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Optimal Unemployment Insurance in Search Equilibrium

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Should unemployment benefits be paid indefinitely at a fixed rate or should the rate decline (or increase) over a worker's unemployment spell? We examine these issues using an equilibrium model of search unemployment. The model features worker-firm bargaining over wages, free entry of new jobs, and endogenous search effort among the unemployed. The main result is that an optimal insurance program implies a declining benefit sequence over the spell of unemployment. Numerical calibrations of the model suggest that there may be nontrivial welfare gains associated with switching from an optimal uniform benefit structure to an optimally differentiated system.

I. Introduction

The economics of unemployment insurance (UI) has attracted considerable attention over the past couple of decades. The research has primarily been concerned with positive analysis, such as the effects of UI benefits

on the duration of unemployment.[1] Much less attention has been devoted to the normative issues: What does an optimal UI system look like? The ultimate rationale for public UI is, after all, to provide income insurance for risk-averse workers. A welfare analysis of UI policies thus requires a unified treatment of the insurance benefits provided by UI as well as of the adverse incentive effects induced by the usual moral hazard problems. The purpose of this article is to contribute to the normative analysis of UI by means of an equilibrium model of search unemployment.

The seminal papers on optimal UI appeared in the late 1970s (Baily 1978; Flemming 1978; Shavell and Weiss 1979). These papers analyzed the problem of UI design in an optimal taxation framework; more generous benefits caused lower search intensity and, hence, longer spells of unemployment. Shavell and Weiss (1979) focused in particular on the optimal sequencing of benefits. Their analysis, based on a model of the individual worker's search behavior, suggested that benefits should decline over the spell of unemployment, provided that the unemployed can influence their job-finding probabilities. Baily's (1978) two-period analysis analogously suggested a case for a redundancy payment, that is, a lump-sum transfer to the worker at the start of the unemployment spell.

Recently, a number of papers have extended the analysis of Shavell and Weiss (1979). One strand of the literature stays within the Shavell and Weiss (1979) framework in the sense that it focuses solely on the behavior of the worker. Hopenhayn and Nicolini (1997) enlarge the set of policy instruments by considering a wage tax after reemployment in conjunction with the sequence of benefit payments. According to their analysis, benefits should decrease throughout the unemployment spell, and the tax at reemployment should increase with the length of the spell. The result that benefits should fall monotonically over the unemployment spell is, however, called into question by Wang and Williamson (1996). They add another source of moral hazard by examining an environment where a worker's employment status depends on the choice of effort. The transition rate from unemployment to employment is increasing in search effort; analogously, the probability of remaining employed is increasing in work effort. The optimal UI in this setting involves a large drop in consumption during the first period of unemployment (so as to discourage shirking), and a large reemployment bonus (so as to encourage search effort). The implied time profile of UI compensation is nonmonotonic; compensation increases initially and then falls throughout the spell.

Another strand of the recent literature on benefit sequencing has approached the issue by incorporating some aspects of firm behavior. Davidson and Woodbury (1997) examine whether benefits should be paid

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[1] See Atkinson and Micklewright (1991) and Holmlund (1998) for two recent surveys and assessments of the literature on UI and unemployment.

indefinitely or for a fixed number of weeks. The analysis is cast in a search and matching framework, albeit with a fixed number of jobs and exogenous wages. They conclude that the optimal UI program should offer risk-averse workers indefinite benefit payments, a conclusion that seems to suggest that most existing UI programs with finite benefit periods are suboptimal.[2] Cahuc and Lehmann (1997) ignore job search but allow for endogenous wage determination through union-firm bargaining. They find that a constant time sequence yields a lower unemployment rate than a program with a declining time profile; the reason is that a decreasing benefit schedule increases the welfare of the short-term unemployed at the expense of the long-term unemployed, and, therefore, wage pressure rises. Based on a "Rawlsian" welfare criterion, they argue that a constant benefit sequence is optimal.

Our article reexamines the question of the optimal sequencing of benefits using an equilibrium model of search unemployment along the lines of Pissarides (1990). We allow for endogenous search effort among unemployed workers. In contrast to Shavell and Weiss (1979), Wang and Williamson (1996), Davidson and Woodbury (1997), and Hopenhayn and Nicolini (1997), we incorporate endogenous wage determination and free entry of new jobs. Search effort as well as the wage bargains are affected by the parameters of the UI program. In a search equilibrium framework, as well as in other models of equilibrium unemployment, there is a link between benefits and wages, which in turn implies a relationship between benefits and job creation.[3] The endogenous response of wages to benefits may potentially prove to be an important channel that affects the optimal design of UI, a point stressed by Mortensen (1996). Indeed, the result of Cahuc and Lehmann (1997) suggests that the sequencing of benefit payments matters for wage setting and equilibrium unemployment.

Our analysis ignores the possibility of smoothing consumption through borrowing and saving. The optimal UI system would presumably offer lower replacement rates if workers had access to the capital market. A few recent papers have addressed the welfare implications of UI using general equilibrium search models that allow for capital markets. For instance, Costain (1997) develops a model with endogenous search effort and precautionary savings. The setup of the wage bargain is greatly sim-

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[2] Davidson and Woodbury (1997) do not offer a formal proof of their conclusion. On the basis of a numerical experiment, where the compensation after benefit exhaustion is arbitrarily set, they argue that the constant benefit sequence is preferable, since it reduces risk as it is defined by Rothschild and Stiglitz (1970). However, we would argue that their result does not hold if the compensation after the expiration of UI benefits is determined optimally as well.

[3] This aspect of UI has been widely discussed in research on European unemployment over the past 10–15 years; see, e.g., Layard, Nickell, and Jackman (1991).

plified by ignoring that the wage in general will depend on outside op-
portunities; hence, the benefit level will not directly affect the outcome
of the bargain. Despite this simplification, the model is much too com-
plicated to solve analytically, so the results are based on numerical cali-
brations. According to Costain (1997), optimal replacement ratios in the
range of 30%–40% arise under plausible assumptions. Valdivia (1996)
reports similar results with a somewhat different wage-setting rule than
the one adopted in Costain's paper. Neither of these papers, however,
consider the optimal sequencing of benefits.

For ease of exposition, and without loss of generality, we mainly focus
on a two-tiered UI system, that is, a program with two benefit levels.
Workers who lose their jobs are entitled to UI benefits. UI benefits may
not be paid indefinitely, however; some workers lose their benefits and
are thereafter entitled to what we refer to as "social assistance." Social
assistance payments have infinite duration but are potentially lower than
regular benefits. We ask whether a two-tiered system dominates, in welfare
terms, a program with indefinite payments of a constant wage replacement
rate. The answer to this question turns out to be an unambiguous yes,
provided that we ignore discounting. We also show that the result gen-
eralizes to the case with a multitiered benefit structure; unemployment
benefits should decline monotonically over the spell of unemployment.

Section II presents the basic model and some of its comparative statics
properties. Some of the results from this positive analysis are well known
from the search literature, whereas others are new. Section III turns to
the normative analysis and shows that search effort is too low in market
equilibrium; the reason is that workers do not internalize the tax burden
they impose on others by reducing their search effort. We also characterize
the optimal sequencing of benefits. A declining sequence always domi-
nates a system with indefinite payment of a constant replacement rate in
the limiting case with zero discounting. The result is driven by a feature
known from models of individual worker search, which implies that the
effect of higher benefits on the individual worker's search behavior de-
pends on whether he is presently qualified for UI or not. A rise in benefits
will in general increase search effort among those not insured, as this will
bring them more quickly to employment, which in turn results in eligi-
bility for future UI payments. A two-tiered UI system exploits this "en-
titlement effect" by providing incentives for active search among workers
not currently entitled to benefits.

With discounting, the optimality of a declining benefit sequence cannot
be established analytically. The reason for the ambiguity lies in the fact
that a declining sequence increases the welfare of the short-term unem-
ployed at the expense of the long-term unemployed, which in turn induces
stronger wage pressure than a flat (or increasing) sequence (see Cahuc
and Lehmann 1997). According to our numerical calibrations, however,

this "wage pressure effect" is dominated by the case for exploiting the entitlement effect. Moreover, the numerical experiments suggest that the optimal degree of differentiation should be substantial. The welfare gains of switching from a uniform benefit structure to a two-tiered one appear to be nontrivial.

II. The Model

A. Job Matching and Labor Market Flows

Consider an economy with a fixed labor force, without loss of generality normalized to unity. Workers are either employed or unemployed, individuals have infinite horizons, and time is continuous. Employed workers are separated from their jobs at the exogenous rate ϕ. When they enter unemployment, they are immediately eligible for UI benefits; the unemployed worker is insured as long as he receives UI benefits.[4] Benefits are time-limited, however. We assume for simplicity that benefits expire at a rate λ that is exogenous to the individual (although endogenously determined as part of the optimal UI program). The expected potential duration of benefit receipt is thus $1/\lambda$. An unemployed worker whose benefits have expired is referred to as "noninsured." The insured worker escapes unemployment and enters employment at the rate α^I, whereas the noninsured worker enters employment at the rate α^N. Figure 1 illustrates the labor market flows.

