THE EFFECT OF SELF-SIMILAR TRAFFIC ON THE PERFORMANCE OF
PLAYTHROUGH RING NETWORKS

by

Stephane Joseph Wantou-Siantou

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George Mason University
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1. **Subject of Interest and Motivation for the Research**

   Many simulation models have been developed for the Playthrough protocol in order to study the performance of networks using the Playthrough protocol [SS82], [SSWS82], [Sil85], [Sil86], [SW87], [Hen98]. None of those models have taken the self-similar nature of traffic into account. Instead, previous models assumed that message interarrival times were exponentially distributed and that message lengths were geometrically distributed. However, it has been shown that traffic in Local Area Networks exhibits self-similar behavior, which is inconsistent with the assumption of exponentially distributed interarrivals and geometrically distributed message lengths. Self-similarity is a topic that has been the subject of considerable attention in recent years [PW00a]. It refers to the fact that traffic displays similar characteristics at different time scales [PW00b].

   Studies have shown that the reliable transfer of files drawn from heavy-tailed distributions, such as the Weibull and the Pareto distribution, can cause self-similarity in network traffic [PKC96a], [PKC96b], [PKC97], [PKC00]. This causal explanation of self-similar traffic is appealing because there is ample evidence that file systems possess heavy-tailed file size distributions [AW96], [CB96], [Irl93], [PF94].

   The objective of this work is to study the performance of the Playthrough ring with heavy-tailed message lengths. Precisely, our goal is find an analytical model for the waiting time using Weibull message lengths, to compare the results given by the analitical model with results given by simulations and to the results given by previous models using geometric message lengths.

2. **Nature of the Study and a Concise Statement of the Proposed Problem**

   Our study is a combination of performance evaluation, survey, and theory study. The objective of this work is to find an analytical model for the waiting time of the Playthrough ring using Weibull message lengths. We will then compare the results given by the analytical model using Weibull message lengths with results given by simulations using Weibull message lengths. We will also compare the results obtained under a Weibull message length assumption to the results obtained under the geometric message length assumption.
3. PLAYTHROUGH Ring

3.1 Description

A PLAYTHROUGH network is a type of ring network. The simplest and most widespread type of ring network is the token ring. On a token ring, a token is passed from station to station. When a station holds the token it may transmit a single packet to any other station on the ring. All other stations just pass the packet around the ring. The source removes the packet upon its return. When a station holding the token is done transmitting a packet it passes the token to the next station. Only one station at a time can transmit a packet on a token ring.

PLAYTHROUGH ring differs from token ring in that it allows stations to transfer packets concurrently [Hen98]. Multiple messages can be transmitted simultaneously over contiguous, but nonoverlapping ring segments. A station participates in a PLAYTHROUGH ring through its ring interface unit (RIU). A station sends and receives user data from the attached component (AC) or host. A station uses its RIU transmitter (respectively receiver) to send (respectively...
receive) data to (respectively from) other stations on the ring. There are 3 classes of messages in a PLAYTHROUGH ring with varying priorities. From the lowest to highest priority these classes are data messages, the synchronizing token (called GO), and update control messages. Update control messages include START/STOP messages and acknowledgement messages. Data messages are user data to be sent at a particular node. The token and update control messages are used to implement the PLAYTHROUGH protocol. A message with a higher priority can preempt, i.e. insert itself ahead of, a message with a lower priority.

At any given time, the status of a station in a PLAYTHROUGH ring can either be a source, a destination, a bridge or an idle node. Messages travel from a source to a destination and nodes between a source and a destination are called bridges. An idle node is neither a source nor a destination nor a bridge. In a ring made of N nodes, stations are number consecutively from 0 to N-1, and a node position number is called its address. Each station’s RIU maintains two registers, a status register and a range register. A status register is used to keep the status of a station, either source, destination, bridge, or idle. A given node keeps another node in its range register if all nodes between the two are idle; in other words, if the link between the two nodes is free. A station can send a data message to any node within the range specified by its range register. At ring start up, all stations are idle and every station’s range register includes all other stations.
3.2 Operation

A control frame bracketed by a leading FLAG and a trailing GO symbol is circulated perpetually around the ring, and each station receives it at regular time intervals.

