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INFLUENCE OF CHEMICAL REACTION ON TRANSIENT MHD FREE CONVECTION OVER A MOVING VERTICAL PLATE

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يقدم هذا البحث دراسة عددية لتأثير مجال مغناطيسي على انتقال الحرارة بالحمل الحر الغير مستقر لمائع ممغنط كهربائيا من صفيحة عمودية متحركة، مع وجود تفاعل كيمائي خطي. وقد تم حل المعادلات التي تحكم المسالة تحت الدراسة وهي معادلات غير خطية وغير مستقرة باستخدام طريقة الفروق المحدودة. وتم دراسة تأثير العديد من العوامل الهامة على توزيع السرعة ودرجة الحرارة والتركيز داخل الطبقة الحدية وعلى انتقال الحرارة والكتلة كذلك. وقد وجد أن أهم العوامل التي تؤثر على المسالة تحت الدراسة هي معامل التفاعل الكيمائي ومعامل المجال المغناطيسي . وبينت نتائج الدراسة أن كلا من السرعة والتركيز تقلان بزيادة معامل التفاعل الكيمائي. وكذلك فقد وجد أن زيادة معامل المجال المغناطيسي تؤدي إلى تقليل السرعة وزيادة التركيز.

The effect of magnetic field on the transient free convection flow of an electrically conducting fluid over an impulsively started isothermal vertical plate is numerically investigated, considering a homogeneous chemical reaction of first order. The transient, nonlinear and coupled governing equations are solved using an implicit finite-difference scheme of Crank–Nicolson type. The effects of various parameters on the transient velocity, temperature, and concentration profiles as well as heat and mass transfer rates are analyzed. The fundamental parameters of the problem are the chemical reaction parameter and the magnetic parameter. It is found that, the velocity as well as concentration decreases with increasing the chemical reaction parameter. While the velocity decreases and the concentration increases as the magnetic parameter increases.

Keywords**:** Transient free convection/ Moving plate/ Chemical reaction/ Magnetic field/ Numerical analysis

NOMENCLATURE

B_o	magnetic field flux density, (tesla)
C	species concentration in the fluid, (kg.m^{-3})
C_p	specific heat of at constant pressure, (J.kg^{-1}.K^{-1})
D	mass diffusion coefficient, (m^2.s^{-1})
g	acceleration due to gravity, (m.s^{-2})
Gr_C	mass Grashof number, defined by Eq. (6)
Gr_T	thermal Grashof number, defined by Eq. (6)
Ha	Hartmann number $\sqrt{M}$, defined by Eq. (6)
K	non-dimensional chemical reaction parameter, defined by Eq. (6)
k	thermal conductivity, (W.m^{-1}.K^{-1})
k_l	chemical reaction parameter, (s^{-1})
M	magnetic parameter, defined by Eq. (6)
Nux	local Nusselt number, defined by Eq. (12)
Pr	Prandtl number, defined by Eq. (6)
Sc	Schmidt number, defined by Eq. (6)
Shx	Sherwood number, defined by Eq. (13)
T	temperature of the fluid, (K)
t	time, (s)
u,v	velocity components along (x,y)-axis, (m.s^{-1})
u_p	velocity of the plate, (m.s^{-1})
X	dimensionless spatial coordinate along the plate, defined by Eq. (6)
x	special coordinate along the plate, (m)
y	spatial coordinate normal to the plate, (m)
Y	dimensionless spatial coordinate normal to the plate

Greek symbols

τ	dimensionless time, defined by Eq. (6)
σ	electrical conductivity of the fluid (Ω.m^{-1})
β_C	thermal expansion coefficient, (m^3.kg^{-1})
β_T	coefficient of expansion with concentration, (K^{-1})
ϕ	dimensionless concentration variable, defined by Eq. (6)
α	thermal diffusivity, (m^2.s^{-1})
θ	dimensionless temperature, defined by Eq. (6)
ν	kinematic viscosity,
ρ	fluid density, (kg.m^{-3})

