MODELING OF SWASH PLATE AXIAL PISTON PUMPS WITH CONICAL CYLINDER BLOCKS


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Abstract

Electrically controlled swash plate axial piston pumps with conical cylinder blocks are recently used in the industry in view of their superior performance. Several studies have been carried out to study the characteristics of such novel pump mechanism. In these studies, partial mathematical modeling is conducted relevant to the points discussed.

In the present study, a comprehensive pump mathematical model is developed and experimentally validated. The model could be used as a design tool in order to fully exploit the advantages of the new design.

Keywords

Axial Piston Pump, Swash Plate, Model

Introduction

Modeling of the conventional swash plate axial piston pumps was the focus of many studies in the last decade. In 1994, Kalafetis and Costopoulos [1] studied the static and dynamic characteristics of a standard variable geometric volume swash plate pump with pressure regulator. They proposed a mathematical model and checked its validity. Their findings revealed that operating conditions are very crucial for the pump dynamic performance. Manring and Jonson [2] in 1996 carried out modeling and design of a variable geometric volume axial piston pump. They discussed the effect of factors such as the volume of the actuator and discharge fluid line, the controller gain and system leakage on the pump performance. In 2001, X.Zhang, J. Cho, S. S. Nair and N.D. Manring [3] introduced a reduced model of a damping swash plate pumping mechanism. They validated the proposed reduced order model by comparing it to a complete nonlinear simulation of the pump dynamics over the entire range of operating conditions.

Swash plate pumps with conical cylinder blocks have generated some interest recently in view of their improved static and dynamic characteristics. Kassem and Bahr [4] studied theoretically the lateral moment acting on the swash plate. They showed that the lateral moment is periodic and contains an average and 27 harmonics, first of them has a frequency equal pump rotation speed. They showed also that the average value of the lateral moment increases linearly with the increase in the load pressure and/or the decrease in the swash plate inclination angle. Bahr, Svoboda and Bhat [5] carried out a vibration analysis of the pump. Results shows that control valve compensate the effect of the lateral moment on the swash plate.

In the present paper, a comprehensive pump model is developed. An experimental setup is built to measure the pump step response and validate the model. Very good agreement was found between the simulation and measured results.

Pump Mathematical Model

Swash plate pump mechanism that has conical shaped cylinder block is shown in Fig.1. The figure shows also the main dimensions of the pump mechanism.
Using the conical cylinder block, each piston is doing complex kinematical general space motion. Due to this complexity, we followed the principle of frame transformation in order to find the coordinates of the piston during one complete cycle. As shown in Fig.2, the initial frame of reference $X_0Y_0Z_0$ is chosen such that its origin coincides with the swash plate pivoting point $O_0$. The axis $Z_0$ coincides with the pump driving shaft axis and $Y_0$ coincides with the axis around which the swash plate is swinging. Five steps of frame transformation, starting from $X_0Y_0Z_0$ and ending by $X_5Y_5Z_5$, are then carried out to get the components of the position vector $[r_{5k}]_0$ that pointing to the kth piston spherical head center relative to the initial frame of reference. Position vector $[r_{5k}]_0$ is then given by

$$
[r_{5k}]_0 = \begin{bmatrix}
x_{5k} \\
y_{5k} \\
z_{5k}
\end{bmatrix} = \begin{bmatrix}
L_{3k}\cos\theta_k \sin\beta + R_2 \cos\theta_k \\
L_{3k}\sin\theta_k \sin\beta + R_2 \sin\theta_k \\
-L_{3k}\cos\beta + L_1
\end{bmatrix}
$$ (1)

The piston spherical head center is guided by a plane parallel to the swash plate surface and passing through the swash plate pivoting point, the kth piston displacement relative to its cylinder is given by [4]

$$
s_k = L_3 - (L_1 / \cos\beta)
$$ (2)

where

$$
L_3 = 0.5(D_2 \cos \theta_0 \tan \alpha + L_3)
$$

$$
\theta_k = \omega_k + 2\pi(k-1)/N, \quad \beta = \tan^{-1}0.5(D_1 - D_2)/L_2.
$$

