The Effective Teaching of Mathematics : A Review of Research

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Abstract

An outline is given of three bodies of knowledge that are in general agreement about the characteristics of the effective teaching of mathematics, and which are reflected in the British government's National Numeracy Strategy. A fourth body of knowledge related to the development of higher order skills is outlined, together with some further research needs.

Introduction

Much controversy has surrounded the issue of the standards of mathematics achievement in the United Kingdom. Reviews of international evidence (Reynolds and Farrell, 1996) suggest a poor performance, with some evidence of deterioration over time, although performance on number has been particularly poor and that on skills such as geometry and data handling/analysis particularly good. The recent Third International Science and Mathematics Study (TIMSS) (Keys, Harris and Fernandes, 1996) also showed that British performance in mathematics was poor, whilst performance in science at ages 9 and 14 was particularly good by comparison with other countries, making it unlikely that socio-economic factors, cultural characteristics such as television viewing or other environmental features of British society were responsible, given that they would have predisposed to low mathematics *and* science scores.

Given the likelihood that educational factors of some kind are implicated, a wide array of solutions have been proposed, ranging from the completely mixed ability, whole class teaching offered in some primary schools in Barking and Dagenham Local Education Authority, to the National Numeracy Strategy that was generated by the British government's Numeracy Task Force (Department for Education and Employment (DfEE), 1998a and b). All these approaches are covered in more detail elsewhere (Thompson, 1999).

Whilst these very heterogeneous solutions have been proposed historically for the British mathematics 'problem', more recently there has occurred the emergence of an agreed 'solid centre' of practice which is widely seen as being effective in generating high mathematics achievement. This position is neither 'progressive' or 'traditional' (to use the over simple labels present in political debate) but represents a 'both/and' philosophy of practice, rather than the mutually exclusive 'either/or' orientation that has existed historically. Thus, adherents to this position would argue for classrooms having whole class interactive teaching *and* differentiated groups *and* some individual work, and would argue for calculators to be used *together* with both mental calculation and mental strategies (see Reynolds, 1999 for an outline of the new concensus and its generation).

In this paper, we outline some of the knowledge bases that have played a part in facilitating the emergence of the new paradigm, and which are now available to practitioners and policymakers in schools to help develop this practice. We deal here particularly with the evidence from three bodies of knowledge:

- the American evidence on effective maths teaching;
- the British knowledge base on effective maths teaching;
- the British 'professional' knowledge base from OFSTED about effective maths teaching.

We then go further than the bodies of knowledge that have helped to facilitate the recent concensus on effective maths teaching and look at the multinational evidence about more advanced teaching strategies that can be used to deliver 'higher order' mathematical skills. In our conclusions, we outline some further thinking about exactly how these kinds of teaching skills can be developed by practitioners within schools.

American Teacher Effectiveness Research on Mathematics Teaching

This follows a 'process-product' model and originated (in the late sixties/early seventies) in large-scale classroom observation studies, from which teacher behaviours emerged that were found to be correlated with higher achievement in mathematics. The majority (but not all) of this research was carried out amongst primary age students. Based on this research a number of ‘active teaching’ models were developed that were tested in a number intervention programmes, the most well known being the Missouri Mathematics programme in the late seventies that we outline later. These models approximate to the whole class ‘interactive’ model of maths teaching that is currently the focus of British national policy (DfEE, 1998a and b), and contain the following elements:

High Opportunity to Learn

One of the factors to most consistently and most strongly affect mathematics test scores is ‘opportunity-to-learn’, whether it is measured as amount of the curriculum covered or the percentage of test items taught (Brophy and Good, 1986; Hafner, 1993, Herman and Klein, 1996). Opportunity-to-learn is clearly related to such factors as length of the school day and year, and to the amount of hours of mathematics taught. It is, however, also related to the quality of classroom management, especially to what is known as time-on-task (i.e. the amount of time children are actively engaged in learning activities in the classroom). Opportunity to learn is also clearly related to the use of homework, which expands available learning time.