Subscripts

w	evaluated at wall conditions
∞	evaluated at free stream condition

1. INTRODUCTION

The study of heat and mass transfer to chemically reacting MHD free convection on a moving vertical plate has received a growing interest during the last decays. This is due to its important in several engineering, industrial, geophysical and astrophysical application, such as polymer production, manufacturing of ceramic, packed-bed catalytic reactors, food processing, cooling of nuclear reactors, enhanced oil recovery, underground energy transport, magnetized plasma flow, high-speed plasma wind, cosmic jets and stellar systems. A clear understanding of the nature of interaction between thermal and concentration buoyancies is necessary to control these processes. The problem of steady and unsteady of combined heat and mass by free convection along an infinite and semi- infinite vertical plate with and without chemical reaction has been studied extensively[1-13]. Ganesan and Loganathan[4] presented numerical solutions of the transient natural convection flow of an incompressible viscous fluid past an impulsively started semi-infinite isothermal vertical plate with mass diffusion, taking into account a homogeneous chemical reaction of first order. Ghaly and Seddeek[5] analyzed the effect of variable viscosity; chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate.

Muthucumaraswamy and Ganesan[6] studied numerically the transient incompressible viscous fluid flow regime past a semi-infinite isothermal plate under conditions of natural convection. Rahman and Mulolani[7] studied the laminar natural convection flow over a semi-infinite vertical plate at constant species concentration. They found that in the absence of chemical reaction, a similarity transform is possible, while when chemical reaction occurs, perturbation expansions about an additional similarity variable dependent on reaction rate must be employed. Ganesan and Loganathan[8] analyzed the development of the free convection boundary layer flow of a viscous and incompressible fluid past an impulsively started semi-infinite vertical cylinder with uniform heat and mass fluxes and chemically reactive species. Muthucumaraswamy and Kulaivel[9] presented a analytical solution to the problem of flow past an impulsively started infinite vertical plate in the presence of uniform heat flux and variable mass diffusion, taking into account the homogeneous chemical reaction of first-order. Mulolani and Rahman[10] obtained similarity solutions for the steady laminar natural convection flow over a semi-infinite vertical plate. Muthucumaraswamy and Ganesan[11] analyzed the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Muthucumaraswamy and Ganesan[12] considered the incompressible fluid flow over an impulsively started vertical plate with constant heat flux and mass transfer. Muthucumaraswamy and Ganesan[13] investigated the diffusion and first-order chemical reaction on impulsively started infinite vertical plate with variable temperature.

There has been a renewed interest in MHD flow and heat transfer due to the important effect of magnetic field on the performance of many systems using electrically conducting fluid. In a review article, Ram[14] presented an account of several MHD heat and mass transfer problems. Chamkha[15] considered the MHD free convection from vertical plate in a thermally stratified porous medium with Hall current effect. Emad and Elsayed[16] studied the influence of Hall current effect on magnetohydrodynamic free-convection flow past a semi-infinite vertical plate with mass transfer. The main objective of this study is to investigate numerically the influence of magnetic field on unsteady combined heat and mass transport by free convection along an impulsively started semi- infinite vertical plate taking into account a homogeneous chemical reaction of first order. The fluid considered in this investigation is air. The boundary layer equations governing the problem under consideration are solved numerically using an implicit finite difference technique of Crank–Nicolson. The effects of magnetic field and the chemical reaction on the velocity, temperature and concentration profiles as well as the local heat and mass transfer rates are presented and discussed in details.

