STOCK INDEX FUTURES MARKETS: STOCHASTIC VOLATILITY MODELS AND SMILES†

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† This piece of research was partially supported by the Austrian Science Foundation (FWF) under grant SFB#10 (Adaptive Information Systems and Modelling in Economics and Management Science). This paper has benefited from comments by Stewart Hodges, Walter Schachermayer, Friedrich Hubalek, Stephan Pichler, Ole Barndorff-Nielsen, Neil Shephard and attendees at the University of Aarhus mini-symposium on volatility and the Austrian Working Group on Banking and Finance. The author would also like to thank the editor of this journal and the two referees for extremely helpful suggestions for improvements. As always, I am responsible for all remaining errors.

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ABSTRACT

This paper examines whether the inclusion of an appropriate stochastic volatility that captures key distributional and volatility facets of Stock Index Futures is sufficient to explain implied volatility smiles for options on these markets.

Two variants of stochastic volatility models related to the Heston (1993) are considered. These models are differentiated by alternative normal or non-normal processes driving log-price increments. For four stock index futures markets examined, models including a negatively correlated stochastic volatility process with non-normal price innovations perform best within the total sample period and for sub-periods.

Using these optimal stochastic volatility models, prices of European options are determined. Comparisons of simulated and actual options prices for these markets find substantial differences. This suggests that the inclusion of a stochastic volatility process consistent with the objective process alone is insufficient to explain the existence of smiles.

JEL classifications: C15, G13

Keywords: Stochastic Volatility, Normal Inverse Gaussian Distributions, Methods of Moments Estimation, Implied Volatility Smiles.

1. INTRODUCTION

Soon after the publication of the Black-Scholes (1973) paper on option pricing, Black (1975) pointed out that the constant volatility assumption may be incorrect; noting that the volatility may be a function of the underlying price level. Subsequent research has identified the existence of different volatilities implied from option prices for different strike prices and terms to maturity [see Jackwerth and Rubinstein (1996) for a review of this literature]. This effect has been commonly referred to as the implied volatility smile (for options with the same term to expiration) and the term structure of volatility (for options with different terms to expiration).

Broadly speaking, two possible reasons have been proposed to explain these effects. The first approach assumes that market imperfections exist that systematically prevent option prices from taking their true Black-Scholes values. Such market imperfections include the introduction of errors associated with discrete hedging, transactions costs, incomplete markets and heterogeneous market agents with diverse expectations. Alternatively, a number of papers have examined the implications for option prices when the underlying asset price process differs from the lognormal diffusion process and/or the volatility is neither constant [as in Black-Scholes (1973)] nor a deterministic function [as in Merton (1973)]. Mayhew (1995) provides a brief review of both approaches. This research examines the latter hypothesis.

This paper examines the nature of the objective dispersion processes for stock index futures and proposes the inclusion of stochastic volatility to explain these empirical facets of the objective price processes. Once this is done, the implications for option pricing are considered. Previous research has examined the effects of stochastic volatility (in the underlying objective price process) on option pricing [see Johnson & Shanno (1987), Hull & White (1987), Scott (1987), Wiggins (1987), Melino and Turnbull (1990), Stein & Stein (1991) and Heston (1993)]. This paper extends this line of research.

The models considered include non-normality in the underlying log-price process and correlated price and volatility processes. Due to the rich nature of these models, they do not lend themselves to parameterisation using standard methods (such as maximum likelihood methods) and a simulated method of moments approach was utilised. The choice of target empirical moments was done to address most stylised results previously pointed out in the literature. Specifically, higher moments of the return series were consided, as were dynamics of the volatility process and leverage effects. Once suitable parameter values for alternative models were found and a feasible risk-neutral drift adjustment was derived, options prices were determined numerically assuming the drift adjustment was unique[1].

To allow comparisons between the simulated and actual option prices, all prices were re-expressed as standardised implied volatilities and plotted as surfaces. Substantial differences between these surfaces were found for the S&P 500 futures market and this suggests that the inclusion of stochastic volatility for the objective process alone is insufficient to explain the existence of implied volatility smiles. However, consistent divergences between simulated and actual implied volatility surfaces were found across for four markets and this may provide insights into the nature of market imperfections and risk premia. Scott (1987) completed similar research on stock options and found similar results. He suggested that further research should test the effects of stochastic volatility on the pricing of options for a larger sample. This paper achieves this by examination of four stock index futures and options markets for a period encompassing the decade of the 1990s.

The paper is organised as follows: the first part briefly reviews previously presented empirical evidence, which indicates that stock index futures prices do not follow an independent and identically distributed (i.i.d.) Geometric Brownian Motion (GBM) process. Two possible models, which have been proposed in the literature to explain these results, were considered. Seven empirical attributes were selected to capture key aspects of non-normality and inter-dependence. A description of the data sources used for this research follows. After this, both possible models were examined using a simulated method of moments approach to assess their ability to explain the key attributes. Having determined the best model, option prices were estimated consistent with these processes and were then expressed as implied volatilities using the Black (1976) formula. These simulated implied volatility surfaces were compared to the implied volatility surfaces from traded options on the S&P 500 futures markets. Finally, conclusions and suggestions for further research appear.

2. PRICE PROCESSES FOR STOCK INDEX FUTURES UNDER THE 'EMPIRICAL' MEASURE

It is well established that the unconditional return series for individual stocks, stock indices and Stock Index Futures do not conform to the assumptions of an i.i.d. lognormal dispersion process [see Stoll & Whaley (1990)]. Returns for stock index futures usually display excess kurtosis and (for many periods) significantly negative skewness compared to a normal distribution. Returns for futures also display inter-temporal dependence. In the examination of absolute returns for Stock Index markets, significant positive autocorrelations have been found. This result has led Ding, Granger and Engle (1993) to conclude, "It is clear that the S&P 500 stock market return process is not an i.i.d. process" (page 87).

Another line of empirical research has examined the unconditional volatility series directly. A typical approach to better understand the volatility process is to examine the statistical moments of the process. Burghardt and Lane (1990) examined the variability of the unconditional volatility process using a volatility cone approach. We extended this by looking at the sampling properties (restricted to the standard deviation) of the unconditional volatility measured at a 20-day time horizon. Using non-overlapping data, we obtained an average estimate of the unconditional volatility and the standard deviation of this average. Under the assumption that an i.i.d. price process is generating these volatilities, the expected coefficient of variation of the 20-day volatility is known. This measures the variability of volatility for a given time horizon.

However, the time varying dynamics of the variability of volatility as the time horizon of estimation is extended, is of additional interest. A simple log-linear form was chosen to capture these dynamics. The rate of decay in the standard deviation of volatility implies that the maturity structure of historical volatility experiences long-term memory (interdependence). This can be seen as complimentary to long-term memory effects for absolute returns identified by Ding, Granger and Engle (1993). An additional and important feature of these price processes is the leverage effect pointed out by Christie (1982) among others.

While a number of theories have been proposed to explain these results, three alternative hypotheses will be examined here. The Constant Elasticity of Variance (CEV) model of Cox and Ross (1976), a non-GBM price process model [such as the Jump diffusion model proposed by Merton (1976)] and Stochastic Volatility Models were considered. These three models can be nested within a stochastic volatility model. Given the wide range of stochastic volatility models proposed in the literature [see Taylor (1994)], it was not obvious which model to select. As models with correlated processes were considered, the Heston (1993) model was the obvious choice. This model has the additional benefit of having a closed form solution for the pricing of options [see Heston (1993), Bakshi, Cao and Chen (1997) and Bates (2000)]. Variants of the Heston (1993) model proposed by Barndorff-Nielsen (1997) and Barndorff-Nielsen and Shephard (1999) are also considered.

Specifically, the following three models were:

MODEL 1 $$dF = \mu F dt + \sigma F dZ(t) \quad \textbf{(1)}$$

Where Z(t) is a standard Wiener Process and μ and σ are constants, the return series r_t is normally distributed with $r_t = \mu + \sigma Z_t$ and $Z_t \sim$ N(0,1). This is the assumption of the Black (1976) model of i.i.d. Geometric Brownian Motion and will be referred to as GBM.

It is clear that Model 1 is a straw man, given the well-known results that Stock Index futures returns display both non-normality and inter-dependence. However, this can serve as a benchmark for the relative effectiveness of the alternative models and is used to assess the sampling properties of the evaluation approach. Subsequent models will consider stochastic volatility ($\hat{\sigma}$) which will be evaluated in terms of a stochastic variance process ($\sqrt{V}$).

MODEL 2 $$dF = \mu F dt + \hat{\sigma} F dZ_1 \quad \textbf{(2.1)}$$

With the variance process defined by:

$$dV = \kappa(\theta - V)dt + \xi\sqrt{V}dZ_2 \quad \textbf{(2.2)}$$

Where Z_1 and Z_2 are standard Wiener processes with correlation ρ. The term κ indicates the rate of mean reversion of the variance, θ is the long-term variance and ξ indicates the volatility of the variance. The terms V and √V represent the variance and the volatility of the process, respectively. This model will be referred to as SV in this paper.

MODEL 3 $$dF = \mu F dt + \hat{\sigma} F d\mathrm{N} \quad \textbf{(3.1)}$$

With the variance process defined by:

$$dV = k(\theta - V)dt + \xi\sqrt{V}dZ \quad \textbf{(3.2)}$$

where N represents a non-normal price process for the underlying price series and in this research is the Normal Inverse Gaussian distribution (NIG). This model is related to that of

Barndorff-Nielsen (1997) who was the first to propose a stochastic volatility model of this form [subsequently extended by Andersson (1999a)]. This approach extends the findings of Bates (1996, 2000) and Ho, Perraudin and Sørensen (1996), who assumed the volatility process is subordinated in a non-normal price process. In this model, the stochastic variance process is assumed to follow a standard Wiener Processes, Z, with correlation ρ existing between the two processes.[2] When correlated processes are considered, this variant of model 3 is related to the model for incorporating a leverage effect in a stochastic volatility model proposed by Barndorff-Nielsen and Shephard (1999). This model will be referred to a NIGSV in this paper. The CEV model of Cox and Ross (1976) which allows for the inclusion of negative correlations between the two stochastic processes is nested in both Models 2 and 3. This allows the leverage effect to be captured.