An Academic Orientation from the Teacher

Another highly important factor that is also connected to children’s time-on-task, is the teacher’s academic orientation. Effective maths teachers emphasise academic instruction, and see learning as the main classroom goal. This means that they spend most of their time on curriculum-based learning activities, and create a task-oriented, businesslike, but also supportive, environment. They spend time on academic activities rather than on personal matters, group dynamics, socialising or free time (Brophy and Good, 1986; Griffin and Barnes, 1986; Lampert, 1988; Cooney, 1994).

Effective Classroom Management

Effective teachers are able to organise and manage classrooms as effective learning environments in which academic activities run smoothly, transitions (between lesson segments) are brief, and little time is spent getting organised or dealing with inattention (Brophy and Good, 1986). For this to happen, good prior preparation of the classroom and the installation of clear rules and procedures (before or at the start of the school year) are essential. During lessons it is better to use positive rather than negative language to cue behaviour. Desired behaviour must be recognised and reinforced, attention must be stimulated, for example by regularly altering teaching strategies, and continuous monitoring during lessons is essential. All in all, effective teachers manage to create a well-organised classroom with minimal disruption and misbehaviour (Secada, 1992; Evertson et al, 1980; Lampert, 1988; Griffin and Barnes, 1986; Brophy and Good, 1986).

High Teacher Expectations of the Pupils

Effective maths teachers must show that teachers believe that all children can master the curriculum (not just a percentage of children). They emphasise the positive (e.g. if a child is not so good in one area such as shape and space, s/he might be good in number) and these positive expectations need to be transmitted to the children. Teachers should also emphasise the importance of effort, clarifying the relationship between effort and outcomes and helping pupils gain an internal locus of control by constantly pointing out the importance of their own work (Borich, 1996).

A High Proportion of Whole-Class Teaching

Moving to the nature of the classroom organisation, American research has found that children learn more in classes where they spend time being taught or supervised by their teacher rather than working on their own. In such classes teachers spend most of their time presenting information through lecture and demonstration. Teacher-led discussion as opposed to individual work dominates. This is not to say that all individual work is negative, individual practice is even necessary and important, but ineffective teachers have been found to rely too much on pupils working on their own, at the expense of lecture-demonstration and class discussion (Evertson et al, 1980; Rosenshine and Stevens, 1986). Research has found that classrooms where more time is spent teaching the whole class, rather than on letting individual pupils work by themselves (e.g. with worksheets), show higher pupil achievement gains. This is mainly because teachers in these classrooms provide more thoughtful and thorough presentations, spend less time on classroom management, enhance time-on-task and can make more child contacts. Teachers giving whole-class instruction have also been found to spend more time monitoring children's achievement. There were also likely to be less child disruptions, thus again increasing time-on-task (Good, Grouws and Ebmeier, 1983; Evertson et al, 1980; Walberg; 1986; Brophy, 1986; Good et al, 1990; Mason and Good, 1993; Borich, 1996).

In this form of classroom organisation the effective teacher carries the content personally to the student rather than relying on curriculum material or textbooks to do so. Information is mainly conveyed in brief presentations, followed by recitation and application opportunities. It is clear that in this type of instruction the teacher takes an active role, conveying information to the students rather than just 'facilitating' student learning. Use of examples is important, and effective teachers strive to make the presentation lively, incorporating an element of performance. Presentations of new concepts are usually brief, followed by opportunities for practice or recitation. This kind of instruction is usually called ‘direct’ or ‘active’ (Brophy and Good, 1986; Lampert, 1988; Brophy, 1986; Borich, 1996; Creemers, 1994).

Achievement is maximised when the teacher not only actively presents the material, but does so in a structured way, by beginning with an overview and/or review of objectives. Effective teachers outline the content to be covered and signal transitions between lesson parts. Attention is drawn to the most important ideas, and subparts of the lesson are summarised as it proceeds. The main ideas are reviewed at the end of the lesson. In this way, the information is not only better remembered by the children, but is also more easily apprehended as an integrated whole, with recognition of the relationship between the parts (Brophy and Good, 1986; Lampert, 1988).

In connection to this approach to teaching the whole class, the question of whether or not setting or streaming is necessary when teaching in this way. The answer to that question has to be a pragmatic one, in that, while most research does not show strong benefits from setting (e.g. Askew and Williams, 1995; Good, Mulryan McCaslin, 1992) setting might be necessary in a context in which the ability levels of children differs strongly.