2. MATHEMATICAL ANALYSIS

Consider transient MHD free convection flow of an electrically conducting, viscous incompressible fluid over an impulsively started isothermal vertical plate with chemical reaction. The x-axis is assumed to be taken along the plate and the y-axis normal to the plate. The wall is maintained at constant temperature T_w and concentration C_w higher than the ambient temperature T_∞ and concentration C_∞, respectively. The viscous dissipation and the Joule heating effects are assumed to be negligible in the energy equation. Also, it is assumed that there exists a homogenous chemical reaction of first-order with rate constant k_l between the diffusing species and the fluid. A uniform magnetic field is applied normal of magnitude B_0 to the plate. It is worth noting that the existence of the boundary layer requires a sufficiently large Reynolds number Re>>1, Also, the variables y, v have been rescaled by the thickness of the boundary layer. Under the Boussinesq approximations and boundary-layer the boundary layer[17], the governing equations for the problem under consideration are:

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial \mathrm{v}}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+ \quad (2)$$
$$g\beta_C(C-C_\infty)-\frac{\sigma B_0^{\,2}}{\rho}u$$

$$\frac{\partial T}{\partial t}+\mathrm{u}\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\alpha\,\frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$\frac{\partial C}{\partial t}+\mathrm{u}\frac{\partial C}{\partial x}+\mathrm{v}\frac{\partial C}{\partial y}=D\,\frac{\partial^2 C}{\partial y^2}-k_l C \quad (4)$$

where (u, v) are the averaged velocity components along the x and y, directions respectively, t is the time, T is the temperature, C is the concentration, β_T and β_c are the thermal expansion coefficient and the concentration expansion coefficient respectively, ν is the fluid kinematic viscosity, σ is the electrical conductivity, B_o is the magnetic field flux density, ρ is the fluid density, α is the effective thermal diffusivity, D is the mass diffusion coefficient and k_l is the chemical reaction coefficient.

The physical problem assumes the following initial and boundary conditions

$$\left.\begin{array}{l}\mathrm{u}(x,y,0)=\mathrm{v}(x,y,0)=0,\\ T(x,y,0)=T_\infty, C(x,y,0)=C_\infty\\ u(0,y,t)=0, T(0,y,t)=T_\infty, C(0,y,t)=C_\infty\\ u(x,0,t)=u_p, \mathrm{v}(x,0,t)=0, T(x,0,t)=T_w, C(x,0,t)=C_w\\ u(x,\infty,t)=0,\quad T(x,\infty,t)=T_\infty, C(x,\infty,t)=C_\infty\end{array}\right. \quad (5)$$

By introducing the dimensionless parameters

$$\left.\begin{array}{l}X=\frac{xu_p}{\nu}, Y=\frac{yu_p}{\nu}, U=\frac{u}{u_p}, V=\frac{\mathrm{v}}{u_p}, \tau=\frac{tu_p^2}{\nu}\\ \theta=\frac{T-T_\infty}{T_W-T_\infty}, \phi=\frac{C-C_\infty}{C_W-C_\infty}, Gr_T=\frac{\nu g\beta_T(T_W-T_\infty)}{u_p^3}, M=\frac{\nu\sigma B_0^{\,2}}{u_p^2},\\ Gr_C=\frac{\nu g\beta_C(C-C_\infty)}{u_p^3}, K=\frac{\nu k_l}{u_p^2}, \mathrm{Pr}=\frac{\nu}{\alpha}, Sc=\frac{\nu}{D}\end{array}\right\} \quad (6)$$

where M is the magnetic parameter (the square root of Hartmann number), Gr_C is the mass Grashof number, Gr_T is the thermal Grashof number, K is the dimensionless chemical reaction parameter, Pr is the Prandtl number, and Sc is the Schmidt number.

The governing equations (1)-(5) can be rewritten in dimensionless form as follows

$$\frac{\partial U}{\partial X}+\frac{\partial \mathrm{V}}{\partial Y}=0 \quad (7)$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+\mathrm{V}\frac{\partial \mathrm{V}}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+Gr_T\,\theta+Grc\phi-MU \quad (8)$$

$$\frac{\partial \theta}{\partial \tau}+\mathrm{U}\frac{\partial \theta}{\partial X}+\mathrm{V}\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial Y^2} \quad (9)$$

