Anytime Temporal Reasoning : Preliminary Report (Extended Abstract)*

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We present a new approach for determining consistency of temporal constraint networks based on Allen's interval-based framework [All83]. The temporal network is first translated into a clausal theory in propositional logic [Men64], whose satisfiability is then determined using an anytime family of tractable reasoners [Dal96].

Anytime reasoners are complete reasoners that provide partial answers even if stopped prematurely; the completeness of the answer improves with the time used in computing the answer. Our anytime family $\vdash_0, \vdash_1, \ldots$ is built upon clausal boolean constraint propagation (BCP) [McA90], a variant of unit resolution. It is known that each $\vdash_i$ is sound and tractable, each $\vdash_{i+1}$ is at least as complete as $\vdash_i$, and each propositional theory has a complete reasoner $\vdash_i$ for reasoning with it.

A straight-forward translation of a network into a propositional theory can be obtained by directly instantiating each entry of Allen's transitivity table by relations among each triple of intervals in the network. We improve upon this by first embedding a tree in the network and then using the edges of this tree to restrict the number of formulas.

A temporal network is a directed graph whose edges are labeled by subsets of 13 basic temporal relations defined by Allen. Consider any maximal tree M embedded in the given connected network N (our approach extends easily to non-connected networks). The edges in M are called *tree edges*, and the rest of the edges in N are called *back edges*. The notions of *parent*, *ancestor*, etc. are defined as usual, with respect to the tree M. For each tree edge (x, y), $Rel(x, y)$ is defined to be the label of edge (x, y), and for sub-branch $x_1, \ldots, x_k$ of nodes in M, where $k > 2$, $Rel(x_1, x_k)$ is defined to be the set $Rel(x_1, x_2) \circ \ldots \circ Rel(x_{k-1}, x_k)$ of relations. Note that $\circ$ is the composition relation defined by Allen. The translated theory $\mathrm{Tr}(N)$ with respect to tree M consists of exactly the following formulas:

I: $xr_1y \vee \ldots \vee xr_ky$ for each edge (x, y) with label $\{r_1, \ldots, r_k\}$ in N;
II: $\neg xry \vee \neg ytz \vee xr_1z \vee \ldots \vee xr_kz$ for each triple x, y, z of nodes such that x is a proper ancestor of y and y is a proper ancestor of z, for each relation r in $Rel(x, y)$, and for each relation t in $Rel(y, z)$, where $r \circ t = \{r_1, \ldots, r_k\}$;
III: similar to formulas II above, except that $z = x$, x is an ancestor of y, and there is a back edge from y to x; and

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IV: zez and $\neg zuz$ for each node z with an incoming back edge and each relation u different from e (equal) such that the atom zuz occurs in some formula III above.

Theorem 1 shows that the above translation is sound and complete:

Theorem 1. *A network N is consistent iff the theory* $\mathrm{Tr}(N)$ *is satisfiable.*

We present an algorithm that obtains a translated theory for any given temporal network N, and prove that the time taken by the algorithm and the size of the translated theory $\mathrm{Tr}(N)$ are both bounded by $O(nm^2)$, where n is the number of nodes in N and m is the length of the longest branch in the embedded tree M. Since m can grow as large as n, the worst-case size of a translated theory is cubic in the size of the network.

Since we are interested only in determining consistency of temporal networks, we use the anytime family $\vdash_0, \vdash_1, \ldots$ to determine the satisfiability of translated theories. The least k needed for obtaining $\mathbf{f}$ from the translated theory using $\vdash_k$ is called the *intricacy* of the network. Since each $\vdash_k$ can be determined in time exponential in k, the intricacy of a network captures the difficulty of detecting its inconsistency using our approach.

We randomly generated thousands of small networks for comparing our approach with three other incomplete reasoners. We found that intricacy is 1 for all networks that were found inconsistent by Allen's path consistency algorithm. However, the other two algorithms, that first translate interval networks into point algebra networks (with and without $\neq$, respectively), could not detect inconsistency in most networks with intricacy 1. We have not yet found any inconsistent network with intricacy greater than 2.

Since the translated theories are quite large, our current approach can be used only for networks with a small number of intervals. We are currently working on further reducing the size of the translated theory, by translating fewer transitivity rules. We are also extending our approach to handle qualitative constraints like "A before B or B before C" that can not be expressed in temporal networks. Our current work also involves inferring implicit temporal constraints from temporal networks, rather than just determining whether a network is consistent.

References

[All83] J.F. Allen. Maintaining knowledge about temporal intervals. *Communications of the ACM*, 26(11):832–843, 1983.

[Dal96] M. Dalal. Anytime families of tractable propositional reasoners. In *Fourth International Symposium on Artificial Intelligence and Mathematics (AI/MATH-96)*, pages 42–45, Florida, 1996.

[McA90] D. McAllester. Truth maintenance. In *Proceedings Eighth National Conference on Artificial Intelligence (AAAI-90)*, pages 1109–1116, 1990.

[Men64] E. Mendelson. *Introduction to Mathematical Logic*. Van Nostrand, Princeton, N.J., 1964.