Heavily Interactive Teaching that Involves Pupils in Classroom Attitudes

This focus on the teacher presenting material in an active way to students should, however, not be equated to a traditional ‘lecturing and drill’ approach in which the students remain passive. Active teachers ask a lot of questions (more than other teachers), and involve students in class discussion. In this way students are kept involved in the lesson and the teacher has the chance to monitor children’s understanding of the concepts taught. Individual work is only assigned after the teacher has made sure children have grasped the material sufficiently to be ready for it. In general, effective teachers have been found to teach a concept, then ask questions to test children's understanding, and if the material did not seem well understood, to reteach the concept, followed by more monitoring. Teachers must provide substantive feedback to students resulting from either pupil questions or answers to teacher questions (Brophy and Good, 1986; Good, Grouws and Ebmeier, 1983; Brophy, 1986; Rosenshine and Stevens, 1986; Borich, 1996).

There is no simple picture of the most functional cognitive level of questions. It would seem that the best strategy is to use a mix of low and higher level questions, the mix depending on lesson goals and subject content, more cognitively complex subjects demanding more higher level questions. The optimal post-question 'wait time' similarly depends on context. When testing basic skills a short wait time is appropriate. However, when asking more cognitively demanding questions, or questions that are meant to elicit group exploration or discussion, a longer wait time is appropriate. Effective teachers have been found to ask more process questions (i.e. questions calling for explanations by the students), though the majority of questions asked were still product questions (i.e. calling for a single response) (Evertson et al, 1980; Brophy and Good, 1986; Borich, 1996).
A well known example of an effective programme of maths teaching noted earlier is the Missouri Mathematics Effectiveness Project conducted by Good and associates in the late 1970s (Good, Grouws and Ebmeier, 1983; Good and Grouws, 1979). On the basis of research findings, a primary school teaching model was designed, which teachers were trained to implement. Lessons were structured as follows for teachers:

a) Daily Review (approx. 10 minutes)
   1. Review concepts and skills associated with previous day's homework
   2. Collect and deal with homework assignments
   3. Ask several mental computation exercises

b) Development (approx. 20 minutes) (introducing new concepts, developing understanding)

1. Briefly focus on prerequisite skills and concepts
2. Focus on meaning and promote student understanding by lively explanations, demonstrations etc.
3. Assess student competence
   a. Using process and product questions (active interaction)
   b. Using controlled practice
4. Repeat and elaborate on the meaning portion as necessary

c) Individual Work (approx. 15 minutes)

1. Provide uninterrupted successful practice
2. Momentum - keep the ball rolling - get everyone involved, then sustain involvement
3. Alerting - let students know their work will be checked at the end of each period
4. Accountability - check the student's work

d) Homework assignment

1. Assign on a regular basis at the end of each maths class
2. Should involve about 15 minutes of work to be done at home
3. Should include 1 or 2 review problems

Teachers were trained to follow the active teaching method as described above throughout. Although some teachers had problems implementing the crucial development part of the lesson, the impact on pupils' mathematics achievement was impressive. A replication study and a study using a slightly revised model in secondary schools were equally effective (Good, Grouws and Ebmeier, 1983). Programs based on the same teacher-effectiveness principles but with modification suited to the particular contexts, have been implemented on a smaller scale in the US, such as Griffin and Barnes' (1986) CTP project, which also showed good results.

The British Teacher Effectiveness Research on Mathematics Teaching

The United Kingdom knowledge base is, by contrast to the American, a highly restricted one, although there is evidence of considerable contemporary policy interest in 'teaching' (Galton, 1995) and some promising new research avenues being explored, particularly in the areas of teacher's conceptual and subject knowledge in mathematics (Askew et al, 1997) and in variation in the effects of teachers' behaviours in lessons (Creemers and Reynolds, 1996). Interestingly, the British results on effective teaching parallel those from America.