$$\frac{\partial \phi}{\partial \tau}+\mathrm{U}\frac{\partial \phi}{\partial X}+\mathrm{V}\frac{\partial \phi}{\partial Y}=D\,\frac{\partial^2\phi}{\partial Y^2}-K\phi \quad (10)$$

and the boundary conditions are transformed to

$$\left.\begin{array}{l}U(X,Y,0)=\mathrm{V}(X,Y,0)=0,\\ \theta(X,Y,0)=0, \phi(X,Y,0)=0\\ U(0,Y,\tau)=0, \theta(0,Y,\tau)=0, \phi(0,Y,\tau)=0\\ U(X,0,\tau)=1, V(X,0,\tau)=0,\\ \theta(X,0,\tau)=\phi(X,0,\tau)=1\\ U(X,\infty,\tau)=0, \theta(X,\infty,\tau)=0, \phi(X,\infty,\tau)=0\end{array}\right. \quad (11)$$

The mass diffusion equation (10) can be adjusted to represent a destructive reaction if $K > 0$ and generative reaction if $K < 0$.

The physical quantities of fundamental interest of heat and mass transfer are the heat and mass coefficients in terms of Nusselt and Sherwood numbers, respectively. The dimensionless heat and mass transfer coefficients can be expressed as

$$Nux=-X\left(\frac{\partial\theta}{\partial Y}\right)_{wall} \quad (12)$$

$$Shx=-X\left(\frac{\partial\phi}{\partial Y}\right)_{wall} \quad (13)$$

3. SOLUTION METHODOLOGY

The unsteady, non-linear, coupled partial differential equations (7-10) along with their boundary and initial conditions (11) have been solved numerically using an implicit finite-difference technique of Crank–Nicolson The computational domain is considered as a rectangle of width 1 ($X_{\max}$) and length of 8 ($Y_{\max}$), where $Y_{\max}$ corresponds to $Y = \infty$, which lies very well outside the momentum, thermal, and concentration boundary layers. The maximum value of Y was chosen after some preliminary investigations so that the boundary conditions are satisfied with a tolerance of 10^{-5}. After experimenting with a few set of mesh sizes, the mesh sizes have been fixed at the level $\Delta X = 0.01$ and $\Delta Y = 0.05$, with a time step $\Delta\tau = 0.005$ The results obtained using a finer grid do not reveal discernible changes in the predicted heat and mass transfer and flow field. Thus, due to computational cost and accuracy considerations the above mesh size was as the optimal. The iteration continues (ε), in percentage form has been defined as follows

$$\varepsilon=\left|\frac{f^{n+1}{}_{i,j}-f^{n}{}_{i,j}}{f^{n+1}{}_{i,j}}\right|\times 100\%$$

where f stands for U, V, θ and, ϕ and n refers to time and i, j refers to space coordinates. The value of ε is chosen as 10^{-5}.

An examination of data for unsteady solution revealed little or no change in U, V, θ and ϕ after $\tau = 5$ for all computations. Thus, the result for $\tau = 5$ are taken as essentially the steady state values.

In order to verify the accuracy of the present computer code, particular results are compared with those available in the literature. The unsteady results without magnetic field are compared with [6]. Figure 1 displays this comparison, it can be seen that the agreement between the results is excellent. This has established confidence in the numerical results reported in this article.

4. RESULTS AND DISCUSSION

It is difficult to study the influence of all parameters involved in the present problem on the flow and thermal field. Therefore, this study is focused on the effects of the chemical reaction parameter and the magnetic field parameter on the transient velocity, temperature and concentration profiles as well as on the local heat and mass transfer coefficients. The fluid considered in this study is air at 20°C ($Pr = 0.71$).

The effects of magnetic parameter (square of Hartmann number) on the transient velocity, temperature, and concentration profiles are shown in Figs. 2-4. These presented profiles are those at $X = 0.5$. It is observed that the velocity increases with time, reaches a chronological maximum. The time of the maximum value decreases with the magnetic parameter. The presence of the transverse magnetic field produces a resistive force the fluid flow. This force is called the Lorentz force, which leads to slow down the motion of electrically conducting fluid, which tends to increase the temperature and concentration. The time required for reaching the steady state increases with increasing the magnetic field parameter. Also, it is noted that hydrodynamics, thermal and concentration boundary layers thicken gradually with time.

