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Fractal analysis of eight glacial cycles from an Antarctic ice core

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Abstract

A rescaled range analysis was performed on recent electrical measurements of a deep ice core obtained from Dome C, Antarctica, representing a continuous 740 000 year record of the Earth's climate. It is shown that two measures of the electrical conductivity of the ice surface, both d.c. conductance and 100-kHz dielectric profile, exhibit fractal behavior over 3 orders of magnitude in ice depth. Positive Hurst exponents of 0.795 and 0.848 were calculated for the d.c. conductance and dielectric profile, respectively.

1. Introduction

Recently, a consortium of 10 nations called the European Project for Ice Coring in Antarctica (EPICA) drilled the deepest ice core to date at Dome C in the East Antarctica ice sheet [1]. 3139 m of the 3190 m-deep ice core have been analyzed so far for various physical characteristics—including deuterium content, ice grain size, dust concentration, and electrical conductivity—representing a 740 000 year record of climate. This latest study nearly doubles the climatic record of the previous ice core obtained at the Russian station Vostok [2]. Here I address the question of whether these markers of global climate exhibit fractal or scale-invariant behavior over the range of depth/time points available. Previous investigations of fractal behavior in climate records have focused on much shorter time intervals, as such an uninterrupted data set stretching back 740 000 years has not been previously available. Table 1 shows a representative sampling of studies of scale-invariance in climate behavior.

In the next section I briefly describe the experimental measurements of electrical conductance on ice core samples performed by the EPICA community members [1]. Next, in Section 3 I present the rescaled range analysis employed to determine whether any fractal behavior exists in the climatic data of the EPICA study. In Section 4 the results of this analysis are shown for the zero- and high-frequency electrical conductance measurements. Finally, these results are discussed and summarized in Section 5.

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Table 1
Some previous studies of scale invariance in climate behavior

Brief description	Time span (years)	Reference
Extreme floods in northern Italy	224	[3]
Extreme minimum temperature in Guanajuato, Mexico	102	[4]
Volcanic eruptions on Kamchatka	10000	[5]
Power spectral analysis of climate	200000	[6]
Daily maximum and minimum atmospheric temperature	70	[7]
Mean daily temperature of the atmosphere	215	[8]
Mean annual temperature records in Hungary	91	[9]
Current study	740000	

