Event-Event Correlation in Seismicity and Aging of Aftershocks

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A recent discovery of the aging phenomenon and scaling law in complex seismic time series is reported. The results suggest that the mechanism of aftershocks is governed by glassy dynamics.

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Seismicity is a field-theoretical phenomenon. Energy released by each earthquake can be regarded as a field amplitude defined at a discrete spacetime point. In contrast to a familiar field theory such as electromagnetic theory, however, both amplitudes and locations are intrinsically probabilistic. Accordingly, seismicity is characterized by remarkably rich phenomenology, which is of extreme interest from the viewpoint of statistical mechanics of complex systems/phenomena.

There are two celebrated empirical laws known in seismology. One is the Gutenberg-Richter law [1]. It relates the cumulative frequency of earthquakes, $n(>M)$, to magnitude M as follows:

$$\log n\,(>M) = A - bM\,, \tag{1}$$

where A and b are constants, and, in particular, $b \approx 0.9$. Since magnitude is the logarithm of released energy, the frequency obeys a power law with respect to energy. This power-law nature makes it difficult or even meaningless to statistically distinguish earthquakes by their values of magnitude since there exist no typical scales for energy. The other one is the Omori law [2, 3]. It states that the rate of the frequency $N(t)$ of aftershocks following a main shock occurred at time $t = 0$ obeys

$$\frac{dN(t)}{dt} = \frac{A}{(1+t/\tau)^p}\,, \tag{2}$$

where p, A and, τ are positive constants and, in particular, the exponent p is considered to vary p=0.8~1.5 (a careful data analysis, however, indicates a wider range: p=0.5~2.5). The Gutenberg-Richter and Omori laws are statistical laws concerning long time scale and medium time scale properties of seismicity, respectively.

An important point of seismicity, which seems less noticed, is that correlation of two successive events is strong, no matter how large their spatial separation is. In fact, there is an observation [4] that an earthquake can be triggered by a foregoing one, which is more than 1000 km away. This implies that the correlation length is enormously large, indicating a strong similarity to phase transitions and critical phenomena. Therefore, it is not appropriate to put spatial windows in analysis of seismicity, in general, and whole data in a relevant area (ideally the whole globe, though still not satisfactorily available) should be treated based on the nonreductionistic standpoint.

In this article, we succinctly explain our recent discovery [5] of a novel and certainly remarkable feature of earthquake aftershocks: aging and scaling. Combined with the physical nature of main shocks and associated aftershocks, the results suggest the existence of a profound similarity between (yet unknown) aftershock mechanism and glassy dynamics.

According to other results we obtained [6, 7], there are definite statistical laws for 3-dimensional distance between the hypocenters and time interval (calm time, or interoccurrence time) between two successive earthquakes. This fact as well as the above-mentioned long-range correlation implies that two successive events are indivisibly interrelated. To mathematically formulate event-event correlation, we have proposed to employ occurring time of the nth event, t_n, associated with a chosen initial event at $t = t_0$, as a fundamental random variable. Then, we have defined the correlation function between the mth and nth events as follows:

$$C(m,n) = \frac{\langle t_m t_n\rangle - \langle t_m\rangle\langle t_n\rangle}{\sqrt{\sigma_m^2\sigma_n^2}}\,, \tag{3}$$

where $\langle t_m\rangle = (1/M)\sum_{k=0}^{M-1} t_{m+k}$, $\langle t_m t_n\rangle =$

$(1/M)\sum_{k=0}^{M-1} t_{m+k}\, t_{n+k}$, and $\sigma_m^2 = <t_m^2> - <t_n>^2$. We notice that, compared to the ordinary autocorrelation function of a dynamical system, the role of time here is played by the event-numbering label, $n = 0, 1, 2, \ldots$. In this article, it is referred to as "event time" (somewhat analogous to the concept of "natural time" [8, 9]). Thus, the symbol $< \bullet >$ denotes the event-time average.