When a station has a message to transmit, it first checks its range register to ensure that the destination of the message is in its range. If it is, the station waits for the control frame. When it arrives, the station inserts an update control message in front of GO and behind the leading flag F. GO is interrupted by the insertion of the update control message. The update control message includes the node number of the station (the source of the message), the destination node number, and a START command to initiate a virtual circuit between the source and the destination.
GO symbol preceded by the control message continues to circulate around the ring until it reaches the destination station, updating status and range registers of nodes on its path and establishing a virtual circuit between the source and the destination. At the destination, the START control message is changed into an acknowledgement message and continues to travel around the ring back to the source, updating registers on its path.

![Diagram of a ring network with stations inserting a START message in a control frame.](image)

Figure 3: Station i inserting a START message in control frame.

When the acknowledged START control message arrives back at the source, the source removes the control message and continues sending the data message characters to the destination right after GO departs the station.
The destination station is in charge of removing the data message from the ring when it receives it. After all characters of the data message have been transmitted, the source node waits for the return of GO. When it arrives, the source station transmits a STOP control message to inform all downstream nodes that it is releasing the virtual path dedicated to the data transfer. When the STOP control message arrives at the destination, it is changed into an acknowledgement message and sent back to source, updating registers on its path. Upon receipt of the STOP control message, the source removes it from the ring.

Figure 4: Destination removal of data messages.
In our study of the PLAYTHROUGH ring, the performance parameters that are of interest to us are the waiting time and the service time. The waiting time or queuing time is measured from the time a message arrives at a particular node until the time the RIU of the node receives the GO and inserts a START message to begin the transmission of the message. The service time is measured from the insertion of the START message until the return of the STOP message to the source for removal.
3.3 Advantages of PLAYTHROUGH ring

PLAYTHROUGH ring is very suitable for multimedia applications for several reasons. One of them is fairness: stations access the medium in a round-robin fashion. Another reason is bounded delay: A station can predict the maximum amount of time for the control frame to return to the station. In addition the fact that PLAYTHROUGH ring allows for multiple message transfer allows for high throughput.

3.4 Disadvantage of PLAYTHROUGH ring

Packets whose source or destination is on the path of another transmission face starvation, wherein the packet is blocked from transmission by the transmission from another station. Blocking duration can be substantial if the packet being transmitted by another station is very large.

4. Self-similarity and heavy-tailed distributions

4.1 What is Self-Similarity?

A self-similar phenomenon displays structural similarities across a wide range of timescales. Objects that possess the self-similar property are sometimes referred to fractals. Traffic that is bursty across a wide range of times scales is described as being self-similar. The traffic measure in question can be packet interarrival, throughput, etc. Self-similar traffic is characterized by high variability at all time scales. Recent examinations have shown that Internet traffic is highly variable over a wide range of time scales [PF95]. The variability over wide time scales implies that bursts do not average out over long enough time scales. The fact that burst do not average out implies that, unlike voice networks, the internet cannot be engineered to reduce ill-effects such as packet loss below any desired threshold [FF01].
4.2 Heavy-tailed Distributions

Studies have shown that distributions with heavy tails such as the Weibull, Pareto, and lognormal distribution yield better models for file size in file systems [AW96], [CB96], [Irl93], [PF94]. Studies have also shown that the TCP transfer of files drawn from heavy-tailed distributions, such as the Weibull and the Pareto distribution causes self-similarity in network traffic [PKC96a], [PKC96b], [PKC97], [PKC00].

A random variable $X$ is said to be heavy-tailed if:

$$1 - F(x) = P(X > x) \sim \frac{1}{x^\alpha} \quad \text{as} \quad x \to \infty$$

Heavy tailed distributions have high or even infinite variance and therefore show extreme variability over all time scales.
More intuitively heavy tailed distributions show a wide range of values, including very large ones, even if almost all values are small.

In our study of PLAYTHROUGH ring we will use the Weibull distribution. The Weibull distribution is a 2-parameter distribution whose probability distribution function is given by:

\[ F(x) = 1 - e^{-\frac{x^c}{a}} \]

Its probability density function is given by:

\[ f(x) = \frac{c}{a} \left(\frac{x}{a}\right)^{c-1} e^{-\frac{x^c}{a}} \]

And its mean and second moment are given by:

\[ E[X] = a\Gamma\left(1 + \frac{1}{c}\right) \]
\[ E[X^2] = a^2\Gamma\left(1 + \frac{2}{c}\right) \]

In our analysis, \( a = 6667 \) and \( c = 0.6 \), so that \( E[X] = 10000 \).