The assumption that benefits have a stochastic rather than a fixed duration is made for tractability.[5] It is not restrictive, however, as the analysis generalizes to the case with an arbitrary number of insured unemployment states, each characterized by a given benefit level (see app. D). Search effort varies by unemployment state but is constant within a state.[6] One might also argue that the practical implementation of the work test in existing UI systems is bound to involve a degree of randomness in benefit receipt from the job searcher's perspective, since the authorities cannot monitor the job acceptance behavior of all unemployed workers.

Unemployed individuals can affect the rate at which they enter employment. Let s^I denote the search intensity of a representative insured worker and s^N the corresponding intensity for a noninsured worker. The

[4] The assumption of immediate eligibility for UI compensation simplifies the analysis considerably, as it implies that all employed workers will receive the same wage.

[5] If benefit duration is fixed and perfectly predictable, one would have to deal with the issue of how search effort changes over the spell of insured unemployment. Such time dependence results from the fact that the value of insured unemployment declines as the worker approaches the date of benefit exhaustion (Mortensen 1977).

[6] Thus, as the number of steps in the benefit ladder becomes sufficiently large, we mimic the search behavior arising in a setup with fixed duration.
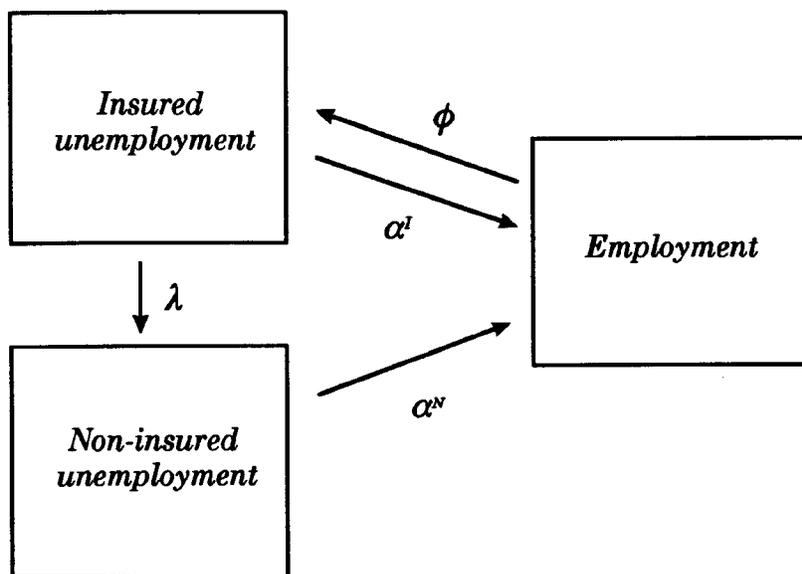
Fig. 1.—Labor market flows

effective number of searchers in the economy is then given as $S \equiv s^I u^I + s^N u^N$, where u^I and u^N, respectively, denote the number of unemployed in the two categories. The matching process is summarized by an aggregate matching function that relates new hires (H) to the number of effective searchers and the number of vacancies (v): $H = H(S, v)$. The probability per unit time that individual i gets an acceptable offer is given by $\alpha_i^j \equiv s_i^j H(S, v)/S = s_i^j \alpha(\theta), j = I, N$, assuming constant returns to scale; $\theta \equiv v/S$ is a measure of labor market tightness, and $\alpha(\theta) = H(S, v)/S$. Firms fill vacancies at the rate $q(\theta) = H(S, v)/v$. Clearly, $\alpha(\theta) = \theta q(\theta)$. Differentiating $\alpha(\theta)$ with respect to θ, we have $\alpha'(\theta) = q(\theta)(1 - \eta) > 0$, where $\eta \in (0, 1)$ is the elasticity of the expected duration of a vacancy with respect to θ. Further, $q'(\theta) < 0$; thus, the tighter the labor market, the more difficult it is to fill a vacancy.

The flow equilibrium conditions for this economy are as follows:

$$\phi e = \alpha^I u^I + \alpha^N u^N \tag{1}$$

$$\alpha^N u^N = \lambda u^I. \tag{2}$$

The first condition pertains to employment (e) and the second to non-insured unemployment. Equations (1) and (2) imply a flow equilibrium condition for insured unemployment as well. The solution for the employment rate takes the form:

$$e = \frac{\mu^I \alpha^I + \mu^N \alpha^N}{\phi + \mu^I \alpha^I + \mu^N \alpha^N}, \tag{3}$$

where we define μ^j as the number of unemployed in state j relative to total unemployment: $\mu^j \equiv u^j/(u^I + u^N)$.

B. Worker Behavior

Workers do not have access to a capital market, so individuals consume all of their income at each instant. The employed worker's income is given by his wage, w, and the insured unemployed worker's income by UI benefits, B. Noninsured unemployed workers receive a transfer, Z, from the government; Z can be thought of as "social assistance" that is available for workers who have run out of benefits. We proceed under the assumption that $B \geq Z$; an objective of the normative analysis will be to determine whether this inequality is socially optimal.

The utility of unemployed workers is decreasing in search effort, as search reduces available leisure time. The utility functions can accordingly be written as $v(B, s^I)$ and $v(Z, s^N)$. An employed individual works a fixed amount of hours $(\bar{h})$ and does not search; hence, we write his utility function as $v(w, \bar{h}) = v(w)$. The utility function is assumed to have the following specific form:

$$v = \begin{cases} \dfrac{[(c\ell^\delta)^\sigma - 1]}{\sigma}, & \sigma \leq 1, \sigma \neq 0, \delta \in (0,1) \\ \ln c + \delta \ln \ell, & \sigma = 0, \delta \in (0,1) \end{cases} \tag{4}$$

where c denotes consumption and ℓ leisure. If T denotes the available time, then consumption and leisure in the three states are given by $c = w$ and $\ell = T - \bar{h}$ if employed; $c = bw$ and $\ell = T - s^I$ if insured unemployed; and $c = zw$ and $\ell = T - s^N$ if noninsured unemployed.[7] The variables B and Z are thus proportional to the aggregate wage, where b and z are the wage replacement rates in the two states.

Let U^I, U^N, and E denote the expected present values of being in insured unemployment, noninsured unemployment, and employment, respectively. The relevant value functions for worker i can be written as asset equations of the form:

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[7] If individuals choose their working time, taking the hourly wage as given, hours during employment will be independent of the wage and given as $h = T/(1 + \delta)$. The specific utility function is commonly used in the growth literature, as it is consistent with a balanced growth rate assumption (King, Plosser, and Rebelo 1988).

$$rU_i^I = v(B, s_i^I) + \alpha_i^I(E - U_i^I) - \lambda(U_i^I - U^N), \tag{5}$$

$$rU_i^N = v(Z, s_i^N) + \alpha_i^N(E - U_i^N), \tag{6}$$

$$rE_i = v(w_i) - \phi(E_i - U^I), \tag{7}$$

where r is the subjective rate of time preference. These present values can be solved in terms of the utilities pertaining to each state and the transition rates. For many purposes it is the differences in present values between employment and unemployment that matter; these are given in appendix A.

An unemployed worker i in state j chooses search intensity, s_i^j, to maximize the value of unemployment, U_i^j. Using $\partial\alpha_i^j/\partial s_i^j = \alpha(\theta)$ and the fact that all workers in state j choose an identical search intensity, the first-order conditions have the following structure:

$$\Lambda_s^j \equiv v_s^j + \alpha(\theta)(E - U^j) = 0, j = I, N, \tag{8}$$

where $v_s^j = \partial v(\cdot, s^j)/\partial s^j$. Thus, in optimum, the marginal cost of increasing search effort is equated to the expected marginal gain of doing so. Assuming that the second-order condition holds ($v_{ss} < 0$), imposing symmetry and taking wages and labor market tightness as givens, we can state the following results concerning individual search behavior:

LEMMA 1. (i) an increase in UI benefits (B) reduces s^I (if $v_{sB}^I = \partial^2 v/\partial B\partial s^I \leq 0$, i.e., $\sigma \geq 0$) but increases s^N; (ii) an increase in the potential duration of UI benefit receipt ($1/\lambda$) reduces s^I but increases s^N; (iii) an increase in social assistance (Z) reduces s^N (if $v_{sZ}^N \leq 0$, i.e., $\sigma \geq 0$) and s^I.