3. CHOICE OF ATTRIBUTES AND FITTING PARAMETER VALUES

A key problem in the empirical testing of stochastic volatility models is the estimation of optimal input parameters into the model. Andersen, Chung and Sørensen (1999) provide a good review of the approaches used to parameterise stochastic volatility models. Many of the models considered here do not lend themselves to estimation by traditional maximum likelihood methods. Andersen, Chung and Sørensen (1999) propose the use of the efficient method of moments approach suggested by Gallant and Tauchen (1996). Their primary contributions are to understand the sample properties of this estimator and show that the method is robust in larger sample sizes. Recently, Andersson (1999b) also examined the maximum likelihood estimator of the NIGSV model of Barndorff-Nielsen (1997) and by Monte Carlo simulation demonstrated that a simulated method of moments approach is equally robust. Due to the complexities of our models, it is not clear whether estimation of parameter inputs by maximum likelihood techniques is feasible. Given that these papers have demonstrated that parameter estimation via simulation can be equally robust and this approach may provide a better intuitive understanding of the estimation procedure, a simulated method of moments approach similar in spirit to General Method of Moments (GMM) was utilised.

This research uses a hybrid between the Generalised Method of Moments (GMM) approach of Melino and Turnbull (1990) and the Simulated Method of Moments (SMM) approach of Duffie and Singleton (1993). This approach subjectively selects essential attributes, simulates price processes consistent with the alternative models and assesses the sum of squared errors between simulated and empirical attributes. Alternative parameterisation of the models was examined and optimised. Similar to Andersen, Chung and Sørensen (1999), we investigated the sample properties of this estimator technique (using Model 1). This allowed comparisons to be made between the attributes of each market and the models and conclusions to be drawn regarding the overall fit of each model.

At the heart of this estimation technique is the judicious choice of key attributes. It is crucial that the choice of the attributes jointly considers the relevant elements of empirical interdependence and non-normality and provides a means by which the salient features of the alternative models can be captured. Given that both alternative price process (to GBM) and stochastic volatility models have been proposed to explain excess kurtosis in returns, the unconditional kurtosis for daily returns is a logical attribute to choose. Furthermore, some research has indicated that negative skewness is an important attribute for describing the returns of stock indices and stock index futures. Both Theodossiou (1998) and Harvey and Siddique (1998) chose to examine the skewness in addition to the excess kurtosis. This attribute would provide evidence for the existence of asymmetric jumps and/or leverage effects.

To examine both of these moments, price changes were estimated based upon continuously compounded returns in the standard manner (using log-price increments). Extreme care was taken to assure that returns were estimated using only futures prices with the same expiration date. With this time series of daily returns, the unconditional skewness and kurtosis statistics were determined.

Clearly, given that this research examined two variants of a stochastic volatility model, attributes had to be selected that would allow the salient features of these models to be captured. This required attributes capturing the volatility of volatility parameter, the rate of mean reversion and the correlation between the processes. For this research, the estimation of the standard deviation of the returns was determined using squared returns and these results were annualised (assuming 252 trading days in a year). A number of attributes were selected in order to capture critical dynamics of the volatility process.

The first attribute examines the volatility of volatility. A time series of unconditional volatilities was estimated on a daily basis for a time horizon of 20 days (using non-overlapping data). From this series, the average and standard deviation were estimated. Given widely different levels of these sample statistics, a coefficient of variation statistic was chosen as the key attribute. As a basis for comparison, the coefficient of variation of volatility measured at the 20th lag would be approximately equal to 0.1622 if the underlying return series were independent and identically distributed [1/√(2*19)].

While this measured the variability of volatility at a single time horizon, the time varying dynamics of the standard deviation of volatility could not be captured. This was achieved by the estimation of the coefficient of variation of volatility with time horizons from 20 days to 200 days in 20-day increments.[3] From statistical theory, the expected decay in the standard deviation of a volatility estimate ($\sigma *$) is a square root function of the number of observations used for estimation ($SE = \sigma^{*} / \sqrt{2N}$). Given that the average level of volatility remains constant (which was found empirically for these four markets), the decay in the coefficient of variation should follow the same functional form. If the first volatility observation is at the 20-day horizon, the decay in the standard deviation of volatility from that point forward can be expressed as $\sigma_N \cdot \sqrt{\frac{N}{N+1}}$. In this formula, σ_N is the standard deviation of the volatility at the Nth observation and N+1 is the number of observations in the next time horizon (initially N=20 and N+1=40). The functional form of this decay can be expressed as a power function of the form $SE = \frac{\sigma^{*}}{\sqrt{2}} \times N^{-0.5}$. A linear regression of (the natural logarithms of) the time horizon of estimation regressed upon the levels of the coefficient of variation was used to capture the empirical rate in decay. This can be expressed as:

$$\ln(\hat{\sigma}_N) = \alpha + \beta(\ln(N)). \tag{4.1}$$

From this regression equation, the decay attribute, follows the following exponential form:

$$e^{\alpha + \beta(\ln(N))} \tag{4.2}$$

Observed divergences in the Beta coefficient of equation 4.2 from -0.5 give an indication of the degree of the volatility of volatility persistence observed in the unconditional process. This also provides an attribute to capture the interaction between the rate of mean reversion and volatility of volatility in the stochastic volatility process.

Another salient feature of empirical return volatility series is that subsequent realisations may not be independent. To capture serial correlation in absolute returns, autocorrelation dynamics were examined directly rather than using alternative methods relying on maximum likelihood. This was achieved by examining the autocorrelograms of absolute returns previously employed by Taylor (1986) and Ding, Granger and Engle (1993). This approach has the additional benefit of known sampling properties. Thus, a simple confidence interval test can be used to reject the null hypothesis of independence.

For the purposes of this research, composite measures of the autocorrelations are required. Given that the markets differ in the manner that the autocorrelations decay, the averages of the autocorrelations from lag 1 to 20 and from lag 51 to 70 were both selected. The first average represents the short-term autocorrelation. The medium-term average provides an indication of how quickly the autocorrelations die out. Unfortunately, such measures may no longer have known sampling properties allowing for a simple parametric confidence interval test. Therefore, to assess the sample characteristics of these composite measures, nonparametric confidence intervals were determined via simulation. These two attributes provide additional information relevant to the calibration of a stochastic volatility model, as they capture both short-term and medium-term evidence of volatility clustering.

The final attribute must measure the leverage effect and provide a means for a correlation between the stochastic processes to be captured. There is, however, one problem with the determination of the leverage effect: Even if the volatility is a stationary series, the prices are not. To solve this problem, a new variable was constructed which measures recent price movements and is stationary.[4] This variable is an exponentially weighted return series, which indicates whether recent price movements are relatively high or low. It can be shown that given some exponential weighting scheme: where $W = 1 - \theta$ and $\theta \approx e^{-W\Delta t}$, we can define a new series, ω_i, that can be expressed as:

$$\omega_i = \omega_{i-1} + W(r_{i-1} - \omega_{i-1}) \quad \textbf{(5)}$$

where ω_i is the exponentially weighted price movement, r_i is the daily return and the initial ω_i is set to zero. W represents the weight used in the weighting scheme. This new variable was created and the series of 20-day unconditional volatility were compared. With an arbitrary weight (W) for all markets imposed at 0.03, the correlation between the two variables was estimated.[5] This correlation coefficient serves as an attribute to measure the leverage effect.

With these seven target attributes, the stochastic volatility models were parameterised via simulation. Following the three models proposed previously, price series of 1500 observations were generated consistent with these models. The return and volatility characteristics of these simulated price series were estimated in exactly the same manner as was done for the four stock index futures. The resulting simulated attributes were then compared to the empirical attributes using a sum of squared errors statistic. To reduce scaling impacts due to different levels of the attributes, the squared errors were divided by the standard deviation of the attributes across the four markets[6]. This test statistic can be written as:

$$\min \sum \left(\frac{M_i - X_i}{\sigma_i} \right)^2 \qquad \textbf{(6)}$$

where M_i is the attribute for the stock index futures market, X_i is the attribute of the price series generated by the model and σ_i is the standard deviation of the attributes across all the stock index futures markets for the relevant period of analysis.

Finally, 500 samples of 1500 prices consistent with Model 1 (GBM) were drawn to better understand the sample properties of all the attributes and of the test statistic. This provided a non-parametric estimation of the standard errors of the attributes and allowed test statistics for comparisons between markets and models.

4. DATA SOURCES

The futures markets examined include the S&P 500 and Nikkei 225 traded at the Chicago Mercantile Exchange (CME), the DAX index futures traded at the Deutsche Terminbörse (DTB) and the FTSE 100 futures traded at London International Financial Futures Exchange (LIFFE). Daily closing futures prices were analysed and were obtained directly from the relevant exchange and the period of analysis was restricted to a period in which all four stock index futures were traded. The underlying assets, the time period of analysis and the number of observations used in the analysis appear in Table I. To assess if results are period specific, the data was also split into roughly equal time periods from 1990-1994 and from 1995-1999. The time periods and number of observations in these split periods also appear in Table I.

[Table I appears here]

Given that this portion of the research is empirical in nature, a major effort was made to assure the validity of the data used in the analysis and to verify that the analytic methods employed were correct. This was achieved in a number of ways. Firstly, the futures price series were compared with the options (on the futures) price series for the same days to identify obvious errors in recording either price series. This comparison was achieved by comparing the put-call parity values of the options with the underlying futures prices for every single date in our database (and for all four markets). A screening procedure was imposed: If futures or options prices diverged by more than the normal bid/offer spread (of one tick), the observations were flagged. Once this was done, each price was compared with the original daily price sheets to confirm if a 'keypunch' error had occurred. We discovered that only 1-2%

of the data had such errors. Nevertheless, these errors were of a sufficient magnitude that they did influence the results and therefore required correction.