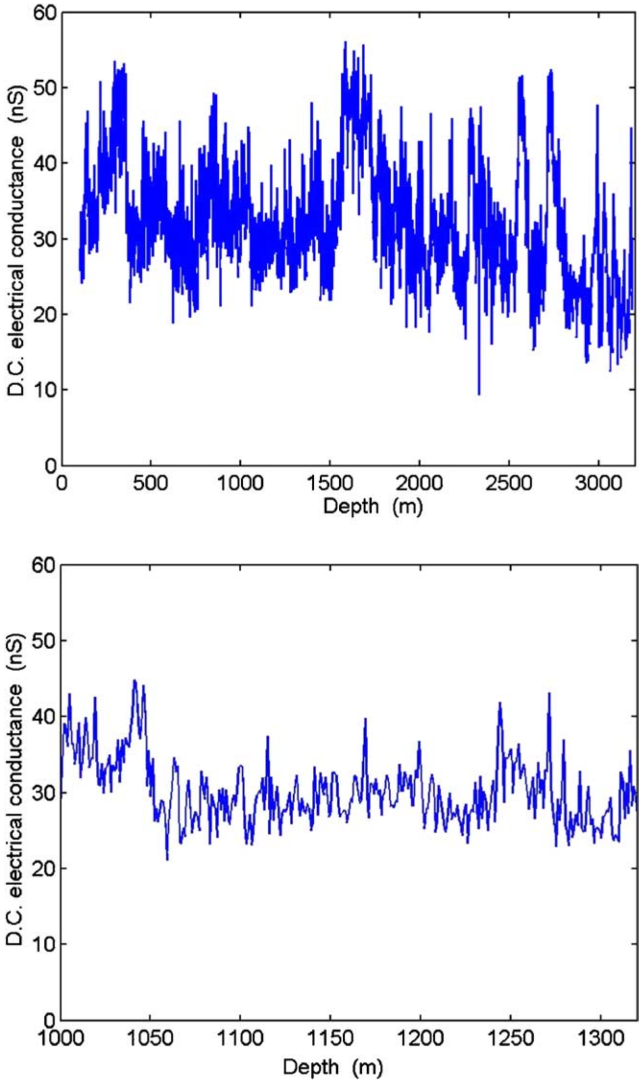

Fig. 1. D.C. electrical conductance measurements as a function of ice core depth, from Ref. [1]. The depth scale is enlarged in the lower figure by a factor of 10. The depth of 3200 m roughly corresponds to climate conditions 740000 years ago.

2. Collection of experimental data

Recently, the EPICA community members described their drilling of a 3190 m deep ice core at the Dome C site in East Antarctica [1]. This represents, by far, the oldest record of world climate change. Various characteristics of the ice samples have already been measured, including (i) deuterium content, (ii) ice grain radius (1–7 nm), (iii) dust content (0–1600 µg/kg), and (iv) electrical conductivity. This latter measurement is the focus of this study, since the other quantities (i)–(iii) were not found to exhibit any significant fractal behavior. Electrical conductivity measurement was conducted in two separate manners, by placing electrodes on a fresh ice surface at a temperature of −20 ± 2 °C. The first method was to measure the d.c. conductance in units of nS. The second method was to measure the dielectric profile conductivity at a frequency of 100-kHz in units of µS/m. As stated in Ref. [1], the electrical data were collected at high resolution and then averaged to 1 m.

3. Rescaled range analysis

The fractal geometry of the ice core electrical conductance was analyzed using a rescaled range analysis [10,11]. Although calculation of fractal dimension has been previously applied to time series data (e.g., [12]), calculation of the Hurst exponent via rescaled range analysis is more appropriate for x-y data of unequal units. The rescaled range analysis is carried out by first calculating the R/S statistic, or rescaled adjusted range, as

$$\frac{R(n)}{S(n)} = \frac{1}{S(n)}\left[\max\left(Y(t) - \frac{t}{n}Y(n)\right) - \min\left(Y(t) - \frac{t}{n}Y(n)\right)\right], \quad 0 \leqslant t \leqslant n, \tag{1}$$

where

$$Y(n) = \sum_{i=1}^{n} X_i,$$

$X_i(t)$ is the original "time series" (in our case, the electrical conductance as a function of depth), and S is the sample standard deviation of X_i. From a given series with N samples, the series is divided into K blocks of size N/K. The rescaled and adjusted range $R(k_i,n)/S(k_i,n)$ is calculated for each block n starting from the point

$$k_i = \frac{iN}{K} + 1, \quad i = 1,2,\ldots \quad \text{such that } k_i + n \leqslant N. \tag{2}$$

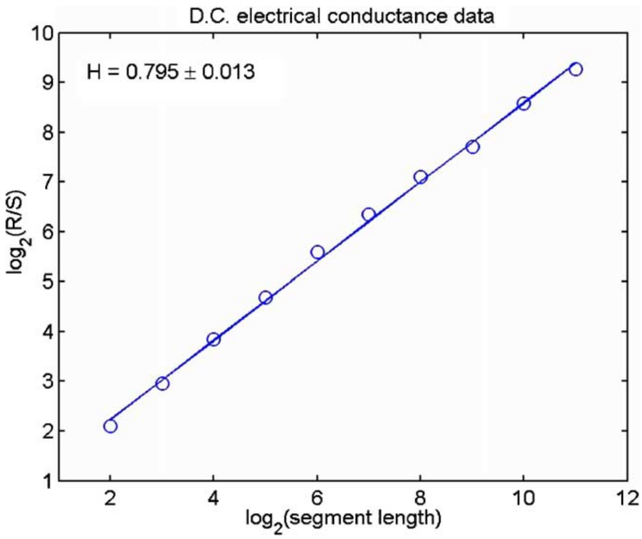

Fig. 2. Rescaled range analysis of the d.c. conductance data of Fig. 1. Note the highly linear relationship with a Hurst exponent of 0.795 ± 0.013, indicating that the conductance data is scale invariant over three orders of magnitude in depth (time).