Proof. Differentiate equation (8) implicitly with respect to B, λ, and Z, using equation (A1): $\Lambda_{sB}^I < 0$ (if $v_{sB}^I \leq 0$, i.e., $\sigma \geq 0$); $\Lambda_{sB}^N > 0$; $\Lambda_{s\lambda}^I > 0$; $\Lambda_{s\lambda}^N < 0$; $\Lambda_{sZ}^I < 0$; and $\Lambda_{sZ}^N < 0$ (if $v_{sZ}^N \leq 0$, i.e., $\sigma \geq 0$). Note that the "if-statements" are ones of sufficiency. Q.E.D.

The result that noninsured search rises when the benefit level is increased is known in the literature as an "entitlement effect" (Mortensen 1977). The entitlement effect arises because higher benefits make employment more attractive relative to noninsured unemployment, as a spell of employment is a prerequisite for benefit eligibility, that is, $E - U^N$ is increasing in B. For the exact same reason, noninsured search is increasing in benefit duration.

According to lemma 1, the effect on insured search of increasing UI benefits, for example, depends on whether consumption and leisure are substitutes or complements in the production sense. With our specific utility function, this provision is translated into a condition on the degree

of relative risk aversion $(1 - \sigma)$.[8] We will mostly discuss the positive results in terms of $\sigma \geq 0$; the sign of σ will not matter for our normative results.

C. Firm Behavior and Wage Determination

The modeling of firms and wage determination follows Pissarides (1990) closely. Let J and V denote the expected present values of an occupied job and a vacant job, respectively. Labor productivity is constant and denoted y. The cost of holding a vacancy open is ky, where $k > 0$. The assumption that the cost of holding a vacancy is proportional to labor productivity is akin to the conventional idea that vacancy costs are proportional to real wage costs (see Pissarides 1990).[9] Wages are taxed at the proportional rate τ. The flow values of having an occupied and a vacant job are given by

$$rJ = y - \omega - \phi(J - V), \tag{9}$$

$$rV = -ky + q(\theta)(J - V), \tag{10}$$

where $\omega \equiv w(1 + \tau)$ is the real labor cost. For simplicity we assume that firms discount the future at the same rate as workers.[10] There are no costs associated with opening and closing a vacancy, so free entry ensures that $V = 0$. From equations (9) and (10), then, we get:

$$J = \frac{y - \omega}{r + \phi} = \frac{ky}{q(\theta)}. \tag{11}$$

We refer to equation (11) as a zero-profit condition. It gives the wage

[8] The importance of the cross-derivative of utility with respect to leisure and consumption for search behavior is common in the literature; see, e.g., Mortensen (1977). Risk aversion will influence whether two goods are complements or substitutes; as Samuelson (1974, p. 1277) notes, "[risk aversion] pushes all complementarity coefficients towards substitutability." Empirical work on search intensity is scarce, but Jones (1989) finds some support for the hypothesis that higher benefits reduce the search effort among benefit recipients. By contrast, Blau and Robins (1990) and Wadsworth (1991) find that unemployed workers eligible for unemployment compensation search more actively than do those not eligible. As suggested by Wadsworth (1992), the latter results may reflect that benefit claimants enter into an environment favorable to search, involving, e.g., regular contacts with officials at employment exchange offices. Our analysis ignores these aspects of the search process.

[9] To rationalize this assumption, think of a world where firms allocate their workforce between production and recruitment activities. In such a setup the cost of recruiting—the vacancy cost—consists of the alternative cost, i.e., the marginal product of labor.

[10] We think of firms as owned by a group of risk-neutral "rentiers" who do not work. Our welfare calculations take their income from profits into account (although this is only relevant as long as $r > 0$; see Sec. III).

cost as proportional to the marginal product of labor, that is, to $\omega = [1 - (r + \phi)k/q(\theta)]y = \omega(\theta)$, $\omega'(\theta) < 0$. According to equation (11), firms react by posting fewer vacancies (θ falls) as ω increases.

Wages are set in decentralized Nash-bargains between workers and firms. Wages can be renegotiated at any time. Hence, the relevant fallback position for the worker is the state of insured unemployment, irrespective of whether he entered employment from insured or noninsured unemployment. The Nash-bargain thus solves the problem:

$$\max_{w_i} [E_i(w_i) - U^I]^\beta [J_i(w_i) - V]^{1-\beta}, \beta \in (0, 1),$$

where the definition of J_i is analogous to equation (9).[11] The first-order condition for the maximization of the Nash-product can be written as:

$$\frac{E - U^I}{v_w w} = \frac{\beta}{1 - \beta} \frac{J}{\omega}, \tag{12}$$

where $V = 0$ and symmetry across bargaining units has been imposed. Note that $E - U^I$ depends on labor market tightness, the various transition rates, and compensation in the three states (see app. A).

D. Equilibrium

To complete the characterization of equilibrium, we need to specify the government's budget constraint, which simply states that taxes on the total wage bill are used to finance UI benefits and social assistance transfers. Since $Z = zw$ and $B = bw$, the budget restriction can be written as:

$$\tau e = bu^I + zu^N. \tag{13}$$

The equilibrium of this model may in principle be very complex. However, the model has a convenient recursive structure, which simplifies the analysis considerably.

LEMMA 2.   The zero-profit condition (11) and the wage-setting equation (12) determine ω and θ, independently of the remaining endogenous variables. With θ determined, we get search behavior from equation (8). Having determined θ and s^j, we get e and u^j from equations (1)–(3). Finally, the tax rate is given by equation (13) and the wage by $w \equiv \omega/(1 + \tau)$.

Proof.   See appendix B.

The assumptions of constant wage replacement rates and the specific utility function thus render search and unemployment independent of the

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[11] Rubinstein and Wolinsky (1985) show that surplus sharing of the kind depicted in eq. (12) and $\beta = 1/2$ is the outcome of a strategic bargaining game if the bargaining parties can costlessly search while negotiating.

payroll tax; they also imply that changes in labor productivity have no effect on search and unemployment.

We arrive at a single equation determining θ by combining the free-entry condition ($J = ky/q(\theta)$) with the solution to the wage bargain:

$$\Psi(\theta, \lambda, b, z) \equiv \frac{1 - \sum_j m^j \rho^j}{\sigma}$$

$$-\left(r + \phi + \sum_j m^j \alpha^j\right) \frac{\beta}{1 - \beta} \frac{k/q(\theta)}{1 - (r + \phi)k/q(\theta)} = 0, \quad (14)$$

where $m^I = (r + \alpha^N)/(r + \alpha^N + \lambda)$, $m^N = 1 - m^I$, and

$$\rho^j = x^\sigma \left(\frac{T - s^j}{T - \bar{b}}\right)^{\delta\sigma}, x \in \{b \text{ if } j = I, z \text{ if } j = N\}.$$

Equation (14) defines $\theta = \theta(\lambda, b, z)$. The properties of this relationship are as follows.

LEMMA 3.   Equilibrium labor market tightness increases if benefit duration, the benefit level, or social assistance is reduced.

Proof.   By implicit differentiation: $\theta_\lambda > 0$, $\theta_b < 0$, and $\theta_z < 0$. Q.E.D.

The intuition for lemma 3 is straightforward: every change that reduces workers' threat point in the wage bargain produces more moderate wage-setting behavior on the part of workers and, consequently, increases equilibrium market tightness.

As a first step toward deriving the equations for equilibrium search intensity, we substitute the Nash-bargaining solution and the free-entry condition into the first-order conditions for optimal search.[12]

$$\frac{\rho^I \delta}{T - s^I} = \frac{\beta}{1 - \beta} \frac{\theta k}{1 - (r + \phi)k/q(\theta)} \qquad (15a)$$

$$\frac{\rho^N \delta}{T - s^N} = \frac{\alpha(\theta)}{r + \lambda + \alpha^N} \frac{\rho^I - \rho^N}{\sigma} \qquad (15b)$$

$$+ \frac{r + \lambda + \alpha^I}{r + \lambda + \alpha^N} \frac{\beta}{1 - \beta} \frac{\theta k}{1 - (r + \phi)k/q(\theta)}.$$

Equations (15a) and (15b) define the following "semireduced forms": $s^I = s^I(\theta, b)$ and $s^N = s^N(\theta, b, z, \lambda)$. Lemma 4 summarizes the properties of these relationships.

LEMMA 4.   When the outcome of the wage bargain is taken into ac-

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[12] Equation (15b) is derived as follows: first decompose $(E - U^N)$ into $E - U^N = (E - U^I) + (U^I - U^N)$; then use $U^I - U^N = (r + \lambda + \alpha^N)^{-1} (v^I - v^N + (\alpha^I - \alpha^N)(E - U^I))$, and eliminate $(E - U^I)$ using eq. (12) and eq. (11).

count, search effort has the following properties: $s_\theta^I > 0$, $s_\theta^N > 0$, $s_b^N > 0$, and $s_\lambda^N < 0 \ \forall \sigma$; $s_b^I \leq 0$ and $s_z^N < 0$ if $\sigma \geq 0$; $s_b^I > 0$; and $s_z^N \gtreqless 0$ if $\sigma < 0$.

