

# Performance Analysis of Contention Based Medium Access Control Protocols

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**Abstract**—We study the performance of contention based medium access control (MAC) protocols. In particular, we provide a simple and accurate method for estimating the throughput of IEEE 802.11 DCF and IEEE 802.11e EDCA. Our method is based on a rigorous analysis of the Markov chain associated with the back-off process at the contending nodes. Our results provide new insights into the operation of IEEE 802.11 DCF and IEEE 802.11e EDCA. Although we focus on IEEE 802.11 MAC protocol in this paper, the techniques developed are applicable to a wide variety of contention based MAC protocols.

## I. INTRODUCTION

Wireless local area networks (WLANs) based on the IEEE 802.11 standard are one of the fastest growing wireless access technologies in the world today. They provide an effective means of achieving wireless data connectivity in homes, public places and offices. The low-cost and high-speed WLANs can be integrated within the cellular coverage to provide hotspot coverage for high-speed data services, thus becoming an integral part of next generation wireless communication networks.

The fundamental access mechanism of IEEE 802.11 MAC is the Distributed Coordination Function (DCF). The DCF is a carrier sense multiple access protocol with collision avoidance (CSMA/CA). In addition to DCF, the IEEE 802.11 standard also defines an optional Point Coordination Function (PCF), which uses a central coordinator for assigning the transmission right to stations, thus guaranteeing a collision free access to the shared wireless medium. While DCF has gained enormous popularity and been widely deployed, the use of PCF has been rather limited.

Whereas the IEEE 802.11 standard was targeted at best-effort service for data transfer, it is expected that in the future WLANs will need to support a mix of QoS-sensitive, multimedia and interactive traffic, in addition to data traffic which is only sensitive to the throughput. Future WLANs must therefore provide service differentiation in order to better support the diverse QoS requirements of the applications running on them. A new standard, namely IEEE 802.11e, has been proposed for this purpose; it defines two new access mechanisms: EDCA (an enhancement to DCF), and HCF (an enhancement to PCF). Of the two, EDCA appears to be gaining more early acceptance.

In this paper we study the performance of contention based MAC protocols, with a specific emphasis on DCF and EDCA. There have been several previous works on the performance of DCF; these include simulation studies [1], [2] as well as analytical studies based on simplified models of DCF [3], [4], [5], [6], [7], [8]. Most of the analytical work is based on a decoupling approximation, first proposed by Bianchi in [3]; we henceforth refer to the simplified model with this decoupling assumption as Bianchi's model. More recently, there have also been several studies [9], [10], [11], [12], [13] evaluating the performance of EDCA, an earlier version of EDCA (see [14]). With the exception of [13], where the authors develop an extension of Bianchi's model for analyzing EDCA, all these studies are simulation based.

The main contribution of this paper is a novel technique for estimating the throughput and other parameters of interest of contention based MAC protocols. Our technique is based on a rigorous analysis of a Markovian model of the system, and does not require the decoupling assumption of Bianchi. In fact, through the insights it yields into the system dynamics, it provides intuitive justification of Bianchi's simplifying assumptions. The technique is easy to apply, and we use it to analyse the IEEE 802.11 DCF as well as IEEE 802.11e EDCA. It can easily be applied to other MAC protocols as well. We now briefly sketch the key ideas behind our approach.

A common feature of all contention based MAC protocols is the concept of *back-off stage* for a station. The stations can be in different back-off stages; the back-off stage for a station depends on the number of collisions that it has encountered since its last successful transmission (and, possibly, other information) and can be thought of as its estimate of the current level of contention at all stations. The stations use this estimate to control their access probabilities. The key observation we make in this paper is that, when the number of stations is large, the Markov chain associated with the back-off process stays close to what we call a *typical state*, which can be obtained as the unique *equilibrium* point of the drift equations associated with the back-off process. We can obtain quite accurate estimates of the throughput and other parameters of interest by assuming the system to be in this typical state at all times.

We find that the accuracy of the throughput estimates obtained using our technique is about the same as those obtained using Bianchi's analysis. But, in addition, we are able to provide some key insights about the system dynamics; in fact, our results provide a justification for Bianchi's approximation, which may be of separate interest.

The rest of the paper is organized as follows. We provide a brief description of DCF and EDCA, and discuss some related work, in the next section. Our technique for performance evaluation is discussed in the context of DCF in section III. An extension of our technique in the context of EDCA is discussed in Section IV. Some concluding remarks are presented in Section V. Due to space constraints, all technical details and proofs have been omitted. They can be found in [15].

## II. DCF, EDCA, AND RELATED WORK

In this section, we provide a brief description of DCF and EDCA, and discuss some related work in the literature. We start with a description of DCF.

### A. IEEE 802.11 DCF

The DCF is a Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) MAC protocol. The collision avoidance scheme of DCF is based on the binary exponential back-off (BEB) scheme [16], [17]. The DCF defines two access mechanisms for packet transmissions: *basic access mechanism*, and *RTS/CTS access mechanism*.

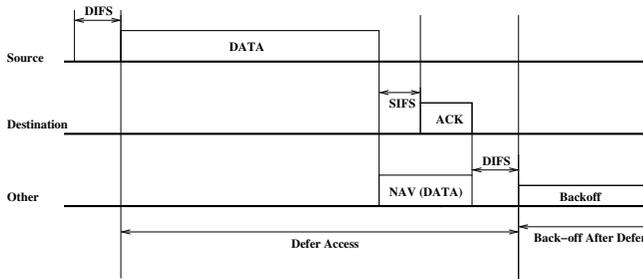


Fig. 1. Basic Access Method.

In the basic access mechanism (see Figure 1), any station, before transmitting a DATA frame, senses the channel for a duration of time equal to the Distributed Interframe Space (DIFS) to check if it is idle. If the channel is determined to be idle, the station starts the transmission of a DATA frame. All stations which hear the transmission of the DATA frame set their Network Allocation Vector (NAV) to the expected length of the transmission, as indicated in the Duration/ID field of the DATA frame. This is called the *virtual carrier sensing* mechanism. The channel is considered to be busy if either the physical carrier sensing or the virtual carrier sensing indicates so, and in that case, the station enters into a wait period determined by the back-off procedure to be explained later. Upon successful reception of the DATA frame, the destination station waits for a SIFS interval following the DATA frame, and then sends an ACK frame back to the source station, indicating successful reception of the DATA frame.

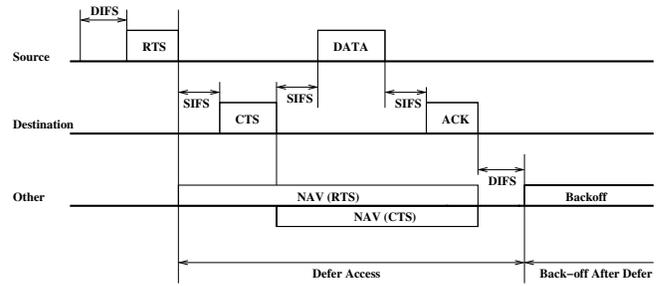


Fig. 2. RTS/CTS Access Method.

The RTS/CTS access mechanism uses a four-way handshake in order to reduce bandwidth loss due to the hidden terminal problem (see, for example, [18]). A station that wishes to send a DATA frame first senses the channel for a DIFS duration. If the channel is determined to be idle, then a RTS frame is sent to the destination. Otherwise, the back-off algorithm is triggered after the end of the current transmission and a further DIFS interval. Upon successful transmission of the RTS frame, the destination waits for a SIFS interval, and then sends a CTS frame back to the source. The source can start sending the DATA frame a SIFS interval after the reception of the CTS frame. As in the basic access mechanism, upon successful reception of the DATA frame, the destination waits for a SIFS interval, and then sends an ACK frame back to the source. A station that hears either the RTS, CTS, or DATA frame updates its NAV based on the Duration/ID field of the corresponding frame (see Figure 2). The four way handshake prevents any DATA-DATA collisions that might occur due to the hidden terminal problem. Since the RTS and CTS frames are very small in size, the RTS/CTS access scheme significantly reduces bandwidth loss due to collisions.