Figures 5-7 display the influence of chemical reaction parameter on the transient velocity, temperature and concentration profiles. It is clear that increasing the chemical reaction parameter tends to decrease the velocity as well as the species concentration. The hydrodynamics and the concentration boundary layer become thin as the reaction parameter increases. However, the influence of reaction parameter on the temperature is insignificant.

Figure 8 illustrates the dimensionless velocity profiles for generative and destructive chemical reactions. It is obvious that, the velocity is increased by the presence of the generative chemical reaction and decreased in destructive chemical reaction.

The effect of magnetic parameter on the time evolution of the local Sherwood and Nusselt numbers are plotted in Figs. 9 and 10, respectively. Both heat mass transfer coefficients have large values at early stages of the double diffusive process, and then it decreases significantly to reach its steady state values. The magnetic field effects are insignificant at early stages of times. Both heat and mass transfer rates are observed to be increasing with the increase of the magnetic parameter.

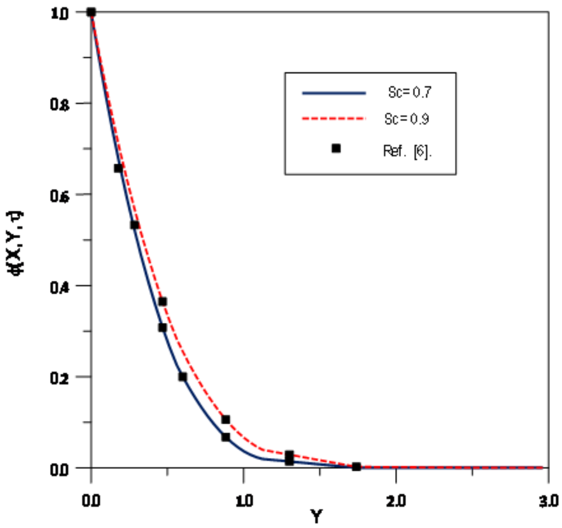

Figure 1. Comparison of dimensionless concentration distributions of the present work and [6] at $\tau = 0.2$.

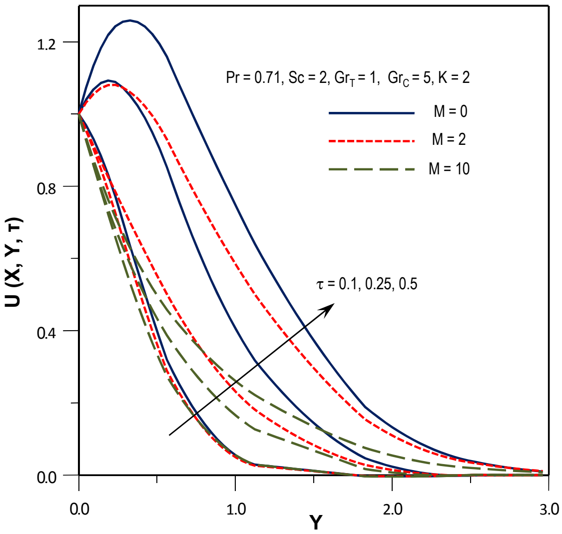

Figure 2. Effect of magnetic parameter on dimensionless velocity profiles

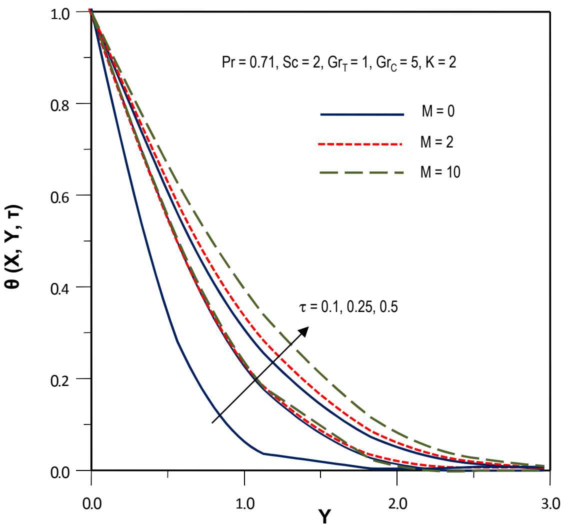

Figure 3. Effect of magnetic parameter on dimensionless temperature distributions