The notable ORACLE study, which involved a 'process-product' orientation similar to the American research just discussed, showed that teachers labelled as 'Class Enquirers' generated the greatest gains in the areas of mathematics and language, but that this finding did not extend to reading. By contrast, the group of 'Individual Monitoring' teachers made amongst the least progress. It is important to note that the more successful 'Class Enquirers' group utilised four times as much time in whole class interactive teaching as the 'Individual Monitors' (Galton and Croll, 1980; Croll, 1996).

Further analyses (Croll, 1996) correlated the academic gain made by different classes with different patterns of class/teacher interactions, finding a moderate positive (0.29) correlation between the percentage of time utilised in whole class and small group interaction and children's progress, showing as Croll (1996, p.23) notes '.. a positive association of progress and non-individualised interaction'. Croll (1996) notes two dangers in any rapid translation of ORACLE findings into recommended practice - that the 'whole class interactive' teachers differed in ways other than in their class teaching techniques, and that teachers utilising other teaching styles (the 'Infrequent Changers') which did *not* have high levels of whole class interaction also scored above average in gain.

The ORACLE study also looked at the children's 'time on task' (or academically engaged time) and found that whole class interaction was positively associated with high levels of time on task, with the 'Class Enquirers' having average time on task 10% higher than other teachers. The PACE study (Pollard et al, 1994) also notes high levels of whole class interactive teaching to be correlated with high pupil 'task engagement'. Further analyses by one of the ORACLE authors (Croll and Moses, 1988) shows a high positive correlation between time in whole class interaction and time on task. Time in group based interaction showed no such association. It is important to emphasise in this context of course the close link between 'time on task' and learning gain, which is one of the most replicated of the America teacher effectiveness findings that we noted earlier.

The second important British teacher effectiveness study is the Junior School Project (JSP) of Mortimore et al (1988), based upon a four year cohort study of pupils from 50 primary schools, which involved collection of a considerable volume of data on children and their family backgrounds ('intakes'), school and classroom 'processes' and 'outcomes' in academic (reading, mathematics) and affective (e.g. self conception, attendance, behaviour) areas. This study reported twelve factors that were associated with effectiveness both across outcome areas and in specific subjects such as mathematics. Significant positive relationships were found with such factors as structured sessions, use of higher-order questions and statements, frequent questioning, restricting sessions to a single area of work, involvement of pupils and the proportion of time utilised in communicating with the whole class. Negative relationships were found with teachers spending a high proportion of their time communicating with individual pupils (Mortimore et al, 1988).

It is important at this point to note the considerable agreement between the American research base and the British. The American literature emphasised the gains produced by active teaching and by interactive teaching, and the positive effect of a high proportion of teacher time on questioning, on communication with the class and on pupil involvement with their work that was the teaching style of the ‘class enquirers’ on the ORACLE study as noted above. The potency of whole class interactive instruction is shown in both the American and British teacher effectiveness literature, and the benefits of whole class teaching, active teaching and of interactive teaching, are all shown in the classic Mortimore (1988) study.

It is also important to note at this point that British research shows the deleterious effect of children working on their own, emphasising the need to reduce unguided practice to optimise gain, as shown in the American research too.

The British Professional Evidence on Effective Mathematics Teaching

There is a third body of knowledge that we can look at to discern ‘good practice’, which is that taken from the publications of OFSTED. This knowledge base has been generated by professionals with considerable experience of mathematics processes at classroom and school level, and of the mathematics outcomes related to them, although it does not necessarily possess the tight research design of our first two bodies of knowledge above.

Three reports from OFSTED seem appropriate here. The first, *Primary Matters* (OFSTED, 1995), outlined a number of general teacher/teaching factors associated with positive outcomes in general. These included factors such as good teacher subject knowledge, good questioning skills, an emphasis on instruction, clear objectives, good time management and good classroom organisation.

The second study was specifically related to the characteristics of the successful teaching of basic skills and, for our purpose here, to achievement in numeracy (OFSTED, 1996). It outlined firstly the characteristics of classroom processes where maths standards were low, which included:

- too much emphasis upon repetitive number work
- too much individualisation of work
- too little fluency in mental calculation.

