

# Factor of Safety of a Slope Subjected to Seismic Load

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## ABSTRACT

A procedure to obtain the factor of safety of the slope subjected to seismic load is developed. The stresses in the soil due to static (self weight) and dynamic load are obtained using finite element method. The critical slip surface and factor of safety is obtained using a Monte Carlo technique. The applicability of the analysis is demonstrated by analyzing a 1:1 slope when it is subjected to dynamic load due to El Centro earthquake and a harmonic ground acceleration. It is concluded from the study that the method of determining the factor of safety is simple. The proposed method can be used to obtain the factor of safety of slope and deformation of the slope.

**Editor's Note:** By the term *El Centro Earthquake*, what is meant in this paper is the *El Centro record* of 1940 Imperial Valley earthquake.

**KEYWORDS:** slope stability; factor of safety; finite element analysis; El Centro earthquake; sinusoidal ground acceleration

## INTRODUCTION

Analysis of stability of slopes in terms of factor of safety is important to define the stability of a slope. Mostly stability analysis is carried out under static loading. However, in a seismically active region, earthquakes are one of the important forces that can cause the failure of slopes. Hence, in these regions, it is also necessary to carry out the analysis of slopes under dynamic condition also. A slope becomes unstable when the shear stresses on a potential failure plane exceed the shearing resistance of the soil. The additional stresses due to earthquake further increases the stresses on these planes and decreases the factor of safety further. Most commonly adopted methods for slope stability analysis in static condition are the limit equilibrium methods and finite element method of analysis. Limit equilibrium methods have been widely adopted for slope stability analysis. A potential sliding surface is assumed prior to the analysis and a limit equilibrium analysis is then performed with respect to the soil mass above the potential slip surface. Many methods based on this approach are available. For example, Bishop (1955), Janbu (1957), Morgenstern and Price (1965), Spencer (1967), Sarma (1979).

Although the approach is straightforward, these methods will not consider the stress – strain behavior of the soil mass while calculating the stresses. It is well known that the stresses in soil mass depend on the stress – strain behavior of soil. Due to the availability of large storage, high speed computers, numerical methods have become popular for the analysis of continuum problems. Finite element method is widely used to compute the stresses within a soil mass. Finite element method utilizes a stress – strain characteristic of a soil to calculate the stresses in the soil mass. Duncan and Dunlop (1969), Donald and Giam (1988) used the finite element method to study the development of failure in a slope. Scott et al. (1993) obtained the stresses in soil using finite element method and a critical slip surface is defined by joining the points of local failure in a slope. Delwyn et al. (1999) combined finite element analysis to obtain the stresses in soil and a limit equilibrium method to obtain the factor of safety.

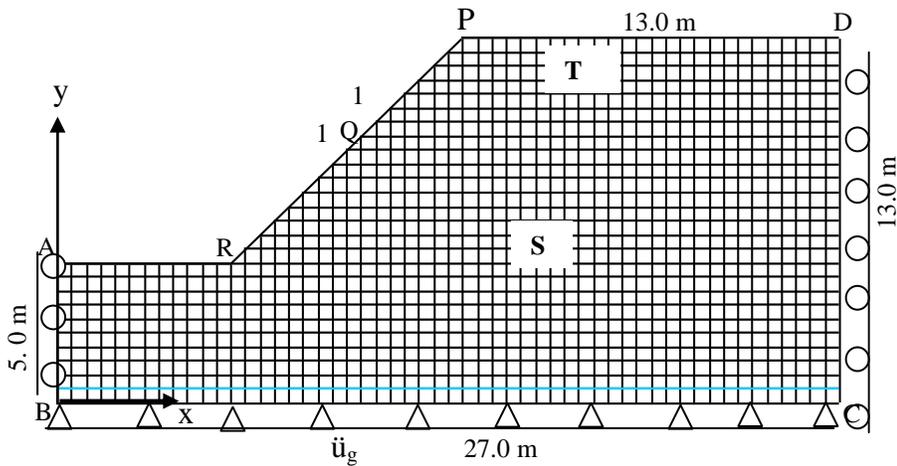
Recently non linear programming based on minimization of factor of safety approach has been used frequently for locating the critical slip surface. Venanzio (1996) proposed a Monte Carlo method of the random walking type for locating critical slip surface of the general shape for slope stability analysis. The simplest approach for dynamic analysis of slopes is the pseudo static analysis. In pseudo static approach the factor of safety against sliding is obtained by including horizontal and vertical forces in the static analysis. These forces are usually expressed as a product of the horizontal or vertical seismic coefficients and the weight of the potential sliding mass. Although the pseudo static approach for dynamic slope stability analysis is simple and straight forward, it cannot really simulate the actual dynamic effects of earthquake force through a constant unidirectional pseudo static acceleration. Newmark (1965) proposed a sliding block method for calculating the permanent slope displacement based on the assumption that the potential failure mass is rigid perfectly plastic.