The most arduous of the data cleaning process was the ongoing examination of the data as results of the analysis were obtained. One reason why four stock index futures and options markets were examined was to allow a cross-sectional comparison. Apart from the benefit of assessing general tendencies across markets, it is also possible to use anomalous results as an additional check on data validity. This assured that the data series employed in this research was as accurate as is humanly possible.

5. ATTRIBUTES FOR INDIVIDUAL MARKETS

For each market, the return statistics for daily returns were determined for the entire period of analysis and for each sub-period. The results of these analyses can be seen in Table II.

[Table II appears here]

The first column describes the market under investigation and indicates the time period examined. The second and third columns present the mean and standard deviation of the return distribution. The fourth and fifth columns present the statistics for the unconditional skewness and these provide a significance level relative to a null hypothesis of normality.[7] All skewness significance statistics that are significantly different from the normal assumption at a 95% level (±1.96) appear in bold type. In the sixth and seventh columns, the unconditional kurtosis statistic and the significance level relative to a null hypothesis of normality appear.[8] When this statistic does not reject the null hypothesis of normality at the 95% level or above, the statistic and the significance levels are in bolded type. The statistic in the eighth column is the Bera-Jarque (BJ) statistic for detecting departures of the data from normality. Under the null hypothesis of normality, the BJ statistic is distributed as χ^2 with 2 degrees of freedom. The critical value at the one-percent level is 9.21. When the BJ statistic exceeds this level, this statistic also appears in bolded type.

For all four markets, the dispersion of returns is not well described by a normal distribution. For many of the four markets, the skewness statistic tends to be significantly different than that of a normal distribution. However, the skewness is neither consistently negative for all markets nor always significant. On the other hand, for all four markets (and for all time periods), the daily returns always display significant excess kurtosis. These two factors lead to all BJ values exceeding their critical values. These results are consistent with previous empirical examination of return series for stock markets by Theodossiou (1998) and Harvey & Siddique (1998).

As the remaining attributes all capture characteristics of the volatility process, these will be summarised in a single table, Table III.

[Table III appears here]

In this table, the first column describes the individual market examined and the time period of the analysis. The next three columns display the average annualised volatility measured at a 20-day time horizon. At the bottom of these columns are the expected attributes from a GBM dispersion process with constant variance. The attribute of interest to this research is the Coefficient of Variation statistic. For all four markets and for all time periods, we can compare this measure of the volatility of volatility to what would be expected under the GBM assumption. By determining a standard error of this attribute by simulation, we can reject the hypothesis that the volatility process conforms to the GBM i.i.d. assumption.

In the fifth column appears the beta of the regression of the relationship between the [natural logarithms of the] time horizon of the estimation period against the coefficient of variation of volatility. If markets conform to a GBM i.i.d. process, a decay coefficient of -.50 (seen at the bottom of the column) would be observed. For each market, the rate of decay is (statistically) significantly less that this decay function (the standard error of this attribute is estimated in a non-parametric manner by simulation). In Column six, the leverage correlation coefficient appears. This measures the relationship between the 20-day unconditional volatility and the recent relative prices. Consistent with the negative leverage effects Christie (1982) pointed out for individual stocks, stock index futures markets also seem to have a significant negative leverage effect. The exceptions are the FTSE 100 and S&P 500 Futures over the first period (1990-1994). These results should be interpreted with care given that the simulated standard error of this attribute is fairly high (at 0.0999). While these effects remain significant, it is only (barely) at the 95% level.

The final two columns represent the average autocorrelations of absolute returns for lagged periods from 1-20 days and 51-70 days. Underlying the assumption of an i.i.d. price process, these have a prior expectation of zero. For all four markets and for all time periods, these averaged autocorrelations are statistically significantly positive. However, the degree of autocorrelations seems to be higher in the second period relative to the first period.

The results from both tables II and III indicate that both the return series and the volatility process are significantly divergent from a prior assumption of a GBM i.i.d. process. With these attributes as target conditions, the proposed alternative models were examined.

6. FITTING ALTERNATIVE MODELS

In previous sections the models to be tested were presented. It only remains to discuss technical issues in the simulation process and the parameterisation of the stochastic volatility models before proceeding directly to the results.

The simulated method of moment's approach was done in a two-stage process. The first stage was to determine representative distributions for the later simulations. Specifically, this entailed the generation of 500 samples of 1500 draws from an independent normal distribution. According to standard procedures, the random number generation process used a standard Box-Muller method and the anti-thetic approach suggested by Boyle (1977). With these 500 samples, price series were constructed that conformed to GBM with constant variance (using equation 1). The distributional and time series attributes of each series were assessed and compared to the theoretical moments of an i.i.d. GBM process. Utilising formula 6, the sum of squared errors between each of the 500 samples and the true attributes of an i.i.d. GBM process were determined. The two distributions with the lowest sums of squared errors relative to the priors were selected as representative normal distributions. For the sake of convenience, the normal disturbances for the underlying price process will be referred to as Z_1 and the normal disturbances for the volatility process will be referred to as $Z_{2.}$ These two series were uncorrelated. Thereafter, whenever simulation used either of these distributional forms, the same sets of random numbers were used (to reduce errors introduced by the selection of random numbers). Table IV details the results of the simulated GBM price series and provides the sample standard deviation of the 500 simulated series.

[Table IV appears here]

In this table, the theoretical attribute values for a GBM process are listed as are the average attribute values and the standard deviations of the attributes across the 500 simulations. Of crucial interest are the sampling properties of the attributes and especially the sum of squared errors (SSE) statistic. The standard deviation of this statistic was found to be equal to 4.5646 and will be used subsequently as a means to establish confidence intervals for the comparison of the alternative models. Finally, the characteristics of the two representative draws of the GBM process appear in the bottom two rows.

To generate the NIG distributions, random numbers were generated using the method suggested by Rydberg (1997). These simulations required the input of the four moments of the distribution. The first moment (mean) was set to 0.0 and the second moment (variance) to 1.0. The third (skew) and fourth (excess kurtosis) moments were chosen to be less than the observed moments for daily returns in Table III. This was done, due to the fact that the stochastic volatility will interact with the NIG distribution and yield simulated moments that are amplified compared to the NIG moments. Given these effects could not be ascertained prior to the simulation, four NIG distributions were simulated and all were examined as potential candidates for the underlying price process. These four possible NIG distributions appear in Table V as NIG#1 to NIG#4.

[Table V appears here]

This approach contains an apparent inconsistency: while the random numbers estimated conform to a NIG distribution, prices were simulated assuming that a risk-neutrality condition exists. When the underlying price process follows an alternative process, such as a NIG distribution, a correction to the drift term is required to allow risk neutral evaluation. An appropriate risk-neutral drift adjustment is:

$$a_t = \frac{\delta}{\Delta t} * \left[\sqrt{\alpha^2 - (\beta^2 + \sigma_{t-1} \cdot \sqrt{\Delta t})^2} - \sqrt{\alpha^2 - \beta^2} \right] - \frac{\mu \cdot \sigma_{t-1}}{\sqrt{\Delta t}} \qquad \textbf{(7)}$$

Where a_t is the risk neutral adjusted drift, the Greek letters, α, β, δ, and μ are the parameters of the NIG distribution and σ_{t-1} is the random volatility (from stochastic volatility process in equation 3.2) for the previous discrete observation. A complete proof of the derivation of this drift adjustment appears in Appendix 1.[9] This drift is then used in equation 3.1 with the drift term (μ) equal to a_{t-1} from equation 7 and the disturbances are drawn from the NIG distribution.

Finally, to simulate correlated processes, we estimated a new set of random numbers (Z') for the volatility process using the usual method for drawing samples from a standardised bivariate distribution:

$$Z' = Z \cdot \rho + \sqrt{(1-\rho^2)} \cdot Z_2 \qquad \textbf{(8)}$$

Where Z represents the random disturbances for the underlying price process (in the case of GBM, the Z_1 set of normal draws, in the case of a NIG process, the draws from one of the four samples). The term, Z_2, refers to the representative normal distribution selected for the volatility process and the term, ρ, refers to the correlation coefficient between the two processes.

Once the distributions were drawn, the second stage of the simulated method of moment's approach was done in three steps. The first step was to simulate price and volatility series that were consistent with the proposed models and secondly, determine the attribute values for this simulated series. The third step entailed varying the paramater inputs into the models to minimise the sum of squared errors relative to the observed empirical attributes. To efficiently perform the third step, a starting point was to examine the sensitivities of the overall sum of squared errors to a small incremental change in each of the parameter inputs (holding the other parameters constant). Of the four variables in both of the stochastic volatility models, it was found that only three of the factors were critical. For a given long-term volatility, θ (taken as the average 20-day volatility in Table III), the crucial factors are the rate of mean reversion, κ, the volatility of the volatility, ξ and the correlation between the price and volatility process, ρ. It was found that small changes in the level of the long-term volatility had almost no effect on the sum of squared errors. Given that only three parameters needed to be varied, the parameterisation simply compared the sum of the squared errors using the initial seeded parameter values to the sum of squared errors for the same model (and random numbers) but varying the three critical parameters (κ,ξ and ρ). By varying only three parameters both up and down, thus, only eight alternatives to the original results had to be compared. If one of the new combinations of parameters achieved a lower sum of squared errors, this model would replace the previous model. The search routine continued to search the "cube" of eight adjacent alternative parameter combinations until no new combination yielded better results. The initial search procedure first used fairly high increments in the adjacent corner search (for example 1.0 for κ and 0.1 for ξ and ρ). When optimal parameters were found, the increments for the search was progressively reduced (for example, as low as 0.01 for κ and 0.0001 for ξ and ρ) until no further reduction in the sum of squared errors was achieved. When no further improvement in the sum of squared errors is possible, the final combination of parameter values is deemed the optimal estimation of the alternative stochastic volatility models.