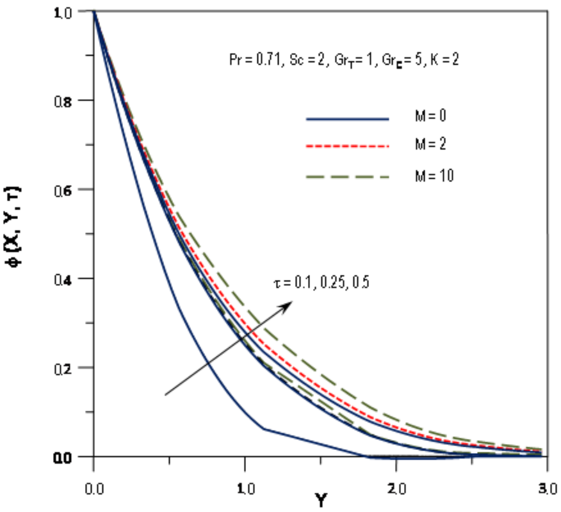

Figure 4. Effect of magnetic parameter on dimensionless concentration distributions

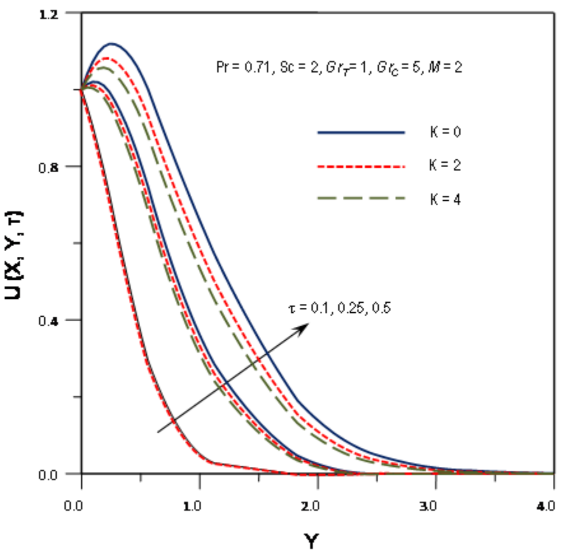

Figure 5. Effect of chemical reaction parameter on dimensionless velocity profiles

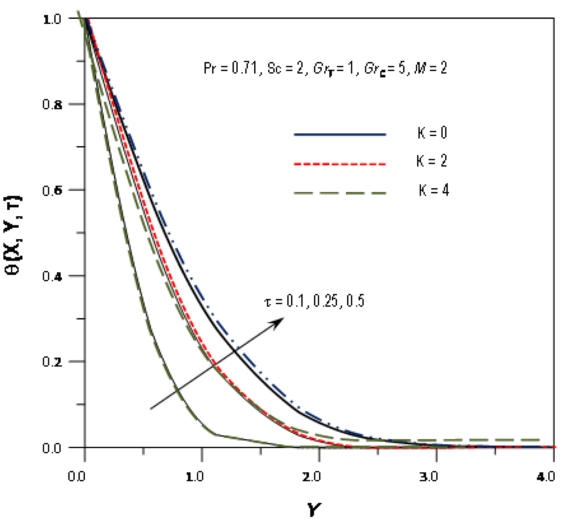

Figure 6. Effect of chemical reaction parameter on dimensionless temperature distributions