Newmark (1965), while introducing the sliding displacements criterion, used a very simple technique that of a block moving on an inclined plane. There have been few attempts to improve the basic Newmark concept to compute displacements along a non-planar surface. Sarma (1981) used a circular surface to compute the displacements along a slip surface. Sarma and Chlimintzas (2001) developed multi block model taking into account the effect of internal deformations and transfer of mass between successive blocks. Permanent displacement of a slope subjected to dynamic force was also obtained using finite element analysis. Various approaches used based on finite element method mainly varies with the constitutive model adopted to model the behavior of soil. Prevost et al. (1985), Daddazio (1987) and Elgamal et al. (1990) used constitutive relationship to model the behavior of soil. Thus in most of the methods available for the dynamic analysis of slopes, the importance is given for finding the displacement of the slope rather than the factor of safety. Even though the displacement of a slope is a very important criterion for the design of a slope, it is also important to know the factor of safety of a slope when subjected to dynamic load. Hence, in the present method, a procedure to obtain the factor of safety and displacement of a slope subjected to dynamic load is developed. The method can be used to obtain the factor of safety, displacement and stresses in soil at all time interval from the beginning to end of the earthquake. In the proposed method, the factor of safety of a slope is obtained using a combination of finite element analysis to obtain the static and dynamic stresses and a Monte Carlo technique proposed by Venanzio (1996) to obtain the critical slip surface. However, the slope is assumed as dry and elastic and hence no constitutive relationship is used to model the nonlinear behavior of soil.

## METHOD OF ANALYSIS

The analysis consists of two parts:

- Finding the stresses at required points using the finite element method.
- Locating the critical slip surface and finding the factor of safety using these stresses.



**Figure 1:** Finite element discretization of slope

Figure 1 shows a finite element discretization of a slope. Four noded isoparametric quadrilateral plane strain element and three noded triangular elements are used for discretization. The displacement along X and Y direction is restrained along the bottom surface of the slope BC and displacements in X direction is restrained along the side CD and AB as shown in figure. The stresses in soil due to self weight of the soil are obtained using the stress – strain relationship of soil. For dynamic analysis, the stiffness matrix  $[k]$  and consistent mass matrix  $[m]$  are obtained for each element and the stiffness matrix and mass matrix of each element are added by direct stiffness method to obtain the overall stiffness matrix  $[K]$  and mass matrix  $[M]$ . The damping of the slope is assumed as Rayleigh type and the damping matrix  $[C]$  is obtained using the equation  $[C] = \alpha[M] + \beta[K]$  where  $\alpha$  and  $\beta$  are the Rayleigh constants. These constants can be determined easily if the frequency of vibration and damping ratio corresponding to any two mode shapes are known. The dynamic equation for the entire slope can then be expressed in matrix form as

$$[M] \{ \ddot{u} \} + [C] \{ \dot{u} \} + [K] \{ u \} = \{ F(t) \}$$

For a slope subjected to earthquake load  $\{ F(t) \} = -[M] \{ I \} \ddot{u}_g(t)$  where  $[M]$  is the mass matrix,  $\{ I \}$  is the influence vector and  $\ddot{u}_g(t)$  is the ground acceleration. The resulting dynamic equation is solved to obtain displacements, velocity and acceleration at each time interval  $\Delta t$ . A time interval  $\Delta t$  equal to 0.04 sec is considered for the analysis. The

displacement at nodes and stresses at required points are obtained at time interval of 0.04 seconds. These stresses are added to the static stresses to obtain the total stresses due to static and dynamic stresses at each time interval.

## Determination of Factor of Safety

A Monte Carlo technique proposed by Venanzio (1996) as explained below is used to obtain the factor of safety of the slope at each time interval.

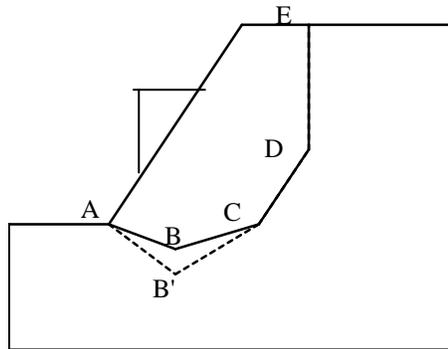
Nonlinear programming procedures for searching for the minimum of a function of several variables start from a trial slip surface  $S^0$  and proceed towards the minimum in an iterative way, generating sequence of feasible slip surfaces  $S^1, S^2, S^3, \dots, S^k, S^{k+1}$  so that the sequence of the associated safety factors decreases

$$\text{i.e. } F(S^0) > F(S^1) > F(S^2) > F(S^3) > \dots > F(S^k) > F(S^{k+1})$$

$$\text{Where } S^k = \{ x_1^k, y_1^k, x_2^k, y_2^k, x_3^k, y_3^k, \dots, x_n^k, y_n^k \}$$

$$S^{k+1} = \{ x_1^{k+1}, y_1^{k+1}, x_2^{k+1}, y_2^{k+1}, x_3^{k+1}, y_3^{k+1}, \dots, x_n^{k+1}, y_n^{k+1} \}$$

