IMPROVED DECISION RULES IN THE FELLEGI-SUNTER
MODEL OF RECORD LINKAGE

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ABSTRACT

Many applications of the Fellegi-Sunter model use simplifying assumptions and ad hoc modifications to improve matching efficacy. Because of model misspecification, distinctive approaches developed in one application typically cannot be used in other applications and do not always make use of advances in statistical and computational theory. An Expectation-Maximization (EMH) algorithm that constrains the estimates to a convex subregion of the parameter space is given. The EMH algorithm provides probability estimates that yield better decision rules than unconstrained estimates. The algorithm is related to results of Meng and Rubin (1993) on Multi-Cycle Expectation-Conditional Maximization algorithms and make use of results of Haberman (1977) that hold for large classes of loglinear models.

Key Words: MCECM Algorithm, Latent Class, Computer Matching, Error Rate

   This paper provides a theory for obtaining constrained maximum likelihood estimates for latent-class, loglinear models on finite state spaces. The work is related to Expectation-Maximization (EM) algorithms by Meng and Rubin (1993) for obtaining unconstrained maximum likelihood estimates. Meng and Rubin generalized the original ideas of Dempster, Laird, and Rubin (1977), hereafter denoted by DLR. The new class of algorithms, denoted by EMH, apply results of Haberman (1977) to constrain estimates to appropriate subregions of the parameter space and assure that the likelihood of successive parameter estimates is nondecreasing. In a variety of cases including the one of this paper, the method of constraining estimates can be expressed in closed form. Thus, constraining estimates need not necessitate iterative fitting methods such as Newton-Raphson or grid search.
   With many applications involving latent class models, computation and modeling is greatly simplified because observed variables are assumed to be independent conditional on unobserved classification variables (e.g., Titterington, Makov, and Smith 1988) or such independence is known to hold (Rubin and Stern 1993). With record linkage problems, conditional independence does not hold (Smith and Newcombe 1975, Thibaudeau 1993). If latent class models have a large, say ten, number of observed variables, modeling the correct set of interactions is considerably more difficult than it is with general loglinear models where it is known to be difficult (e.g., Bishop, Fienberg, and Holland 1975). Conventional statistics such as chi-square do not yield accurate indications of the fit of estimates to the truth (Winkler 1989a, 1992, Rubin and Stern 1993).
   Instead of modeling the precise set of interactions, it may be suitable to include an easily specified, say all 3-way, set of interactions and restrict the solutions to a subregion of the parameter space based on prior knowledge. If the constraints based on a priori knowledge are appropriate, then the resultant parameter estimates and decision rules may be nearly as good as

those obtained through detailed modeling of specific sets of interactions. The key is that the constraints need to be easily specified, must be sufficiently weak that they provide sensible restriction in a variety of similar situations, and that 3-way interactions and suitable parameter-space restrictions may yield reasonable approximations to correct models that use interactions of orders higher than 3-way.

   The main example involves a record linkage problem having known matching status. Computation is straightforward because each of the successive components of the Maximization step are in closed form and the restriction to a subregion of the parameter space is also in closed form using simple constraints. To motivate the concepts, basic parameter-estimation in record linkage and the successive types of EM-type estimation procedures that have so far been applied are described.

   Fellegi and Sunter (1969) gave a formal model for record linkage that involves optimal decision rules that divide a product space A×B of pairs of records from two files A and B into matches and nonmatches, denoted by M and U, respectively. The main issue is the accuracy of estimates of probability distributions used in a crucial likelihood ratio. When estimates are sufficiently accurate, decision rules are (nearly) optimal. The optimality is in the sense that, for fixed bounds of the proportions of false matches and false matches, the size of the set of pairs on which no decision is made is minimized.

   Fellegi and Sunter (1969, pp. 1194-1197) considered the following probability decomposition

$$P(pat) = P(M)\ P(pat|M) + P(U)\ P(pat|U), \tag{1.1}$$

where *pat* represents an agreement pattern on characteristics such as surname, house number, and phone. They observed that, in the case for which *pat* represents the eight patterns of simple agreement/disagreement on three fields, and the agreements are conditionally independent given M and U, (1.1) represents seven equations and seven unknowns that can be solved directly. In more general situations, the set of equations (1.1) can be solved by least squares, methods of moments, or maximum likelihood. Because methods of moments do not yield solutions that are as pleasing as those via maximum likelihood (Titterington, Smith, and Makov, 1988), and least squares has shown numerical instability in record linkage situations (Jaro 1989), Expectation-Maximization (EM) Algorithms (e.g., DLR) are used to get maximum likelihood estimates.