If a time series is stationary in terms of event time, then $C(m, n)$ is a function only of the difference, m–n, whereas in a nonstationary time series it individually depends on m and n. In the latter case, it is useful to introduce "waiting event time", n_w, as $C(n+n_w, n_w)$ in Eq. (3). Clearly, $C(n+n_w, n_w) = 1$ for $n = 0$. $C(n+n_w, n_w)$ has complicated n_w-dependence, in general. However, a special class of nonstationary time series may exhibit specific n_w-dependence, which is the aging phenomenon discussed below.

We have analyzed the seismic data provided by the Southern California Earthquake Data Center (http://www.scecdc.scec.org/catalogs.html). Through extensive analyses, we have confirmed that the following results are universal, though we here present only one case. The main shock selected is the Landers Earthquake with M7.3 occurred at 11:57:34.10 on June 28, 1992 (34°12.01′N latitude, 116°26.20′W longitude, and 0.97 km in depth). This event has, in fact, been followed by a swarm of aftershocks.

We have identified the time interval termed the *Omori regime*, in which the events following the main shock obey the Omori law. This has been done in the following way. First, integrate Eq. (2) to obtain $N(t) = [A\tau/(1-p)]\times[(1+t/\tau)^{1-p}-(1+t_0/\tau)^{1-p}]$ ($p\neq 1$), $A\tau \ln[(t+\tau)/(t_0+\tau)]$ ($p=1$), where $N(t_0)$ is set equal to zero with t_0 being the occurring time of the main shock. $N(t)$ contains three parameters, A, τ and p. Fix a time interval $[t_0, t_0 + T]$ after the main shock and use the method of least squares for the data and the model to perform the parameter search. Then, change T, do the parameter search each time, and find the value of T^*, with which the best-fit regression is achieved.

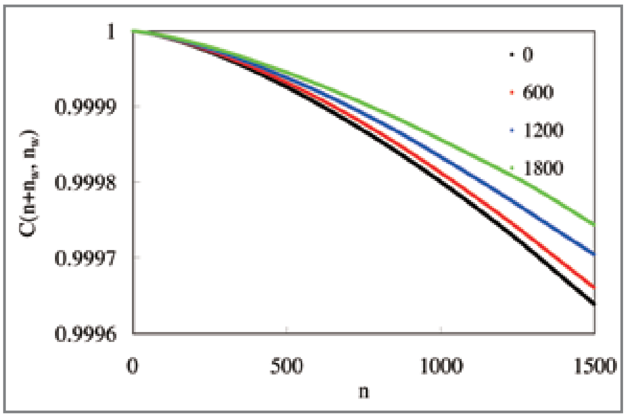

Fig. 1: The plots of the event-event correlation function with respect to event time for several values of waiting event time inside the Omori regime. All quantities are dimensionless.

The Omori regime $[t_0, t_0 + T^*]$ is identified in this way. In the above procedure, no windows should be put on spatial region as well as magnitude, due to the reason mentioned earlier. Thus, the Omori regime associated with the Landers Earthquake has been determined: between 11:57:34.10 on June 28, 1992 and 07:07:44.27 on August 15, 1992. The values of the set of the parameters are $A = 7.58\times10^{-3}[s^{-1}]$, $\tau=1.80\times10^{6}[s]$ $p=0.80$, with the maximum value of the correlation coefficient $\rho_{max}=0.999923$ for the data and the model.

In Fig. 1, we present the plots of the correlation functions inside the above-mentioned Omori regime with respect to event time for several values of waiting event time. First 15000 events are analyzed, that is, M=15000 in Eq. (3). One can clearly observe the aging phenomenon with respect to waiting event time: that is, the smaller waiting event time is, the faster correlation decays. This implies that the Omori regime has very peculiar nonstationarity in terms of event time.