5. Performance Analysis

5.1 Traffic Assumptions:

We assume that the ring has \( N \) stations. We also assume uniform symmetric traffic on the ring, that is, every station has the same arrival rate and stations send packets to other stations with the same probability. If we assume that the aggregate arrival rate to the ring is \( \lambda \), if we designate by \( \lambda_{i,j} \) the arrival rate of packets sourced by station \( i \) destined for station \( j \), \( j \) stations away from station \( i \), and if we designate by and \( f_{i,j} \), the probability that a packet sourced by station \( i \) is destined for station \( j \), we have:

\[ \lambda_{i,j} = \frac{\lambda}{N(N-1)}, \quad f_{i,j} = \frac{1}{N-1}, \quad 1 \leq i \leq N, \quad 1 \leq j \leq N-1 \]

Where \( \oplus \) refers to addition modulo \( N \).

The total arrival rate to station \( i \) is:

\[ \lambda_i = \sum_{j=1}^{N-1} \lambda_{i,j} = \frac{\lambda}{N} \]

We have, \( \lambda = \sum_{i=1}^{N} \lambda_i \).

Packet lengths are given in character units. All time units are given in character-time units. A character-time is the time it takes for one character to go through a station or node.

We assume that packets arriving at a station are served on a first-in, first-out basis (FIFO).
5.2 Minipacket Statistics

Let:
- \( k_R \): delay caused by a station receiver; we assume \( k_R = 2 \).
- \( k_T \): delay caused by a station transmitter; we assume \( k_T = 2 \).
- \( \tau \): round trip time for empty control frame; we assume \( \tau = (k_T + k_R)N \).
- \( M \): random variable representing message length.
- \( k_G \): size of empty control frame = 2.
- \( n_G \): random variable representing the number of times GO “plays through” source node \( i \) during a data message transfer.

We assume that \( M \) follows a Weibull distribution with parameters \( a \) and \( c \). The distribution of \( M \) is given by:

\[
F_M(x) = 1 - e^{-\frac{x}{a}}
\]

A station \( i \) transmitting a packet of size \( M \) requires \( n_G = (M)/m \), \( \mu = \tau - k_G \),

minipackets to transmit its payload. If we assume that \( M \) follows a Weibull distribution, \( n_G \) follows a Weibull distribution with mean \( E[M]/m \) and second moment \( E[M^2]/m \).

\[
E[M] = a \Gamma(1 + \frac{1}{c})
\]

\[
E[M^2] = a^2 \Gamma^2(1 + \frac{2}{c})
\]

where the \( \Gamma \)-function is given by:

\[
\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt
\]

\[
E[n_G] = \frac{a \Gamma(1 + \frac{1}{c})}{m}
\]

\[
E[n_G^2] = \frac{a^2 \Gamma^2(1 + \frac{2}{c})}{m}
\]

5.3 Control Frame Interruption Rate

Packets sourced by station \( i \) and destined by station \( i \oplus j \) arrive to station \( i \) with arrival rate \( \lambda_{i,i\oplus j} \). Each packet generates two control messages, a \( \text{START} \) and a \( \text{STOP} \) control message. Those control messages interrupt GO when they are inserted in the control frame at a rate \( \nu \) that we approximate by:

\[
\nu = \sum_{i=1}^N \sum_{j=1}^{N-1} 2\lambda_{i,i\oplus j}
\]
4.4 Control Frame Round Trip Time

In the absence of control message interruption, the control frame round trip time is \( \tau \). Each control frame interruption adds an additional \( k_D \) character-times (we assume that \( k_D=3 \)) to the duration of the control frame round trip time. If we designate by \( \eta \) the number of times update control messages interrupt GO, the control frame round trip time is: \( S_{GO}=\tau+\eta k_D \).

We assume that the number of control frame interruptions follows a Poisson distribution with rate \( \nu \) calculated above.

In order to find the moments of \( S_{GO} \), we find the Laplace transform of \( S_{GO} \):

\[
S_{GO}^*(s) = E[e^{-sS_{GO}}] = E[e^{-s(\tau+\eta k_D)}]
\]

\[
S_{GO}^*(s) = E[E[e^{-S_{GO}} | S_{GO}]] = e^{-s\tau} E[E[e^{-\eta k_D} | S_{GO}]] = e^{-s\tau} E[\sum_{k=0}^{\infty} P[\eta = k | S_{GO}] e^{-sk_D}]
\]

\[
= e^{-s\tau} E\left[\sum_{k=0}^{\infty} \frac{(\nu S_{GO})^k}{k!} e^{-\nu S_{GO}} e^{-sk_D}\right] = e^{-s\tau} E\left[\sum_{k=0}^{\infty} \frac{(\nu S_{GO} e^{-sk_D})^k}{k!} e^{-\nu S_{GO}} \right] = e^{-s\tau} E[\exp(\nu S_{GO} e^{-sk_D})].\exp(-\nu S_{GO})\]

\[
= e^{-s\tau} E[\exp(-\nu S_{GO} (1-e^{-sk_D}))]
\]