The back-off procedure is implemented by means of the *back-off counter* and *back-off stages*. Initially, upon receiving a new frame to be transmitted, the station starts in back-off stage 0, window ( $CW$ ) size set to  $CW_{min}$ . Following an unsuccessful transmission attempt (collision), the back-off stage is incremented by 1 and the contention window size is doubled until the maximum size of the contention window,  $CW_{max}$ , is reached, after which the back-off stage and the contention window size remain unchanged on subsequent collisions. The back-off window size as well as the back-off stage are set back to their initial values of  $CW_{min}$  and 0 after a successful transmission attempt or if the *retry count* limit for the frame is reached. At the start of each back-off stage, the back-off counter is set to an integer chosen uniformly at random between zero and the value  $CW - 1$  of the contention window for the current back-off stage. The back-off counter is decremented by 1 in every subsequent slot, as long as the channel is sensed idle in that slot. (Here, a slot is an interval of fixed duration specified by the protocol.) If a transmission by some other station is detected, then the station freezes its back-off counter, and resumes its count from where it left off after the end of the transmission plus an additional DIFS interval.

When the back-off counter reaches 0, the station transmits\*.

The scheme described above treats all stations equally. The IEEE 802.11e task group (TGe) is working on an extension to the current IEEE 802.11 standard in order to provide QoS in WLANs via service differentiation. We now describe the enhanced distributed channel access (EDCA) mechanism, which is an extension of the DCF mechanism, as specified in the most recent draft [20] published by the TGe.

### B. IEEE 802.11e EDCA

The EDCA has been designed from the perspective of providing QoS in WLANs. The notion of an *access category* (AC) is used to achieve service differentiation. The EDCA defines four different ACs, each maintaining its own channel access function (an enhanced variant of the DCF). Each AC on a station contends for a transmission opportunity (TXOP) with its own set of EDCA parameters. A TXOP is the interval of time when a station has the right to initiate transmissions. The main differences between the EDCA and DCF are:

- 1) The minimum specified idle duration time, called the *arbitration inter frame space* (AIFS), is not a constant value (DIFS) as in the DCF. Each AC has its own value for the AIFS, denoted AIFS[AC], which is calculated as follows:

$$AIFS[AC] = SIFS + AIFSN[AC],$$

where AIFSN denotes the arbitration inter frame space number and is different for different ACs (see Table I).

- 2) The contention window limits,  $CW_{min}$ <sup>†</sup> and  $CW_{max}$ , are different for different ACs (see Table I).

The collisions between the contending channel access functions within a station are resolved within the station such that the data frames from the higher priority AC receives the TXOP and the data frames from the lower priority colliding AC(s) behave as if there were an external collision on the wireless medium. The motivation is to provide low delay and high throughput to multimedia and other real time traffic. Note that EDCA does not provide any service guarantees, but it establishes a probabilistic priority mechanism to allocate bandwidth based on the access categories. In Section IV, we develop and analyze a simple model for EDCA.

### C. Related Work

One of the earliest analyses of the throughput of DCF was carried out in [4] using a greatly simplified back-off model, namely that the back-off counter value is geometrically distributed with *constant* parameter  $p$ , irrespective of the current back-off stage of the station. A more realistic model was proposed in the seminal paper of Bianchi [3].

\*As in [3] and majority of the related literature, in our analysis, we ignore the facts that (i) the back-off procedure is not invoked immediately after a successful transmission or during the transmission of the first data packet, and (ii) the back-off counter is not decremented if the channel is sensed to be busy. For a more accurate model of the back-off procedure, we refer the reader to [19].

<sup>†</sup>The parameters  $aCW_{min}$  and  $aCW_{max}$  depend on the physical layer.

TABLE I  
DEFAULT EDCA PARAMETER SET [20]

Category (AC)/Info.	$CW_{min}$	$CW_{max}$	AIFSN
AC_BK/Background	$aCW_{min}$	$aCW_{max}$	7
AC_BE/Best Effort	$aCW_{min}$	$aCW_{max}$	3
AC_VI/Video	$aCW_{min}/2$	$aCW_{min}$	2
AC_VO/Voice	$aCW_{min}/4$	$aCW_{min}/2$	2

Here, the evolution of the back-off stage at each node is described by a Markov process; the Markov chains at different nodes evolve independently, but in an *environment* specified by the collision probability  $p$  for any transmission attempt. The parameter  $p$  is a constant derived from the *mean* transmission probability in the associated Markov chains. This formulation leads to a fixed point equation for  $p$ . Note that the model is analogous to mean-field models in statistical physics; the only interaction between the Markov processes at different nodes is through the parameter  $p$ , which represents a mean value of the environment. It is not a goal in [3] to provide a rigorous justification of the mean-field assumption. The assumption is justified through simulations, which show that the model predictions are quite accurate.

Several subsequent studies have built on the work of Bianchi. In [7], the authors obtain similar fixed point equations using the same decoupling assumption but without the Markovian assumptions of Bianchi; extensions of this fixed point formulation are studied in [8].<sup>‡</sup> In [6], the authors present an approximate delay analysis based on Bianchi's model, and also extend the model to account for channel errors.

Recently, Proutiere et al. [21] have shown that the mean field analysis of Bianchi is asymptotically (in the infinite station limit) accurate. In particular, they have used ideas from the theory of propagation of chaos to show that Bianchi type decoupling holds asymptotically as number of stations is allowed to increase to infinity.

Several works have evaluated the performance of EDCA, an earlier version of EDCA (see [14]). Most of these have employed simulation [9], [10], [11], [12]. An exception is [13], where the authors use an extension of Bianchi's model to analyze the performance of IEEE 802.11e MAC protocol. More recently, the performance of EDCA has been analyzed in [22], [23], using theoretical models based on Bianchi type assumptions.

Our approach differs fundamentally from the work described above in that we do not make the decoupling assumption introduced by Bianchi, and common to all of them except [21]. Instead, starting from a Markov chain description that explicitly takes into account the interactions between stations, we show that in a large system, namely one with a large

<sup>‡</sup>In order to avoid confusion arising from the superficially similar terminology, we emphasize that the *fixed points* we talk of in this work are different from the fixed points in [3], [7], [8]. Their fixed points are for the 1-dimensional coupling parameter  $p$ ; our fixed points are for the  $n$ -dimensional state descriptor in a joint Markovian representation of the back-off stages at all  $n$  stations. The details are provided in the next section.

number of stations, the Markov chain converges to a *typical* state. Thus, one can approximate the collision probability seen by any single station by that seen in the typical state. Our analysis therefore provides a rigorous justification for Bianchi’s model, which has been the basis of much subsequent work. In addition, it provides an alternative approach to performance analysis of MAC protocols; performance measures of interest can be derived directly from analysis of the typical state. We validate this approach by showing that the performance predictions thus obtained are close to those seen in simulations, both for the IEEE 802.11 DCF and IEEE 802.11e EDCA protocols.

Finally, we point out that we focus on DCF and EDCA protocols in this paper because they are likely to be the two most widely deployed wireless MAC protocols in the near future; however, we do not specifically advocate their use. Several works (see, for example, [24], [25], [26], and the references therein) have identified the limitations of these protocols, and proposed alternative MAC protocols that can provide better performance. The techniques developed in this paper are very general, and can be applied to evaluate the performance of these alternative MAC protocols as well.

### III. PERFORMANCE EVALUATION OF IEEE 802.11 DCF

In this section, we present a performance analysis of DCF. We start with a description of our model.