   The second section of this paper contains background on EM theory and the generalizations that are possible for loglinear models on finite state spaces. The third section describes specific record linkage concepts more fully and the succession of EM-type algorithms that have been applied previously. The results in the fourth section compare estimates computed via the EMH Algorithm of this paper with estimates computed via prior methods. In the fifth section, the limitations of current EM-type methods involving loglinear models of latent classes are described. The final section consists of a summary and conclusions.

2. **EM BACKGROUND AND THEORY**

   This section contains background needed for how EM-type procedures can be applied to loglinear models for latent classes. While the existing EM-type procedures are known to hold for general distributions, in the latter part of this section, only distributions from curved exponential families on finite state spaces will be considered. The restriction on the distributions

is necessary because some of the general optimization theory applied by Wu (1973; also Meng and Rubin 1993) is not readily applicable to the procedures described in this paper. This is particularly true when restraints are imposed that cause some solutions to reside on the boundary of a convex subregion of the linear manifold associated with loglinear models for latent classes.

2.1. Expectation-Maximization of Dempster, Laird, and Rubin

   DLR presented the EM Algorithm as a means of obtaining maximum likelihood estimates for incomplete data. In the context of the latent classes of this paper, the missing data are the indicators for the classes into which the pairs in A×B are partitioned. To make the ideas more precise, the basic ideas of DLR are summarized but details are left out. There exists two sample spaces X and Y and a many-to-one mapping x->y(x) from X to Y. Observed data are from Y. Unobserved x∈X are observed indirectly through Y. In the case of the contingency tables associated with the latent classes, each observed count would represent the sum of the counts from the unobserved counts in the contingency tables corresponding to the classes into which A×B (in the more general notation of this section, Y) is divided.

   A family of sampling densities $f(x|\varphi)$ depending on parameters φ in a general parameter space Ω is postulated. The corresponding family of sampling densities $g(y|\varphi)$ is obtained via

$$g(y|\varphi) = \int_{X(y)} f(x|\varphi)\, dx. \qquad (2.1)$$

The following equations

$$L(\varphi) = \log g(y|\varphi),$$

$$k(x|y,\varphi) = f(x|\varphi)/g(y|\varphi),$$

$$Q(\varphi'|\varphi) = E(\log f(x|\varphi')|y,\varphi), \text{ and}$$

$$H(\varphi'|\varphi) = E(\log k(x|y,\varphi')|y,\varphi).$$

yield

$$L(\varphi') - L(\varphi) = Q(\varphi'|\varphi) - Q(\varphi|\varphi) + H(\varphi|\varphi) - H(\varphi'|\varphi).$$

Q is the log-likelihood function with the missing x values filled in with expected values under the distribution $f(x|\varphi)$. Because Q is a complete data log-likelihood, φ' can be chosen that maximizes Q. For such a φ',

$$Q(\varphi'|\varphi) \geq Q(\varphi|\varphi).$$

By Jensen's inequality (e.g., DLR, p. 6),

$$H(\phi|\phi) \geq H(\phi'|\phi) \quad \text{for all } \phi'.$$

Given an initial estimate ϕ_0, data could be completed using $Q(\phi'|\phi_0)$ and a new value ϕ_1 that maximizes Q and increases L could be obtained. By iterating E and M steps, a sequence ϕ_p is obtained that increases the log-likelihood L. When L is bounded over ϕ_p, a subsequence of ϕ_p can be chosen that converges to either a stationary point or a local maximum (Wu, 1983). DLR observed that there can be different parameter formulations $f(x|\phi)$ that yield that same $g(y|\phi)$. Thus, the family $f(x|\phi)$ might be chosen that is most natural to maximizing the likelihood of $g(y|\phi)$ or for simplifying computation. Also, in certain situations, such as with latent classes for contingency tables, X might include what are traditionally called parameters. Use of equation (2.1) will be referred to as *filling in missing data* or *completing the data*.