Proof. Implicit differentiation of equations (15a) and (15b), recognizing that the right-hand side of equation (15b) is independent of s^j by the envelope theorem. Q.E.D.

According to lemma 4, the entitlement effect remains when wage adjustments are taken into account, but θ is held constant. To find the general equilibrium effects on search behavior of changing b, z, or λ, we must also invoke $\theta = \theta(\lambda, b, z)$.

E. Employment Effects of Introducing Benefit Differentiation

From the point of view of positive analysis, we are ultimately interested in the employment effects of changing the parameters of the two-tiered benefit system. In general, these effects are ambiguous, the reason being that the parameters of the UI system may have opposite effects on the search behavior of the insured and noninsured. However, the case where we introduce a two-tiered benefit system yields some conclusive results; we summarize these in proposition 1.

PROPOSITION 1. A reform that introduces marginally different benefit levels, b and z, while holding unemployment expenditure per unemployed constant will:

a) decrease employment if the wage cost and, hence, θ adjust but search is constant,

b) increase employment if search adjusts but the wage cost is constant,

c) increase employment if $r \to 0$.

Proof. Differentiate employment with respect to b, assuming log utility for convenience.[13]

$$\frac{de}{db} = \left(e_\theta + e_{s^I} s_\theta^I + e_{s^N} s_\theta^N \right)\left(\theta_b + \theta_z \frac{dz}{db} \right) + e_{s^N}\left(s_b^N + s_z^N \frac{dz}{db} \right),$$

where $e_x = \partial e / \partial x$, $x = \{\theta, s^I, s^N\}$. We evaluate this derivative at the initial point, that is, $b = z$. The reform we consider thus satisfies $\mu^I db + \mu^N dz = 0$. At $b = z$, $\theta_z = (m^N/m^I)\theta_b$ and $s_z^N = -s_b^N$. Hence

$$\frac{de}{db} = \left(e_\theta + e_{s^I} s_\theta^I + e_{s^N} s_\theta^N \right)\left(1 - \frac{\mu^I}{\mu^N} \frac{m^N}{m^I} \right)\theta_b + \frac{e_{s^N}}{\mu^N} s_b^N.$$

a) If search is exogenous then

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[13] This is only for convenience. All results hold for a more general utility function.

$$\frac{de}{db} = e_\theta \left(1 - \frac{\mu^I}{\mu^N} \frac{m^N}{m^I} \right) \theta_b < 0,$$

since $e_\theta > 0$, $1 - (\mu^I/\mu^N)(m^N/m^I) = r/[r + s\alpha(\theta)] > 0$, and $\theta_b < 0$.

b) If wage costs are fixed, then θ is constant by equation (11). In this case, we must use the partial equilibrium relationships for search: equation (8). To facilitate comparison, we write the employment effect in terms of s_b^N, as defined in lemma 4.[14] We derive:

$$\frac{de}{db} = \frac{e_{s^N}}{\mu^N} s_b^N \frac{\phi + s\alpha(\theta)}{r + \phi + s\alpha(\theta)} > 0.$$

c) If $r \to 0$, then $1 - (\mu^I/\mu^N)(m^N/m^I) = 0$, and so

$$\frac{de}{db} = \frac{e_{s^N}}{\mu^N} s_b^N > 0.$$

Q.E.D.

Part *a* of proposition 1 is the pure "wage pressure effect" highlighted by Cahuc and Lehmann (1997). When search is exogenous, "front-loading" the benefit system has adverse employment effects; shifting benefit payments in favor of the insured implies that wage claims rise, as it is the welfare of the insured that is of direct relevance for wage setting. Part *b* of proposition 1 provides the essence of the result in Shavell and Weiss (1979). They maximize the expected utility of a newly unemployed individual holding the expected size of the UI budget constant. When wage costs are exogenous, front-loading the benefit system restores search incentives and, hence, reduces the expected cost of providing benefits. For marginal differences between *b* and *z*, the effects on expected utility are of the second order, but, since employment increases, compensation in all states can be raised, and so individuals in all labor market states gain. Part *c* of proposition 1 will be crucial for our welfare analysis. When $r \to 0$, the timing of benefits does not matter for wage setting. Thus a reform that holds average compensation constant is beneficial for employment since search incentives are restored.

III. Optimal Unemployment Insurance

A. The Optimal Policy

We are now ready to address the welfare economics of unemployment insurance. What does a socially optimal UI system look like? Determining the welfare optimizing policy involves an optimal choice of the policy

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[14] This is just to say that the expressions are algebraically equivalent. Lemma 1 still defines behavior.

variables (b, z, λ). We take the welfare objective to be utilitarian, that is,

$$W = erE + u^I rU^I + u^N rU^N + erJ + vrV. \tag{16}$$

Substituting the explicit expression for the value functions into equation (16), invoking the flow equilibrium conditions and taking the limit of the resulting expression as $r \to 0$, yields the following simple expression:

$$W = ev(w) + u^I v(bw, s^I) + u^N v(zw, s^N). \tag{17}$$

Thus, the flow of steady state welfare simplifies to a weighted average of workers' instantaneous utilities. We ignore discounting in order to be able to compare alternative steady states without considering the adjustment process. As we illustrate in Section III.B, the general case of a positive rate of discount is more complicated, and we are restricted to numerical analysis on this account.

Before we characterize the optimal policy, let us consider the derivative changes of equation (17) with respect to s^j. This yields the following result.

LEMMA 5.   Search intensity is too low in market equilibrium.

Proof.   Differentiate equation (17) with respect to s^j, while recognizing that $w \equiv \omega(\theta)/(1 + \tau)$, $e + u^I + u^N \equiv 1$, $e = e(\theta, \lambda, s^I, s^N)$, $u^N = u^N(\theta, \lambda, s^I, s^N)$, and $\tau = \tau(b, z, e, u^N)$:

$$\frac{\partial W}{\partial s^j} = u^j[v_s^j + \alpha(\theta)(E - U^j)] - v_w w\Big(e + \sum_j u^j \rho^j\Big)\frac{d\tau}{ds^j}(1 + \tau)^{-1}.$$

According to the first-order conditions for optimal search, equation (8), we get:

$$\frac{\partial W}{\partial s^j} = -v_w w\Big(e + \sum_j u^j \rho^j\Big)\frac{d\tau}{ds^j}(1 + \tau)^{-1} > 0, \, j = I, N,$$

as $d\tau/ds^j = (\partial\tau/\partial e)(\partial e/\partial s^j) + (\partial\tau/\partial u^N)(\partial u^N/\partial s^j) < 0$. Q.E.D.

Increases in search reduce equilibrium unemployment and, hence, required unemployment expenditures. Since these gains are external to the unemployed, search intensity is too low in market equilibrium.

Now, let us proceed to the optimal choice of the parameters of the benefit system: b, z, and λ. Maximizing equation (17), taking into account that $\theta = \theta(b, z, \lambda)$, $s^I = s^I(\theta, b)$, $s^N = s^N(\theta, b, z, \lambda)$, and the rest of the constraints listed above, we have:

$$\frac{dW}{db} = \frac{\partial W}{\partial b} + \sum_j \frac{\partial W}{\partial s^j} s_b^j + \theta_b \frac{dW}{d\theta} = 0, \qquad (18a)$$

$$\frac{dW}{dz} = \frac{\partial W}{\partial z} + \frac{\partial W}{\partial s^N} s_z^N + \theta_z \frac{dW}{d\theta} = 0, \qquad (18b)$$

$$\frac{dW}{d\lambda} = \frac{\partial W}{\partial \lambda} + \frac{\partial W}{\partial s^N} s_\lambda^N + \theta_\lambda \frac{dW}{d\theta} = 0, \qquad (18c)$$

where $dW/d\theta = \partial W/\partial\theta + \sum_j (\partial W/\partial s^j) s_\theta^j$. We can establish the following result.

PROPOSITION 2. The optimal benefit system involves $b > z$, provided that an interior solution to equation (18b) exists.

Proof (sketch; see app. C for details): pick an arbitrary $\lambda^0 \in (0, \infty)$ and consider the trial solution $b = z$. A uniform benefit structure cannot be optimal if $dW/db > 0$ at $b = z$. Making use of equation (14), equation (18a), and equation (18b), we derive

$$\frac{dW}{db} = \frac{1}{\mu^N} \frac{\partial W}{\partial s^N} s_b^N > 0.$$

According to lemma 4, we have $s_b^N > 0$ ($\forall\sigma$). So, at $b = z$, $dW/db > 0$. Q.E.D.

COROLLARY. The potential duration of UI benefit receipt is finite and positive: $\lambda \in (0, \infty)$.