#### A. The Model

We consider a wireless LAN with  $n$  stations employing the IEEE 802.11 DCF. Every station can hear every other station in the network, i.e., there are no hidden stations. Our discussion covers both *ad hoc networks*, where there is no central access point (AP) through which all the traffic must pass, as well as *infrastructure based networks*, where an AP connects the wireless network with the wired infrastructure. In order to simplify the analysis, we assume, in common with most related work, that all stations always have a packet to send. The throughput obtained under such saturation conditions is commonly referred to as the *saturation throughput*. In some cases (see, for example, [27]), it can be shown that the queues at all the nodes are stable if the arrival rate at each node is less than the saturation throughput.

We make the following additional assumptions:

- **(A1)** The back-off durations are geometrically distributed, i.e., when a station is in back-off stage  $i$ , it makes a transmission attempt in the next slot with a probability  $p_i$ . In order to maintain the same average waiting time as in the IEEE 802.11 DCF, we set  $p_i = \frac{2}{W_i+1}$ , where  $W_i$  is the contention window size in back-off stage  $i$ .
- **(A2)** The back-off stage is reset to 0 only after a successful transmission, i.e., the retry count limit, as defined in Section II-A, is infinite. This assumption is not necessary for our analysis, but simplifies the exposition considerably.

All stations use the same back-off parameters. There are  $M+1$  back-off stages, labeled 0 to  $M$ . We adopt a discrete time model indexed by the slot number  $t$ . To avoid confusion,

note that the term “slot” in our usage refers to a different quantity from the slot in the IEEE 802.11 protocol description. We use the term to denote the time period at the end of which stations may modify their back-off counters. In particular, the duration of a slot is not a fixed physical layer parameter, but varies depending on whether it represents an idle slot, a successful transmission or a collision.

The state of the system at time  $t$  can be represented by a vector  $X_n(t) = (X_{n0}(t), \dots, X_{nM}(t))$  denoting the number of stations in each of the back-off stages 0 through  $M$ . It is easy to see that  $X_n(t)$ ,  $t = 0, 1, \dots$  forms an irreducible and aperiodic Markov chain on the state space

$$S_n \triangleq \left\{ x \in \mathbb{Z}^{M+1} : \sum_{i=0}^M x_i = n; x_i \geq 0 \text{ for all } i \right\}.$$

In principle, one could solve for the stationary distribution of  $X_n(t)$  and thereby obtain parameters of interest about the system. However, the number of states,  $n^{M+1}$ , is too large to make this feasible for systems of practical interest. The key insight we provide in this paper is that, when  $n$  is large (and exact computation expensive), the Markov chain  $X_n(t)$  stays close to what we call a *typical state*. Moreover, accurate estimates of various parameters such as throughput can be obtained by assuming that  $X_n(t)$  is in this typical state at all times.

We remark for purposes of comparison that Bianchi [3] models the system as a Markov chain with (typically) an even larger state space of size  $(M+1)^n$  by considering the back-off stage at each station. The analysis is simplified by replacing this  $n$ -dimensional Markov chain by  $n$  1-dimensional Markov chains (with  $M+1$  states each) which are *assumed* to be conditionally independent, conditional on the collision probability  $p$ . We do not make any such independence assumptions.

We now proceed with the analysis of the Markov chain  $X_n(t)$ . Let us look at the expected change in  $X_n(t)$  over one time slot. For  $x^{(n)} \in S_n$ , let

$$\begin{aligned} f^{(n)}(x^{(n)}) &\triangleq \mathbb{E}\{X_n(t+1) - X_n(t) | X_n(t) = x^{(n)}\} \\ &= \sum_{l: x^{(n)}+l \in S_n} l P_l^{(n)}(x^{(n)}), \end{aligned}$$

where  $P_l^{(n)}(x^{(n)})$  is the probability of making a transition from  $x^{(n)}$  to  $x^{(n)} + l$  over one time slot. We now compute  $f_i^{(n)}(x^{(n)})$  for  $i \in \{0, 1, \dots, M\}$ .

First consider  $i = 0$ . Let  $I(x^{(n)}) \triangleq \prod_{i=0}^M (1-p_i)^{x_i^{(n)}}$ , where  $p_i$  denotes the transmission probability for a station in back-off stage  $i$ . Note that  $I(x^{(n)})$  is the probability of an idle slot when the system is in state  $x^{(n)}$ . The following events can result in a change in the number of stations in back-off stage 0:

- A successful transmission by a station in back-off stage  $i$ ,  $i \in \{1, 2, \dots, M\}$ , resulting in an increase in the number of stations in back-off stage 0.
- An unsuccessful transmission attempt by one or more stations in back-off stage 0, resulting in a decrease in the

number of stations in back-off stage 0.

For the former event to occur, the station itself must transmit and no other station in the network should transmit; this has probability  $\frac{p_i}{1-p_i}I(x^{(n)})$ . Noting that there are  $x_i^{(n)}$  stations in the back-off stage  $i$  to choose from, and summing over  $i$ , we obtain

$$\sum_{i=1}^M x_i^{(n)} p_i \frac{I(x^{(n)})}{1-p_i} \quad (1)$$

to be the expected increase in the number of stations in back-off stage 0 due to successful transmissions by stations in other back-off stages. Likewise, we find

$$x_0^{(n)} p_0 \left(1 - \frac{I(x^{(n)})}{1-p_0}\right) \quad (2)$$

to be the expected decrease in the number of stations in back-off stage 0 due to unsuccessful transmission attempts by stations in back-off stage 0. Combining Eqs.(1) and (2), we obtain

$$f_0^{(n)}(x^{(n)}) = \sum_{i=0}^M x_i^{(n)} p_i \frac{I(x^{(n)})}{1-p_i} - x_0^{(n)} p_0. \quad (3)$$

Next, let  $i \in \{1, 2, \dots, M-1\}$ . We now need to consider the following events:

- A transmission attempt by a station in back-off stage  $i$ .
- An unsuccessful attempt by a station in back-off stage  $i-1$ .

A station in back-off stage  $i$  attempts to transmit with probability  $p_i$ , following which, it either moves to back-off stage 0 (successful transmission) or to back-off stage  $i+1$  (collision). Thus, the expected decrease in the number of stations in back-off stage  $i$  at time  $t$  is

$$x_i^{(n)} p_i. \quad (4)$$

A station in back-off stage  $i-1$  transmits with probability  $p_{i-1}$  and moves to back-off stage  $i$  if it suffers a collision, i.e., if one or more others station in the network also transmit, which happens with probability  $\left(1 - \frac{I(x^{(n)})}{1-p_{i-1}}\right)$ . Thus,

$$x_{i-1}^{(n)} p_{i-1} \left(1 - \frac{I(x^{(n)})}{1-p_{i-1}}\right). \quad (5)$$

is the expected increase in the number of stations in back-off stage  $i$  due to unsuccessful transmission attempts by stations in back-off stage  $i-1$ . Combining Eqs.(4) and (5), we obtain

$$f_i^{(n)}(x^{(n)}) = x_{i-1}^{(n)} p_{i-1} \left(1 - \frac{I(x^{(n)})}{1-p_{i-1}}\right) - x_i^{(n)} p_i \quad (6)$$

for  $i \in \{1, 2, \dots, M-1\}$ .

Finally, let  $i = M$ . In this case, we need to consider the following events:

- A successful transmission attempt by a station in back-off stage  $M$ .
- An unsuccessful transmission attempt by a station in back-off stage  $M-1$ .