2.2. Meng-Rubin ECM and MCECM

Meng and Rubin considered the ϕ_p obtained through successive expectations and then maximizations of the complete data expected log-likelihood Q. They observed that each successive maximization ϕ_p might be replaced by a series of conditional maximizations $\{\phi_{(p-1) + t/S}$, $t = 1, ..., S\}$ where the conditional maximizations are based on a set of restraint functions $G = \{g_s(\phi), s = 1, 2, ..., S\}$. The vectors g_s are preselected. The sth conditional maximization (CM) involves maximizing $Q(\phi|\phi_p)$ over $\phi \in \Omega$ subject to the constraint $g_s(\phi)=g_s(\phi_{p+(s-1)/S})$. That is, for s = 1, ..., S, the sth CM step in the pth iteration of the ECM finds $\phi_{t+s/S}$ such that

$$Q(\phi_{p+s/S}|\phi_p) \geq Q(\phi|\phi_p)$$

$$\text{for all } p \in \Phi_s(\phi_{p+(s-1)/S}) \equiv$$

$$\{\phi \in \Phi : g_s(\phi) = g_s(\phi_{p+(s-1/S)})\}.$$

Meng and Rubin provided examples showing how a set of conditional maximizations can involve less computation than a full maximization that might require an iterative procedure such as Newton-Raphson. With the Meng and Rubin ECM, each individual conditional maximization is done only once before going to the E-step. Thus, each CM-step only increases the likelihood. Unlike the M-step in the original EM, it does not necessarily produce a maximum likelihood estimate given the E-step data. When G is "space-filling" in the sense of allowing unconstrained maximization over Ω in its parameter space, Meng and Rubin showed ECM converged under the same general conditions that Wu (1983) used for the EM. They also noted that the E and individual (or sets of) CM steps could be alternated in a multi-cycle ECM (MCECM). If G is not "space-filling," then the maximization is over a subspace of Ω which they specify explicitly using a dual space characterization. For a large class of loglinear models, the same dual space characterization has been given by Haberman (1977).

2.3. EMH Algorithm

This section contains background on how EM-type procedures can be applied to latent class models. While the existing EM-type procedures are intended for unconstrained maximization on the parameter space Ω (DLR, Wu 1983, Meng and Rubin 1993), the EMH algorithm is intended to constrain solutions to a closed, convex subregion R of Ω. The key idea needed for the EMH

algorithm is the following inequality due to Haberman.

   Theorem. (Haberman 1975, 1977). Let the parameters being estimated via an EM-type procedure be products of multinomial or Poisson distributions. If ϕ_p and ϕ_{p+1} are successive estimates, then for all, $0 \leq \alpha \leq 1$, the log-likelihood L satisfies

$$L(\phi_p) \leq L(\alpha\,\phi_p + (1-\alpha)\,\phi_{p+1}). \tag{2.1}$$

Inequality (2.1) states that all parameters on the line segment between ϕ_p and ϕ_{p+1} yield nondecreasing likelihood. If ϕ_p lies in the interior of a convex subregion R of the parameter space, then it is possible to obtain an α such that $\alpha\,\phi_p + (1-\alpha)\,\phi_{p+1}$ lies on the boundary of R. If the constraints defining R are simple, then such an α can be represented as a closed form solution of an equation; otherwise, such an α may have to be obtained via an iterative procedure such as Newton-Raphson. Inequality (2.1) does not hold for general EM-type procedures. In the following, the restraint functions $\{g_i, i = 1, ..., S\}$ can be assumed to be the same as those given by Meng and Rubin (1993, also 1991, pp. 245-246). While additional restraint functions may determine the closed, convex subregion R of the parameter space Ω, they are not explicitly needed in the statement of the algorithm. Replacing the missing data with expected values is referred to as *completing* or *filling-in* the data (DLR).