Tables VIa, VIb, VIc and VId display the empirical attributes (taken from Tables II and III) and the results of the models for each of the stock index futures markets. These results are split into the three periods of analysis. In each of these tables, the three time periods of analysis appear. On the left-hand side, the parameterisation of each of the three models appears. Only the parameter values and the NIG distribution that has the lowest SSE are presented. On the right-hand side, the simulated attributes of each model appear with the empirical attributes directly above for comparison's sake. In the column immediately to the right of this the sum of squared errors (SSE) for each model appears. The final column indicates a T-test statistic comparing models 2 and 3 to the GBM assumption. This is computed by taking the difference in the SSE for the models compared to the GBM case and dividing this by the simulated standard error of the SSE statistic found in Table IV.

[Tables VIa, VIb, VIc and VId appear here]

As was expected, the GBM model with constant variance is rejected in favour of models 2 and 3. The T-statistics indicate a rejection at well above a 99% confidence interval. If we assume that the model with the lowest SSE is optimal, of twelve data sets, Model 3 (NIGSV) is the best in nine and Model 2 (SV) is the best for the remaining three. For all twelve data sets, correlated processes were indicated.

These comparisons provide insights into which attributes are addressed by the facets of the proposed models. The pure stochastic volatility model (SV) addresses the volatility clustering effects measured by the two autocorrelation attributes as well as other volatility dynamics [the volatility of volatility (CoV attribute) and the decay of this over time (Line fit)]. The inclusion of correlated processes allows the leverage effect and the negative skewness in the returns to be captured. However, this model still fails to generate sufficient excess kurtosis. Model 3 (NIGSV) is able to capture all seven attributes.

Comparisons between Models 2 and 3 can be made by examination of the T-test statistics in Tables VIa to VId. As the T-test statistics provide an indication of the improvement in these models relative to the GBM case, taking the differences between these statistics provides insights into marginal improvement of Model 3 to Model 2. In nine of the twelve cases, Model 3 has a higher T-statistic. However, the differences are small. For all attributes except the excess kurtosis, both models perform equally well. What appears to be most critical is the inclusion of correlations between the underlying price and volatility processes. These capture the leverage and a portion of the skewness effect.

7. IMPLICATIONS FOR OPTION PRICING

From the preceding section, a more realistic price process for the four stock index futures markets has thus been uncovered. This will serve as a prior process for the estimation of option values. The next step is to examine the implications for options based upon these assets. Given that the parameter estimation of the stochastic volatility models relies upon simulation, it is a simple matter to use a similar simulation technique to estimate option prices. This simulation approach determines European call and put options numerically (a Monte Carlo approach) over a variety of strike prices and times to expiration. This is similar in spirit to the approach used by Johnson and Shanno (1987).

In the immediately preceding section, it was demonstrated that in almost all instances Model 3 (NIGSV) was optimal, therefore, only this was considered for the estimation of option values.[10] The choice of the appropriate input parameters for this model is taken from the previous section and is based on the results for the entire period of analysis.

An apparent inconsistency for this approach is that the use of Monte Carlo simulations to price options assumes that a risk-neutrality condition exist. This suggests that the state space is continuous and spanned across that space by existing securities. However, for the model examined, stochastic volatility, correlated processes and (negative) jumps have been introduced into the state space. Given that no securities exist allowing the state space to be spanned when the volatility displays such dynamics, these models do not permit us (in the strictest sense) to use the risk-neutrality argument to price the options. This is an apparent theoretical inconsistency. However, the determination of the risk-neutral drift adjustment for the NIGSV process in equation 7 does allow limited comparisons of the simulated options prices to be made actual option prices, which are evaluated under the risk neutral measure. While, the equivalent risk-neutral measure in equation 7 is certainly feasible, it may not be unique. This research examines whether this measure is unique by first assuming that both simulated and actual option prices are evaluated under an equivalent risk-neutral measure. If this is case, we can assess if this alternative price process alone is sufficient to explain the existence of implied volatility smiles. If this is not the case, this may suggest that the equivalent risk-neutral measure is not unique and markets are incomplete.

In this simulation, price series of three months in length were determined. Given that the estimation of the unconditional (historical) dispersion processes was completed for trading days, options were also priced using trading time instead of calendar time. The assumed number of trading days in a year is 252. Option prices were estimated at time horizons from one week (five trading days) to three months (in 5-day increments). Such options would correspond to typical terms to maturity of actively traded options on Stock Index Futures.

To gain a better understanding of the impacts of the alternative models across strike prices, fifteen strike prices were examined. The median strike price was centred at the starting value of the simulation and was equal to 100. As the assumed underlying assets were futures or forward contracts, the interest rate was assumed to zero. This corresponds to an at-the-money option relative to the forward price.

The impacts of the model on options with different strike prices were of additional interest. The analysis was restricted solely to out-of-the-money strike prices. Thus, when the strike price was equal to or below the starting value of 100, the option evaluated was an European put and when the strike price was above 100, the option evaluated was an European call. A non-trivial problem is the choice of strike prices so that as maturities of options vary, meaningful comparisons can be drawn. In previous papers on the impacts of stochastic volatility on option prices, strike price determination has taken one of two forms. Authors have either chosen to fix a single maturity and vary the strike prices in terms of "moneyness" [see Hull & White (1988)] or fixed the degree of moneyness (or strike prices) and examined the impacts across different maturities [see Henker and Kazemi (1998)].

Unfortunately, both methods do not allow meaningful conclusions to be drawn regarding the impacts of the models on option prices across time and a consistent measure of moneyness. Natenberg (1994) and Tompkins (1997) have proposed a more consistent measure of strike price. This was slightly modified to:

$$\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau / 252}} \qquad \textbf{(9)}$$

where X is the strike price of the option, F is the underlying futures price and the square root of time factor reflects the percentage in a trading year of the remaining time until the expiration of the option. The sigma (σ) is the at-the-money volatility.

This adjustment notes that the distance of an option strike price to the level of the underlying asset is relative, both in respect to the current price of the underlying, the time to expiration and the level of expected volatility. This adjustment converts all strike prices into a metric that can be interpreted as a standard deviation. Thus, in this analysis, strike price ranges ± 3.5 standard deviations away from the at-the-money level in 0.5 standard deviation increments were examined. This change in measure will allow more direct comparison of model impacts on option prices where the time to maturity varies but the relative strike prices remain the same. [11]

For the simulations, the volatility parameter chosen was equal to the level of volatility used in the parameter determination of all models for each of the four markets for the entire period of analysis. Given the extremely wide range of volatilities across the four markets, the standardisation of the strike prices allows direct comparisons to be made and allows subsequent comparisons to be made with actual implied volatility surfaces.

For the Monte Carlo simulation, random numbers consistent with a GBM process were determined using a Box-Muller technique and employed the anti-thetic approach suggested by Boyle (1977) for both. This series was later used to determine the bivariate distribution used to estimate the stochastic volatilities. This series of random numbers were stored and used for all subsequent estimations of stochastic volatility. For each of the three NIG distributions, 10000 were drawn using the method suggested by Rydberg (1997). The same approach was used for the estimation of the optimal stochastic volatility parameters in the previous section. These were also stored and used for all analysis using that particular NIG distribution. Then, the bivariate distribution was estimated for the volatility series using formula 7 and the optimal parameters of Model 3 for each stock index futures in Tables VIa, VIb, VIc, and VId (for the entire period of analysis).

With the appropriate NIG distribution and the estimated bivariate distribution for the stochastic volatility, volatilities and prices were estimated using an Euler approach (discrete form of formula 3.1 and 3.2). With the prices of each model estimated, the payoffs of the fifteen options (at each point in time) were determined and the result averaged. As interest rates were assumed to be zero, there is no need to discount the result to present value.

In parallel, we estimated the prices of all the options for each market using the Black (1976) model with the same strike prices as the simulation, the same term to expiration and the volatility equal to the same long-term volatility used in the simulations. The underlying futures prices used in the determination of the Black (1976) price were equal to the average futures price in the simulation at the same point in time. Given that the underlying asset is a futures contract, the interest rate and dividend yield was set to zero (the same assumptions were made when estimating the Model 3 option prices).

Although, standardisation of the strike prices simplifies comparisons between the four markets, the sheer amount of information makes such comparisons cumbersome. Thus, only the results for a single market, the S&P 500, are presented. Furthermore, to simplify comparisons between simulated and actual implied volatility surfaces; the simulated option prices were expressed as implied volatilities. These implied volatilities were further standardised by indexing them to the constant volatility assumed in the Black (1976) model (dividing each of the implied volatilities by the assumed constant volatility and multiplying by 100). The indexed implied volatilities are then presented as a continuous surface relative to the standardised strike prices and time to expiration. This surface for the S&P 500 futures appears in the middle panel of Figure 1 and is titled: Simulated Implied Volatility Surface. This figure has been scaled to allow direct comparison to actual implied volatility surfaces of options on the S&P 500 futures (which appears in the upper panel).

[Figure 1 appears here]

The simulated implied volatility surface for the S&P 500 appears to display some of the features reported by Derman & Kani (1994), and Corrado & Su (1996). The characteristic curvature of volatility smiles is found and there is some degree of negative skewness (especially for longer maturity options). While we have not explicitly discussed the impacts of this model across a cross-section of option prices, this figure implicitly displays these impacts. For all the markets, the Black (1976) pricing model overvalues options that are at-the-money and within a significant range around (and above) the at-the-money level. The shaded areas below 100 represent this in the graphs. For out-of-the-money options, the Black (1976) model tends to undervalue option prices (especially for options with lower strike prices) and the overpricing bias tends to increase the longer the term to expiration. These results are consistent with the biases of stochastic volatility on option prices found elsewhere [Hull & White (1988)]. Similar results are found for the other three stock index futures markets

In the top panel of Figure 1, the actual implied volatility surface associated with options on S&P 500 futures appears. To construct this surface, implied volatilities were estimated using the Black (1976) model for all (out of the money) options on the S&P 500 futures. The period of analysis was from November 1990 to December 1998 (contemporaneous with the period of analysis of the S&P 500 Futures). Analysis was restricted solely to options with the same terms to expiration as were used for the simulated option prices (5 days to 3 months in 5-day increments) and excluded all options prices allowing arbitrage [see Jackwerth & Rubinstein (1996)]. The strike prices were then converted to the same standardised form as was done for the simulated implied volatility surfaces (using formula 9) and the levels of implied volatility were indexed to the level of the at-the-money implied volatility. Then, the indexed implied volatilities were grouped by the same term to expiration. Finally, for each term to expiration, a polynomial fitting procedure [see Shimko (1993) and Tompkins (1999)] was implemented to yield an implied volatility smile. These were then combined and the volatility surface was drawn.[12]

For the actual implied volatility surface associated with options on the S&P 500, it is clear that the degree of curvature is most extreme when the options are closest to expiration and becomes more linear with longer expirations. For the simulated surfaces, the effect is the opposite. Furthermore, the implied volatility surfaces for the actual options market are much more curved that for the simulated surfaces.