Let: \( f(s) = e^{-s\tau} \), \( g(s) = S_{GO}^*(s) = E[e^{-sS_{GO}}] \), \( \omega(s) = \nu (1-e^{-sk_D}) \)

We have: \( S_{GO}^*(s) = f(s)g(\omega(s)) \)

\( \omega(0) = 0; \quad g(0) = S_{GO}^*(0) = 1; \)

\[
\omega^*(s) = \nu(k_D) e^{-sk_D} \quad \omega^*(0) = \nu(k_D)
\]

\[
\omega^*(s) = \nu(k_D)^2 e^{-sk_D} \quad \omega^*(0) = \nu(k_D)^2
\]

\[
f^*(s) = (-\tau) e^{-s\tau} \quad f^*(0) = (-\tau)
\]

\[
f^*(s) = (-\tau)^2 e^{-s\tau} \quad f^*(0) = (-\tau)^2
\]

\[
g^*(s) = E[-S_{GO} e^{-S_{GO}^*}] \quad g^*(0) = -E[S_{GO}]
\]

\[
g^*(s) = E[S_{GO}^2 e^{-S_{GO}^*}] \quad g^*(0) = E[S_{GO}^2]
\]

The first two moments of \( S_{GO} \) are found by calculating the first two derivatives of \( S_{GO}^* \) and evaluating them at 0. To do this, we find the derivatives of \( g(s) \).

\[
S_{GO}^*(s) = f(s)g(\omega(s))
\]

\[
S_{GO}^{**}(s) = f^*(s)g(\omega(s)) + f(s)\omega^*(s)g^*(\omega(s))
\]
\[ S_{GO}(0) = f'(0)g(\omega(0)) + f(0)\omega'(0)g'(\omega(0)) = (-\tau) + \nu(k_d)(-S_{GO}^*(0)) \]

\[ S_{GO}^*(0) = \frac{-\tau}{1 - \tau \nu k_d} \]

\[ E[S_{GO}] = \frac{\tau}{1 - \nu k_d} \]

\[ S_{GO}^*(s) = f'^*(s)g(\omega(s)) + f'(s)\omega'(s)g'(\omega(s)) + f'(s)\omega'(s)g'(\omega(s)) + f(s)\omega'(s)g'(\omega(s)) + f(s)[\omega'(s)]^2 g'(\omega(s)) \]

\[ S_{GO}(s) = f'^*(s)g(\omega(s)) + 2f'(s)\omega'(s)g'(\omega(s)) + f(s)\omega'(s)g'(\omega(s)) + f(s)[\omega'(s)]^2 g'(\omega(s)) \]

\[ S_{GO}(0) = f'^*(0)g(\omega(0)) + 2f'(0)\omega'(0)g'(\omega(0)) + f(0)\omega'(0)g'(\omega(0)) + f(0)[\omega'(0)]^2 g'(\omega(0)) \]

\[ S_{GO}(0) = E[S_{GO}^2] = \tau^2 + 2(-\tau)\nu(k_d)(-E[S_{GO}]) - \nu(k_d)^2(-E[S_{GO}]) + \nu(k_d)^2 E[S_{GO}] \]

\[ (1 - [\nu(k_d)]^2) E[S_{GO}^2] = \tau^2 + 2(-\tau)\nu(k_d)(-E[S_{GO}]) - \nu(k_d)^2(-E[S_{GO}]) \]

\[ E[S_{GO}^2] = \frac{1}{(1 - [\nu(k_d)]^2)} (\tau^2 + 2(-\tau)\nu(k_d)(-\frac{\tau}{1 - \nu k_d}) - \nu(k_d)^2(-\frac{\tau}{1 - \nu k_d})) \]

\[ E[S_{GO}^2] = \frac{1}{(1 - [\nu(k_d)]^2)(1 + [\nu(k_d)])} (\tau + 2\nu(k_d)(\frac{\tau}{1 - \nu k_d}) + \nu(k_d)^2(-\frac{1}{1 - \nu k_d})) \]

\[ E[S_{GO}^2] = \frac{\tau}{(1 - [\nu(k_d)]^2)(1 + [\nu(k_d)])} \]

**5.5 Service Time**

A message of length M requires \( n_G \) minipackets (and therefore \( n_G \) round trip times) to be transmitted to its destination. The service time is the time for all the \( n_G \) minipackets to be transmitted and for the STOP control message (of \( k_D \) characters) to be transmitted and to return.