Applying the continuity equation to the piston chamber control volume (C.V.) shown schematically in Fig.3, piston chamber variation is governed by the following equation

$$
Q_{ak} + A_k \dot{\phi}_k = Q_{ak} + p_k / R_k + V_{ak} \dot{\theta}_k / B
$$ (3)

where:

$$
Q_{ak} = C_d A_{ak} \left( \frac{2|p_k - p_d|}{\rho} \right)^{1/2} \text{sgn} (p_k - p_d),
$$

$$
Q_{ak} = C_d A_{ak} \left( \frac{2|p_k - p_d|}{\rho} \right)^{1/2} \text{sgn} (p_k - p_d),
$$

$$
V_{ck} = A_p (0.5L_c - s_k) + V_o.
$$

Pressure force due to single piston that acts on the swash plate along $Z_k$ axis is denoted by $[F_{jk}]_0$, and has, relative to the 5th frame of reference, the components of 0, 0 and $\text{ApPk}$. This force can be redefined, relative to the inertial frame of reference, as follows

$$
[F_{jk}]_0 = A_p p_k \begin{bmatrix}
-\cos\theta_k \sin\beta \\
-\sin\theta_k \sin\beta \\
\cos\beta
\end{bmatrix}
$$ (4)

Fig.2 Frames of reference

Fig.3 Piston chamber parameters and variables

As shown in Fig.4, position vector $[r_{ck}]_0$ that points to kth piston center of mass, is given by

$$
[r_{ck}]_0 = [r_{5k}]_0 + [r_{ck}]_6
$$ (5)

where vector $[r_{ck}]_6$ is the fixed magnitude position vector $[r_{ck}]_5$ as redefined relative to 6th frame of reference that has an origin coincided with the 5th frame origin, and axes parallel to those of the initial frame of reference. It can be calculated as follows

$$
[r_{ck}]_6 = \begin{bmatrix}
-\cos\theta_s \sin\beta \\
-\sin\theta_s \sin\beta \\
\cos\beta
\end{bmatrix}
$$ (6)

Hence, the piston absolute acceleration, due to its general space motion relative to the initial frame of reference, is given by

$$
[a_{ck}]_0 = \ddot{\theta}_s \times [r_{ck}]_6 + \dot{\omega} \times (\omega \times [r_{ck}]_6) + 2\dot{\omega} \times \dot{[\phi]}_6 + \dot{[\phi]}_6
$$ (7)

The first two terms are due to the piston rotation while the last two terms are due to its translation. The first term is the piston tangential acceleration that equals zero due to constant pump driving speed. The second term is the piston normal (centripetal) acceleration. The third term is the piston coriolis acceleration. The last term is the second derivative of the position vector $[r_{ck}]_5$. 
The total force by which the \( k \)th piston acts on the swash plate is the algebraic sum of the pressure force and piston inertia and is given by

\[
[F_k]_0 = [F_{pk}]_0 + m_p [a_{pk}]_0 \tag{8}
\]

This force has the components \( F_{xk}, \ F_{yk} \) and \( F_{zk} \) that cause accordingly moment relative to the inertial frame of reference, given by

\[
[M_k]_0 = [r_{sk}]_0 \times [F_k]_0 =
\begin{bmatrix}
i_0 & j_0 & k_0 \\
x_{sk} & y_{sk} & z_{sk} \\
x_{sk} & y_{sk} & F_{zk}
\end{bmatrix} \tag{9}
\]

The total moment components acting on the swash plate can be found by superimposing \([M_k]_0\) components due to the whole piston group. The total moment has thus the components \( M_x, \ M_y \) and \( M_z \). The moment \( M_y \) tends to change the swash plate inclination angle and should be overcome by the control system. The moment \( M_z \) equals the pump driving torque, while the resultant of the two moment components \( M_x \) and \( M_z \) acts on the swash plate bearing system.

Figures 5 and 6 show the layout and schematic representation of the pump control unit, respectively. It consists of a proportional valve drives a symmetric hydraulic cylinder, which is mechanically attached to the swash plate. When the valve solenoid receives a control current \( i_v \), a force proportional to this current; namely \( k_{ij_v} \), acts on the valve spool and causes it to move. Motion of the valve spool is described by the following second ordered differential equation.

\[
m \ddot{x} + f \ddot{x} + k x = k_{ij_v} \tag{10}
\]

Flow rate through the valve control gaps are

\[
Q_a = C_d w(s_{v_{max}} - s_v) \sqrt{\frac{2(p_{c1} - p_t)}{p}} \text{sgn}(p_{c1} - p_t) \tag{11}
\]

\[
Q_b = C_d w s_v \sqrt{\frac{2(p_1 - p_{c1})}{p}} \text{sgn}(p_1 - p_{c1}) \tag{12}
\]

\[
Q_c = C_d w s_v (s_{v_{max}} - s_v) \sqrt{\frac{2(p_1 - p_{c1})}{p}} \text{sgn}(p_1 - p_{c1}) \tag{13}
\]

\[
Q_d = C_d w s_v \sqrt{\frac{2(p_{c1} - p_t)}{p}} \text{sgn}(p_{c1} - p_t) \tag{14}
\]
Applying the continuity equation to the side chambers of the control piston resulted in

\[
p_{c1} = \frac{B}{V} \int (Q_b - Q_a - A_{cp} \Delta p_a - p_c / R_c) dt,
\]

\[
p_{c2} = \frac{B}{V} \int (Q_c - Q_d + A_{cp} \Delta p_d + p_c / R_c) dt
\]

where \( V_{c1} = V_c + A_{cp} x_{cp} \) and \( V_{c2} = V_c - A_{cp} x_{cp} \).

