Assessment and the graphics calculator*

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Abstract: As graphics calculators become more accessible, issues of assessment will become more important. The main reason for this is that graphics calculators provide students with significant mathematical capabilities, some of which are described here. Some of the issues that need to be addressed in formal assessment are identified. These issues include the appropriateness of some traditional kinds of questions, the significance of programming, the relevance of differences between calculators, the desirability of assessing efficient calculator use and the need to consider what students should be expected to record in an examination.

A major advantage of the graphics calculator over more complex forms of technology, and particular the microcomputer, is the possibility that students will be able to have one with them at all times, including in particular when completing assignments and during formal examinations. A second major advantage is the diminishing cost of purchasing a calculator. It has been suggested recently that these two advantages together render the use of graphics calculators much more attractive to schools than computers (Kissane, 1995). This paper identifies some areas that require attention by mathematics educators in the likely event that graphics calculators will be permitted for student use in formal examinations.

Background

The purpose of assessment is to find out what students know, understand and can do. There are a variety of opportunities to find out such things in educational settings, and for teachers to react appropriately to what they find. In many respects, informal means of assessment, relying on careful observation and listening in classrooms, provide the most helpful forms of information for teachers and students, since the information can be acted upon straight away and it has been obtained in a fairly natural setting. In recent years, there has been a welcome climate of experimentation in Australia with new forms of assessment to overcome some of the well-known defects of formal traditional examinations (Clarke, 1988). However, in practice, formal assessment settings, particularly tests and examinations, continue to retain major significance to students and their teachers, and consequently are the focus of this paper.

At present, graphics calculators are not yet widespread in secondary schools and universities in Australia. In part for this reason, external examination authorities are uneasy about permitting their use in examinations. An analysis of some equity and other aspects of this unease is given in Kissane, Bradley & Kemp (1994). Of course, official prohibitions send a clear signal to teachers and their students about the level of importance of acquiring and learning to use well a graphics calculator, which in turn serves to impede the penetration of calculators into senior mathematics, in a well-understood cycle. As David Clarke has observed:

> Irrespective of the purposes we might have for assessment, it is through our assessment that we communicate most clearly to students which activities and learning outcomes we value. (1988, p 1)

A similar situation existed some years ago with regard to scientific calculators, which were treated with some suspicion in schools until their use was permitted in examinations. By the time of the AAMT Calculator Policy (1986), scientific calculators were widespread in schools and generally expected to be used by students in assessment situations, including formal examinations.

Adjustments to curricula and their associated examinations were needed to accommodate the use of scientific calculators, but in hindsight, these were surprisingly small. A reconsideration of the importance of computational arithmetic was necessary, some parts of the curriculum became less important (such as reading tables and using logarithms for calculation), and other parts of the curriculum increased in significance (such as iterative solution of equations).

It is now time to undertake similar analyses with respect to graphics calculators and examinations, and this paper is a tentative first step in that direction. The paper is based in part on our experiences, reported in Bradley, Kemp & Kissane (1994) and will be augmented at the Conference by some practical examples, some observations from abroad and our experiences with a project based at Murdoch University and funded by CAUT (the Committee for the Advancement of University Teaching). Analyses of relationships between graphics calculators and assessment are likely to be helped by foreign developments. For example, in the United States, the *NCTM Standards* published in 1989 were based on an assumption that all students had access to a graphics calculator from the ninth grade onwards. From 1995, the Advanced Placement examinations in calculus conducted by The College Board (and used to give advanced college credit to high school students) expect that students will have a graphics calculator with them in the examination. In the United Kingdom, all Examination Boards now permit the use of graphics calculators at A-level, and some innovative curricula are being developed on the assumption that students have graphics calculator access. It is not yet clear how closely classroom realities match these kinds of assumptions, but individual student calculator access seems inevitable, with the main variable being time.

When graphics calculators are regarded as an integral part of the toolkit of mathematics students, and curricula are adjusted to accommodate this, it will also be important to assess how well students can use such technology, which is not quite the same as assessing how well they understand the mathematics itself. It might well be argued that learning to make thoughtful use of a graphics calculator is an important outcome in itself of a senior school mathematics education, and so attention ought be paid to assessing how well it has been achieved.

Capabilities of graphics calculators

An important first step in considering the implications of graphics calculators for assessment is to understand what mathematical capabilities they provide students with, and the ways in which they need to be operated to take advantage of these capabilities. A recent AAMT publication (Andrews & Kissane, 1994) provides a great deal of detailed information of this kind; also, Kissane (1994a, 1994b) describes and analyses some recent developments. In this section, a very brief summary of typical current capabilities is given.

Computation	All graphics calculators operate well as computational devices, with typically many more capabilities than scientific calculators. All use natural algebraic syntax, unlike many scientific calculators. It is unnecessary for students to have a scientific calculator as well as a graphics calculator.
Graphing	Graphs of several functions can be drawn, using cartesian, polar or parametric definitions. Once drawn, graphs can be manipulated (by zooming in or out) to study closely particular aspects, such as extreme values, points of intersection and asymptotic behaviour. Graphs of derivative functions can also be drawn.
Finding zeroes and solving equations	Zeroes of elementary functions and thus solutions of elementary equations in a single variable can be obtained by graphical or numerical iteration, or by a direct numerical iterative procedure (such as a ‘solve’ command). At least one model routinely gives complex solutions to quadratic and cubic equations, when necessary.
Statistical analysis	Both numerical procedures and graphical procedures (such as histograms, box plots and bivariate regression lines) are available. Unlike scientific calculators, data sets are stored in the calculator for editing, transformation and later use.
Simultaneous equations	Systems of simultaneous linear equations can be solved directly or through the use of elementary row operations or by means of matrix arithmetic. Numerical solutions to nonlinear systems are available through graphical means.
Sequences and series	Numerical sequences can be defined either recursively or explicitly, and individual terms accessed. Series can be obtained by commands to add successive terms of a sequence, or by more direct procedures.
Differentiation and integration	Numerical derivatives of functions at a point, to a specified level of accuracy are available, and definite integrals can be evaluated to a specified level of accuracy. On some models, graphical versions of derivatives and integrals are also available.