A trial slip surface with six or seven trial points is selected as trial surfaces. As shown in figure 2, A, B, C, D and E is one such trial surface with point A, point B, point C, point D and point E as trial points. Each of these trial points A, B, C, D and E is then shifted to a new position in eight directions. A, B', C, D and E is the surface obtained after shifting B to B'. For each shift the factor of safety is computed. If the factor of safety of the new slip surface (A, B', C, D, E) thus obtained is lesser than the factor of safety of the previous slip



**Figure 2:** Trial slip circle ABCDE

surface (A, B, C, D, E) then the point is fixed to this new position (B'). Otherwise, it is returned to the previous position (B). The process of shifting the trial points to new position based on comparison of old and new factor of safety is repeated for all trial points A, B, C, D and E. As trial points are shifted to new position a new slip surface is obtained. The factor of safety of the new slip surface, thus obtained is compared with the factor of safety of the old surface. If the factor of safety of the new surface is less than the factor of safety of the old surface then the procedure is repeated for the new surface. Thus shifting of trial points of the slip surface in eight directions as explained above is repeated until the factor of safety of new slip surface is more than the factor of safety of the old surface.

### Equation for factor of safety

The trial slip surface is divided into 'n' number of segments each of length  $\Delta L$ . The overall factor of safety for a particular slip surface is obtained using the equation

$$F.S = \frac{\sum \tau_f \Delta L_i}{\sum \tau_i \Delta L_i}$$

where  $\tau_i$  is the mobilized shear stress and  $\tau_f$  is the shear strength of the material  $\Delta L_i$  is the length of  $i^{\text{th}}$  segment. For the  $i^{\text{th}}$  segment on a particular slip surface, the values of  $\tau_i$  and  $\tau_f$  may be expressed as

$$\tau_f = c + \sigma_{ni} \tan \phi$$

$$\tau_i = 0.5 (\sigma_{yi} - \sigma_{xi}) \sin 2\alpha_i + \tau_{xyi} \cos 2\alpha_i$$

$$\sigma_{ni} = \sigma_{xi} \sin 2\alpha_i + \sigma_{yi} \cos 2\alpha_i - \tau_{xyi} \sin 2\alpha_i$$

where  $c$  and  $\phi$  are the cohesion and angle of internal friction of the soil.  $\sigma_{ni}$  is the normal stress acting on segment  $i$ .  $\sigma_{xi}$ ,  $\sigma_{yi}$  and  $\tau_{xyi}$  are the effective stresses on  $i^{\text{th}}$  segment.  $\alpha_i$  is the inclination of the  $i^{\text{th}}$  segment with horizontal.

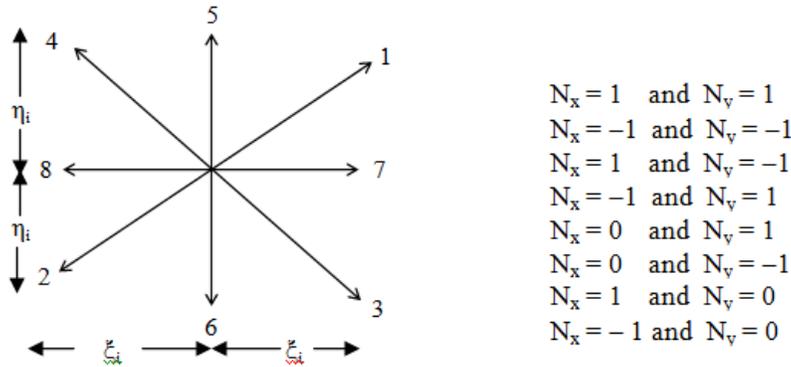
### Criteria for moving the vertex

Each trial point of the current slip surface is randomly moved to a new position to obtain the reduction in safety factor. The point  $i$  is moved from point  $(x_i^k, y_i^k)$  to point  $(x_i^{k+1}, y_i^{k+1})$  where