   EMH Algorithm for loglinear models constrained to a closed, convex subregion
      R of parameter space Ω.
   1. Beginning with an initial set of parameters ϕ_0 in R, complete data with expected
      values using ϕ_0.
   2. Using the restraints imposed by the completed data and g_1, find the maximum
      likelihood estimate ϕ_1 in Ω. If $\phi_1 \notin R$, find the α so $\alpha\,\phi_0 + (1-\alpha)\,\phi_1$ is
      on the boundary of R and use $\alpha\,\phi_0 + (1-\alpha)\,\phi_1$ as the estimate. If $\phi_1 \in R$,
      use it as the estimate. Complete the data according to the new estimate ϕ_1.
   3. Using the restraints imposed by the completed data and g_2, find the maximum
      likelihood estimate ϕ_2 in Ω. If $\phi_2 \in R$, use it as the estimate. If ϕ_1 is on
      the boundary of R and $\phi_2 \notin R$, use ϕ_1 as the estimate of ϕ_2. If ϕ_1 is in the
      interior of R and $\phi_2 \notin R$, find the α so $\alpha\,\phi_1 + (1-\alpha)\,\phi_2$ is on the boundary of R
      and use $\alpha\,\phi_1 + (1-\alpha)\,\phi_2$ as the estimate. Complete the data according to the new
      estimate ϕ_2.
   4. Continue constrained E and M steps by successively cycling through all restraints
      in the manner of Step 3.

   By the theorem, the EMH algorithm yields nondecreasing likelihood. If the restraint functions g_i, $i = 1, 2, ..., S$, are the usual constraints associated with iterative proportional fitting, then each conditional maximization is in closed form. If the constraints defining the subregion R are simple, then each of the αs that pull successive estimates back to the boundary of R are also in closed form. The initial estimate ϕ_0 must be in R. If there is no restriction to R, then the EMH algorithm corresponds to the MCECM algorithm of Meng and Rubin (1993). Rather than find a maximum at each CM-step, it is sufficient to find ϕ_{p+1} different from ϕ_p so that the likelihood

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increases. To assure that some ϕ_{p+1} are different from ϕ_p, computer code should monitor that the likelihood strictly increases on some of the CM-steps.

3. **RECORD LINKAGE BACKGROUND**

This section contains background on the previous applications of the EM to give insight into why the new methods were developed. The first subsection summarizes earlier EM-applications to record linkage and the second describes the empirical data.

3.1. Previous Applications of EM

In most record linkage applications, parameter estimates associated with equation (1.1) have been obtained under a conditional independence assumption. When conditional independence is assumed (Winkler 1988, Jaro 1989, Winkler 1989a,b), the EM algorithm does not yield accurate estimates of the parameters associated with the underlying probability distributions. If such parameters are used, then decision rules are not optimal and error rates can not be accurately determined. To improve decision rules, independent probabilities have been adjusted based on prior knowledge of similar files (Winkler and Thibaudeau 1991).

The main reason that existing parameter-estimation procedures fail to yield optimal decision rules is that the conditional independence assumption is not valid. Thibaudeau (1993) observed that, when files contain name and address information, strong dependencies between agreements on fields such as surname, house number, street name, and phone number occur on the set on nonmatches U. Winkler (1992) showed that, instead of A×B naturally dividing into the desired two classes M and U, A×B can be naturally partitioned into three classes: C_1- matches agreeing on name and address, C_2- nonmatches agreeing on address, and C_3- nonmatches not agreeing on address. In the decision rules, classes C_2 and C_3 are combined into U. The primary reason the 2-class EM procedure fails is that it divides the set of pairs into those agreeing on address and those not. If the 3-class EM algorithm is applied under the independence assumption, reasonable decision rules are obtained when matching decision thresholds are obtained manually (Winkler, 1992). Error rates, however, cannot be estimated accurately. An alternative error-rate estimation procedure (Belin and Rubin 1993, Rubin and Belin 1991) can yield accurate estimated false match rates (Winkler and Thibaudeau 1991) in some cases but is not applicable to the situations of this paper.

Winkler (1992) applied 3-class EM algorithms under models in which all 3-way interactions were allowed. While the interactions gave dramatically lower chi-square values and the overall fits as given by the estimated cumulative probability distributions appeared acceptable, both probability estimates for individual agreement patterns and the estimated proportion of pairs in class C_1 could differ substantially from the true values. The EM-type procedures arbitrarily classify sets of pairs into classes according to the variables that are agreeing in the patterns and according to interactions being fit. When a set of matching variables contains many associated with addresses, general EM-type procedures can yield probability estimates and resultant decision rules that give primary weight to address information and secondary weight to name and demographic information. When all 3-way interactions are fit, spurious agreements of nonessential variables may be given too much weight by the EM algorithm.