The report went on to outline in detail the characteristics of successful maths teaching. This:

- provides clear structure for lessons and makes good use of time, maintaining challenge, pace and motivation;

- includes sessions of direct teaching, with the teacher involved pro-actively and not just when pupils are stuck;

- involves regular interaction with pupils, with the teacher using perceptive questioning, giving careful attention to misconceptions and providing help and constructive response;

- rehearses existing knowledge and skills in order to keep them sharp and enhance them, and encourages quick recall of as many number facts as possible;

- uses a variety of activities on a topic in order to consolidate and extend understanding;

The third study was that undertaken by OFSTED on *The Teaching of Number in Three Local Education Authorities* (1997). Some key conclusions of this report were (Italics added):

> ‘In all the schools there was a mix of whole class, group and individual work in varying proportions during the lesson. Some teachers were much more successful than others in the way they used these organisational strategies to extend and direct their teaching and to increase pupils’ number knowledge, understanding and skills. *In the best lessons there was usually a higher proportion of time spent teaching the class together, often at the start and sometimes at the end of the lesson, with individual and group work closely linked to the whole-class work.* The pupils worked on tasks individually or in groups which reinforced or extended what they had been taught in the whole-class time and consolidated their learning by coming together again at the end of the lesson. The effect of this was an obvious common gain in core knowledge and skills which made it easier for the pupils to help one another and progress together. For the teacher, too, this well-structured mix of whole-class, group and individual work not only made the lesson more manageable, it also established a climate and a common language for talking about mathematics which benefited more children for more of the time.

> These good lessons stood in sharp contrast to the *poorer work which suffered from a distinct, common organisational weakness, notably a debilitating over-use of individual work, and to a lesser extent, group work.* Where these weaknesses occurred there was often an over-reliance on worksheets and published schemes. In other words, more often than not complex arrangements for individual work were self-defeating; they dissipated rather than intensified the quality of the teaching and reduced the opportunities for children to learn.

A further advantage of whole-class teaching identified by Ofsted in their recent review of early results of the National Numeracy Project (Ofsted, 1998) is that it is more successful in bringing pupils whose first language is not English up to the level of their peers, thanks to the direct teaching of mathematics terms with which they might not otherwise be as familiar as other children.

A Fourth Area : Research on Problem Solving and Co-operative Group Work in Mathematics

While programmes based on a direct instruction/active teaching model have been effective in improving mathematics achievement in basic skills, doubt has been cast on whether this approach is sufficient for teaching higher-order mathematical problem-solving or mathematical thinking skills, the importance of which has received increasing emphasis in recent years (Galton, 1995). According to Peterson (1988), while direct instruction focusing on basic skills may be a necessary condition for being able to develop higher order thinking and problem-solving skills, it is not sufficient. A number of additional classroom processes may be needed to enhance higher order thinking: a focus on meaning and understanding in mathematics, direct teaching of higher level cognitive strategies and problem solving, and co-operative small group work, which we deal with here.

The advantages of co-operative small group work to developing problem solving skills lie partly in the 'scaffolding' process, whereby pupils help each other learn in the 'zone of proximal development'. Giving and receiving help and explanation may develop children's thinking skills, as well as helping them to verbalise and structure their thoughts (Peterson, 1988; Leikin and Zaslavsky, 1997). By co-operating in small groups children can share their own ways of thinking and reflect on them and on the thinking and ideas of others. This exchange may encourage students to engage in more higher order thinking (Becker and Selter, 1996). Pupils thus provide assistance and support to each other. Co-operative small groups force the accommodation of the opinions of various members, and students must therefore search, engage in problem solving and take one another's perspective. In this way students can develop an enhanced understanding of self and other, and learn that others possess both strengths and weaknesses. This may help students who are less able problem solvers to overcome their insecurity about problem solving because they can see more able peers struggling over difficult problems. The fact that a group contains more knowledge than an individual means that problem solving strategies can be more powerful. This may help students see the importance of co-operation. Group members may serve as models to one another, thus enhancing learning-to-learn skills. Students also receive practice in collaboration, a skill they will require in real life. (Daniels, 1993; Good, McCaslin and Reys, 1992; Mevarech and Kramarski, 1997).