$$x_i^{k+1} = x_i^k + \xi_i$$

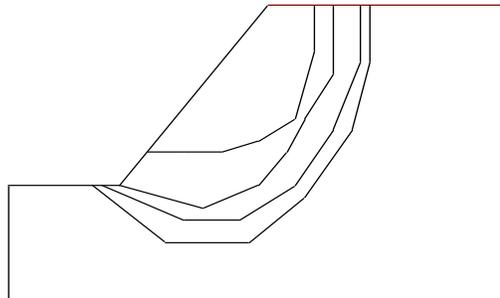
$$y_i^{k+1} = y_i^k + \eta_i.$$

$\xi_i = N_x D x_i$  and  $\eta_i = N_y D y_i$ ,  $D x_i$  and  $D y_i$  are the widths of the search steps in directions  $x$  and  $y$  for various points  $i$ . The following eight combinations are given for parameters  $N_x$  and  $N_y$ . In this way eight random displacements are tried for every trial points of the slip surface. In the present analysis, the values of  $D x_i$  and  $D y_i$  are taken as 0.5 m.



## NUMERICAL APPLICATIONS

A homogeneous, dry 1:1 slope with a unit weight of  $18.5 \text{ kN/m}^3$ , modulus of elasticity of  $5000 \text{ kN/m}^2$ , and Poisson's ratio of 0.3 is analyzed. The height of the slope is equal to 8.0 m. The slope is discretized using four noded quadrilateral element of size  $0.5 \text{ m} \times 0.5 \text{ m}$  and triangular element as shown in Figure 1. Four trial surfaces considered for the analysis is shown in Figure 3. Various combination of cohesion  $c$  and angle of internal friction  $\phi$  is considered. The factors of safety obtained from the analysis are tabulated in Table 1. The factor of safety obtained by Scott et al. (1993) based on the local minimum factor of safety method and by using Bishop's (1952) method of analysis for the same slope are also



**Figure 3:** Various trial slip surfaces considered for the study

tabulated in the same table for various values of  $c$  and  $\phi$ . It can be observed from the table that the results obtained from the present analysis and that presented by Scott et al. (1993) based on local minimum factor of safety method and by using Bishop's (1952) method agree very well for all values of  $c$  and  $\phi$ .

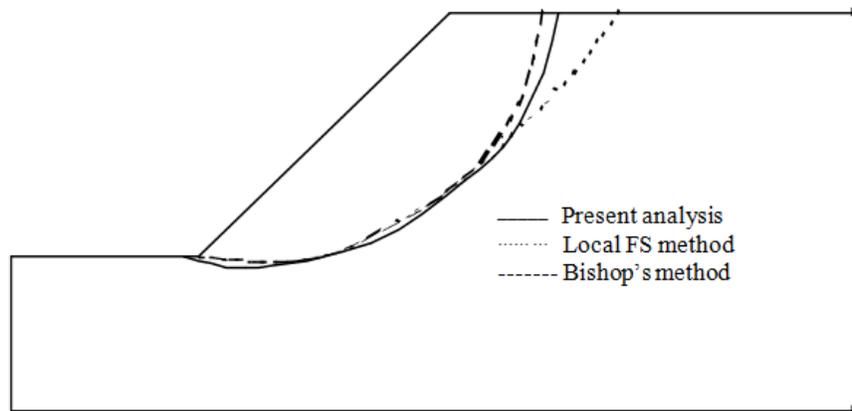
**Table 1:** Factor of safety obtained by the present method and by Scott et al. (1993)

c (MPa)	$\phi$ (degrees)	FS by proposed method	FS by local minimum FS method	FS by Bishop's (1952) method
25	20	1.89	1.87	1.71

20	20	1.67	1.68	1.50
15	20	1.39	1.46	1.29
10	20	1.08	1.0	1.05
30	15	1.88	1.85	1.75
25	15	1.65	1.65	1.53
20	15	1.43	1.45	1.32
15	15	1.21	1.24	1.11
10	15	0.94	1.0	0.89
25	10	1.46	1.42	1.35
20	10	1.25	1.23	1.15
15	10	1.02	1.0	0.97

Figure 4 shows the critical slip surface obtained by the present analysis and by Scott et al. (1993) using local minimum factor of safety method and by using Bishop's (1952) method of analysis. The critical slip surface obtained by the present analysis also matches closely with the critical slip surface presented by Scott et al. (1993).

Hence, the proposed method of combining the finite element method to obtain stresses and a Monte Carlo method to find the factor of safety can be used to analyze the embankment slopes. Also, since the present method requires few trial slip surfaces, the method of finding factor of safety is simple and efficient compared to other methods.