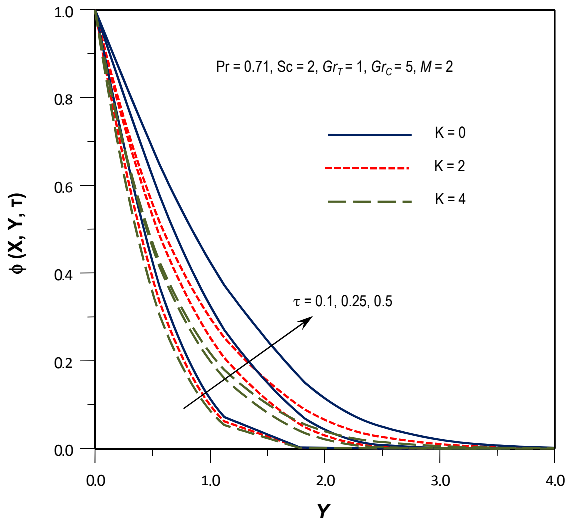

Figure 7. Effect of chemical parameter on dimensionless concentration distributions

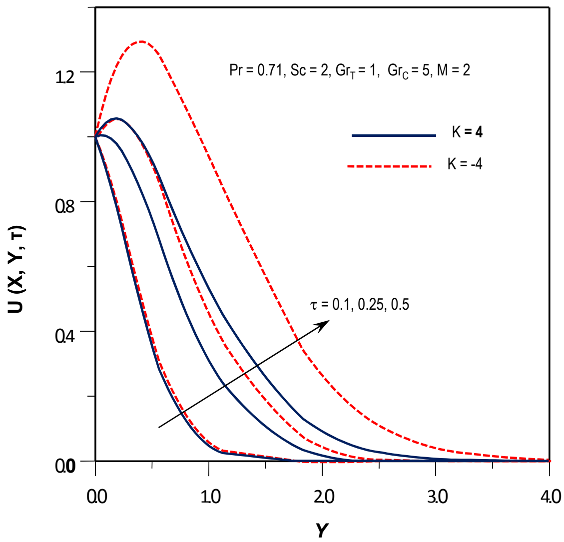

Figure 8. Dimensionless velocity profiles for generative and destructive chemical reactions

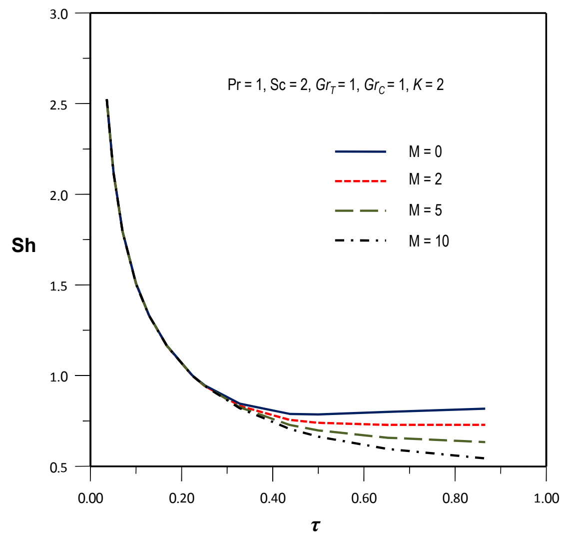

Figure 9. Effect of magnetic parameter on local Sherwood number (Sh) evolution with time

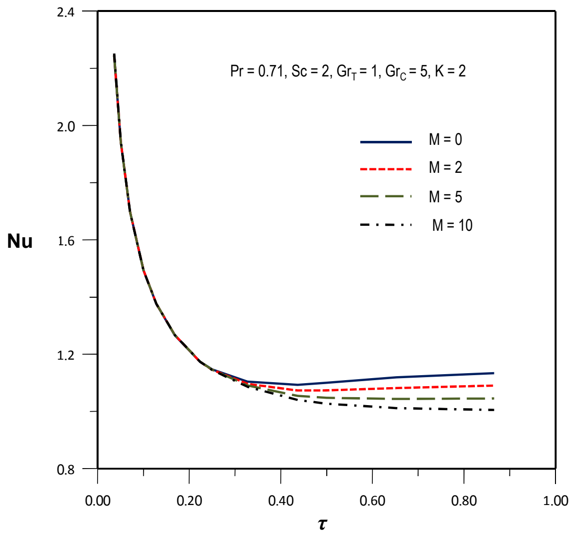

Figure 10. Effect of magnetic parameter on Local Nusselt number (Nu) evolution with time