To better make use of prior information, specific sets of interactions can be modeled using new algorithms first used by Armstrong (1992). The difficulty with using specific interactions is that there are far fewer available degrees of freedom (dofs) with latent class models than with

ordinary loglinear models. For instance, with ten variables there are insufficient dofs to model all 4-way interactions; with eight variables, insufficient for all 3-way. If use of specific sets of interactions is not sufficient, then convex constraints can be used to predispose solutions to convex subregions that are more likely to yield accurate estimates.

3.2. <u>Data used in Results</u>

The pairs are taken from two files having known matching status and 12,000 and 15,000 records, respectively. Only 116,305 pairs agreeing on a geographic identifier and the first character of the surname are used. There are 9800 matches. The analysis evaluates nearly all matches because less than 4 percent of the true matches disagree on the geographic identifier or on the first character of the surname. The matching fields that are: surname, first name, house number, street name, phone, age, relationship to head of household, marital status, sex, and race. To simplify computation, all comparisons are considered agree/disagree. The ten data fields yield 1024 data patterns for which frequencies are calculated. If one or both identifiers of a pair are blank, then the comparison (blank) is considered a disagreement. This only substantially affects age (15% blank) and phone (35% blank). Name and address data are never missing.

4. **RESULTS USING EM-DERIVED PROBABILITIES**

This section presents results from fitting using under five models: (1) independent, 3-class EM, (2) dependent, 3-class EMH with all 3-way interactions of variables, (3) dependent, 3-class EMH with a selected subset of interactions, and (4) dependent, 3-class EMH with all 3-way interactions and selected convex constraints, and (5) dependent, 3-class EMH with a selected subset of interactions and selected convex constraints.

When the number of interactions is increased, chi-square values will typically decrease. The selected interaction patterns (Table 4.1) are chosen as a compromise that allows a number of degrees of freedom so that the statistics can be tested. The first set of patterns was chosen with knowledge of some of the true statuses. The set of patterns is described as ja because combinatorial algorithms due to Armstrong (1992) are used in the fitting. The second set consisting of all 3-way interactions was chosen as a general exploratory tool that left some degrees of freedom for testing. Because the interaction patterns do not necessarily yield estimated probabilities that give good decision rules, the convex constraints of Table 4.2 are used to predispose estimates into subsets of the parameter space. The second set of convex constraints use twice the true underlying probabilities as an upper bound on the estimated probabilities. Some of the convex constraints will lead to solutions that are on the boundary imposed by the constraints. Others will initially constrain the solutions to certain subsets but final limiting solutions will not hit the boundary imposed by the constraints.

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Table 4.1.  Interactions Used in Fitting Hierarcharical Models
_____

   ja      1. last, first, hsnm, stnm, phone, age, rel, marit
           2. first, hsnm, phone, sex
           3. last, race

 3-way     1. all 3-way
_____
```

```
Table 4.2.   Convex Constraints Use in Fitting
───────────────────────────────────────────────────
  ja
   14 complicated restraints

  3-way
    P({agree last, disagree first} ∩ C₁)    ≤ 0.0070
    P({disagree last, disagree first} ∩ C₁) ≤ 0.0014
───────────────────────────────────────────────────
```

The statistics associated with the fits of the different models yield somewhat contradictory information (Table 4.3). With the exception of the chi-squares associated with fits of the models that include all 3-way interactions, all chi-square values are much too high. The log-likelihood associated with fitting using the first set of interactions (denoted by ja) is closer to the theoretical maximum of -4.1072 than the log-likelihood from all 3-way interactions. The Z-statistic is the standard normal approximation to the chi-square statistic and is given as a reference. The reason that the Z-values associated with the ja models are higher than those associated with 3-way models is that many interactions are fit in those models and the Z-values are associated with considerably fewer degrees of freedom. The P_1-value associated with Class C_1 is the estimated proportion of pairs that are matches M. The P_1-value 0.0878 associated with the first set of interactions (denoted by ja) is the second closest to the true P_1 value of 0.0869. The detailed constraints in ja, convex were chosen to force the estimated proportion P_1 close to true proportion.