Proof. By construction, the policies $(\lambda^0, b^0 = z^0)$, zero potential duration $(\lambda \to \infty, z^0)$, and infinite potential duration of UI benefit receipt $(\lambda = 0, b^0)$ all yield identical welfare, since they effectively pay the same uniform wage replacement rate. According to proposition 2, however, $W(\lambda^0, b > z^0) > W(\lambda^0, b^0 = z^0)$; therefore, an interior λ is optimal. Q.E.D.

The gist of the proof of proposition 2 is the following. By invoking equation (18b) on equation (18a), we evaluate equation (18a) at the optimal uniform insurance scheme. Hence, the costs associated with an unequal flow of income are of the second order for marginal differences between b and z. The relevant first-order effects are the effects on wage setting and search, but as the timing of benefits does not matter for wage setting when $r = 0$, the only remaining first-order effect is that of restoring search incentives. Therefore, the entitlement effect, $s_b^N > 0$, in conjunction with the fact that the search effort is too low in equilibrium, $\partial W/\partial s^N > 0$, drives the result that a two-tiered benefit structure dominates a uniform one.[15]

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[15] The provision of the proposition translates into a restriction on σ. If agents are sufficiently risk averse, they will demand some insurance, and then it will always be optimal to differentiate the benefit system.

B. Discounting and Optimal Differentiation
of the Benefit System

In this section we consider the case where $r > 0$. Discounting complicates the analysis for two reasons: first, we must take the adjustment process toward the new steady state into account; second, front-loading the benefit system ($b > z$) has adverse employment effects when search effort is exogenous.

With discounting, the flow of steady state utility equals

$$W = ev(w) + u^I v(bw, s^I) + u^N v(zw, s^N) + er\frac{ky}{q(\theta)}. \qquad (19)$$

If $r \to 0$, this expression simplifies to equation (17). Equation (19) depends on time (t) because of the matching technology; the relevant equations of motion are:

$$\dot{e} = \alpha^I u^I(t) + \alpha^N u^N(t) - \phi e(t), \qquad (20a)$$

$$\dot{u}^N = \lambda u^I(t) - \alpha^N u^N(t). \qquad (20b)$$

The proper objective is

$$\Omega(t) = \int_t^\infty \exp[-r(\hat{t} - t)]W(\hat{t})d\hat{t},$$

as the timing of gains and losses matters when $r > 0$. Since $\Omega(t)$ satisfies the standard equation of asset equilibrium,

$$r\Omega(t) = W(t) + \dot{\Omega}. \qquad (21)$$

In order to find the optimal benefit system, we calculate the effects of permanent and unexpected differential changes in b, z and λ, starting from an initial steady state, defined by equations (1) and (2), while taking into account that the economy must obey the laws of motion (eqs. [20a] and [20b]). We do this using the methodology in Diamond (1980).[16] Budget balance is assumed to hold in every period.

We focus on the case of a logarithmic utility function. Log utility implies additive separability between consumption and leisure, which simplifies the analysis somewhat; nevertheless, the fundamental properties of the more general problem are preserved. With log utility: $s^I = s^I(\theta)$ and $s^N = s^N(\theta, b/z, \lambda)$; thus, the relative compensation in the two unemployment states determines noninsured search.

Evaluating $dr\Omega/db$ at $dr\Omega/dz = 0$, we have that $dr\Omega/db > 0$ implies

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[16] Notice that if we take the policy experiment to be unexpected and permanent, $\dot{\theta} = 0$ and eq. (14) always holds. The reason for this result is that it is free to open and close vacancies (see Pissarides 1990).

$b > z$ and vice versa. The time-sequencing of benefits is therefore determined by

$$\text{sign}\left(\frac{dr\Omega}{db}\right) = \text{sign}\left\{-r\left[1 - \frac{z}{(1 + \tau)e}\right]\right. \tag{22}$$

$$\left.-\alpha^N\left(\frac{b}{z} - 1\right)\frac{z}{(1 + \tau)e} + \frac{\partial r\Omega}{\partial s^N}\frac{s^N}{u^N}\varepsilon_b^N(r + \lambda + \alpha^N)\right\},$$

where $\varepsilon_b^N = \alpha^N g(s^N)/(r + \lambda + \alpha^N) > 0$, with $g(s^N) = \delta^{-1}[(T - s^N)/s^N]^2$, is the elasticity of noninsured search with respect to b. The first term in equation (22) captures the wage pressure effect of front-loading the benefit system. The second term is the direct "tax cost" of having $b > z$. The last term is a consequence of the externalities associated with search. Differentiating welfare with respect to noninsured search, we get

$$\frac{\partial r\Omega}{\partial s^N} = \frac{\alpha u^N}{eA}\left\{(r + \lambda + \alpha^I)\left[er\frac{ky}{q(\theta)} + \frac{z}{(1 + \tau)e}\right]\right. \tag{23}$$

$$\left.+ru^I\left(\frac{b}{z} - 1\right)\frac{z}{(1 + \tau)e}\right\} > 0,$$

where $A = (r + \lambda + \alpha^N)(r + \phi + m^I\alpha^I + m^N\alpha^N)$. In this case, the search externality not only reflects the fact that taxes can be lowered when search is higher, it also includes the gain for capital owners ($J = ky/q(\theta)$) when employment increases. For given λ and benefit parameters, the externality has increased in size in comparison to the case that has no discounting.

It is ambiguous whether a declining benefit sequence improves welfare. Evaluating equations (22) and (23) at $b = z$, we deduce that the virtue of having $b > z$ is determined by the size of (the negative) wage-pressure effect vis-á-vis (the positive) search externality. In contrast, if $r \rightarrow 0$, we can derive an explicit expression for benefit differentiation. From equations (22) and (23), we have

$$\mu^I\left(\frac{b}{z} - 1\right) = \varepsilon_b^N = \mu^I g(s^N) > 0, \tag{24}$$

implying $b > z$.

Before proceeding to the numerical analysis, we wish to emphasize two things related to the special cases we analyzed in proposition 1. First, if we ignore search effort, a uniform or increasing benefit structure would be optimal, since only the effect on wage setting matters. Second, if we fix wage costs and ignore discounting, we can in fact derive a condition for differentiation that is identical to equation (24). This does not imply that the extent of benefit differentiation will be identical in the partial and general equilibrium setting. The reason is that the case for exploiting

the entitlement effect is more potent in a situation when overall benefit generosity is high. When wages are endogenous, the optimal policy has to take an additional source of moral hazard into consideration; this would tend to reduce benefit generosity. Consequently, we should expect that our general equilibrium analysis delivers less benefit differentiation than the partial equilibrium analysis. However, as we show numerically in Section III.C, the case for differentiation will still be valid, even if we take discounting into account.

C. Numerical Results

To provide some rough indications of the numbers involved, we have calibrated the model numerically. We begin with the simpler case of no discounting. Then we proceed to analyze the case with discounting; when $r > 0$ distributional issues are relevant, and, hence, we can also address the political economy of UI.

The matching function is taken to be Cobb-Douglas, that is, $H = aS^{\eta}v^{1-\eta}$. The day is the basic time unit. We impose the "Hosios-condition," $\beta = \eta$, implying that equilibrium labor market tightness is efficient absent policy interventions.[17] The rationale for doing so is that we do not want our results to be influenced by particular assumptions about whether market tightness is efficient or not; moreover, Moen (1997) finds that $\beta = \eta$ is the endogenous outcome in a framework in which employers can credibly announce the pay associated with vacant jobs. We set $\eta = 0.5$, which is in the upper range of the estimates according to Blanchard and Diamond (1989). Other parameters imposed are $a = 0.023$ and $y = 1$. Hours of work are set to $\bar{h} = T/(1 + \delta)$, which is the optimal working time if workers were free to choose.