The service time is given by: \( S = n_G S_{GO} + k_D \)

For the sake of simplifying the analysis, we assume that \( S_{GO} = E[S_{GO}] \), that is \( S_{GO} \) is constant.

\[ E[S] = E[n_G]E[S_{GO}] + k_D \]

\[ E[S^2] = E[n_G^2]E[S_{GO}] + k_D \]
5.6 Waiting Time

The queue at station \(i\) is modeled as an \(M/G/1\) queue. The Waiting time for packets at station \(i\) is given by:

\[
E[W_i] = \frac{\lambda_i E[S^2]}{2(1 - \rho)} \quad [\text{Kle75}]
\]

where, \(\rho = \lambda_i E[S]\)

![Mean Waiting Time vs Offered Load](image)

Figure 7: Mean waiting time vs. offered load for a 10-station ring.
APPENDIX: PROBABILITY DISTRIBUTIONS

1. Definition of Several Probability Distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability Density p(x)</th>
<th>Cumulative Probability F(x)</th>
<th>Mean E(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>( \exp(-x/\lambda) / \lambda )</td>
<td>1 - ( \exp(-x/\lambda) )</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Weibull</td>
<td>( \frac{1}{a} \left( \frac{x}{a} \right)^{a-1} e^{-(x/a)^a} )</td>
<td>1 - ( e^{-(x/a)^a} )</td>
<td>( a \Gamma \left( \frac{1}{c} + 1 \right) )*</td>
</tr>
<tr>
<td>Pareto</td>
<td>( \frac{ak^a}{x^{a+1}} )</td>
<td>1 - ( \left( \frac{k}{x} \right)^a )</td>
<td>( \frac{ak}{a-1} ) if ( a &gt; 1 )</td>
</tr>
<tr>
<td>(k &gt; 0, a &gt; 0; x ≥ k)</td>
<td></td>
<td></td>
<td>( \infty ) if ( a ≤ 1 )</td>
</tr>
<tr>
<td>Lognormal with trans</td>
<td>( \frac{1}{x\sqrt{2\pi}\sigma} e^{\left[\log(x-\zeta)/\sigma\right]^2/2\sigma^2} )</td>
<td>No closed form</td>
<td>( e^{\zeta^2/2} )</td>
</tr>
<tr>
<td>Geometric</td>
<td>( pq^{x-1} )</td>
<td></td>
<td>( 1/p )</td>
</tr>
</tbody>
</table>

\* \( \Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \)

2. Derivations of the Expressions for the Exponential and the Weibull Distributions Used in the Simulators

For the exponential distribution [Gre72]:

The cumulative distribution of the exponential distribution in the table above is given by:

\[ F(y) = 1 - e^{-\lambda y}, \text{ where } \lambda = 1/\rho \quad (1) \]

Since \( y \) ranges from 0 to \( \infty \), \( F \) varies from 0 to 1. So we can substitute \( RN \) for \( F \) and then solve equation (1) for \( y \) to obtain:

\[ y = -\frac{1}{\lambda} \ln(1 - RN) \quad (2) \]

Letting \( m = 1/\lambda \), and \( FN = y \), we can write:

\[ FN = -m \times \ln(1 - RN) \]
For the Weibull distribution:

The cumulative distribution of the Weibull distribution in the table above is given by:

$$F(y) = 1 - e^{-(\lambda y)^a}, \text{ where } \lambda = 1/a \quad (4)$$

Since $y$ ranges from 0 to $\infty$, $F$ varies from 0 to 1. So we can substitute $RN$ for $F$ and then solve equation (4) for $y$ to obtain:

$$y = -\frac{1}{\lambda} [\ln(1 - RN)]^\frac{1}{a} \quad (5)$$

Letting $m=1/\lambda$, and $FN=y$, we can write:

$$FN = -m \times [\ln(1 - RN)]^\frac{1}{a} \quad (6)$$

Gamma Function

$$\Gamma(x) = \int_0^\infty t^{x-1} \exp(-t) dt$$
REFERENCES