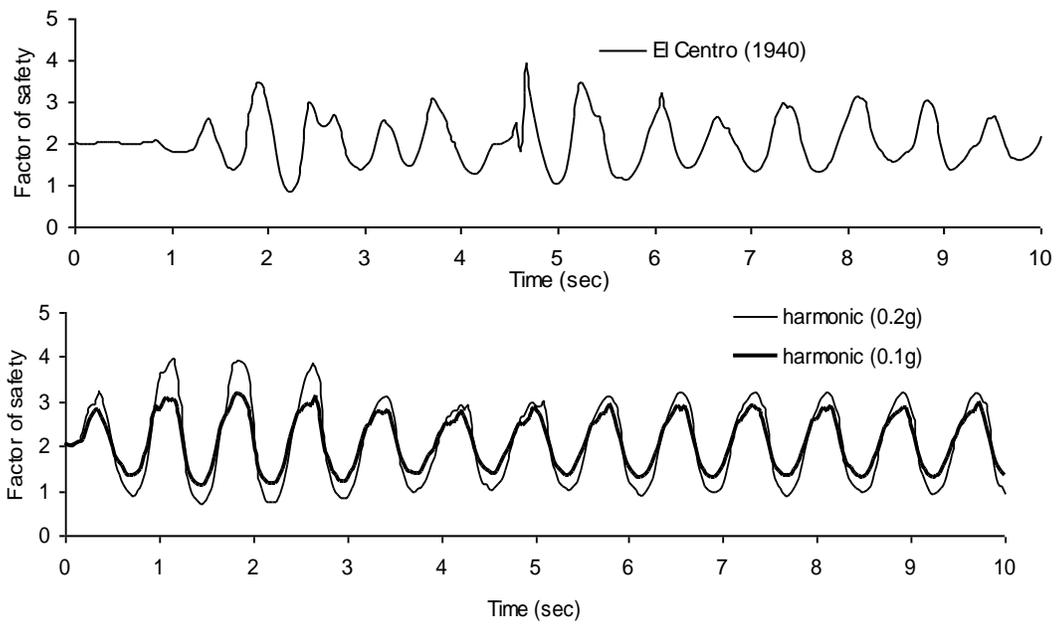
**Figure 4:** Critical slip surface obtained by present method and by Scott et al. (1993)

## ANALYSIS OF SLOPE SUBJECTED TO DYNAMIC LOAD

The slope shown in Figure 1 is subjected to a ground acceleration in horizontal direction due to El Centro (1940) earthquake and harmonic ground acceleration. To obtain the frequency of the slope, a method based on simultaneous iteration is developed. The first two natural frequencies of the slope obtained from this method is equal to 8.2 rad/sec and

9.4 rad/sec. The values of Rayleigh constants  $\alpha$  and  $\beta$  for 5% damping of slope is equal to 0.437 and 0.00568 respectively. These values are used to obtain the damping matrix as already explained. The value of cohesion  $c = 30 \text{ kN/m}^2$  and angle of internal friction  $\phi$  is equal to  $20^\circ$ . The factor of safety of the slope without the earthquake load (static analysis) is equal to 2.068. The slope is subjected to ground acceleration at base in horizontal direction. The input ground acceleration considered for the analysis are NS component of El Centro (1940) earthquake and harmonic ground acceleration of intensity  $0.2g \sin(\omega t)$  and  $0.1g \sin(\omega t)$ . ( $\omega$  is the excitation frequency and  $g$  is the acceleration due to gravity). Since the response of the slope is maximum when natural frequency,  $\omega_n$  is equal to the excitation frequency,  $\omega$ , an excitation frequency,  $\omega$ , equal to 8.2 rad/sec is used for harmonic ground acceleration. The ground acceleration is applied at time interval of 0.04 seconds up to 10 seconds and the factor of safety of the slope, displacement of the slope and stresses in soil are obtained at interval of 0.04 seconds up to 10 seconds.

Figure 5 shows the variation of factor of safety of the slope with time when the slope is subjected to earthquake load and to harmonic ground acceleration of intensity  $0.2g \sin(8.2t)$  and  $0.1g \sin(8.2t)$ . It can be observed from the figure that the factor of safety of the slope varies with time. The dynamic factor of safety is more than static factor of safety at some time interval whereas it is less than the static factor of safety at some other time interval. The maximum reduction in factor of safety occurs at time interval equal to 2.52 rad/sec. The factor of safety at this time interval is equal to 0.98. Thus the factor of safety of the slope reduces from 2.08 to 0.98 when subjected to earthquake load due to El Centro earthquake. Similarly, the factor of safety of the slope also varies with time when subjected to harmonic ground acceleration. The maximum factor of safety of the slope is equal to 3.98 and minimum factor of safety is equal to 0.58 when the slope is subjected to a harmonic ground acceleration of intensity  $0.2g \sin(8.2t)$  whereas the maximum and minimum factor of safety when the slope is subjected to a harmonic ground acceleration of intensity  $0.1g \sin(8.2t)$  is equal to 2.8 and 1.1 respectively.