```
Table 4.3.   Statistics Associated with Various Models
──────────────────────────────────────────────────────────────
                log-likelihood   chi-square      Z         P₁
──────────────────────────────────────────────────────────────

independent        -4.2206       26,382.9      570.4      0.0910
ja                 -4.1084          294.0        8.7      0.0878
ja, convex         -4.1086          340.4       11.4      0.0869
3-way              -4.1088          375.0       -2.8      0.1015
3-way, convex      -4.1100          660.1        5.2      0.0886
──────────────────────────────────────────────────────────────
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Plots of the cumulative probability distributions of the five models versus the truth (given by the 45 degree line) are presented in Figures 1-5 for matches; in Figures 6-10 for nonmatches. Only the regions in which the error proportions are less than 0.30 are shown. With the exception of the independence model (Figures 1 and 6) which deviates substantially from the truth, all plots show reasonable fits to the true distributions. While the curves associated with the chosen subset of interactions (ja, Figures 4 and 9) appear acceptable, they mask the fact that some probability estimates at individual points deviate substantially from the truth. The final two models that involve convex constraints give better fits at individual points (and thus in each subrange of the distribution) than the others.
   To get a better feel for the accuracy of the distributions estimated under the different models, it is necessary to provide plots of alternative curves. The curves are the cumulative false classification rates against matching weight. The false match classification rate for a given

threshold weight is the ratio of the distribution of nonmatches having weight above a threshold to the distribution of all pairs above the same threshold. In other words, among the pairs actually classified, it is the proportion that are in error. More precisely,

$$\text{fmcr(threshold)} = \sum_{\{wgt \geq threshold\}} P(\text{nonmatch} \cap wgt) \, / \, \sum_{\{wgt \geq threshold\}} P(wgt).$$

The fmcr curves are given in Figures 11-14 and analogous false nonmatch classification curves are given in Figures 15-18, for ja, ja-convex, 3-way, and 3-way-convex curves, respectively. The curves show that, even though 3-way has a better chi-square fit than ja, the misclassification error rates are much more accurately estimated by ja. While ja-convex has a lower log-likelihood and a worse chi-square fit than ja, ja-convex provides more accurate misclassification error rates than ja.

5. **DISCUSSION**

The results show that conventional chi-square statistics describing the fits give no valid indication of the quality of the estimated probabilities when they are used in decision rules. In decision rules, matches need to be distinguished from nonmatches. Often there is no clear demarcation between Class C_1, matches within the same household, and Class C_2, nonmatches within the same household. For instance, in one set of pairs, husband-wife pairs in which age agrees and sex agrees due to miskeying are placed in Class C_1; in other sets such pairs may be placed in Class C_2. This section describes limitations on the general applicability of convex constraints and the extension of the EMH algorithm to general statistics.

5.1. Basic Limitations on Use of Convex Constraints

With other similar data bases having the same matching variables, representing similar types of geographic characteristics, and for which true matching status was known, the following improving relative accuracies of estimated probabilities were observed

independent < all 3-way < ja selected interactions
        < all 3-way + convex

where the ja selected interactions are the same as those given in Table 4.1 and the convex constraints are the same as those given in Table 4.2. Generally, the last two models had about the same accuracy and were much better than the first two. In one set of files in which some of the demographic variables had substantially higher typographical variation than in other files, the last model yielded more accurate estimated probabilities.

On an absolute basis, the estimated probabilities under the last two models were not as accurate as those given Figures 3-5 and 8-10 but were still reasonable. The estimated cumulative probabilities given a nonmatch were typically much more accurate than the corresponding cumulative probabilities given a match.

No set of convex constraints or set of interactions has been found that consistently yield highly accurate estimates. This is due to the fact that the underlying true probabilities vary significantly from data set to data set. With ten matching variables, some true conditional probabilities associated with individual data patterns vary by an order of magnitude. The estimates associated with the ja selected interaction model exhibited more variation than estimates under the all 3-

way, convex model. Severe or unusual typographical variation in only 0.1 percent of the pairs (associated with Classes C_1 and C_2) were sufficient for significant changes in estimates.