This yields up to K values of R/S for small values of n, and one sample value for n approaching N. The Hurst exponent H is then estimated by plotting

$$\log\left[\frac{R(k_i,n)}{S(k_i,n)}\right] \quad \text{vs.} \quad \log(n) \tag{3}$$

and performing a linear regression for the slope of the resulting line.

4. Results

Fig. 1 shows "time series" plots of the d.c. electrical conductance measurements as a function of ice depth, collected by the European Project for Ice Coring in Antarctica (EPICA). Much structure is evident in the data, with small-scale and large-scale apparently random fluctuations. When comparing the complete data set with a portion of the data enlarged by a factor of 10 (Fig. 1), an apparent scale invariance to the data is seen. To quantify and more rigorously test

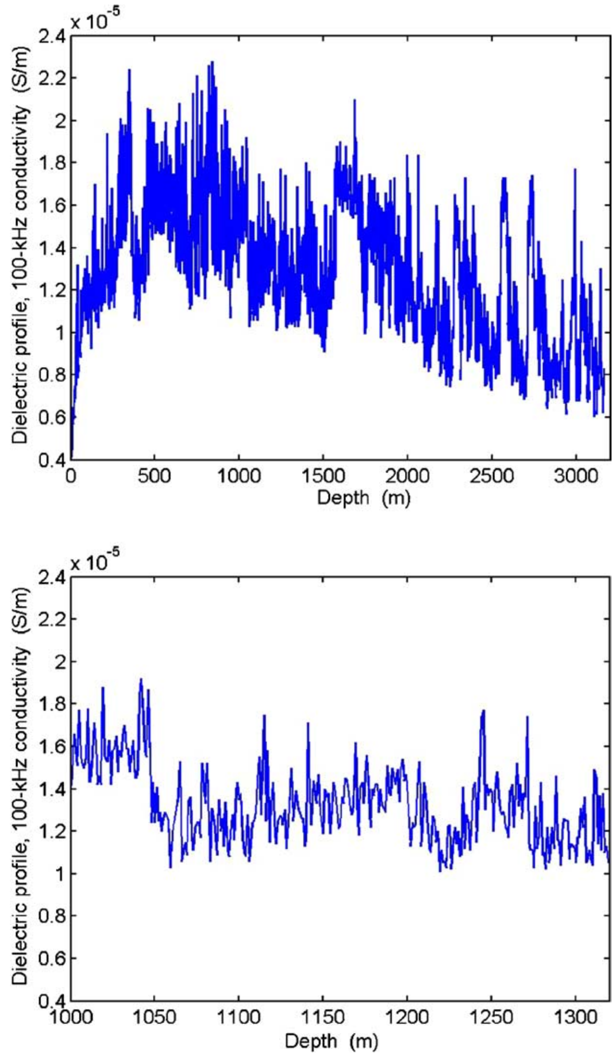

Fig. 3. Dielectric 100-kHz conductance measurements as a function of ice core depth, from Ref. [1]. The depth scale is enlarged in the lower figure by a factor of 10. The depth of 3200 m roughly corresponds to climate conditions 740 000 years ago.

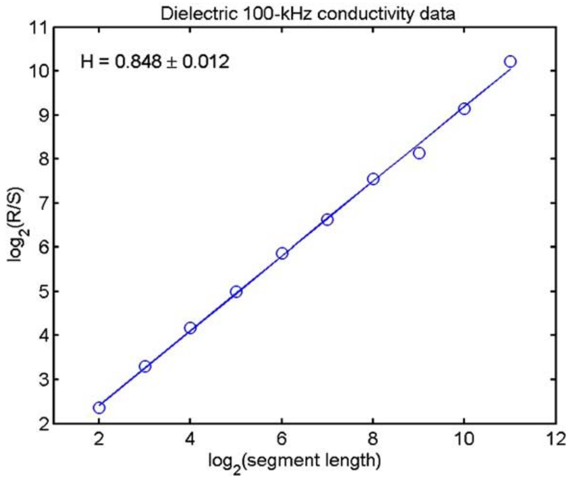

Fig. 4. Rescaled range analysis of the high frequency conductance data of Fig. 3. Note the highly linear relationship with a Hurst exponent of 0.848 ± 0.012, indicating that the 100-kHz conductance data is scale invariant over three orders of magnitude in depth (time).