A station in back-off stage  $M$  transmits with probability  $p_M$

and, if no other station in the network transmits, an event of probability  $\frac{I(x^{(n)})}{1-p_M}$ , then the station moves to back-off stage 0; otherwise it stays in the back-off stage  $M$ . The expected decrease in the number of stations in back-off stage  $M$  at time  $t$  due to a successful transmission is thus:

$$x_M^{(n)} p_M \frac{I(x^{(n)})}{1-p_M}. \quad (7)$$

A station in back-off stage  $M-1$  transmits with a probability  $p_{M-1}$  and, if at least one other station in the network also transmits, an event of probability  $\left(1 - \frac{I(x^{(n)})}{1-p_{M-1}}\right)$ , then the station enters into back-off stage  $M$ . The expected increase in the number of stations in back-off stage  $M$  at time  $t$  due to collisions is thus

$$x_{M-1}^{(n)} p_{M-1} \left(1 - \frac{I(x^{(n)})}{1-p_{M-1}}\right). \quad (8)$$

Combining Eqs.(7) and (8), we obtain

$$f_M^{(n)}(x^{(n)}) = x_{M-1}^{(n)} p_{M-1} \left(1 - \frac{I(x^{(n)})}{1-p_{M-1}}\right) - x_M^{(n)} p_M \frac{I(x^{(n)})}{1-p_M}. \quad (9)$$

Let

$$B_n \triangleq \{x \in \mathbb{R}^{M+1} : \sum_{i=0}^M x_i = n; x_i \geq 0\},$$

and  $E \triangleq B_n/n$ . Let  $f^{(n)} : \mathbb{R}^{M+1} \rightarrow \mathbb{R}^{M+1}$  be the function with components  $f_i^{(n)}$  specified by Eqs.(3), (6), and (9). It is essentially the one-step drift of the Markov chain  $X_n(t)$ . We have so far defined the function  $f^{(n)}(x)$  for  $x \in S_n$  only; we now extend the definition of  $f^{(n)}$  to  $x \in B_n$  by using the same equations on the extended domain.

In [15], we analyze an appropriately scaled version,  $Y_n(t) = X_n(\lfloor nt \rfloor)/n$ , of the process  $X_n(t)$  for  $n = 1, 2, \dots$ , and show that for all  $t \geq 0$ , it satisfies:

$$\lim_{n \rightarrow \infty} \sup_{0 \leq s \leq t} \|Y_n(s) - Y(s)\| = 0 \text{ a.s.},$$

where  $Y(t)$  is a deterministic process given by the unique solution of the differential equation

$$\frac{dY(t)}{dt} = F(Y(t)) \text{ for } t \geq 0,$$

with initial condition  $y^0 = \lim_{n \rightarrow \infty} Y_n(0) = X(0)/n$ , where  $F(x) = \lim_{n \rightarrow \infty} f^{(n)}(nx)$  for  $x \in E$ . In words, we prove a functional ‘law of large numbers’ limit theorem for the process  $Y_n(\cdot)$ . We also show that the error involved in approximating  $X_n(t)$  with  $nY(t/n)$  is (almost surely)  $O(n^\beta)$  for all  $\beta > 1/2$ .

In [15], we also show that the equation  $F(x) = 0$  has a unique solution. For  $M = 1$ , we further show that  $Y(t)$  converges to  $x$  from all possible initial states. We conjecture that such a result holds for all  $M$  (as observed in our simulations).

In view of the results in [15], we can expect that, for large  $t$ , the process  $X_n(t)$  remains close to the unique point  $x^{(n)} \in B_n$  satisfying  $f^{(n)}(x^{(n)}) = 0$ , which will henceforth be referred

to as the equilibrium point of the system.

### B. Throughput Calculation

We now estimate the throughput of IEEE 802.11 DCF, assuming that the system stays close to its equilibrium point  $x^{(n)}$  at all times. Let

- $T \triangleq$  The normalized throughput of the system.
- $P_c \triangleq$  The conditional collision probability.
- $I \triangleq$  The probability of an idle slot in state  $x^{(n)}$ .
- $P \triangleq$  The payload duration<sup>§</sup>.
- $T_c \triangleq$  The average time the channel is sensed busy during a collision.
- $T_s \triangleq$  The average time the channel is sensed busy because of a successful transmission.
- $\sigma \triangleq$  The duration of an idle slot.

Note that some of the above defined quantities may vary with  $n$ . For the sake of brevity, we do not make explicit this dependence.

To calculate the throughput, observe that a station in back-off stage  $i$ , transmits with a probability  $p_i$ , and the transmission is successful if no other station in the network transmits, an event of probability

$$\frac{I}{1 - p_i}.$$

Since there are  $x_i^{(n)}$  stations in back-off stage  $i$ , the probability that a station in back-off stage  $i$  transmits successfully is

$$x_i^{(n)} p_i \frac{I}{1 - p_i}.$$

Summing over all possible back-off stages, we obtain the probability of a successful transmission to be

$$\sum_{i=0}^M x_i^{(n)} p_i \frac{I}{1 - p_i}.$$

Since the probability that at least one station transmits in a given slot is  $1 - I$ , we have

$$P_c = 1 - \frac{\sum_{i=0}^M x_i^{(n)} p_i \frac{I}{1 - p_i}}{1 - I} \quad (10)$$

The normalized throughput of the system can be expressed as

$$T = \frac{\text{Expected Payload duration per slot}}{\text{Slot duration}}. \quad (11)$$

The expected payload duration per slot is  $(1 - I)(1 - P_c)P$ . The expected duration of a slot is readily obtained considering that, with a probability  $I$  a slot is idle; with a probability of  $(1 - I)(1 - P_c)$  it contains a successful transmission, and with a probability of  $(1 - I)P_c$  it contains a collision. And plugging this is Eq.(5),we obtain

$$T = \frac{(1 - I)(1 - P_c)P}{(1 - I)(1 - P)T_s + (1 - I)P_c T_c + I\sigma} \quad (12)$$

<sup>§</sup>In this paper, we consider the payload duration to be fixed. Variable payload duration can also be analyzed as in [3].

TABLE II  
IEEE 802.11 DSSS PHY PARAMETER SET [28] AND OTHER  
PARAMETERS USED TO OBTAIN NUMERICAL RESULTS

PARAMETER	VALUE
Basic Bit Rate (BBR)	1Mb/s
Bit Rate (BR)	11Mb/s
PHY Header (PH)	192 bits
MAC Header (MH)	272 bits
H	PH/BBR + MH/BR
ACK	112/BR + PH/BBR
RTS	160/BR + PH/BBR
CTS	112/BR + PH/BBR
Propagation Delay ( $\delta$ )	$1\mu\text{s}$
SIFS	$10\mu\text{s}$
Slot Time ( $\sigma$ )	$20\mu\text{s}$
DIFS	$50\mu\text{s}$

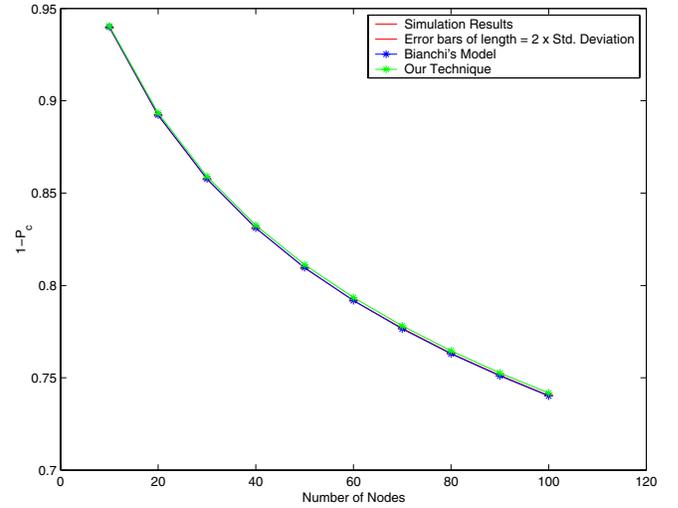


Fig. 3. Success probability  $(1 - P_c)$  for  $M = 5$  and  $W_0 = 128$ .