The pressure difference on the two sides of the control piston drives the swash plate to a new equilibrium position. Considering the moment acting on the swash plate due to piston group, swiveling of the swash plate is described by the following second order differential equation

\[
I_{\alpha} \dddot{\alpha} = (p_{c1} - p_{c2}) A_{cp} r_s + M_y - f_{\mu} \Delta \mu - k_\alpha (\alpha + 0.09)
\]

Pump Performance Investigation

Software based on Matlab was developed in order to solve the foregoing equations numerically and simulate the pump dynamic performance. Figure 7 shows the layout of the simulation program main blocks. It contains subsystems represents valve and pump dynamics, in addition to the electronic controllers. An arithmetic unit is added for setting the maximum limits of the pump working condition that should be respected by the control scheme. This unit and the controller represent the function of the electronic control card shown in Fig. 5.

Figure 7 shows the control scheme used currently with the pump. It consists of double negative feedback control loops. The inner feedback control loop is used for the accurate positioning of the proportional valve spool using PID controller. The outer feedback control loop is used to control the swash plate angle using only PD controller due to the control cylinder integration effect. The empirical-analytical method, “Ultimate Sensitivity” introduced by Ziegler–Nichols [6] is used to parameterize the system controller.

Successive step inputs are fed to the program. Pump response is simulated when the swash plate inclination angle increases from initial zero position to different percentages of its maximum value and vice versa. An experimental setup is built to measure the pump step response. It consists of a hydraulic test bed interfaced with real time control and data acquisition system. Simulation and measured results are compared in Figs. 8 and 9. The evident agreement between the theoretical and experimental results fairly validates the developed model.

![Fig.7 Block diagram of the simulation program](image)

![Fig.8 Swash plate time response when the swash plate inclination angle increases](image)

![Fig.9 Swash plate time response when the swash plate inclination angle decreases](image)
Conclusion

A comprehensive mathematical model has been developed to describe the dynamic performance of swash plate axial piston pumps with conical cylinder blocks. The model could be also used for the conventional cylindrical cylinder block by considering zero cone angle.

The model is used to simulate the dynamic performance of electrically controlled pumps. Very good agreement was found between the simulation and the experimental results.