the invariance of these data, a rescaled range analysis was performed as described in Section 2. As can be seen in Fig. 2, the d.c. conductance data is highly fractal over three orders of magnitude in ice depth ($2^{11} = 2048$). The resulting Hurst exponent was found to be $H = 0.795 \pm 0.013$, the significance of which will be discussed in the next section.

The dielectric, high frequency conductance data exhibits a similar fractal behavior. Fig. 3 shows the 100-kHz data collected by the EPICA team, plotted at two different depth scales. Again, a scale invariance in the conductance fluctuations is apparent. A rescaled range analysis of these data produce a Hurst exponent of $H = 0.848 \pm 0.012$ (Fig. 4). This value is similar to that obtained from the d.c. data (0.795), however, based on the 68% confidence intervals reported for each regression parameter, the difference between the two would appear to be statistically significant ($>2\sigma$).

The other physical parameters measured by the EPICA team and reported in Ref. [1], including deuterium content, ice grain radius, and dust concentration, were also examined for fractal characteristics using the rescaled range analysis. No notable fractal or scale-invariant behavior was found for these variables. It was pointed out by the EPICA authors that both measures of the electrical conductivity are unlike the other variables measured in that the conductivity is affected by the acidity of the ice, and does not vary with respect to climate in a straightforward manner as do the other measures. Specifically, ice acidity increases in both extreme cold and warm stages, and reaches a minimum for intermediate climates. This presumably introduces an additional nonlinearity to the response of conductivity to climate. Thus, in light of this it is perhaps not surprising that among the variables measured by the EPICA team, the conductance alone was found to exhibit clearly fractal behavior.

5. Discussion and summary

For both the d.c. and high-frequency conductivity measurements of the deep ice core taken from Dome C, Antarctica, the Hurst exponent was found to be positive and less than unity. A physical interpretation of the Hurst exponent can be borrowed from the mathematical analysis of oceanic coastline [13]. The value of H divides random surfaces into three categories: (i) $H < 0$: monoscale surfaces which resemble a landscape with hills of approximately the same size, with a bounded surface height; (ii) $0 < H < 1$: rough surfaces which resemble rocky mountains with smaller and higher peaks and an unbounded height; and (iii) $H > 1$: smooth surfaces in which the largest scale defines the slope of the surface. In this view, we may view the "peaks" and "valleys" of the climatic landscape to represent the extreme maxima and minima in temperature or other climate determinant. It has been established that over that past 500000 years the climate has exhibited a strong periodicity occurring with a frequency of 100000 years [14]. This 100000-year periodicity arises from the eccentricity of the Earth's orbit. A second periodicity of 40000 years was discovered in the occurrence of ice ages over one million years ago, arising from the timescale of change in the Earth's tilt. These long wavelength oscillations, when superimposed with the yearly seasonal change of the global climate, must result in the complex "rocky mountain" structure in the climatic record as evidenced by a Hurst exponent between 0 and 1.

Here I have shown for the first time via rescaled range analysis that the conductivity of deep ice core samples are scale invariant over three orders of magnitude in core depth. As more detailed data from the Dome C, East Antarctica facility become available in the coming months and years, these should also be analyzed for any inherently fractal characteristics that may shed light on the temporal evolution of the glacial cycles. Previous studies have correlated the information obtained from Antarctic ice cores with historic climate data obtained from marine sediment collected from the deep sea. Perhaps fractal analysis of these existing data, such as the rescaled range analysis used here, may shed light and make new connections between climatic data from widely separated geographic locations.

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