The values of  $T_c$  and  $T_s$  depend on the access mechanism being used. Let  $\delta$  be the propagation delay, then one can readily obtain (for details, see [3])

$$\begin{aligned} T_s^{rts} &= RTS + CTS + H + P + ACK + \\ &\quad 3SIFS + 4\delta + DIFS \\ T_c^{rts} &= RTS + DIFS + \delta \\ T_s^{bas} &= H + P + ACK + SIFS + 2\delta + DIFS \\ T_c^{bas} &= H + P + DIFS + \delta \end{aligned} \quad (13)$$

where  $T_c^{rts}$  (correspondingly,  $T_c^{bas}$ ) and  $T_s^{rts}$  (correspondingly,  $T_s^{bas}$ ) represent the  $T_c$  and  $T_s$  values for the RTS/CTS based access (correspondingly, basic access) mechanism, respectively; the parameters  $RTS$ ,  $CTS$ ,  $H$ ,  $ACK$ ,  $DIFS$ , and  $SIFS$  are all physical layer dependent. We will use the values of these parameters as defined in the DSSS PHY (see Table II).

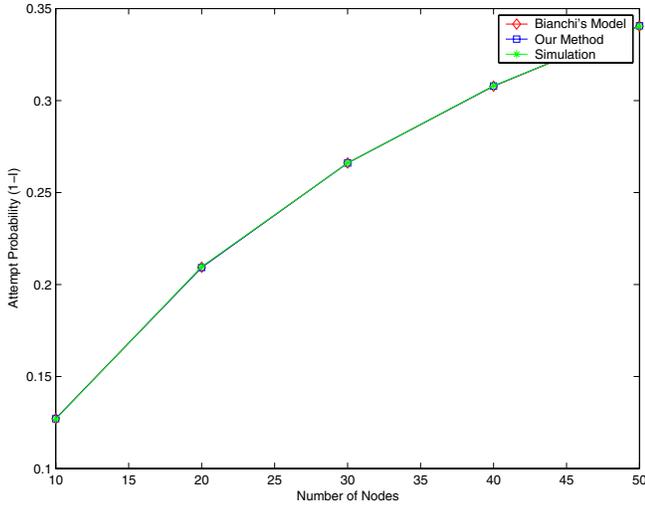


Fig. 4. Attempt probability ( $1 - I$ ) for  $M = 5$  and  $W_0 = 128$ .

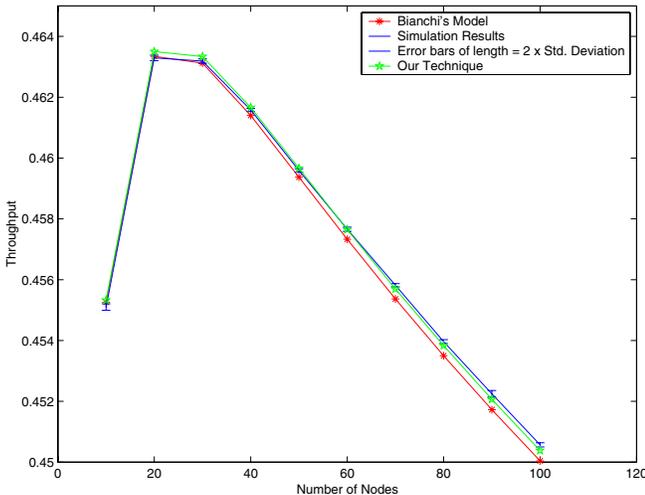


Fig. 5. Throughput ( $T$ ) for  $M = 5$  and  $W_0 = 128$ .

### C. Performance Comparison

We have performed extensive simulations with different values of  $M$  and  $W_0$ . The simulation results match extremely well with the numerical results obtained using our technique and Bianchi's model. The results for the RTS/CTS access mechanism with  $M = 5$  and  $W_0 = 128$  are shown in Figures 3-5. As is evident in these figures (error bars are barely visible), the variation of results across various simulation runs is quite small, thereby showing the high confidence level of the simulation results. An interesting thing to note is that although our technique and Bianchi's model are fundamentally different, they both result in (roughly) the same fixed point (in terms of  $P_c$  and  $I$ ), and correspondingly, the estimates of throughput obtained using the two techniques are very close. Similar results have been obtained for the basic access mechanism as well. An interesting special case ( $M = 1$ ) in

which the exact throughput of DCF can easily be calculated is discussed in the Appendix.

## IV. PERFORMANCE EVALUATION OF IEEE 802.11E EDCA

In this section, we analyze a simplified version of the IEEE 802.11e EDCA mechanism [20]. We start with a description of our model.

### A. The Model

We consider a WLAN with  $n$  stations using the IEEE 802.11e EDCA protocol. Each station can hear every other station in the network, i.e., there are no hidden stations. Our discussion covers both *ad hoc networks* and *infrastructure based networks*. There are  $K$  different access categories (ACs), each maintaining its own set of back-off parameters. Each station maintains a separate transmit queue for each AC. All queues are assumed to be saturated, i.e., they always have a packet to send. We make the following additional assumptions:

- **(A1)** The back-off durations are geometrically distributed, i.e., the type- $k$  AC at a station, when in back-off stage  $j$ , transmits with probability  $p_{k,j}$ . In order to keep the average waiting time the same as in IEEE 802.11e EDCA, we set  $p_{k,j} = \frac{2}{W_{k,j}+1}$ , where  $W_{k,j}$  is the contention window size of the type- $k$  AC in back-off stage  $j$ .
- **(A2)** The back-off stage is reset to 0 only after a successful transmission.
- **(A3)** The minimum idle duration time is the same,  $DIFS$ , for all ACs.
- **(A4)** The internal collision avoidance mechanism (see section II-B) is not used. Note that the throughput under this assumption provides a lower bound on the actual saturation throughput. It is possible to carry out the analysis without this assumption but, for ease of exposition, we do not do so.

Let  $M_k + 1$ ,  $W_{k,0}$ , denote the number of back-off stages and minimum contention window size, respectively, for the type- $k$  AC,  $k \in \{1, 2, \dots, K\}$ . Let  $X_{k,j}(t)$ ,  $j \in \{0, 1, \dots, M_k\}$ , denote the number of type- $k$  ACs in back-off stage  $j$  at time  $t$ . Let

$$\mathcal{S} \triangleq \{(k, j) : k \in \{1, 2, \dots, K\}; j \in \{0, 1, \dots, M_k\}\}$$

and  $M = \sum_{k=0}^K (M_k + 1)$ . Then  $X_n(t) = \{X_{k,j}(t)\}_{(k,j) \in \mathcal{S}}$ , represents the state of the system at time  $t$ . Clearly,  $X_n(t)$  for  $t = 0, 1, \dots$ , is a Markov chain on  $\{0, 1, \dots, n\}^M$ , and satisfies:

$$\sum_{j=0}^{M_k} X_{k,j}(t) = n \text{ for } k \in \{1, 2, \dots, K\}.$$

It can easily be shown that the Markov chain  $X_n(t)$  is irreducible (see [7, Theorem 8.1], for a similar proof). Since it has only finitely many states, it follows that  $X_n(t)$  is positive recurrent and possesses a stationary distribution. However, it does not appear possible to obtain a closed form expression for the stationary distribution of  $X_n(t)$ . Therefore, we proceed as in the previous section.