References


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_d(A_s)$</td>
<td>Delivery (Suction) port area</td>
<td></td>
<td>m²</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Area of the control piston</td>
<td>$8.1 \times 10^{-4}$</td>
<td>m²</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Piston cross-section area</td>
<td>$2.27 \times 10^{-4}$</td>
<td>m²</td>
</tr>
<tr>
<td>$\dot{a}_{k0}$</td>
<td>$k^{th}$ piston absolute acceleration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>Effective bulk modulus</td>
<td>$1 \times 10^9$</td>
<td>Pa</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Coefficient of discharge</td>
<td>0.611</td>
<td></td>
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<tr>
<td>$D_1/R_1$</td>
<td>Pitch circle diameter/radius of the cylinder arrangement at the base of the cylinder block</td>
<td>0.07175/0.0359</td>
<td>m</td>
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<td>$D_2/R_2$</td>
<td>Pitch circle diameter/radius of the cylinder arrangement at the top of the cylinder block</td>
<td>0.0602/0.0301</td>
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<tr>
<td>$F_{xk,yk,zk}$</td>
<td>Components of the resultant force acting on the swash plate due to the $k^{th}$ piston</td>
<td></td>
<td>N</td>
</tr>
<tr>
<td>$[F_{pk0}]/[F_{k0}]$</td>
<td>Pressure / resultant force vector due to single piston</td>
<td></td>
<td></td>
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<tr>
<td>$f_v$</td>
<td>Proportional valve viscous friction coefficient</td>
<td>90</td>
<td>N.s/m</td>
</tr>
<tr>
<td>$f_a$</td>
<td>Control piston viscous friction coefficient</td>
<td>1.5</td>
<td>Nm/(rad/s)</td>
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<tr>
<td>$I_e$</td>
<td>Equivalent moment of inertia of the swash plate</td>
<td>0.0039</td>
<td>kg.m²</td>
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<tr>
<td>$i_s$</td>
<td>Proportional valve solenoid current</td>
<td>A</td>
<td></td>
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<tr>
<td>$k$</td>
<td>Piston number in the piston group arrangement</td>
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<td></td>
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<tr>
<td>$k_s$</td>
<td>Proportional solenoid force-current constant</td>
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<td>N/A</td>
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<tr>
<td>$k_v$</td>
<td>Proportional valve spring stiffness</td>
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<td>N/m</td>
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<tr>
<td>$k_{v0}$</td>
<td>Control piston spring stiffness</td>
<td>72</td>
<td>N/m.rad</td>
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<tr>
<td>$L_1/L_2$</td>
<td>Lengths, referred in Fig.1</td>
<td>0.7666/0.6661</td>
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<tr>
<td>$L_3$</td>
<td>Variable length</td>
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<td>$L_c$</td>
<td>Cylinder length</td>
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<td>$L_p$</td>
<td>Piston length</td>
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<td>m</td>
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<tr>
<td>$m_p$</td>
<td>Piston mass</td>
<td>0.118</td>
<td>kg</td>
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<tr>
<td>$m_v$</td>
<td>Proportional valve spool mass</td>
<td>0.1</td>
<td>kg</td>
</tr>
<tr>
<td>$[M_{k0}]$</td>
<td>Moment acting on the swash plate in vector form</td>
<td></td>
<td>N.m</td>
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<tr>
<td>$M_{x,y,z}$</td>
<td>Components of the resultant moment acting on the swash plate</td>
<td></td>
<td>N.m</td>
</tr>
</tbody>
</table>
\( N \) Number of pistons 9
\( p_c \) Pressure difference across the control piston Pa
\( p_{c1,2} \) Pressure at the two sides of the control piston Pa
\( p_d \) Pump delivery pressure Pa
\( p_k \) Piston chamber pressure Pa
\( p_s \) Pump suction pressure \( 0.05 \times 10^4 \) Pa
\( p_t \) Tank line pressure \( 1 \times 10^7 \) Pa
\( p_r \) Control pressure Pa
\( Q_{d,b,c,d} \) Flow rates through proportional valve ports \( \text{m}^3/\text{s} \)
\( Q_d \) Delivery flow of one cylinder \( \text{m}^3/\text{s} \)
\( Q_s \) Suction flow rate into one cylinder \( \text{m}^3/\text{s} \)
\( R_k \) Leakage resistance \( 1 \times 10^{13} \) \( \text{Pa}/(\text{m}^3/\text{s}) \)
\([r_{ek}]/[r_{ek}]_0\) Position vector of the \( k \)th piston center of mass relative to inertial \( /6 \)th frame of reference
\([r_{sk}]/[r_{sk}]_0\) Position vector of the \( k \)th piston' spherical head center relative to the inertial frame of reference
\( r_s \) Radius of swash plate swinging \( 0.055 \) m
\( s_k \) Piston displacement m
\( s_{v(max)} \) Proportional valve spool displacement (maximum) \( 0.001 \) m
\( t \) Time \( \text{s} \)
\( V_{c1,2} \) Control volume on the two sides of the control piston \( 13 \times 10^6 \) \( \text{m}^3 \)
\( V_{ci} \) Initial control volume \( 1 \times 10^6 \) \( \text{m}^3 \)
\( V_{ck} \) Instantaneous cylinder volume of the \( k \)th piston \( 1 \times 10^6 \) \( \text{m}^3 \)
\( V_o \) Additional piston chamber volume \( 4.8 \times 10^{-3} \) m
\( w \) Proportional valve area factor \( 3 \)
\( x_{sp \text{ max}}, x_{sp \text{ max}} \) Control piston displacement (minimum, maximum) \( (0, 0.015) \) m
\( x_{sk}, y_{sk}, z_{sk} \) Cartesian coordinates of piston spherical head center relative to the inertial frame of reference
\( \dot{x}_{sk}, \dot{y}_{sk}, \dot{z}_{sk} \) Velocity of \( k \)th piston, control piston, valve spool \( \text{m/s} \)
\( \dot{\alpha} \) Swash plate angular velocity \( \text{s}^{-1} \)
\( \alpha_{(sp)}(e) \) Swash plate angle of inclination (set point value) (error)
\( \beta \) Cylinder block cone angle 5 deg
\( \theta_k \) Angular position of the \( k \)th piston
\( \rho \) Oil density 850 kg/m\(^3\)
\( \omega/\omega^* \) Pump angular velocity/acceleration \( \text{s}^{-2}/\text{s}^{-2} \)