Another important difference is that the actual implied volatility surface displays much greater asymmetry than the simulated surface. Options with strike prices below the current futures price have higher implied volatilities than options with higher strike prices. Similar results are found for the other three markets.

These differences are a consequence of the model chosen under the objective measure. Let us consider each facet of the NIGSV model separately. The NIG process for returns captures the effects of non-normality in daily returns. Therefore, if subsequent disturbances are independent, this process will rapidly approach that of a normal distribution from the Central Limit Theorem. If the higher moments of the chosen NIG process were sufficiently extreme, a more curved (and skewed) shape would be observed for options with short terms to expiration; becoming more linear as the time horizon of the option were extended. In the instance of S&P 500 futures, the optimal NIG price process for the S&P 500 (NIG #1) was insufficiently non-normal to produce the biases found with actual smiles. One interpretation of this result is that the higher moments from the risk-neutral process are different (more extreme) than those for the (risk-neutral adjusted) objective process.

The positive relationship between the degree of curvature in the simulated implied volatility surfaces and the time to expiration is most probably due to the inclusion of the stochastic volatility process. As was pointed out by Hull & White (1988), the degree of bias in option pricing increases with the time to expiration of the option. This would produce an implied volatility surface that would display more curvature the longer the term to expiration. As the volatility process in equation 3.2 assumes a Gauss-Wiener process, this effect should be symmetrical. However, the simulated surface displays an asymmetry relationship with the time to expiration. This asymmetry is due to effects of the negative correlation between the processes. Clearly, the longer the period, the greater the impact of the negative correlation. Similar results are found for the other three stock index options markets.

The divergence between the patterns of implied volatility surfaces suggests that the inclusion of our proposed stochastic volatility models in option pricing is insufficient to explain the existence of implied volatility smiles. The most obvious conclusion is that we have the wrong model and that some alternative approach may better describe the unconditional price process for the underlying futures. However, it is unclear what the alternative approaches would be. The models tested here included non-normality in the return process, stochastic volatility and correlated processes. Most models proposed in the literature to explain the objective price and volatility processes are nested in the model examined here.

A second possibility is that models we have examined are correct, but that our method of parameter estimation has yielded incorrect model inputs. The potential problems with parameter estimation could be due either to inadequacies of the simulated method of moments approach or the selection of inappropriate (or incomplete) target moments. As the former has been extensively examined in the literature and dismissed, this is unlikely. The choice of target moments was determined including all relevant facets of non-normality and volatility that have been pointed out in the empirical literature; it is unclear what additional attributes would be included.

The third possibility is even if the price process of the underlying asset was correctly modelled, it is not obvious that option prices would conform to this process. The Cox & Ross (1976) link between the objective and risk-neutral processes would be incomplete in the presence of non-traded sources of risk. For the models examined here, the introduction into the state space of non-constant volatility, jump-processes and correlations between the two processes would introduce just such non-traded sources of risk. Thus, it is clear that our assumption of a unique martingale measure is incorrect. Given this is the case, a logical next step would be to better understand the nature of this risk. Scott (1987) explicitly examined the dynamics of the risk premium when volatility is stochastic and is not a traded security. We will examine the nature of the risk premium implicitly by examination of the differences in the simulated and actual implied volatility surfaces.

The simple difference between these two surfaces appears in the bottom panel of Figure 1. Standardisation of both surfaces simplifies interpretation: as the percentage difference in the (standardised) level of implied volatility. The actual implied volatilities are between 10% and 70% higher than those associated with the implied volatilities consistent with the objective price process. For the shortest to expiration options, almost none of the curvature in the actual implied volatility surface is explained by model of the objective process. Even so, the divergences between the surfaces are relatively symmetrical relative to the ATM level for options with a short time period to expiration (out to 15 days to expiration). One possible interpretation of this is that the skew effect for options on stock index futures with short time periods to expiration might be due primarily to an alternative price process (a NIG process with negative skewness). For longer terms to expiration, approximately one half of the curvature in the actual implied volatility surface is addressed by the objective price process model. However, for longer dated options the divergence in the surfaces is asymmetric. The inclusion of correlated processes should address this. Even so (and using an appropriate measure of the objective leverage effect), this appears insufficient to completely explain this effect. For options with higher strike prices and a longer term to expiration (35 days to 90 days), the divergence between the two surfaces is small. However, the fact that everywhere the difference between the surfaces is positive could be interpreted as evidence for the existence of a risk premia. Alternatively this difference could be associated with transaction costs or could be caused by other sources of market imperfections or frictions. For the other three stock index futures markets, the differences in the simulated and actual implied volatility surfaces display similar patterns. With the exception that for two of the other markets, the divergence seems relatively time invariant (Nikkei and FTSE 100 Futures).

8. SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

This paper examines the nature of the objective dispersion processes, which can be observed for futures contracts on four stock indices. To capture the multi-faceted non-normality and inter-dependence of the empirical dispersion processes, seven attributes were identified. With these attributes for each of the four markets, two alternative stochastic volatility models were examined. It was found that all four markets are best understood with a model which assumes the price process follows a Normal Inverse Gaussian process and the stochastic process driving volatilities is negatively correlated to this NIG process.

Based upon an optimal parameterisation of this model, European options were determined numerically for each market. Significant divergences from the Black (1976) values were observed. For all markets, this option pricing model tended to increase the prices of options with lower strike prices relative to Black (1976) prices and decrease the value of ATM and higher strike price options. This research points out for the first time the relative biases associated with the option pricing model proposed by Barndorff-Nielsen and Shephard (1999). This extends the research of Hull & White (1988) to consider a richer class of stochastic volatility models.

Option prices consistent with the model were then expressed as standardised implied volatility surfaces and compared to actual implied volatility surfaces from options on the S&P 500. While the simulated implied volatility surfaces display some stylised facts similar to actual surfaces, differences are observed that are similar across all four markets. For the entire surfaces, the actual option prices have higher implied volatilities than for option prices consistent with the model. Therefore, we reject the hypothesis that the stochastic volatility models proposed are sufficient to explain the existence of implied volatility smiles. Of interest is that the differences between the implied volatility surfaces consistent with the objective and risk-neutral processes appear to display similar dynamics for all four markets. The nature of this difference should provide a useful starting point for future research into the nature of the risk premium in options prices and provide insights into the relationship between the objective and risk-neutral processes.

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FOOTNOTES:

[1] A number of previous papers have made similar assumptions. Scott (1987), Hull & White (1987), and Johnson and Shanno (1987) either assumed the market price of volatility risk was zero or the existence of a marketable asset perfectly correlated to volatility.

[2] Barndorff-Nielsen (1997) actually assumed that the underlying price process follows GBM and the volatility process follows an Inverse Gaussian distribution. However, it can be shown that this is equivalent to the underlying price process following a NIG distribution and the volatility process follows GBM.

[3] The estimation of the volatilities was done using overlapping data. This introduces a bias in the estimation of the standard deviation of volatility. This bias was corrected using the Hodges and Tompkins (2000) approach.

[4] The author would like to thank Stewart Hodges (University of Warwick) for suggested the use of this variable.

[5] To simplify the selection of the attribute, a fixed weight of 0.03 was applied to all markets and for all periods of analysis, to allow the leverage correlation factor to not be subject to differing weights. This was found to be close to the optimal weights for each market using a maximum likelihood estimation procedure.

[6] In addition, the natural logarithm of the unconditional kurtosis was examined rather than the absolute levels. This was done due to wide variations in this statistic across the four markets. However, all results are presented as absolute levels.

[7] Under the null hypothesis of normality, the skewness statistic is asymptotically normally distributed with standard errors: $se = \sqrt{(6/T)}$, where T is the sample size.

[8] Under the null hypothesis of normality, the excess kurtosis statistic is asymptotically normally distributed with standard errors: $se = \sqrt{(24/T)}$, where T is the sample size. This statistic is equal to the kurtosis statistic appearing in the table minus 3.0.

[9] In Appendix 1, it is noted there that this is only one manner to adjust this drift to achieve risk-neutrality. Given that this model implies markets are incomplete, there may not exist a unique martingale measure. However, this is certainly a feasible adjustment and will allow relative comparisons to be drawn.

[10] While for three of the twelve data sets, the SV model was marginally superior to the NIGSV model, this difference was found statistically significant.

[11] This manner of expressing the strike price is similar to the d_2 term that appears in the Black Scholes formula. It is common market practice in the currency options market to express strike prices in terms of the delta $[N(d_2)]$ and quote implied volatilities relative to this. This approximately expresses equation 1 as a probability.

[12] Tompkins (1999) used a model expanding Shimko (1993) to include higher moments and time/strike price interactions. When implied volatility surfaces were standardised (relative to the ATM volatility). This model was able to explain 95% of the variance in the implied volatility surface of the S&P 500 and Tompkins (1999) demonstrated that this degree of explanatory power is retained outside of sample.

APPENDIX 1

Derivation of the Risk Neutral Drift Adjustment for a
Normal Inverse Gaussian Process.