Let  $\mathbb{Z}_+$  denote the set of non-negative integers, and let

$$S_n \triangleq \left\{ x = \{x_{k,j}\} : x_{k,j} \in \mathbb{Z}_+, \sum_{j=0}^{M_k} x_{k,j} = n, 1 \leq k \leq K \right\}.$$

We denote the one-step drift of  $X_n(t)$  by

$$\begin{aligned} f^{(n)}(x^{(n)}) &\triangleq \mathbb{E}\{X_n(t+1) - X_n(t) | X_n(t) = x^{(n)}\} \\ &= \sum_{l: x^{(n)}+l \in S_n} l P_l^{(n)}(x^{(n)}), \end{aligned}$$

for  $x^{(n)} \in S_n$ ; here  $P_l^{(n)}(x^{(n)})$  is the probability of making a transition from  $x^{(n)}$  to  $x^{(n)} + l$  over one time slot. Set  $f^{(n)}(x^{(n)}) = 0$  for  $x^{(n)} \notin S_n$ . Arguing as in Section III-A, we obtain for  $k \in \{1, 2, \dots, K\}$ :

$$\begin{aligned} f_{k,0}^{(n)}(x^{(n)}) &= \sum_{j=0}^{M_k} x_{k,j}^{(n)} p_{k,j} \frac{I(x^{(n)})}{1 - p_{k,j}} - x_{k,0}^{(n)} p_{k,0}, \\ f_{k,j}^{(n)}(x^{(n)}) &= x_{k,j-1}^{(n)} p_{k,j-1} \left(1 - \frac{I(x^{(n)})}{1 - p_{k,j-1}}\right) - x_{k,j}^{(n)} p_{k,j}, \\ &\quad j \in \{1, 2, \dots, M_k - 1\}, \\ f_{k,M_k}^{(n)}(x^{(n)}) &= x_{k,M_k-1}^{(n)} p_{k,M_k-1} \left(1 - \frac{I(x^{(n)})}{1 - p_{k,M_k-1}}\right) \\ &\quad - x_{k,M_k}^{(n)} p_{k,M_k} \frac{I(x^{(n)})}{1 - p_{k,M_k}} \end{aligned} \quad (14)$$

where  $I(x^{(n)}) = \prod_{(k,j) \in \mathcal{S}} (1 - p_{k,j})^{x_{k,j}^{(n)}}$ . Let

$$B_n \triangleq \left\{ x = \{x_{k,j}\} : \sum_{j=0}^{M_k} x_{k,j} = n, 1 \leq k \leq K; x_{k,j} \geq 0 \right\},$$

and  $E \triangleq B_n/n$ .

The results derived in [15] for IEEE 802.11 DCF can easily be extended to IEEE 802.11e EDCA. In particular, under a similar set of assumptions, we can show that the sequence of scaled stochastic processes

$$Y_n(t) = X_n(\lfloor nt \rfloor)/n, \text{ for } n = 1, 2, \dots,$$

converges (in the same sense, and with the same error bounds, as discussed for DCF earlier) to the deterministic limit  $Y(t)$  given by the unique solution of the differential equation

$$\frac{dY(t)}{dt} = F(Y(t)) \text{ for } t \geq 0,$$

with initial condition  $y^0 = \lim_{n \rightarrow \infty} Y_n(0) = X(0)/n$ , and  $F(x) = \lim_{n \rightarrow \infty} f^{(n)}(nx)$  for  $x \in E$ . The only difference from the DCF case earlier is that  $f^{(n)}$  is now given by Eq.(14) instead of by Eqs.(3), (6), and (9)

Likewise, following the line of analysis in [15], it can be shown that here is a unique point  $x \in E$  that satisfies  $F(x) = 0$ ; we call it the equilibrium point. Using the intuition that for large  $t$ , the process  $X_n(t)$  should remain close to the point  $x^{(n)} \in B_n$  satisfying  $f^{(n)}(x^{(n)}) = 0$ , we obtain a throughput analysis of IEEE 802.11e EDCA.

## B. Throughput Calculation

We now estimate the throughput of IEEE 802.11e EDCA, assuming that the system stays fixed at its equilibrium point  $x^{(n)}$  at all times. Let  $P_c, I$ , and  $T$  be as defined in Section III-B. Further, let

- $T(k) \triangleq$  The normalized throughput of type- $k$  AC.
- $P_c(k) \triangleq$  The conditional collision probability for type- $k$  AC.
- $T \triangleq$  The normalized throughput of the system.

Arguing as in the derivation of (10), for  $k \in \{1, 2, \dots, K\}$ , we obtain

$$P_c(k) = 1 - \frac{\sum_{j=0}^{M_k} x_{k,j} p_{k,j} \frac{I}{1 - p_{k,j}}}{1 - I}$$

and

$$T(k) = \frac{(1 - I)(1 - P_c(k))P}{(1 - I)(1 - P_c)T_s + (1 - I)P_c T_c + I\sigma} \quad (15)$$

where

$$P_c = 1 - \frac{\sum_{(k,j) \in \mathcal{S}} x_{k,j} p_{k,j} \frac{I}{1 - p_{k,j}}}{1 - I}$$

Summing  $T(k)$  over  $k \in \{1, 2, \dots, K\}$ , we obtain

$$T = \sum_{k=0}^K \frac{(1 - I)(1 - P_c(k))P}{(1 - I)(1 - P_c)T_s + (1 - I)P_c T_c + I\sigma} \quad (16)$$

Note that the values of  $T_s$  and  $T_c$  depend on the access mechanism in use (see (13)).

## C. Performance Comparison

In this section, we compare the throughput estimates obtained using our technique with the ones obtained using simulations. We consider the parameter values given in Table II-B, with  $aCW_{min} = 128$ . We have performed extensive simulations under three different settings:

- (A) *AIFSN* is set to  $\{2, 2, 2, 2\}$  and internal collision resolution is disabled.
- (B) *AIFSN* is set to  $\{2, 2, 2, 2\}$  and internal collision resolution is enabled.
- (C) *AIFSN* is set to  $\{7, 3, 2, 2\}$  and internal collision resolution is enabled.

Note that the scenario considered in setting (A) is the one that is captured by the analytical model. By comparing the results obtained under settings (A) and (B), we can determine the effect of internal collision resolution on the throughput of ACs. Likewise, by comparing the results obtained under settings (B) and (C), the effect of variable *AIFS* on the throughput of ACs can be singled out.

The results are shown in Figures 6-10. Note that the  $y$ -axis is broken in Figures 8-10 in order to make the differences between the curves more visible; the theoretical predictions in these cases match the simulation results very closely. Note that the error bars are barely visible, showing the high confidence level of the simulation results. As expected, we find that the throughput estimates obtained using our technique match very well with the simulated results under setting (A).

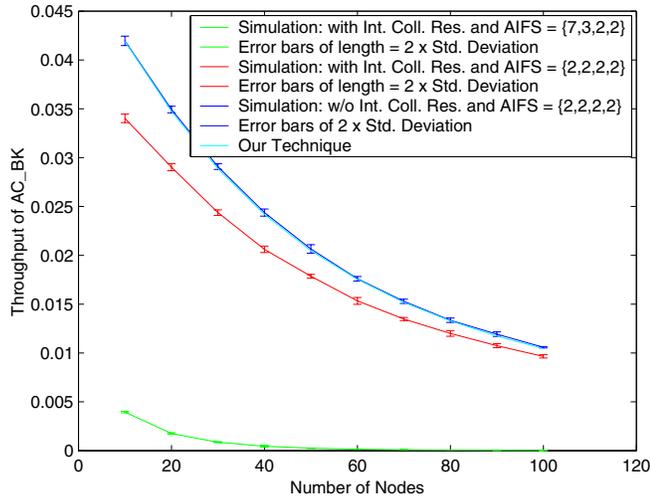


Fig. 6. Throughput of AC\_BK for  $aCW_{min} = 128$ .

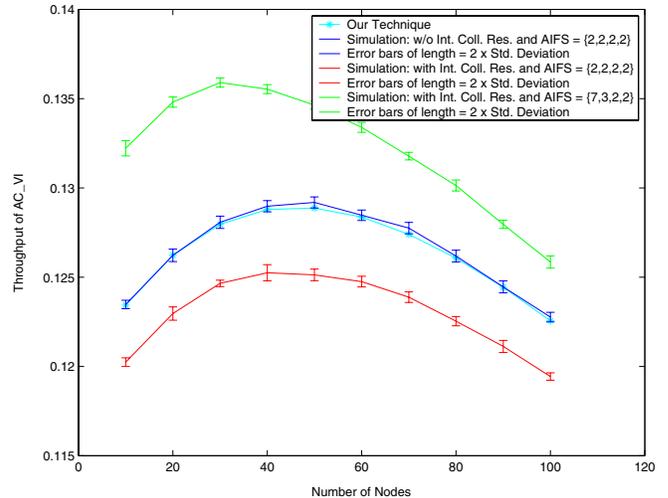


Fig. 8. Throughput of AC\_VI for  $aCW_{min} = 128$ .