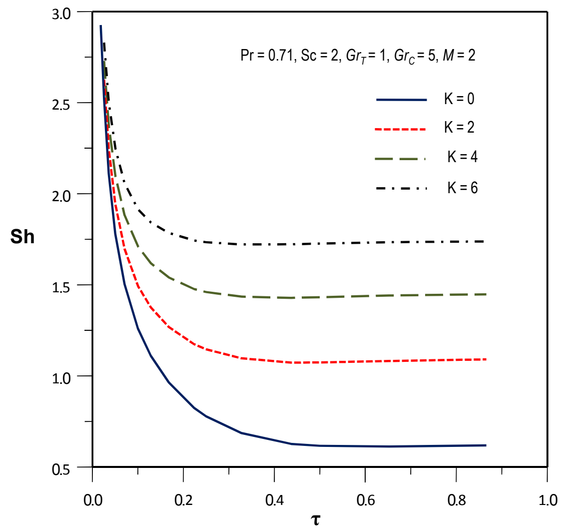

Figure 11. Effect of chemical parameter on local Sherwood number (Sh) evolution with time

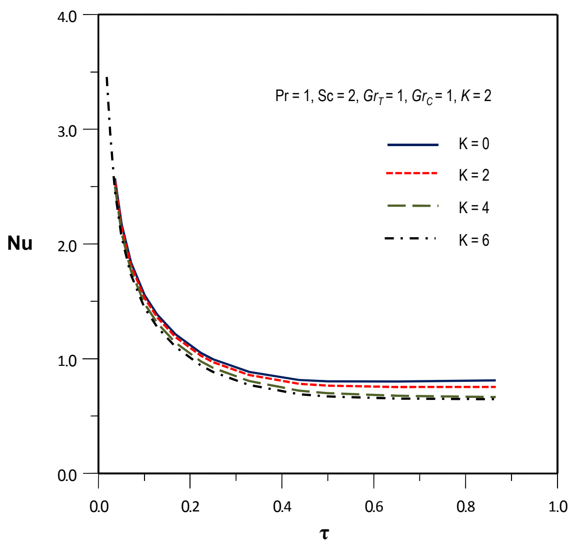

Figure 12. Effect of chemical parameter on local Nusselt number (Nu) evolution with time

Figures 11 and 12 illustrate the effect of chemical reaction parameter on the time evolution of the local Sherwood and Nusselt numbers, respectively. The mass transfer rate increases with increasing the chemical reaction parameter. The effect of chemical reaction on mass transfer coefficient grows with time. However, the reaction parameter has a more significant effect on the Sherwood number than it does on Nusselt number.

5. CONCLUSIONS

In this study, a numerical analysis is presented to investigate the influence of magnetic field on the unsteady combined heat and mass flow of an electrically conducting fluid by free convection along an impulsively started semi- infinite vertical plate taking into account a homogeneous chemical reaction of first order. The transient, nonlinear and coupled governing equations are solved by the finite-difference technique. Transient velocity, temperature, and concentration profiles as well as local Nusselt and Sherwood numbers are presented and analyzed. The fundamental parameters found to affect the problem under consideration are the chemical reaction parameter and the magnetic field parameter. It is found that, the velocity as well as concentration decreases with increasing the chemical reaction parameter. Whereas, increasing the magnetic parameter leads to increase the temperature and the concentration and decrease the fluid velocity. Additionally, the velocity is increased by the presence of the generative chemical reaction and decreased in destructive chemical.

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