For the remaining parameters, T, δ, ϕ, and k, we calibrated the model assuming that utility is logarithmic, the benefit structure is uniform, and $z = b = 0.3$. A wage replacement rate of 30% approximately corresponds to a uniform characterization of the present generosity of UI and welfare

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[17] As in all models of search and matching equilibrium, there are externalities associated with firm and worker entry into the market; hence, tightness (θ) is not necessarily efficient. Provided that the matching function is constant returns to scale, however, there exists a rule for sharing the total surplus of a match, which yields the efficient θ (see Hosios 1990). The efficient outcome occurs if workers' share of the total surplus (β) equals the elasticity of the expected vacancy duration with respect to tightness (η). When there are no policy interventions, we also arrive at the conclusion that θ is efficient if and only if $\beta = \eta$.

benefits in the United States.[18] Also, it is reasonably close to the Organization for Economic Cooperation and Development average replacement ratio in 1995 (see Martin 1996). The variables T, δ, and k were then chosen such that we obtained an unemployment duration of 12 weeks, $s^I = s^N = s = 1$, and a partial equilibrium elasticity of unemployment duration with respect to benefits of 0.5—which is in the middle range of the available estimates (see Layard, Nickell, and Jackman 1991). This procedure resulted in values of $T = 1.60$, $\delta = 0.72$, and $k = 4.13$.[19] The value of k implies that the expected vacancy cost ($ky/q(\theta)$) amounts to around 14 weeks of employers' labor cost (ω).[20] The separation rate, finally, was set at $\phi = 0.000828$, which implies an annual separation rate of around 30% and an unemployment rate of 6.5% (given the above value of unemployment duration).[21]

Table 1 presents the outcome in terms of some key variables. We measure the welfare gain associated with a particular policy in the following way. Let the welfare associated with two kinds of policies be denoted by W^1 and W^0. A measure of the welfare gain of policy 1 relative to policy 0 is, then, given by the value of ξ that solves

$$W^1[(1 - \xi)c\ell^\delta, \cdot] = W^0. \tag{25}$$

Equation (25) gives the consumption tax (ξ) that would make a representative individual indifferent between living in policy regime 1 and 0,

[18] A uniform characterization of the U.S. benefit system would be: $\gamma b[1 - \exp(-s\alpha/\lambda)] + (1 - \gamma)z$, where γ denotes the fraction of unemployed workers eligible for UI. According to Blank and Card (1991), γ roughly equals 0.5. The value of UI (the first term) is corrected to take its finite nature into account. Setting the expected duration of unemployment to 12 weeks, the potential duration of UI to 26 weeks, $\gamma = 0.5$, $b = 0.5$, and $z = 0.17$ (as suggested by Wang and Williamson 1996) yields a replacement rate of around 0.3.

[19] The careful reader will note that our baseline calibration has the unrealistic feature that the unemployed search more than the employed work. We can change the relationship between search and working time by introducing a "death risk." However, we do not pursue this extension since we think that it will not change the essentials of our results.

[20] Admittedly, expected vacancy costs are on the high side compared to the few estimates available. One could argue that vacancy costs also reflect training costs, but these costs are distinct and should probably not be treated as synonymous. Note, however, that studies similar in spirit to ours (e.g., Valdivia 1996; Costain 1997) encounter the same problem; an extreme case is Valdivia (1996), where expected vacancy costs equal 4.5 times the quarterly producer wage.

[21] The average rate of unemployment in the United States during 1983–96 was 6.5%; see OECD (1997). The inflow rate into unemployment averaged 30.8% per year and the average duration of completed unemployment spells was 11.4 weeks during 1984–89 (see Layard, Nickell, and Jackman 1991). Our calibration corresponds to the one in Mortensen (1994). We have also conducted calculations for low values of ϕ (an annual separation rate of around 10%) and for high values (an annual rate of 50%); the results are only marginally changed.

Table 1
Optimal UI and Risk Aversion (No Discounting)

	$\sigma = 0$			$\sigma = -1$		
	$b = z = 0.3$	$b = z$	$\{b, z, \lambda\}$	$b = z = 0.3$	$b = z$	$\{b, z, \lambda\}$
	(Base Run)	Optimal	Optimal	(Base Run)	Optimal	Optimal
Policy variables:						
Social assistance (z) (%)	30	37.91	33.84	30	41.81	36.16
Benefits/social assistance (b/z)	1	1	1.69	1	1	2.04
Benefit duration ($1/\lambda$) (weeks)	…	…	8.4	…	…	17.4
Market variables:						
Tightness (θ)	.268	.216	.199	.489	.263	.203
Employment (e) (%)	93.50	91.64	91.61	93.35	88.61	89.54
Fraction insured (μ^I) (%)	…	…	36.27	…	…	45.37
Noninsured search (s^N/T) (%)	63	53	59	45	34	41
Insured search (s^I/T) (%)	s^N/T	s^N/T	49	s^N/T	s^N/T	45
Welfare and spread:						
Welfare gain, % of consumption (ξ_{base})	…	.43	.65	…	2.91	4.08
Welfare gain, % of consumption, relative to U.S. system (ξ_{US})	…	…	.38	…	…	3.55
Coefficient of variation (CV)	.503	.402	.415	.439	.279	.304

NOTE.—$CV = \text{std}(c\ell^\delta)/E(c\ell^\delta)$ denotes the coefficient of variation of the utility index, $c\ell^\delta$, where c is consumption, ℓ is leisure, and δ a positive parameter. The parameters are set as follows: the matching function $a = .023$, $\eta = .5$, the worker's bargaining power $\beta = .5$, the separation rate $\phi = .000827604$, time endowment $T = 1.5998$, the utility function $\delta = .719514$, labor productivity $y = 1$, the vacancy cost $k = 4.13297$.

respectively. We let ξ_{base} denote the welfare gain relative to the base run, which has a replacement rate of 30%. We have conducted two types of experiments: first, choosing the optimal uniform benefit system; and second, choosing the optimally differentiated benefit system. The welfare gain of moving from an optimal uniform to an optimally differentiated system is given by the difference between the two entries for ξ_{base}.

The simulations reveal, unsurprisingly, that optimal benefit generosity increases in the degree of relative risk aversion $(1 - \sigma)$.[22] With a uniform

[22] Note that we apply a broad definition of "consumption," since it refers to

benefit structure, the optimal wage replacement rate varies between 38% and 42%. These numbers are of the same order of magnitude as results obtained by others using related models (e.g., Valdivia 1996). Although the computed replacement rates may seem low compared to most real-world UI systems, it is important to remember that effective replacement rates typically are much lower than the statutory ones because of various restrictions on eligibility (Martin 1996). We also note that the implied employment rates in the uniform system decline from 91.6% to 88.6% as risk aversion increases, an implication of the fact that higher risk aversion calls for a more generous unemployment compensation.

The potential duration of unemployment benefits is increasing in risk aversion, as insurance arguments would suggest. More surprising, perhaps, is another result. It turns out that the optimal degree of benefit differentiation increases in risk aversion. With "low" risk aversion, we have $b/z = 1.69$, and with "high" risk aversion $b/z = 2.04$; these numbers suggest that the degree of differentiation should be substantial. The key to understanding this result is to note that the time devoted to search declines with risk aversion, partly because of risk aversion per se but also because of the adjustments of the benefit system induced by risk aversion.[23] When risk aversion is high, the optimal uniform system calls for increasing benefit generosity, which has adverse effects on search incentives. When the unemployed devote a small fraction of their time to search, there is great scope for increasing noninsured search via the entitlement effect (see eq. [24]). Thus, increasing the wedge between b and z is an efficient way of restoring search incentives when risk aversion is high; agents are willing to pay the price of increased dispersion in order to increase search and reduce taxes.

The welfare gains implied by moving from an optimal uniform benefit system to an optimal two-tiered system appear to be fairly substantial: agents would be willing to pay between 0.2% ($\sigma = 0$) and 1.2% ($\sigma = -1$) of consumption to live in the optimal two-tiered system as opposed to the optimal uniform one.[24] The gains of designing the benefit system optimally per se are of course even greater. The effects on employment of moving from the optimal uniform to the differentiated system, by contrast, seem to be negligible.

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goods as well as leisure; hence, risk aversion implies that individuals would like to smooth the flow of the consumption index: $c\ell^\delta$. See Hansen and Imrohoroglu (1992) for a similar procedure.

[23] To see this, hold θ constant and differentiate eq. (15a) and eq. (15b) with respect to σ.

[24] Empirical studies–based union wage-setting models usually find that the coefficient of relative risk aversion ($1 - \sigma$) ranges between 1 and 4 (see, e.g., Farber 1978; Carruth and Oswald 1985). It should be noted, though, that risk aversion refers to income only in these studies.

Table 2
Optimal UI and Discounting (Log Utility)

	Annual Discount Rate			
	0	.05	.10	.20
z (%)	33.84	33.74	33.65	33.49
b/z	1.69	1.70	1.71	1.74
$1/\lambda$ (weeks)	8.45	8.27	8.09	7.79
ξ^e (%)	.22	.27	.32	.44
ξ^u (%)	.22	.15	.08	−.06

NOTE.—See table 1 for parameter values and definitions of variables. Variables ξ^k, $k = e, u$, are measures of the welfare gain, analogous to eq. (25), of a move from the optimal uniform to the optimal two-tiered benefit system. Let "*" and "0" index the optimal two-tiered and uniform system, respectively. We then have $\xi^e = 1 - \exp(rE_0 - rE^*)$, for the employed, and $\xi^u = 1 - \exp(rU_0 - r\overline{U}^*)$, for the unemployed, where $r\overline{U}^*$ is a weighted average of rU^{I*} and rU^{N*}. The calculations of rE^* and $r\overline{U}^*$ take the transition path into account.