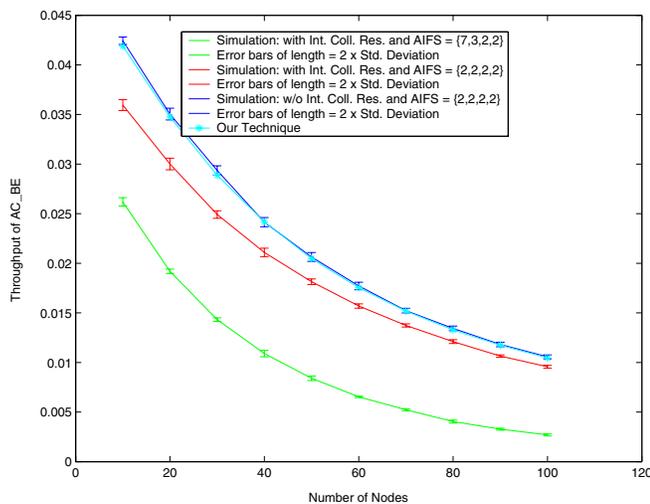


Fig. 7. Throughput of AC\_BE for  $aCW_{min} = 128$ .

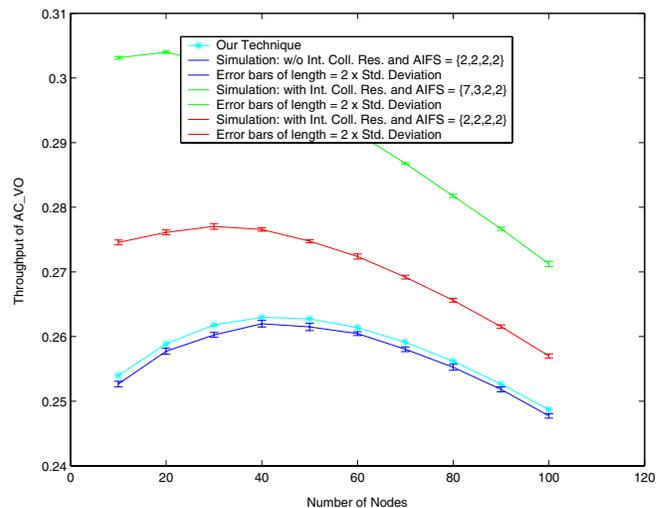


Fig. 9. Throughput of AC\_VO for  $aCW_{min} = 128$ .

Furthermore, with internal collision resolution enabled, the aggregate throughput rises in setting (B) as expected, although there is some decrease in the throughput of low priority ACs. We also find that increasing the AIFS values of low priority ACs in setting (C) results in a further improvement of the aggregate throughput but that it reduces the throughputs of low priority ACs nearly to zero. The inference to be drawn is that differences in AIFS have a greater impact on service differentiation than differences in congestion window sizes. Moreover, the proposed values for AIFS in the IEEE 802.11e EDCA protocol run the risk of starving, or being excessively unfair, to low priority traffic classes. This aspect merits further investigation. An interesting future research challenge would be account for the different AIFS values of different ACs in the analytical model. At present, accounting for the different AIFS values appears to be difficult in the Markovian framework that we have developed in this paper.

## V. CONCLUDING REMARKS

We studied the performance of contention based medium access control (MAC) protocols. We developed a novel technique for estimating the throughput, and other parameters of interest, of such protocols. Our technique is based on a rigorous analysis of a Markovian framework developed in the paper. The analysis shows that in a limiting regime of large system sizes, the stochastic evolution of the back-off stages at different stations converges to a deterministic evolution; moreover, this deterministic process has a unique fixed point. Thus, our analysis provides insight into the dynamics of the MAC protocols, showing that they guide the system to a *typical operating point*. This then allows us to obtain the saturation throughput and other performance measures of interest without having to calculate the stationary distribution of the Markov chain, which would be infeasible for systems

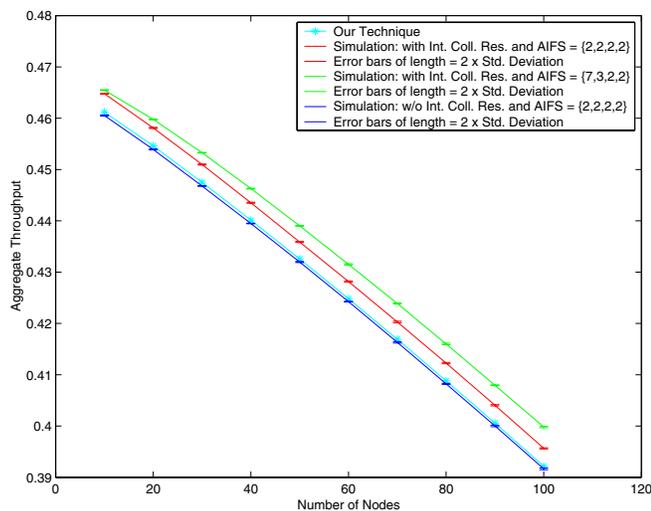


Fig. 10. Aggregate throughput for  $aCW_{min} = 128$ .

of realistic size.

To the best of our knowledge, our technique for performance analysis of MAC protocols is the first one of its kind with a quantifiable accuracy. Our results provide a rigorous justification for the decoupling approximation of Bianchi [3]. Finally, although we focused on two representative MAC protocols (IEEE 802.11 DCF and IEEE 802.11e EDCA), the techniques developed in the paper are quite general and are applicable to a wide variety of MAC protocols.

Our performance analysis is based on the assumption that the system remains at its equilibrium point at all times. A natural refinement is to consider fluctuations around this point, which will typically be small. A mathematical framework for studying such fluctuations is provided by the diffusion approximation (a functional central limit theorem for the Markov process). This is a topic for future research. Secondly, we observed the importance of different AIFS values in achieving service differentiation. It remains to extend our analysis techniques to deal with this, and with other forms of heterogeneity. Finally, we have assumed throughout that all nodes can hear each other; accounting for the hidden node problem remains an important research challenge.

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## APPENDIX

Here we evaluate the throughput of DCF under a special case, namely  $M = 1$ . In this case, the stationary distribution of the Markov chain  $X_n(t)$  (see section III-A) can be computed, and thereby, one can compute the exact throughput of DCF. We start with the computation of the stationary distribution of  $X_n(t)$ .

### Computation of Stationary Distribution

Let  $S_n$  denote the set of the system states for  $M = 1$ , i.e.,

$$S_n = \{(x_1, x_2) : x_1, x_2 \in \mathbb{N}; x_1 + x_2 = n; x_1, x_2 \geq 0\}$$

where  $\mathbb{N}$  denotes the set of integers. Observe that  $S_n$  contains  $n + 1$  states. More precisely,  $S_n = \{(0, n), (1, n - 1), \dots, (n, 0)\}$ . Let  $P_S^i$  be the steady state probability of the system being in state  $(i, n - i)$ . We now formulate the set of global balance equations that can be solved to obtain the stationary distribution of  $X_n(t)$ .