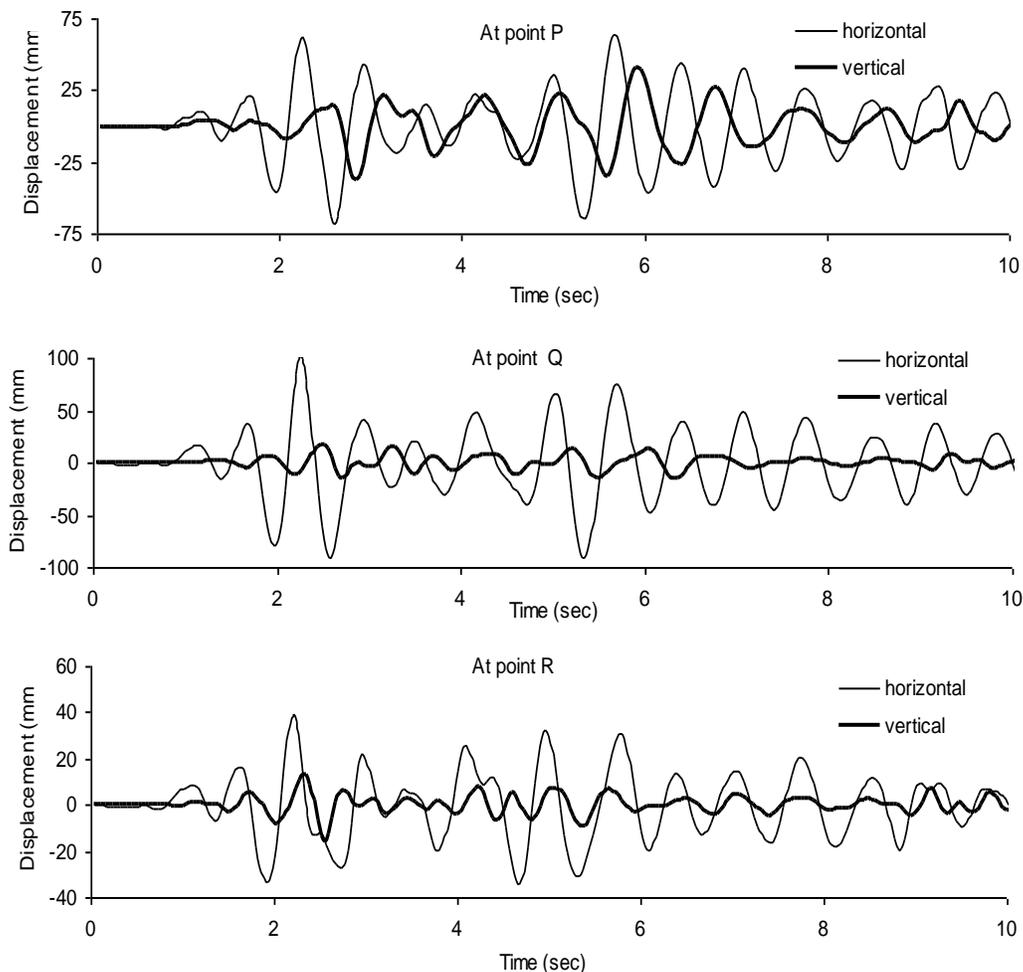


**Figure 5:** Variation of factor of safety with time for a slope subjected to ground acceleration

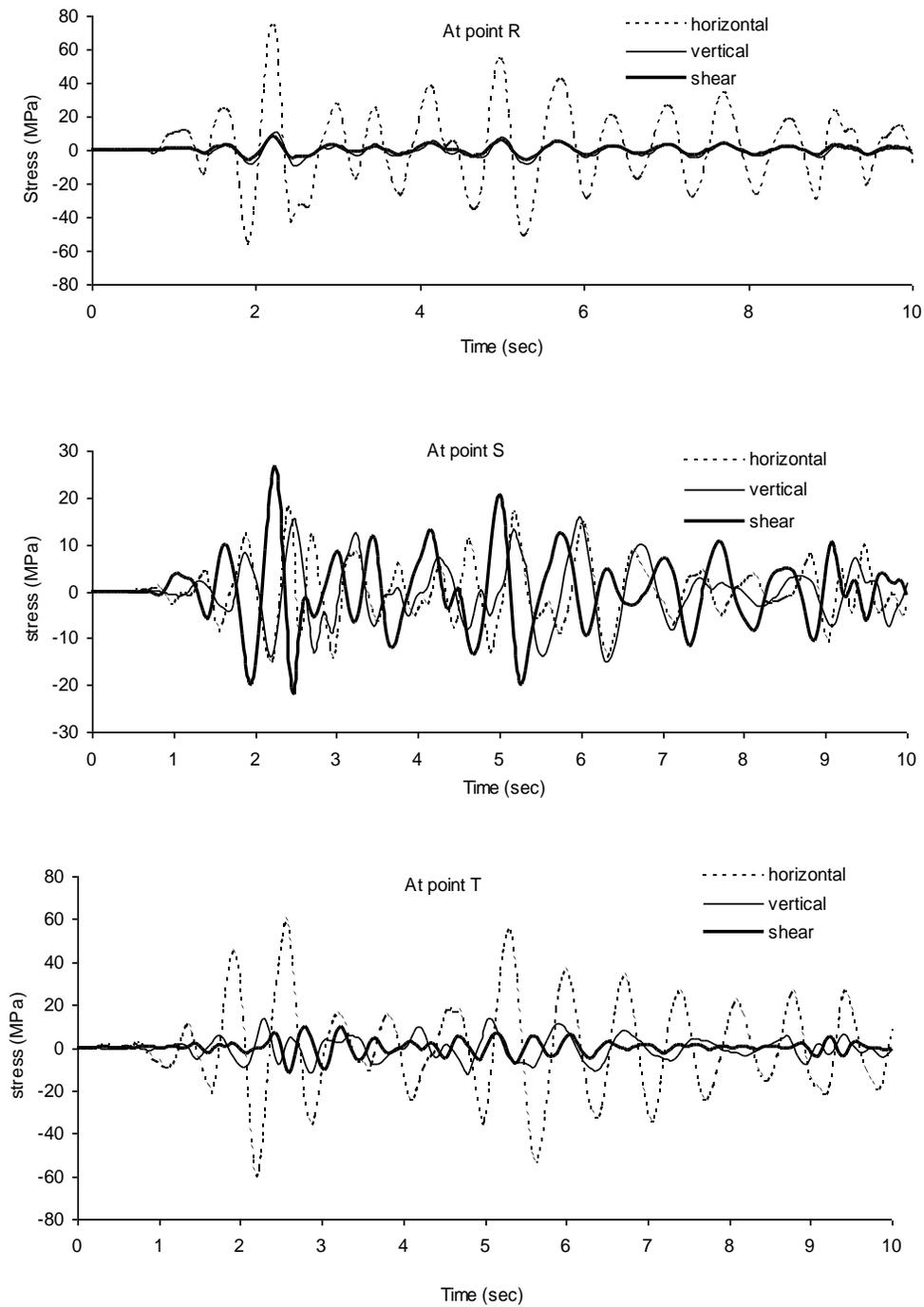
Figure 6 shows the variation of the vertical and horizontal displacement of the slope at point P, Q and R (these points are shown in Figure 1) when the slope is subjected to ground acceleration due to El Centro earthquake. It can be seen from the figure that the displacement varies with time and the horizontal displacement of the slope is more than vertical displacement at all these three points. Also, the displacement of the slope is maximum at center (point Q) as compared with the bottom of the slope (at point R) or at top of slope (at point P).

The variation of horizontal stress, vertical stress and shear stress with time at point R, S and T when the slope is subjected to El Centro earthquake is shown in Figure 7. The horizontal stress is more than the vertical stress or shear stress at point R and T where as at point S; the shear stress is more than the vertical or horizontal stress. The slope is subjected to a harmonic ground acceleration of intensity  $0.2g \sin(\omega t)$  for a period of 10 seconds for various values of  $\omega$  ranged from 1 rad/sec to 20 rad/sec. For each value of  $\omega$ , the factor of safety, horizontal displacement and stresses are obtained at time interval of 0.04 seconds and minimum factor of safety, maximum displacement and maximum stresses from these values are noted. Figure 8 shows the variation of factor of safety with excitation frequency when a slope is subjected to ground acceleration of intensity  $0.2g \sin(\omega t)$  at base.