We have also asked the following question: What are the welfare effects of moving from the current U.S. benefit system to the optimal two-tiered one? The U.S. unemployment insurance is crudely characterized by $b = 0.5$ and a potential duration of 26 weeks; finally, we set $z = 0.17$, and the average social assistance payments per recipient amounted to 17% of average earnings in 1991 (see Wang and Williamson 1996). The welfare gains (ξ_{US}) range from 0.4% to 3.6%; these gains are obtained through more generous levels of compensation and shorter potential duration of regular benefits.

Finally, we have calculated the benefit parameters that would be the outcome if θ (and hence the wage cost) was constant. To facilitate comparison, we fix θ at the solution of the optimal two-tiered benefit system when utility is logarithmic. As expected, this partial equilibrium analysis yields more benefit differentiation; in particular, the parameters of the benefit system would be set as follows: z would equal 36.82%, the potential duration of UI benefit receipt would be 7.6 weeks, and $b/z = 1.91$.

D. Discounting and the Political Economy of UI

We conclude Section III by analyzing the case with discounting. We do this for two reasons. First, we want to resolve the ambiguity illustrated in Section III.B. Second, when $r > 0$ distributional issues are relevant; thus, we can ask whether a proposal to move to a differentiated benefit system is politically viable, in the sense that it pleases the (presumably) employed majority.

Table 2 presents the results of varying the annual discount rate from 0% to 20%. We look only at the case of log utility and use the same parameter values as in table 1.

For a given benefit system, market tightness and aggregate search effort

are decreasing in the discount rate. To see this, first note that the value of employment relative to unemployment decreases with r, so workers opt for higher wages in the bargain; second, the present value of an occupied job falls, given the wage cost. These two effects imply that θ and aggregate search effort decrease (since θ falls). The reduction in search together with the fact that the search externality now also involves the value of operating firms, in addition to the taxation externality, suggests that the case for exploiting the entitlement effect is strengthened. As shown in table 2, the benefit system becomes more differentiated when the discount rate increases. It seems that the adverse effects on tightness of front-loading the benefit system are compensated by reducing the period of UI benefit receipt.

We have also calculated the consumption taxes that would make different categories of individuals indifferent between the optimal uniform system and the optimal two-tiered one. The implied welfare gain for the employed is given by ξ^e, and ξ^u gives the welfare change for the unemployed. The value of ξ^u is the consumption tax that would make an unemployed individual indifferent between, on the one hand, a weighted average of the values of insured and noninsured unemployment in the two-tiered system and, on the other hand, the value of unemployment in the uniform system. These welfare measures are evaluated at the policy that maximizes welfare for society as a whole. The move to a differentiated benefit system always represents a gain for the employed. Thus, a proposal to introduce an optimal two-tiered (or multitiered) benefit system is a politically viable one. The political support received from the unemployed, by contrast, varies with the discount rate: with low rates of time preference they are in favor of the policy, but when they discount the future heavily (20% annually), they actually oppose the proposal.

IV. Concluding Remarks

The main result of this article is that the socially optimal UI program is characterized by a declining time profile of benefit payments over the job searcher's spell of unemployment. A two-tiered benefit structure is optimal because it exploits the differential impact of higher benefits on search incentives among insured and noninsured unemployed workers; raising the compensation offered to the insured induces additional search effort among the noninsured (the so-called entitlement effect). The numerical calibration of the model suggests that the welfare gains obtained from optimal differentiation may well be substantial if individuals are sufficiently risk averse. The numerical results also indicate that a proposal to move from a uniform to a differentiated benefit structure has the virtue of being politically viable, assuming that the employed comprise the majority of the electorate.

The absence of savings is an unrealistic feature of our analysis, although it should be kept in mind that an analytical treatment of equilibrium search models with endogenous savings has proven to be extremely difficult. Would the optimality of the two-tiered benefit structure carry over to a model where workers were allowed to save? It is not obvious why it should not. The driving forces, in particular the entitlement effect, would still be present and motivate some differentiation of benefits.

A complete welfare analysis of UI policies would also have to consider the eligibility rules and how these affect behavior on both sides of the labor market. Existing systems require that workers have demonstrated some attachment to the labor force, for example, in the form of a minimum number of weeks in employment over the past year. A normative analysis of these issues in an equilibrium framework remains a topic for future research.

Appendix A

Present Value Differences

In a symmetric equilibrium, the differences between the expected present values are:

$$E - U^I = A^{-1}\{(r + \alpha^N)[v(w) - v^I] + \lambda[v(w) - v^N]\}$$

$$E - U^N = A^{-1}\{(r + \lambda + \alpha^I)[v(w) - v^N] + \phi(v^I - v^N)\} \tag{A1}$$

$$U^I - U^N = A^{-1}\{(r + \phi + \alpha^I)(v^I - v^N) + (\alpha^I - \alpha^N)[v(w) - v^I]\},$$

where $A \equiv (r + \lambda + \alpha^N)(r + \phi + m^I\alpha^I + m^N\alpha^N)$, with $m^I = (r + \alpha^N)/(r + \alpha^N + \lambda)$ and $m^N = 1 - m^I$. Moreover, $v^I = (B, s^I)$ and $v^N = v(Z, s^N)$.

Appendix B

Proof of Lemma 2

From equation (11), the right-hand side of equation (12) depends on ω and θ but not on s^j and w. Here, we establish that the left-hand side of equation (12) is also independent of s^j and w.

We begin by showing that $E - U^I$ is independent of s^j. Imposing symmetry in equation (7), we have $(r + \phi)E = v(w) + \phi U^I$. Hence, $\partial E/\partial s^j = [\phi/(r + \phi)]\partial U^I/\partial s^j$, $j = I, N$. By the envelope theorem it follows immediately that $\partial E/\partial s^I = 0$. From equations (5) and (7) (and symmetry), we get $[r + \lambda + r\alpha^I/(r + \phi)]U^I = v(B, s^I) + [\alpha^I/(r + \theta)]v(w) + \lambda U^N$. Therefore, $\partial U^I/\partial s^N = 0$ and $\partial E/\partial s^N = 0$, since $\partial U^N/\partial s^N = 0$.

Next, we show that the left-hand side of equation (12) does not depend on w given our utility function, $B = bw$, and $Z = zw$. According to equation (A1), $[v(w) - v^j(xw, s^j)]/v_w w$, where $x \in \{b$ if $j = I, z$ if $j = N\}$, should not depend on w if $(E - U^I)/v_w w$ is to be independent of w. From equation (4), we have $v_w w = w^\sigma(T - h)^{\delta\sigma}$, given that $c = w$ and

$\ell = T - \bar{h}$ for an employed person. The difference between instantaneous utilities is given by

$$v(w) - v^j(xw, s^j) = \frac{w(T - \bar{h})^{\delta\sigma} - (xw)^\alpha (T - s^j)^{\delta\sigma}}{\sigma}.$$

Hence,

$$\frac{v(w) - v^j(xw)}{v_w w} = \frac{1}{\sigma}\left[1 - x^\sigma\left(\frac{T - s^j}{T - \bar{h}}\right)\right],$$

which demonstrates that $(E - U^I)\big/v_w w$ is independent of the wage. Q.E.D.

Appendix C

Proof of Proposition 2

The proof is by contradiction. A uniform benefit structure cannot be optimal if $dW/db > 0$ at $b = z$. Pick an arbitrary $\lambda^0 \in (0, \infty)$ and consider the trial solution $b = z$. At $b = z$, equation (18c) is irrelevant since there is no difference between insured and noninsured unemployment. For the trial solution to be well defined, the value of λ must be interior, otherwise either b or z is not determined. First, we want to eliminate the term $\theta_b dW/d\theta$ in equation (18a). From equation (14): $\theta_b = (\mu^I \rho^I)/(b\Psi_\theta)$ and $\theta_z = (\mu^N \rho^N)\big/(z\Psi_\theta)$, where $\Psi_\theta = \partial\Psi/\partial\theta < 0$. Consequently, $\theta_b dW/d\theta = [(\mu^I \rho^I z)/(\mu^N \rho^N b)]\theta_z dW/d\theta$. Suppose that equation (18b) holds at $b = z$. Solving equation (18b) for $\theta_z dW/d\theta$ and substituting into the expression for dW/db,

$$\frac{dW}{db} = \frac{\partial W}{\partial b} - \frac{\mu^I}{\mu^N}\frac{\partial W}{\partial z} + \frac{\partial W}{\partial s^I}s_b^I - \frac{\mu^I}{\mu^N}\frac{\partial W}{\partial s^N}s_z^N + \frac{\partial W}{\partial s^N}s_b^N,$$

where we have used $\rho^I/\rho^N = 1$ at $b = z$. Also, $\partial W/\partial b = (\mu^I/\mu^N)\partial W/\partial z$, and $\partial W/\partial s^I = (\mu^I/\mu^N)\partial W/\partial s^N$. We then have

$$\frac{dW}{db} = \frac{\partial W}{\partial s^N}\left[\frac{\mu^I}{\mu^N}\left(s_b^I - s_z^N\right) + s_b^N\right].$$

To determine the sign of $(s_b^I - s_z^N)$, we make use of equations (15a) and (15b). Implicit differentiation yields:

$$s_z^N \frac{z}{T - s^N} = s_b^I \frac{b}{T - s^I} - \frac{\rho^N}{\rho^I}s_b^N \frac{b}{T - s^N}.$$

So,

$$s_b^I\big|_{b=z} - s_z^N\big|_{b=z} = s_b^N,$$

and we obtain the desired result:

$$\frac{dW}{db} = \frac{1}{\mu^N}\frac{\partial W}{\partial s^N}s_b^N > 0,$$

since $s_b^N > 0(\forall\sigma)$, according to lemma 4. Hence, at $b = z$, $dW/db > 0$. Q.E.D.