The process S, generated by the NIG distribution should be a martingale. In the discrete time setting this means:

$$E\left[S_{t_j} \mid \Im_{t_{j-1}}\right] = S_{t_{j-1}}, \; j = 1,2,3,\ldots,n \qquad 1.1$$

with $\Im_{t_{j-1}}$ denoting the information at time t_{j-1}. Since both the drift term, a, and the volatility, σ, are known at time t_{j-1}, and the variance of the series V_j is independent from the history leading up until t_{j-1}, this is equivalent to:

$$e^{a_{t_{j-1}}\Delta t} m\left(\sigma_{t_{j-1}}\sqrt{\Delta t}\right) = 1, \qquad 1.2$$

where $m(x) = E[e^{xVj}]$ is the moment generating function of the $NIG(\mu,\delta,\alpha,\beta)$ distribution.

This function is:

$$m(x) = \exp(\delta(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + x)^2}) + \mu x) \qquad 1.3$$

So the appropriate choice of drift is:

$$a_{t_{j-1}} = \frac{1}{\Delta t}\ln\left(\frac{1}{m\left(\sigma_{t_{j-1}}\sqrt{\Delta t}\right)}\right) \qquad 1.4$$

Combining equations 1.3 and 1.4 leads to

$$a_t = \frac{\delta}{\Delta t} * \left[\sqrt{\alpha^2 - (\beta^2 + \sigma_{t-1} \cdot \sqrt{\Delta t}})^2 - \sqrt{\alpha^2 - \beta^2}\right] - \frac{\mu \cdot \sigma_{t-1}}{\sqrt{\Delta t}} \qquad 1.5$$

As a final note, adjusting the drift is only one way to obtain a martingale (for risk-neutral valuation). Given the models examining suggest an incomplete market, there are alternative approaches. For example, for the constant volatility NIG process in continuous time, the natural approach would be to estimate the parameters, μ,δ,α *and* β from the observations of S and adjust β to obtain a risk neutral valuation.

Underlying Asset	Time Period of Analysis		Number of Observations
(Entire Period)	First Date	Last Date	
S&P 500 Futures	23/11/90	17/12/98	2042
FTSE 100 Futures	23/11/90	18/12/98	2042
DAX Futures	23/11/90	18/12/98	2020
NIKKEI-225 Futures	23/11/90	11/12/98	2037
(First Period)	First Date	Last Date	
S&P 500 Futures	23/11/90	15/12/94	1029
FTSE 100 Futures	23/11/90	16/12/94	1029
DAX Futures	23/11/90	16/12/94	1016
NIKKEI-225 Futures	23/11/90	08/12/94	1023
(Second Period)	First Date	Last Date	
S&P 500 Futures	19/12/94	17/12/98	1013
FTSE 100 Futures	19/12/94	18/12/98	1013
DAX Futures	19/12/94	18/12/98	1004
NIKKEI-225 Futures	09/12/94	11/12/98	1014

Table I, Markets Included in Research, Time Period of Data, Number of Observations

Futures Prices are the closing daily levels for the Nearest to Expiration Contract

Underlying Asset	Mean	Standard Deviation	Unconditional Skewness	Skewness Significance Level	Unconditional Kurtosis	Kurtosis Significance Level	Bera-Jarque Statistic	Observations
S&P 500 Futures								
Overall Period	0.00055	0.00907	**-0.4397**	**-8.11**	**11.401**	**77.47**	**6067.18**	2041
First Period	0.00030	0.00717	**0.2074**	**2.72**	**6.078**	**20.14**	**413.14**	1028
Second Period	0.00079	0.01066	**-0.6477**	**-8.42**	**10.884**	**51.22**	**2694.11**	1013
FTSE 100 Futures								
Overall Period	0.00038	0.01000	-0.0944	-1.74	**5.979**	**27.47**	**757.87**	2041
First Period	0.00014	0.00950	0.1322	1.73	**4.457**	**9.54**	**93.94**	1028
Second Period	0.00063	0.01048	**-0.2779**	**-3.61**	**6.983**	**25.88**	**682.76**	1013
DAX Futures								
Overall Period	0.00043	0.01272	**-0.6466**	**-11.86**	**11.744**	**80.20**	**6572.19**	2019
First Period	0.00011	0.01200	**-0.8826**	**-11.48**	**19.675**	**108.44**	**11891.68**	1015
Second Period	0.00075	0.01340	**-0.4865**	**-6.29**	**6.408**	**22.04**	**525.49**	1004
Nikkei 225 Futures								
Overall Period	-0.00037	0.01538	**0.1161**	**2.14**	**4.919**	**17.68**	**317.04**	2036
First Period	-0.00039	0.01516	**0.1943**	**2.54**	**4.447**	**9.44**	**95.58**	1022
Second Period	-0.00035	0.01560	0.0437	0.57	**5.352**	**15.29**	**234.01**	1014

Table II, Statistics of the Daily Returns for Four Stock Index Futures Markets (1990-1998)

These returns are based upon closing futures prices for the nearest to expiration futures contracts. The first column describes the market under investigation and the period of analysis (entire period 1990-1998) (first period 1990-1994) (second period 1994-1998) The second and third columns present the first (mean) and second (standard deviation) moments of the return distribution. The fourth and sixth columns present the statistics for the unconditional skewness and kurtosis. Under the null hypothesis of normality, the skewness statistic is normally distributed with standard errors: se =sqrt(6/T), where T is the sample size. This appears in the furthest right-hand column of the table.The fifth column indicates a significance statistic testing a normality hypothesis. If the skewness statistic rejects this at a 95% level or above, this is indicated in ***BOLDED*** *text.Under the null hypothesis of normality, the excess kurtosis statistic is normally distributed with standard errors: se =sqrt(24/T). This statistic is equal to the kurtosis statistic appearing in the table minus 3.0. Column 7 indicates a significance statistic testing the null hypothesis of normality. If the kurtosis statistic rejects this at a 95% level or above, this is indicated in BOLDED text. The statistic in the sixth column is the Bera-Jarque (BJ) statistic for detecting departures of the data from normality. Under the null hypothesis of normality, the BJ statistic is distributed as Chi squared with 2 degrees of freedom. The critical value at the one-percent level is 9.21. When the BJ statistic is above this level, this statistic appears in* ***BOLDED*** *text.*

Markets	20 Day Average Volatility	20 Day SD Volatility	Coefficient of Variation	Line Fit of SD of Volatility vs. Time Horizon	Leverage Correlation (20 Day Volatility vs. Recent Relative Prices)	Average Autocorrelation of Absolute Returns (Lags 1-20)	Average Autocorrelation of Absolute Returns (Lags 51-70)
S&P 500 Futures							
Entire Period	12.50%	6.29%	**0.503**	**-0.1359**	**-0.2174**	**0.1465**	**0.0903**
First Period	10.54%	3.35%	**0.318**	**-0.1943**	**0.2049**	**0.0563**	**0.0391**
Second Period	14.46%	7.74%	**0.535**	**-0.1191**	**-0.3999**	**0.1541**	**0.0834**
FTSE 100 Futures							
Entire Period	14.60%	5.72%	**0.392**	**-0.1600**	**-0.3010**	**0.1330**	**0.0679**
First Period	14.93%	4.12%	**0.276**	**-0.1584**	-0.1227	**0.0615**	**0.0274**
Second Period	14.35%	6.54%	**0.455**	**-0.1001**	**-0.4536**	**0.1725**	**0.0954**
DAX Futures							
Entire Period	17.50%	9.08%	**0.519**	**-0.2000**	**-0.3534**	**0.1806**	**0.0817**
First Period	17.30%	8.50%	**0.492**	**-0.2650**	**-0.2395**	**0.0714**	**0.0282**
Second Period	17.77%	9.19%	**0.517**	**-0.1001**	**-0.4614**	**0.2512**	**0.1158**
Nikkei 225 Futures							
Entire Period	23.40%	9.22%	**0.394**	**-0.1934**	**-0.2390**	**0.1308**	**0.0397**
First Period	24.63%	9.26%	**0.376**	**-0.1958**	-0.1946	**0.1366**	**0.0355**
Second Period	22.42%	9.73%	**0.434**	**-0.1491**	**-0.2561**	**0.1421**	**0.0574**
Expected GBM Attributes	20.000%	3.244%	0.162	-0.5000	0.0000	0.0000	0.0000
Standard Error of Attributes	3.244%	0.032%	0.010	0.0778	0.0999	0.0051	0.0061

Table III, Characteristics of the Unconditional Volatility (Standard Deviation) of Returns for Four Stock Index Futures Markets.

The furthest left column, the Stock Index Futures market and the time period of analysis appears. In the bottom two rows appears the attribute values expected from an Independent and Identically Distributed (i.i.d.) Price process associated with Geometric Brownian Motion (GBM). The standard error of the attributes is determined for a series of 500 draws from an i.i.d. GBM process of 1500 observations. These non-parametrically estimated standard errors allow significance testing between the empirical attributes and the assumption of an i.i.d. GBM process. When a T-statistic rejects this assumption at a 95% level or above, the attributes are BOLDED. In Columns Two, Three and Four, analysis of the 20 day volatility esimated on an non-overlapping basis appears. Using all available observations, daily returns (differences in the logarithm of daily closing Stock Index futures prices) were estimated. With these returns, the standard deviation was estimated for a fixed time horizon of 20 days and then annualised using the sqrt(252).The estimation of the 20 day volatility was done on a non-overlapping basis. In the fifth column appears the relationship between the standard deviation of volatility and the time horizon of estimation. The Time Factor term is determined by a regression of the logarithm of the time horizon (T) on the logarithm of the unbiased standard deviation of volatility. Only the beta coefficients of the regression appears. Observed divergences in the Beta coefficient from -0.5 give an indication of the degree of the volatility of volatility persistence observed in the unconditional process. The sixth column provides information about the leverage relationship between the levels of Stock Index futures prices and volatility. This is the correlation between the unconditional volatility estimated with daily log price increments and estimated on a 20 day rolling time horizon and an exponentially weighted return series, which indicates whether recent price movements are relatively high or low. It should be noted that for the FTSE 100 Futures in the first period and the Nikkei 225 Futures in the first period, we cannot reject the assumption of a leverage effect.In the two furthest right columns appear information about the autocorrelations in absolute returns (and volatility clustering) composite measures of autocorrelograms were required. Given the relatively rapid decay of the autocorrelations, we averaged the autocorrelations for each lag from lag 1 to 20 and separately averaged the autocorrelations for each lag from lag 51 to 70.