A number of authors have pointed to possible problems with this method, however. Thus, as Good, McCaslin and Reys (1992) point out, shared student misconceptions can be reinforced by group work. Furthermore, students might be tempted to engage in off-task social interaction (Goods and Galbraith, 1996). Students may also receive differential status in groups. Some may start to perceive themselves as having little to contribute to the group, or may find that their contributions are not greatly valued. This may lead them to become passive in the group. Small group work may then advantage high ability students over lower ability ones. Studies have found low ability students to be less active in small groups, in part because they understand the task less well, and in part because student talk can also express low expectancies of certain other students (Good, Mulryan and McCaslin, 1992; Good and Biddle, 1988; Leikin and Zaslavsky, 1997). One of the main problems with small group work is the fact that small groups require far more classroom management skills from the teacher. If not well prepared, small group work can significantly increase the time the teacher spends on direction, transition and nonmathematical managerial activities (Good et al, 1990; Brophy and Good, 1986). To be effective, teachers also need to have access to a large number of problems, further exacerbating the difficulties involved with group work. All in all, small group work requires a lot of preparation time for the teacher (Doyle, 1986).

However, programmes that have attempted to develop problem-solving skills through small-group work report good results, such as enhanced conceptual understanding and higher achievement on problem-solving tasks (e.g. Wood and Sellers, 1997; Maher, 1991; Verschaffel and De Corte, 1993; Leikin and Zaslavsky, 1997; Townsend and Hicks, 1997; Goods and Galbraith, 1996).

While the benefits of co-operative small group work for enhancing students' problem solving skills seem important, it is clear that for group work to be effective it is insufficient to put students into groups and let them get on with it. A number of conditions need to be met. The most important of these, according to Stevens and Slavin (1995), who base their findings on a large number of studies carried out by Slavin, are group goals and individual accountability. Group goals are essential to motivate students to work co-operatively and thus help their groups' mathematics learning. Individual accountability increases the engagement of individual students and decreases the probability of ‘free-rider’ effects, whereby certain students choose to remain passive while letting other group members do all the work.

According to Good, McCaslin and Reys (1992), to make small group work effective, teachers need to design tasks that encourage the group to work together. The use of shared manipulatives (e.g. graphs, calculators) can be useful. The task should engage all members of the group, focusing on processes. Each student should think about the problem in a meaningful way, and then participate in other students’ problem solving strategies. The task should be sufficiently challenging for it to require collective problem solving, but not so challenging that it causes group members to give up prematurely. Work-group tasks that introduce a concept are often more effective than tasks that review a mathematical idea.

As with the active teaching model discussed above, a number of models have been proposed that incorporate co-operative small group work. Good, McCaslin and Reys (1992) propose a model which still relies largely on whole class teaching, as the teacher spends thirty to forty minutes directly teaching the class. Part of this time is spent on developing new concepts and skills with a stress on meaning, by explanation, demonstration, illustrations and discussion. After that the work-group task is described, using explanation, investigation of the task, generalisation and application. Five to ten minutes is then spent on the actual work-group task, which should reinforce or extend concepts explained during whole class teaching, while the last five to ten minutes are spent on review. Mevarech and Kramarski’s (1997) model leaves more time for the actual group work. The teacher starts by introducing a new concept to the whole class, but then proceeds immediately to group work, wherein the pupils have to answer 3 kinds of metacognitive questions: comprehension questions, strategy questions (select, describe and justify strategy appropriate to solving the problem) and connection questions (describe similarities and differences between the problem at hand and previously solved problems).

Overall, it is clear that effective small group work is a structured activity that requires a lot of teacher effort. If the subject worked on requires higher order thinking and problem solving skills, and is used under the conditions described above, the effort will be worth it, however, in terms of learning gains in 'higher order' areas.

In no sense, though, is the development of higher order skills to be seen as unrelated to that of basic computational and other skills, since performance on the two sets of skills is related (Hembree, 1992). What is required for an optimal level of achievement across a range of mathematical skills is both whole class interactive *and* collaborative group based teaching, with perhaps Key Stages One and Two more heavily orientated to the former and later Key Stages towards the latter.