Appendix D

Several Unemployment States

Let us introduce the following notation. Let $X_j = x_jw$, $j = 1, \ldots, J$ denote compensation in state j; s_j denote search effort; $\alpha_j = s_j\alpha(\theta)$ denote the outflow rate from unemployment; u_j denote unemployment; and $\mu_j = u_j/(1 - e)$ denote the fraction of the unemployed in state j.

We restrict attention to the case where λ is identical across unemployment states. The flow equilibrium conditions then take the form

$$\phi e = \alpha(\theta)\sum_j s_j u_j,$$

$$(\lambda + \alpha_j)u_j = \lambda u_{j-1}, j = 2, \ldots, J-1 \tag{A2}$$

$$\alpha_j u_j = \lambda u_{J-1}.$$

The flow values of being in each of the possible labor market states are given by

$$rE = v(w) - \phi(E - U_1),$$

$$rU_j = v(X_j, s_j) + \alpha_j(\theta)(E - U_j) - \lambda(U_j - U_{j+1}), j = 1, \ldots, J-1,$$

$$rU_J = v(X_J, s_J) + \alpha_J(\theta)(E - U_J).$$

The equilibrium conditions are straightforward generalizations of the case of two unemployment states. Equilibrium labor market tightness is determined by

$$\Psi(\theta, x_1, \cdots, x_J, \lambda) \equiv \frac{1 - \sum_j \mu_j\rho_j}{\sigma} \tag{A3}$$

$$-\left(\phi + \sum_j \mu_j\alpha_j\right)\frac{\beta}{1 - \beta}\frac{k/q(\theta)}{1 - \phi k/q(\theta)} = 0,$$

where $\rho_j = x_j^\sigma[(T - s_j)/(T - \bar{b})]^{\delta\sigma}$ and $r = 0$ for simplicity. From equation (A3) we have that market tightness is decreasing in x_j:

$$\frac{\partial\theta}{\partial x_j} = \frac{\mu_j\rho_j}{\Psi_\theta x_j} < 0 \text{ (as } \Psi_\theta < 0). \tag{A4}$$

Taking wage bargaining into account, search is given by

$$\frac{\rho_1 \delta}{T - s_1} = \frac{\beta}{1 - \beta}\frac{\theta k}{1 - \phi k/q(\theta)}$$

$$\frac{\rho_j \delta}{T - s_j} = \frac{D_j}{\mu_j}\left[\mu_1(\lambda + \alpha_1)\frac{\beta}{1 - \beta}\frac{\theta k}{1 - \phi k/q(\theta)}\sum_{l=j}^{J}\mu_l \right. \qquad \text{(A5)}$$

$$\left. +\alpha(\theta)\left(\sum_{k=1}^{j-1}\mu_k\sum_{l=j}^{J}\mu_l\frac{\rho_k - \rho_l}{\sigma}\right)\right], \, j = 2, \, \dots, J,$$

where D_j denotes the duration of state j: $D_j = 1/(\lambda + \alpha_j)$, $j = 1, \dots, J - 1; D_J = 1/\alpha_J$.

LEMMA D1.   Search intensity in state $j \geq 2$ is increasing in x_{j-1}, decreasing in x_j (sufficient condition: $\sigma \geq 0$), and decreasing in x_{j+1}.

Proof.   Implicit differentiation of equation (A5), recognizing that the right-hand side of equation (A5) is independent of s_j. Q.E.D.

The welfare objective and the budget constraint are given by $W = ev(w) + \sum_j u_j v(x_j w, s_j)$ and $\tau e = \sum_j x_j u_j$, respectively. Consider the derivative of W with respect to s_j, taking equation (A5) and the budget constraint into account:

$$\frac{\partial W}{\partial s_j} = -v_w w\left(e + \sum_j u_j \rho_j\right)\frac{d\tau}{ds_j}(1 + \tau)^{-1} > 0, \, j = 1, \, \dots, J. \qquad \text{(A6)}$$

Thus, search intensity is too low in market equilibrium, because of the taxation externality alluded to in the main text.

Consider the fully optimal policy, that is, a choice of $(x_1, \dots, x_J, \lambda)$. To simplify the exposition, we only consider the case of log utility. This does not change any of the fundamental properties of the problem. Thus, maximizing $W(\cdot)$ subject to equation (A2), the budget constraint, $\theta = \theta(x_1, \dots, x_J, \lambda)$, $s_1 = s_1(\theta)$, and $s_j = s_j(x_1, \dots, x_J, \lambda, \theta)$, $j \geq 2$, yields the optimality conditions:

$$\frac{dW}{dx_j} = \frac{\partial W}{\partial x_j} + \sum_{k=2}^{J}\frac{\partial W}{\partial s_k}\frac{\partial s_k}{\partial x_j} + \frac{dW}{d\theta}\frac{\partial \theta}{\partial x_j} = 0, \, j = 1, \, \dots, J, \qquad \text{(A7)}$$

$$\frac{dW}{d\lambda} = \frac{\partial W}{\partial \lambda} + \sum_{k=2}^{J}\frac{\partial W}{\partial s_k}\frac{\partial s_k}{\partial \lambda} + \frac{dW}{d\theta}\frac{\partial \theta}{\partial \lambda} = 0, \qquad \text{(A8)}$$

where $dW/d\theta = \partial W/\partial \theta + \sum_j (\partial W/\partial s_j)(\partial s_j/\partial \theta)$.

PROPOSITION D1.   Given $\lambda \in (0, \infty)$, the optimal benefit policy involves $x_1 > x_2 > \dots > x_J$, provided that $x_J > 0$.

Proof.   Suppose that the Jth condition in equation (A7) holds with equality. Let us consider the $(J - 1)$th. Using equation (A4), we can rewrite the latter condition as follows:

$$\frac{dW}{dx_{J-1}}x_{J-1} = \frac{\partial W}{\partial x_{J-1}}x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial W}{\partial x_J}x_J$$

$$+ \sum_{k=2}^{J}\frac{\partial W}{\partial s_k}\left(\frac{\partial s_k}{\partial x_{J-1}}x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial s_k}{\partial x_J}x_J\right) = 0.$$

Now, $(\partial s_k/\partial x_{J-1})x_{J-1} - (\mu_{J-1}/\mu_J)(\partial s_k/\partial x_J)x_J = 0$ for $k = 2, \ldots, J-2$. Equation (A6) and $(\partial W/\partial x_j)x_j = u_j\{1 - x_j/[e(1 + \tau)]\}$, $\forall j$, yields:

$$\frac{dW}{dx_j}x_j = (1 + \tau)^{-1}\left[\frac{u_{J-1}}{e}(x_J - x_{J-1})\right.$$

$$\left. - \sum_{k=J-1}^{J}\frac{d\tau}{ds_k}\left(\frac{\partial s_k}{\partial x_{J-1}}x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial s_k}{\partial x_J}x_J\right)\right] = 0.$$

Since $d\tau/ds_j < 0$, $\forall j$, and s_J is increasing in x_{J-1} but decreasing in x_J, we know that $d\tau/ds_J[(\partial s_J/\partial x_{J-1})x_{J-1} - (\mu_{J-1}/\mu_J)(\partial s_J/\partial x_J)x_J] < 0$. There is left to show that raising x_J has more adverse effects on s_{J-1} than raising x_{J-1}. Differentiation of equation (A5) gives:

$$\frac{\partial s_{J-1}}{\partial x_{J-1}}x_{J-1} - \frac{\mu_{J-1}}{\mu_J}\frac{\partial s_{J-1}}{\partial x_J}x_J = D_{J-1}\alpha(\theta)\frac{(T - s_{J-1})^2}{\delta}\mu_{J-1} > 0.$$

Therefore, $x_{J-1} > x_J$. Proceeding analogously one can establish that $x_{J-2} > x_{J-1}$; indeed, for any $j = 2, \ldots, J$, we have $x_{j-1} > x_j$. Q.E.D.

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