure 6. Second order distortions are reduced with at most 15 % around 900 Hz, independent of the driving level, see Figure 6 (a) and (c). Third order harmonics are only hardly reduced, see Figure 6 (b) and (d). This is caused by a discrepancy between the real nonlinear parameters and the ones used in the model.

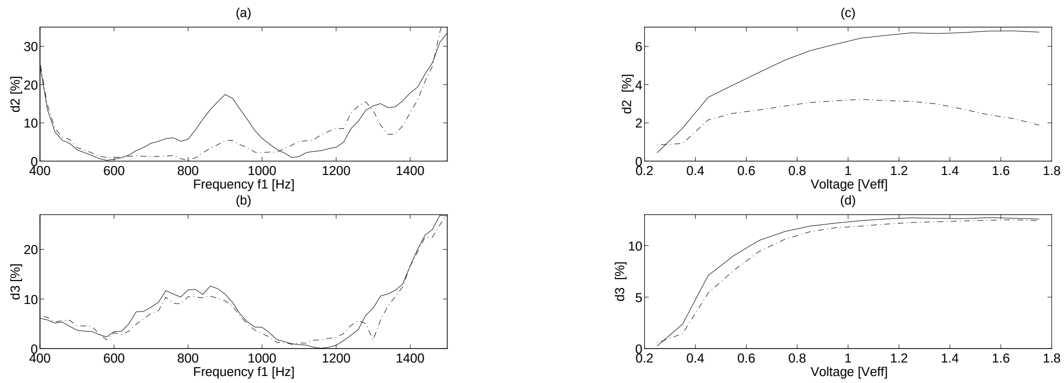


Figure 6: *Measured distortion without (solid) and with (dash-dotted) horn compensator; (a) and (b) depict achieved reduction of second ($d_2$) and third order ($d_3$) harmonics as function of the driving frequency $f_1$ at 2.5Vpeak, while (c) and (d) are measured at a fixed frequency of 800 Hz but at different driving levels.*

# 5 Conclusions

It is concluded that digital compensation of transducer nonlinearity using a digital feedforward compensation circuit is possible. With the growing need for small and cheap loudspeakers with good bass reproduction and decreasing prizes of DSP hardware, this is a realistic design solution. Results with an improved second order Volterra compensator implementation for a direct radiator loudspeaker show a significant reduction of second order harmonic distortion leaving higher order distortions unchanged. To obtain reduction with the horn loudspeaker in the frequency spans around 400 and 1500 Hz, where distortions are still very high, it is necessary to expand the model with more nonlinearities. Modeling of the nonlinear wave propagation inside the horn using lumped parameters is under investigation. Also it is investigated if application to other transducers (e.g. hearing aid receivers) is feasible.

# References


[1] A.J.M. Kaizer, *On the design of broadband electro-dynamical loudspeakers and multiway loudspeaker systems* , Ph.D. thesis Technical University of Eindhoven, 1986

[2] H. Schurer, A.P. Berkhoff, C.H. Slump and O.E. Herrmann, *Modelling and Compensation of Nonlinear Distortion in Horn Loudspeakers*, SIGNAL PROCESSING VII: Theories and Applications, Proceedings of the EUSIPCO-94 Seventh European Signal Processing Conference, Edinburgh, Scotland, U.K. 13-16 September 1994, vol. 3: pp. 1468-1471

[3] M. Schetzen, *The Volterra and Wiener theories of nonlinear systems*, John Wiley, New York, 1980

[4] W. Klippel, *The Mirror filter, a new basis for reducing nonlinear distortion and equalizing response in woofer systems*, Journal of the Audio Engineering Society, vol.40 no.9, pp. 675-691, September 1992

[5] M.A Al-Alaoui, *Novel IIR differentiator from the Simpson integration rule*, IEEE transactions on circuits and systems-1: Fundamental theory and applications, vol.41, no.2, pp. 186-187, February 1994

[6] R. de Vries, A.P. Berkhoff, C.H. Slump, O.E. Herrmann, *Digital compensation of nonlinear distortion in loudspeakers*, Proc. ICASSP-93, vol. I pp. 165-168, April 27-30 Minneapolis, April 1993