To see the situation better, in Fig. 2, we also present the plots of the correlation function outside the Omori regime: the period between 04:43:56.70 on June 14, 2000 and 14:21:09.95 on April 10, 2001, which is chosen after the end of the Omori regime associated with the main shock, the Hector

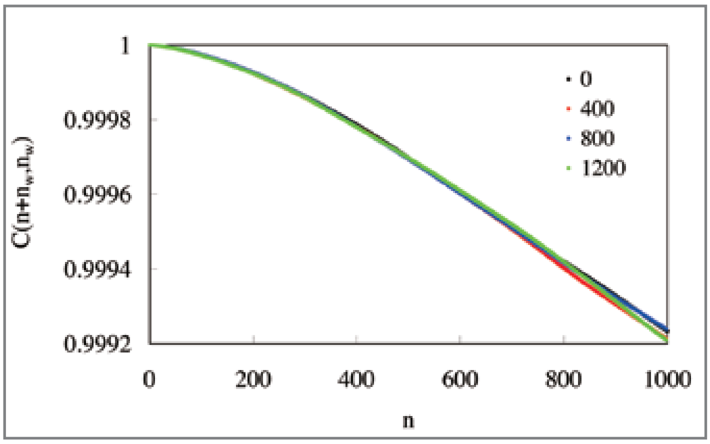

Fig. 2: The event-event correlation function with respect to event time for several values of waiting event time outside the Omori regime. All quantities are dimensionless.

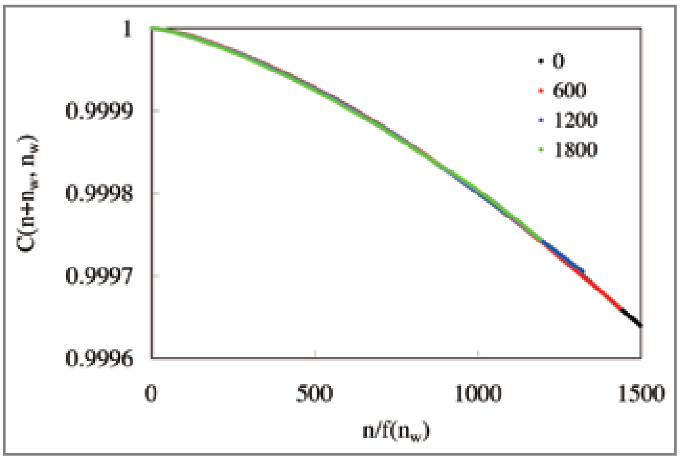

Fig. 3: The data collapse of the event-event correlation function in Fig. 1 by rescaling of event time. All quantities are dimensionless.

Mine Earthquake with M7.1 at 09:46:44.13 on October 16, 1999. (In Ref. [5], we have ascertained that the Omori regime of this main shock exhibits the same trend of the aging phenomenon as that of the Landers Earthquake discussed above.) First 10000 events (i.e., M=10000) are analyzed. In this case, no aging phenomenon is observed, and therefore the part of the seismic time series outside the Omori regime is stationary in terms of event time.

Another important feature of event-event correlation in the Omori regime is existence of the scaling law. For Fig. 1, it is possible to realize the data collapse by rescaling event time with the help of a certain function of waiting event time, $f(n_w)$, which satisfies the obvious initial condition, $f(0)=1$. In fact, such a manipulation transforms Fig. 1 to Fig. 3, showing the scaling law

$$C(n+n_w, n_w) = \tilde{C}(n/f(n_w)) \ , \qquad (4)$$

where $\tilde{C}$ is a scaling function. The functional form of $f(n_w)$ was found to be [5]

$$f(n_w) = a\,(n_w)^{\gamma} + 1, \qquad (5)$$

where a and γ are dimensionless constants, and, in the present case, they are $a = 1.37\times10^{-6}$ and γ=1.62, respectively.

These results characterize the feature of aftershocks in a novel and remarkable manner. It is of crucial importance to notice that a main shock can be considered as a quenching process and the relaxation is very slow (i.e., the power law nature of the Omori law). Combining these facts with the present aging phenomenon and scaling law, we conclude that the mechanism of earthquake aftershocks may be governed by glassy dynamics.

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