GBM (I.I.D) Process	CoV 20-day Vol.	Unconditional Skewness	Unconditional Kurtosis	Leverage Correlation	Auto-Corr. (1-20 lags)	Auto-Corr. (51-70 lags)	Time Decay Line Fit	SSE
Expected Results	0.1622	0.0000	3.0000	0.0000	0.0000	0.0000	-0.5000	0.0000
Average Result of 500 Simulations	0.1617	0.0000	3.0020	0.0000	-0.0014	0.0001	-0.5085	6.9806
Standard Deviation of 500 Simulations	0.0010	0.0607	0.1358	0.0999	0.0051	0.0061	0.0778	4.5646
Representative GBM Price Process (Z_1)	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	0.3636
Representative GBM Volatility Process (Z_2)	0.1586	-0.0160	3.0529	-0.0182	-0.0020	-0.0002	-0.5002	0.3647

Table IV, Attribute Values for Simulated GBM Processes, Standard Deviation of Attributes, and Two Representative Processes

The furthest left column indicates various simulations of an Independent and Identically Distributed (I.I.D.) Geometric Brownian Motion (GBM) price process. The row titled "Expected Results" is not based upon simulation but is what is expected from statistical theory. The next two rows titled "Average Results of 500 Simulations" and "Standard Deviation of 500 Simulations" indicates the sampling properties of 500 series of simulated prices of 1500 observations based upon random draws from an I.I.D. GBM process. The next two rows represent the two draws from the I.I.D. GBM process which has the lowest sum of squared errors relative to the expected theoretical results. These will be used in the second stage of the analysis as representative normal distributions and will be referred to as Z1 and Z2. Columns 2 to 8 indicate the actual attibute values. The furthest right column (the ninth column) indicates the sum of squared errors (SSE) statistic for that row. Of importance to a later stage of analysis is the average standard deviation of the SSE. This result will allow comparisons to be made between alternative models.

NIG Distribution	Mean	Standard Deviation	Skewness	Kurtosis
NIG #1	0.01821	1.00088	-0.0049	4.7807
NIG #2	0.00078	0.93955	-0.9289	8.7576
NIG #3	-0.00665	1.01818	-0.1936	3.4129
NIG #4	0.0035	0.99176	-0.0794	7.627

Table V, Sample Statistical Moments of Simulations of Four Normal Inverse Gaussian (NIG) Distributions.

We simulated four NIG distributions using the method suggested by Rydberg (1996b). This method requires the four moments of the distribution to be input. The first moment (mean) was set to 0.0 and the second moment (variance) to 1.0. The third (skew) and fourth (excess kurtosis) moments were chosen to be less than the observed moments for daily returns of the four Stock Index Futures markets. This was done, due to the fact that the stochastic volatility will interact with the NIG distribution and amplify the resultant simulated moments

Models (Parameters) — Empirical and Simulated Attributes

S&P 500 Futures	(Whole Period)					CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit		
Empirical Attributes						0.5033	-0.4397	11.4006	-0.2174	0.1465	0.0903	-0.1359		
	Distribution	LTV	K	VoV	Correl	Simulated Attributes							SSE	T-Statistic
Model 1	GBM	0.125	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	255.781	
Model 2	GBM	0.125	3.10	0.890	-0.25	0.4471	-0.4685	5.9502	-0.2742	0.1763	0.0765	-0.1086	6.571	**54.651**
Model 3	NIG-T1	0.125	1.80	0.380	-0.35	0.5239	-0.3702	9.6509	-0.2165	0.1597	0.0759	-0.1819	3.387	**55.349**

S&P 500 Futures	(First Period)					CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit		
Empirical Attributes						0.3180	0.2074	6.0778	0.2049	0.0563	0.0391	-0.1943		
	Distribution	LTV	K	VoV	Correl	Simulated Attributes							SSE	T-Statistic
Model 1	GBM	0.105	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	123.538	
Model 2	GBM	0.105	2.39	0.172	0.035	0.2928	0.0161	3.5894	0.1824	0.0852	0.0371	-0.1456	2.686	**26.503**
Model 3	NIG-T3	0.105	2.05	0.170	-0.09	0.2961	-0.1488	4.2690	0.1607	0.0701	0.0377	-0.1875	1.020	**26.868**

S&P 500 Futures	(Second Period)					CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit		
Empirical Attributes						0.5350	-0.6206	11.2081	-0.3999	0.1541	0.0834	-0.1191		
	Distribution	LTV	K	VoV	Correl	Simulated Attributes							SSE	T-Statistic
Model 1	GBM	0.145	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	393.627	
Model 2	GBM	0.145	4.00	0.700	-0.4	0.5072	-0.6220	6.5818	-0.3921	0.2139	0.0796	-0.1171	4.899	**85.247**
Model 3	NIG-T1	0.145	1.99	0.495	-0.61	0.5769	-0.5503	10.6853	-0.4214	0.1833	0.0872	-0.1568	2.885	**85.689**

Table VI a, Empirical Attributes of S&P 500 Futures and Attributes of Five Alternative Models

This table displays the empirical attributes (taken from Tables II and III) and the results of the models for the S&P 500 Futures. These results are split into the three periods of analysis. In each of these tables, the three time periods of analysis appear. On the left-hand side, the parameterisation of each of the three models appears. Only the parameter values and the NIG distribution that has the lowest SSE are presented. On the right-hand side, the simulated attributes of each model appear with the empirical attributes directly above for comparison's sake. In the column immediately to the right of this the sum of squared errors (SSE) for each model appears. The final column indicates a T-test statistic comparing models 2 and 3 to the GBM assumption. This is computed by taking the difference in the SSE for the models compared to the GBM case and dividing this by the simulated standard error of the SSE statistic found in Table IV.

Models (Parameters) — Empirical and Simulated Attributes

(Whole Period)

FTSE 100 Futures	Distribution	LTV	K	VoV	Correl	CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit	SSE	T-Statistic
Empirical Attributes						0.3917	-0.0944	5.9793	-0.301	0.1330	0.0679	-0.1600		
Simulated Attributes														
Model 1	GBM	0.146	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	211.355	
Model 2	GBM	0.146	3.01	1.001	-0.29	0.4858	-0.4720	5.7005	-0.2829	0.1965	0.0122	-0.1969	3.100	**45.670**
Model 3	NIG-T3	0.148	3.45	0.549	-0.64	0.3869	-0.2845	5.1239	-0.3226	0.1439	0.0553	-0.1599	1.101	**46.108**

(First Period)

FTSE 100 Futures	Distribution	LTV	K	VoV	Correl	CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit	SSE	T-Statistic
Empirical Attributes						0.2760	0.1322	4.4571	-0.1227	0.0615	0.0274	-0.1584		
Simulated Attributes														
Model 1	GBM	0.149	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	102.921	
Model 2	GBM	0.149	2.43	0.251	-0.2	0.2723	-0.0962	3.5016	-0.1034	0.0684	0.0278	-0.1795	0.570	**22.445**
Model 3	NIG-T3	0.149	1.76	0.223	-0.266	0.2646	-0.1864	3.9877	0.0522	0.0526	0.0308	-0.2062	2.710	**21.976**

(Second Period)

FTSE 100 Futures	Distribution	LTV	K	VoV	Correl	CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit	SSE	T-Statistic
Empirical Attributes						0.4555	-0.2613	6.5835	-0.4536	0.1725	0.0954	-0.1001		
Simulated Attributes														
Model 1	GBM	0.144	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	394.747	
Model 2	GBM	0.144	4.10	0.750	-0.5	0.4706	-0.6577	6.5588	-0.4462	0.1847	0.0832	-0.1088	2.468	**86.026**
Model 3	NIG-T3	0.144	2.59	0.549	-0.86	0.4253	-0.3877	4.8504	-0.4268	0.1821	0.1072	-0.1047	1.977	**86.134**

Table VI b, Empirical Attributes of FTSE 100 Futures and Attributes of Five Alternative Models

This table displays the empirical attributes (taken from Tables II and III) and the results of the models for the FTSE 100 Futures. These results are split into the three periods of analysis. In each of these tables, the three time periods of analysis appear. On the left-hand side, the parameterisation of each of the three models appears. Only the parameter values and the NIG distribution that has the lowest SSE are presented. On the right-hand side, the simulated attributes of each model appear with the empirical attributes directly above for comparison's sake. In the column immediately to the right of this the sum of squared errors (SSE) for each model appears. The final column indicates a T-test statistic comparing models 2 and 3 to the GBM assumption. This is computed by taking the difference in the SSE for the models compared to the GBM case and dividing this by the simulated standard error of the SSE statistic found in Table IV.