Evaluation of functions	Values of functions at a point are available once the function is defined. On some models, composite functions, transformations of functions and tabular representations of functions are accessible.
Complex arithmetic	Most calculators handle arithmetic computations with complex numbers.
Programmability	All graphics calculators are programmable, so that facilities not readily available on a particular model can usually be added by writing a small program for the purpose. The program can be written by teachers and inserted into the calculator by students, who then need to know only how to operate the program.

Of course, calculator capabilities are constantly changing with the development of new models and considerable competition in the marketplace. There are differences between models, too, as suggested in some places above. However, it seems prudent to regard this set of capabilities as the likely minimum to which students with a graphics calculator would have access. It is of interest that The College Board, in specifying graphics calculators for Advanced Placement examinations, gave *minimum* rather than maximum capabilities (to do with graphing, numerical solution of equations, differentiation and integration), and published calculator programs that students could use to augment the capabilities of particular models .

Examination choices and their consequences

There are many kinds of mathematics examination questions that have been used in the past but which would require careful thinking in the light of graphics calculators. In some cases, allowing students to use a graphics calculator would undermine the capacity of a particular assessment task to provide insightful information about student thinking. Rather, the question would become much easier for students who knew how to use their graphics calculator than it was for students without a graphics calculator. Broadly speaking, there would seem to be three choices regarding graphics calculators and examinations: they can be completely banned, as seems typically to be the case in Australia at present; they can be permitted for use in examinations at the discretion of the student, with minimal control over which graphics calculators can be used and for which questions; or they can be permitted into examinations with some kinds of restrictions.

A complete ban on the use of graphics calculators in assessment, especially in formal examinations, appears to resolve problems of differential student access and differences between the capabilities of calculator models. However, an important principle of validity is compromised by such an approach: an increasing number of students will learn to make good use of graphics calculators for learning and for practical uses of mathematics, but will be artificially prevented from integrating this learning in assessment. Another negative consequences is that some (possibly many) students will be denied access to this powerful technology if it is not permitted in assessment, since it will be regarded as an unnecessary frill. Finally, such a response will impede the appropriate adaptation of curricula to technological change.

The unrestricted approach overcomes the negative consequences for the student and the curriculum, although at first sight it appears to introduce problems of inequity of access and calculator capabilities. Some of the latter problems can be diminished by ensuring that all calculators have a minimum suite of mathematical capabilities, in the way that the College Board has, through the use of programs to supplement the less powerful calculators. Problems of differential access within a school or university need to be addressed by ensuring that enough calculators are available for students to use for long enough and often enough so that students who are unable or unwilling to purchase a graphics calculator can nonetheless attain proficiency in using one. A judicious combination of careful resource allocation and appropriate hire schemes is needed, and is increasingly feasible as the retail price of graphics calculators diminishes.

One problem associated with unrestricted graphics calculator access in assessment is that students will then be able to use the calculators with any assessment task, including those for which they were not intended. Although a test designed for students with graphics calculators may (indeed, should) contain items for which it is expected that students will make good use of the technology, there will also be questions for which it is intended that students will not use a calculator at all. Students may subvert the intention of a question by using their calculator when it is expected that they will rely on analytical methods, or they may get themselves into unnecessary trouble by using a calculator when it is not of assistance. This problem needs to be anticipated in the design of the assessment tasks, a difficult task which requires a thorough understanding of calculator operations.

Another problem is that we have not yet developed a good sense of what should be *recorded* by students who respond to a mathematical situation using a calculator. It would seem inappropriate for students to respond to an equation with nothing more than a numerical solution or to a definite integral with a numerical value, although each is technically correct. Without technological help, students have been expected to ‘show their working’, which is of diagnostic value and has allowed for the allocation of partial credit. It has usually been necessary for students to write down some things anyway (since, at the senior school level, most mathematical tasks are too complex to be performed mentally), so generally we have had little trouble persuading students of the importance of displaying the details of their efforts. However, there is an element of putting the cart before the horse to insist on such recording *only* to permit partial grading in an examination or an assignment, and asking students to write down keystrokes does not seem a sensible solution. Of course, in some situations, we would expect that a calculator would *contribute* to a convincing mathematical argument, and would still expect students to justify their results with an adequate written record.

Until we learn more about some of these sorts of problems, a middle path of allowing some graphics calculator use in assessment, but with some restrictions, may be the best course of action. One relatively easy kind of restriction is to design one part of assessment to include the use of a graphics calculator, while another part excludes them. Annotations on particular questions can serve this kind of purpose, although it is probably better to *design* the questions so that technology is not involved. One way of doing this is to focus on general cases, rather than specific cases, such as evaluating indefinite integrals rather than definite integrals, or solving for x an equation like $ax^3 + b = c$, rather than $2x^3 + 1 = 4$. To find out important things about student use of technology, it may be desirable to include questions directly concerned with calculator use, such as the interpretation of calculator screens for a particular mathematical purpose.

Conclusion

The realities of mathematics education demand that we come to terms with the curriculum and assessment implications of the graphics calculator. A good deal of work remains to be done to find acceptable solutions to some of the problems that are raised in this paper. Some of this work can be analytical in nature, but it is also important to learn from practical experience.

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