Conclusions

Teachers and policymakers may be excused from a degree of cynicism as to why they should believe the research and recommendations about the importance of whole class interactive teaching and the 'active teaching' model we have outlined here, when for perhaps two decades they have been encouraged to pursue other, quite radically different, policies. Whatever the reason for these past 'non rational', ineffective policies may have been, teachers and policymakers should be clear that all the bodies of knowledge that we have converge upon the same set of effective policies. In international surveys, it is societies utilising high proportions of whole class interactive teaching that have the highest test scores (Reynolds and Farrell, 1996). In national programmes, such as the National Numeracy Project, those based upon whole class interactive teaching have obtained impressive gains (DfEE, 1999). From the within Britain and the within American research, it is clear that the active, whole class teaching that involves pupils is associated with higher learning gains in basic skills. When all evidence available converges on the same practice in terms of 'what works', it would be foolish to ignore it.

It is important, though, to recognise some areas where we urgently require further research. Firstly, there is growing evidence of 'context specificity' in the precise factors associated with learning gains, originally shown in interesting research from California, where highly effective schools in poor catchment areas pursued policies *discouraging* parental involvement in the school, in contrast to the effective schools in more advantaged catchment areas that encouraged the practice (Hallinger and Murphy, 1986). Whilst some factors apply across all social contexts (such as having high expectations of what children can achieve or 'lesson structure'), it may be that certain factors apply only in certain environmental contexts. At classroom level an example might be that the factor of 'proceeding in small steps with consolidation if necessary' is important for *all* children who are learning to read for the first time in all contexts, whilst in the contexts inhabited by lower social class or lower attaining children it may be necessary to ensure high learning gain through the use of small 'steps' for teaching *all* knowledge and not just knowledge that is new, before moving on to other approaches.

Borich (1996) gives the following summary of teacher factors that may be necessary to achieve high achievement gains in classrooms in two different social settings, those of low socio-economic status and middle/high socio-economic status. Effective practices within low socio-economic status contexts involve the teacher behaviours of:

- generating a warm and supportive affect by letting children know help is available;

- getting a response, any response, before moving on to the next bit of new material;
- presenting material in small bits, with a chance to practice before moving on;
- showing how bits fit together before moving on;
- emphasising knowledge and applications before abstraction, putting the concrete first;
- giving immediate help (through use of peers perhaps);
- generating strong structure and well-planned transitions;
- generating strong structure, ground-flow and well-planned transitions;
- the use of individual differentiated material;
- the use of the experiences of pupils.

Effective practices within middle socio-economic status contexts involve the teacher behaviours of:

- requiring extended reasoning;
- posing questions that require associations and generalisations;
- giving difficult material;
- the use of projects that require independent judgement, discovery, problem solving and the use of original information;

- encouraging learners to take responsibility for their own learning;

- very rich verbalising.

We do not know as yet the extent of any ‘context specificity’ in the precise factors associated with gains in mathematics achievement. However, there are enough hints of the existence of this factor to make one wary of using undifferentiated methods in highly differentiated school contexts.

Secondly, it may be that there will be difficulties in relating the above bodies of knowledge to practitioners, in the following ways:

- maximum take up of such knowledge is usually held to occur when there is ownership of the process of individual/school improvement by individuals and schools (Fullan, 1991; Hopkins et al, 1996). How can this be maximised in the case of mathematics teaching, when practice has been so different to that which research suggests is optimal?

- given the long term developmental needs of the profession, how can teachers be enabled to be active, reflexive practitioners involved in knowledge creation about effective practices, whilst at the same time being given defined ‘good practice’? How can the provision of educational foundations of good practice also enable long term development?

Given the complexity of these issues, it is essential that the new national programmes involved in the National Literacy Strategy and the National Numeracy Strategy, are evaluated as ‘experiments of nature’ in order to inform these issues. The likely variation between teachers and schools in their utilisation of the programmes can furnish an authentic British knowledge base concerning teacher effectiveness, looking at universal characteristics that may be necessary to optimise achievement in all types of catchment areas, and at context specific characteristics that may be necessary for effectiveness within certain school environments. Evaluation of the National Numeracy Strategy can further test out the validity of the effectiveness factors outlined in this paper.

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