It can be observed from the figure that the factor of safety of the slope varies with excitation frequency and the factory of safety is minimum when excitation frequency is equal to the natural frequency of the slope. However, for all the values of excitation frequency, the dynamic factor of safety of slope is considerably less than the static factor of safety.



**Figure 6:** Variation of displacement with time for a slope subjected to El Centro earthquake



**Figure 7:** Variation of stresses with time for a slope subjected to El Centro earthquake

Figure 9 shows the variation of maximum horizontal displacement with excitation frequency at points P, Q and R. The maximum horizontal displacement also varies with excitation frequency and reaches a maximum value when natural frequency of the slope is equal to the excitation frequency (at 8.2 rad/sec). The maximum displacement varies from 30 mm at excitation frequency equal to 1 rad/sec to about 275 mm when excitation frequency is equal to 8.2 rad/sec. The variation of maximum horizontal stresses at point R, S and T with excitation frequency is shown in Figure 10. The maximum horizontal stresses vary from 20 MPa at 1 rad/sec to 180 MPa at 8.2 rad/sec at point R and T. The maximum horizontal stress at point R and T is more than the maximum horizontal stress at point S when excitation frequency is up to 12 rad/sec whereas when excitation frequency is more than 12 rad/sec, the maximum horizontal stresses at S is more than the maximum horizontal stresses at points R and T.