For the sake of brevity, let  $\bar{p}_i \triangleq 1 - p_i$ ,  $i \in \{0, 1, \dots, M\}$ . Now consider the state  $(n, 0)$ : The system leaves this state if there is a collision, an event of probability

$$(1 - \bar{p}_0^n - np_0\bar{p}_0^{n-1}).$$

The system can enter the state  $(n, 0)$  only from the state  $(n - 1, 1)$ , provided the station in back-off stage 1 transmits successfully, an event of probability  $p_1\bar{p}_0^n$ . Balancing the probability flux entering and leaving the state  $(n, 0)$ , we have

$$P_S^n(1 - \bar{p}_0^n - np_0\bar{p}_0^{n-1}) = P_S^{n-1}p_1\bar{p}_0^n. \quad (17)$$

Now consider the state  $(n - i, i)$ : The system leaves this state if there is a successful transmission by a station in back-off stage 1, an event of probability

$$ip_1\bar{p}_1^{i-1}\bar{p}_0^{n-i};$$

or if there is an unsuccessful transmission involving at least one station in back-off stage 0, an event of probability

$$1 - \bar{p}_0^{n-i} - (n - i)p_0\bar{p}_0^{n-i-1} + (n - i)p_0\bar{p}_1^{n-i-1}(1 - \bar{p}_0^i).$$

The system can enter the state  $(n - i, i)$  from the state

- $(n - i - 1, i + 1)$ : Following a successful transmission by a station in back-off stage 1, an event of probability

$$(i + 1)p_1\bar{p}_1^{i-1}\bar{p}_0^{n-i-1};$$

- $(n - i + 1, i - 1)$ : Following a collision involving exactly one station in back-off stage 0, and one or more stations in back-off stage 1, an event of probability

$$(n - i + 1)p_0\bar{p}_0^{n-i}(1 - \bar{p}_1^{i-1});$$

- $(n - j, j)$  for  $0 \leq j \leq i - 2$ : Following a collision involving  $i - j$  stations in back-off stage 0, an event of probability

$$\binom{n-j}{i-j} p_0^{i-j} \bar{p}_0^{n-j}.$$

Balancing the probability flux leaving and entering the state  $(n - i, i)$ , we get

$$\begin{aligned} P_S^i(ip_1\bar{p}_1^{i-1}\bar{p}_0^{n-i} + 1 - \bar{p}_0^{n-i} - (n - i)p_0\bar{p}_0^{n-i-1}\bar{p}_1^i) = \\ P_S^{n-i-1}(i + 1)p_1\bar{p}_1^{i-1}\bar{p}_0^{n-i-1} + \sum_{j=0}^{i-2} P_S^{n-j} \binom{n-j}{i-j} p_0^{i-j} \bar{p}_0^{n-j} + \\ P_S^{n-i+1}(n - i + 1)p_0\bar{p}_0^{n-i}(1 - \bar{p}_1^{i-1}), \quad 0 < i < n \end{aligned} \quad (18)$$

Note that the summation term in Eq.(18) exists only for  $i \geq 2$ . Since the sum of stationary probabilities across all the system states must equal one, we have

$$\sum_{i=0}^n P_S^i = 1. \quad (19)$$

Observe that we have  $n + 1$  equations in  $n + 1$  unknowns. We leave it for the reader to verify that these equations are linearly independent, and therefore the stationary probabilities can be obtained by solving these equations.

### Throughput Calculation

Once we have the stationary probabilities, we can calculate the throughput and other parameters of interest about the system. For  $k \in \{0, 1, \dots, M\}$ , let:

- $T^k \triangleq$  The expected system throughput given the system is in state  $(k, n - k)$ .
- $P_c^k \triangleq$  The collision probability given the system is in state  $(k, n - k)$ .
- $I^k \triangleq$  The probability of an idle slot given the system is in state  $(k, n - k)$ .
- $T \triangleq$  The system throughput.
- $P_c \triangleq$  The conditional collision probability.
- $I \triangleq$  The probability of an idle slot.

Observe that  $I^k = (1 - p_0)^k(1 - p_1)^{n-k}$ . Arguing as in the derivation of Eq.(10), we obtain

$$P_c^k = 1 - \frac{kp_0 \frac{I^k}{1-p_0} + (n-k)p_1 \frac{I^k}{1-p_1}}{1 - I^k} \quad (20)$$

and the expected system throughput when the system is in state  $(k, n - k)$  is given by:

$$T^k = \frac{(1 - I^k)(1 - P_c^k)P}{(1 - I^k)(1 - P_c^k)T_s + (1 - I^k)P_c^k T_c + I^k \sigma} \quad (21)$$

Since the probability that the system is in state  $(k, n - k)$  is given by  $P_S^k$ , we have

$$T = \sum_{k=0}^n P_S^k T^k \quad (22)$$

Similarly, we have  $I = \sum_{k=0}^n P_S^k I^k$  and  $P_c = \sum_{k=0}^n P_S^k P_c^k$ .

### Performance Comparison

We now compare the exact results obtained by using the above approach, with the numerical results obtained using our technique and Bianchi's model. Note that our technique relies on the fact that for sufficiently large  $t$ , the process

$X_n(t)$  stays close to the equilibrium point  $x^{(n)}$  that satisfies  $f^{(n)}(x^{(n)}) = 0$ . To demonstrate the effectiveness of our technique, we compare the random sample paths of the system with the deterministic trajectory obtained using:

$$x(k+1) = x(k) + f^{(n)}(x(k)),$$

for  $n = 50$ , with  $x(0) = (50, 0)$ . As shown in Figure 11, not only does the system converge to a neighborhood of the equilibrium point for large  $t$ , but also the random trajectory of the system stays close to the above deterministic trajectory at all times (see Theorem 3 in [15] for such a result). Further, we see that the convergence to a neighborhood of the equilibrium point is quite rapid (within 100 slots).

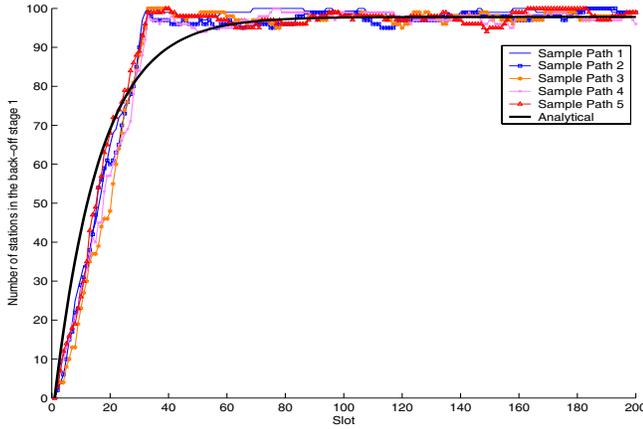


Fig. 11. Random system trajectories converging to a neighborhood of the equilibrium point.

TABLE III  
THROUGHPUT:  $W_0 = 32, M = 1$ .

Stations	Throughput (T)		
	Exact	BM	OT
5	0.4664	0.4666	0.4669
15	0.4486	0.4484	0.4487
25	0.4229	0.4228	0.4230
55	0.3348	0.3348	0.3348
80	0.2543	0.2544	0.2543
100	0.1918	0.1918	0.1918

TABLE IV  
CONDITIONAL COLLISION PROBABILITY:  $W_0 = 32, M = 1$ .

Stations	Conditional Coll. Probability ( $P_c$ )		
	Exact	BM	OT
5	0.1008	0.1022	0.1008
15	0.2713	0.2727	0.2717
25	0.3961	0.3970	0.3965
55	0.6528	0.6530	0.6531
80	0.7879	0.7880	0.7881
100	0.8611	0.8611	0.8612

TABLE V  
IDLE SLOT PROBABILITY:  $W_0 = 32, M = 1$ .

Stations	Idle Slot Probability (I)		
	Exact	BM	OT
5	0.7692	0.7689	0.7681
15	0.5245	0.5244	0.5231
25	0.3782	0.3781	0.3771
55	0.1544	0.1544	0.1541
80	0.0743	0.0743	0.0742
100	0.0411	0.0411	0.0410

Tables III-V show various parameters of interest obtained using the exact analysis, Bianchi's model (BM), and our technique (OT). The results shown are for RTS/CTS access mechanism with  $W_0 = 32$ . It is clear that both our technique and Bianchi's model are extremely accurate even for small  $n$ ; and, as expected, their accuracy increases as  $n$  increases.