Models (Parameters) — Empirical and Simulated Attributes

DAX Futures	(Whole Period)					Empirical Attributes								
						CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit		
						0.5186	-0.6466	11.7437	-0.35339	0.1806	0.0817	-0.2000		
	Distribution	**LTV**	**K**	**VoV**	**Correl**	Simulated Attributes							**SSE**	**T-Statistic**
Model 1	GBM	0.175	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	250.919	
Model 2	GBM	0.175	3.60	1.090	-0.35	0.4714	-0.5806	6.3360	-0.3619	0.1880	0.0721	-0.1337	7.652	**53.348**
Model 3	NIG-T1	0.175	1.93	0.431	-0.51	0.5036	-0.4030	9.3509	-0.3560	0.1478	0.0722	-0.1772	2.226	**54.538**

DAX Futures	(First Period)					Empirical Attributes								
						CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit		
						0.4916	-0.8826	19.6753	-0.2395	0.0714	0.0282	-0.2650		
	Distribution	**LTV**	**K**	**VoV**	**Correl**	Simulated Attributes							**SSE**	**T-Statistic**
Model 1	GBM	0.173	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	95.863	
Model 2	GBM	0.173	6.00	1.400	-0.7	0.4105	-0.6305	5.7878	-0.5376	0.1212	0.0217	-0.2302	9.856	**18.861**
Model 3	NIG-T1	0.173	6.33	0.708	-0.4884	0.4569	-0.6443	9.3737	-0.4505	0.0959	0.0274	-0.2703	3.006	**20.363**

DAX Futures	(Second Period)					Empirical Attributes								
						CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit		
						0.5172	-0.4372	6.2881	-0.4614	0.2512	0.1158	-0.1001		
	Distribution	**LTV**	**K**	**VoV**	**Correl**	Simulated Attributes							**SSE**	**T-Statistic**
Model 1	GBM	0.178	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	434.946	
Model 2	GBM	0.178	3.99	0.795	-0.51	0.4657	-0.6172	6.0690	-0.4912	0.1876	0.0733	-0.1242	2.937	**94.739**
Model 3	NIG-T3	0.178	2.30	0.700	-0.85	0.4669	-0.3921	5.1255	-0.4106	0.2117	0.1314	-0.0933	2.984	**94.728**

Table VI c, Empirical Attributes of DAX Futures and Attributes of Five Alternative Models

This table displays the empirical attributes (taken from Tables II and III) and the results of the models for the DAX Futures. These results are split into the three periods of analysis. In each of these tables, the three time periods of analysis appear. On the left-hand side, the parameterisation of each of the three models appears. Only the parameter values and the NIG distribution that has the lowest SSE are presented. On the right-hand side, the simulated attributes of each model appear with the empirical attributes directly above for comparison's sake. In the column immediately to the right of this the sum of squared errors (SSE) for each model appears. The final column indicates a T-test statistic comparing models 2 and 3 to the GBM assumption. This is computed by taking the difference in the SSE for the models compared to the GBM case and dividing this by the simulated standard error of the SSE statistic found in Table IV.

Models (Parameters)   Empirical and Simulated Attributes

(Whole Period)

Nikkei Futures	Distribution	LTV	K	VoV	Correl	CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit	SSE	T-Statistic
Empirical Attributes						0.3940	0.1161	4.9192	-0.239	0.1308	0.0397	-0.1934		
Simulated Attributes														
Model 1	GBM	0.234	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	169.494	
Model 2	GBM	0.234	5.35	1.016	-0.315	0.3946	-0.3448	4.9679	-0.2255	0.1409	0.0199	-0.1815	3.021	**36.507**
Model 3	NIG-T3	0.234	3.63	0.813	-0.54	0.3666	-0.2570	5.0156	-0.2674	0.1292	0.0394	-0.1835	1.405	**36.862**

(First Period)

Nikkei Futures	Distribution	LTV	K	VoV	Correl	CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit	SSE	T-Statistic
Empirical Attributes						0.3761	0.1943	4.4470	-0.1946	0.1366	0.0355	-0.1958		
Simulated Attributes														
Model 1	GBM	0.246	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	125.754	
Model 2	GBM	0.246	4.48	0.762	-0.21	0.3698	-0.1919	4.2910	-0.0708	0.1353	0.0358	-0.1588	1.604	**27.226**
Model 3	NIG-T1	0.246	4.10	0.732	-0.121	0.3773	-0.1568	5.1243	0.0423	0.1227	0.0333	-0.2005	2.008	**27.137**

(Second Period)

Nikkei Futures	Distribution	LTV	K	VoV	Correl	CoV	Skewness	Kurtosis	Leverage	Corr(1-20)	Corr(51-70)	Line Fit	SSE	T-Statistic
Empirical Attributes						0.4338	0.0528	5.6695	-0.2561	0.1421	0.0574	-0.1491		
Simulated Attributes														
Model 1	GBM	0.224	0.00	0.000	0	0.1648	0.0004	3.0705	-0.0042	-0.0011	-0.0082	-0.5000	287.723	
Model 2	GBM	0.224	4.28	0.750	-0.31	0.4139	-0.3291	4.8356	-0.2273	0.1643	0.0567	-0.1418	2.612	**62.524**
Model 3	NIG-T3	0.224	2.31	0.851	-0.56	0.4243	-0.2431	5.4862	-0.2007	0.1722	0.0683	-0.1560	1.992	**62.660**

Table VI d, Empirical Attributes of Nikkei 225 Futures and Attributes of Five Alternative Models

This table displays the empirical attributes (taken from Tables II and III) and the results of the models for the Nikkei 225 Futures. These results are split into the three periods of analysis. In each of these tables, the three time periods of analysis appear. On the left-hand side, the parameterisation of each of the three models appears. Only the parameter values and the NIG distribution that has the lowest SSE are presented. On the right-hand side, the simulated attributes of each model appear with the empirical attributes directly above for comparison's sake. In the column immediately to the right of this the sum of squared errors (SSE) for each model appears. The final column indicates a T-test statistic comparing models 2 and 3 to the GBM assumption. This is computed by taking the difference in the SSE for the models compared to the GBM case and dividing this by the simulated standard error of the SSE statistic found in Table IV.

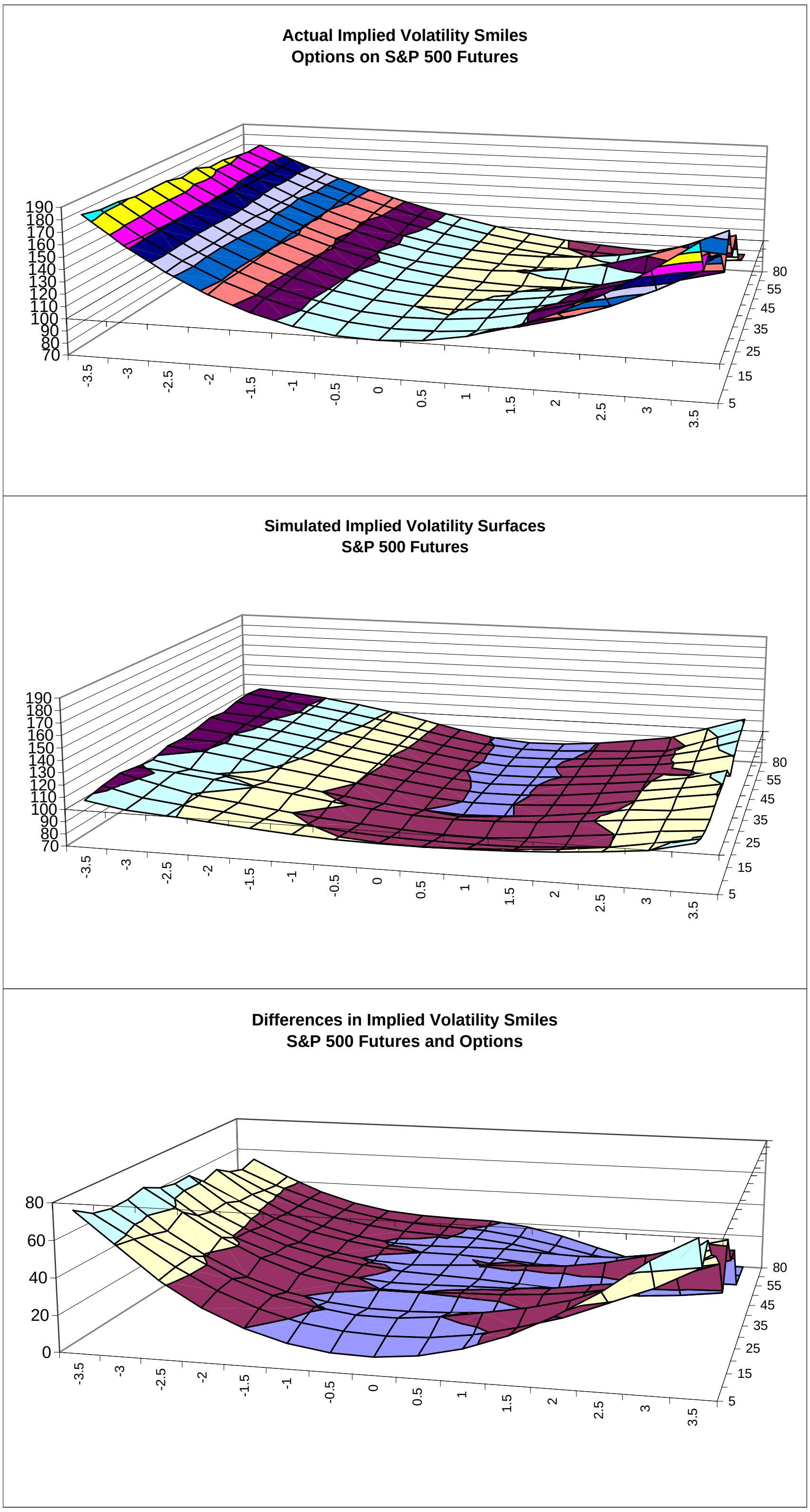

Figure 1, Actual, Simulated and Differences Between Implied Volatility Surfaces for S&P 500 Futures and Options

The top panel presents the actual implied volatility surface associated with options on S&P 500 futures for the period from November 1990 to December 1998. Implied volatilities were estimated using the Black (1976) model for all (out of the money) options on the S&P 500 futures. Analysis was restricted solely to options from 5 days to 3 months in 5-day increments (trading days) The strike prices were expressed in standard deviation terms (away from the underlying futures price) and the volatilities were indexed to the level of the at-the-money volatility. All indexed implied volatilities were grouped with the same term to expiration and a polynomial fitting procedure was implemented to construct the volatility surface.

The middle panel represents a simulated implied volatility surface consistent with the NIGSV model with parameter values determined using the entire period of analysis (1990-1998). Option prices were estimated solely to periods from 5 days to 3 months in 5-day increments (trading days). Implied volatilties were estimated using the Black (1976) model. The strike prices were expressed in standard deviation terms (away from the average simulated futures price) and the volatilities were indexed to the constant level of 20%.

The bottom panel displays the differences between the actual and simulated implied volatility surfaces for the same relative strike prices and terms to expiration.