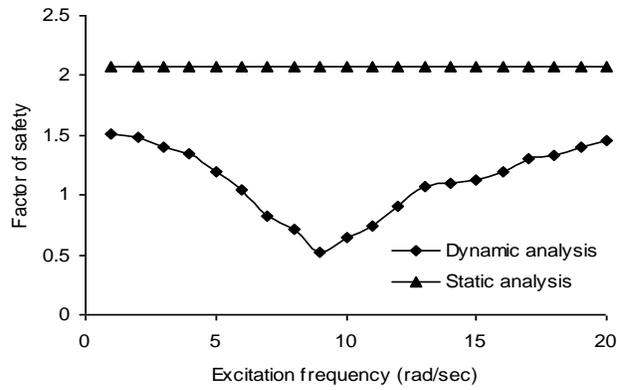
## SUMMARY AND CONCLUSIONS

A procedure to obtain the factor of safety of the slope subjected to dynamic load is developed. The stresses in soil are obtained using finite element method and the critical slip surface is obtained using a Monte Carlo technique. The factor of safety obtained from the present analysis is compared with the factor of safety obtained by Scott et al. (1993). The slope is subjected to a horizontal ground acceleration at base due to El Centro earthquake and harmonic ground acceleration and the factor of safety of the slope, stresses in soil and displacement of slope at time interval of 0.04 seconds is obtained. The factors of safety obtained by the present analysis agree well with the factor of safety obtained by Scott et al (1993). The method of locating critical slip surface is simple as compared to other methods since it requires only very few trial slip surfaces. The factor of safety of the slope subjected to dynamic load like earthquake varies with time and it is more than the static factor of safety at some time interval where as it is less than the static factor of safety at some other time interval. The factor of safety of the slope reduces from 2.08 to 0.98 when the slope is subjected to El Centro earthquake whereas it reduces from 2.08 to 0.58 when subjected to harmonic ground acceleration of intensity  $0.2g \sin(\omega t)$  at excitation frequency equal to the natural frequency of the slope. The factor of safety, displacement in the soil and stresses in soil also varies with excitation frequency and the dynamic factor of safety is considerably less than the static factor of safety at all excitation frequencies. The proposed method can be used to obtain the factor of safety at all time interval from the beginning to end of earthquake. The exact time history of earthquake ground acceleration and realistic soil properties can also be considered in the analysis. The analysis can also be used to obtain the displacement of the slope at required location with in the slope. The slope is assumed as dry and elastic and hence the liquefaction of soil and non linear characteristic of soil is not considered in the analysis. However, this can be incorporated easily by suitably modifying the stress – strain relationship of soil at each time interval by using a realistic constitutive model.

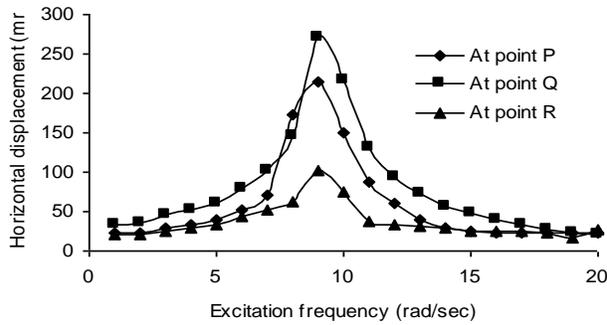
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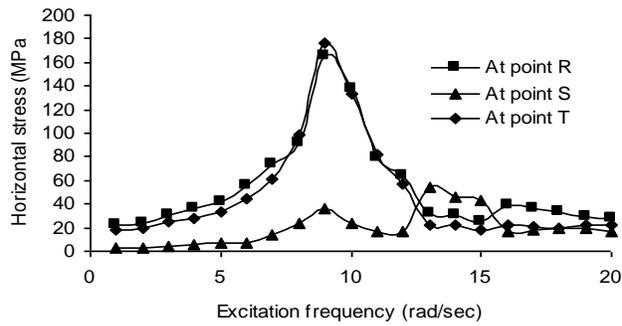
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**Figure 8:** Variation of factor of safety with excitation frequency



**Figure 9:** Variation of horizontal displacement with excitation frequency



**Figure 10:** Variation of horizontal stress with excitation frequency