5.2. EMH Algorithm for General Statistics

The idea of conditional maximization in which parameters are constrained to convex subregions of the parameter space can be extended from latent class models to general statistics. The basic idea is still the one originally emphasized by Meng and Rubin. It is reducing dimensionality to make computation associated with maximizations faster or more stable. In general, it is possible to search for the α_0 so that $\alpha_0 \, \varphi_p + (1-\alpha_0) \, \varphi_{p+1}$ lies in the subregion R imposed by a set of constraints and (approximately) maximizes the likelihood over all α, $0 \leq \alpha \leq 1$. As it is only necessary to increase the likelihood, it may often be possible to find a simple algorithm to obtain an $\alpha_0 > 0$ that yields increasing likelihood. In general, such an α_0 can be found via one-dimensional Newton-Raphson or grid-search methods.   The results basically show that conventional chi-square statistics that describe the fits give no valid indication of the quality of the estimated probabilities when they are used in decision rules. This section contains the justification for use of convex constraints and other extensions.

6. **SUMMARY AND CONCLUSIONS**

This paper describes general theory and algorithms for fitting loglinear models for latent classes. The algorithms do not require an independence assumption on the estimated conditional probabilities associated with different latent classes, are related to ideas of Haberman (1977) and Meng and Rubin (1993), and allow convex constraints to be imposed on the estimated probabilities. Because conventional chi-square and other statistics describing the fits do not give good indication of the accuracy of the estimated probabilities, the convex constraints can be used to predispose solutions to subregions of the parameter space that are consistent with prior knowledge.

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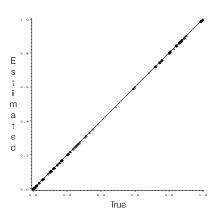 Computer Matching," *Proceedings of the 1991 Census Annual Research Conference*, 657-668.

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Figure 1. Estimates vs Truth
Cumulative Distribution of Matches
3−Class, Independent EM

Figure 2. Estimates vs Truth
Cumulative Distribution of Matches
3−Class, 3−way Interaction EM

Figure 3. Estimates vs Truth
Cumulative Distribution of Matches
3−Class, 3−way Interaction EM, Convex

Figure 4. Estimates vs Truth
Cumulative Distribution of Matches
3−Class, ja Interaction EM

Figure 5. Estimates vs Truth
Cumulative Distribution of Matches
3−Class, ja Interaction EM, Convex

Figure 6. Estimates vs Truth
Cumulative Distribution of Nonmatches
3−Class, Independent EM

Figure 7. Estimates vs Truth
Cumulative Distribution of Nonmatches
3−Class, 3−way Interaction EM

Figure 8. Estimates vs Truth
Cumulative Distribution of Nonmatches
3−Class, 3−way Interaction EM, Convex

Figure 9. Estimates vs Truth
Cumulative Distribution of Nonmatches
3−Class, ja Interaction EM

Figure 10. Estimates vs Truth
Cumulative Distribution of Nonmatches
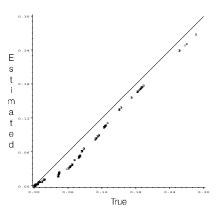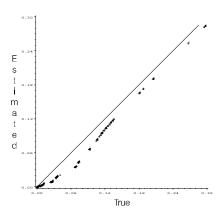3−Class, ja Interaction EM, Convex

Figure 11. Estimates vs Truth
Cumulative False Match Classification Rates
3−Class, 3−way Interaction EM

Figure 12. Estimates vs Truth, convex
Cumulative False Match Classification Rates
3−Class, 3−way Interaction EM, Convex
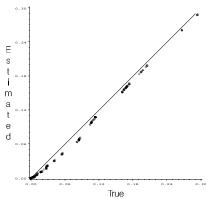
Figure 13. Estimates vs Truth
Cumulative False Match Classification Rates
3−Class, ja Interaction EM

Figure 14. Estimates vs Truth
Cumulative False Match Classification Rates
3−Class, ja Interaction EM, Convex

Figure 15. Estimates vs Truth
Cumulative False Nonmatch Classification Rates
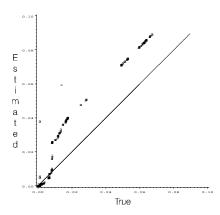3−Class, 3−way Interaction EM

Figure 16. Estimates vs Truth
Cumulative False Nonmatch Classification Rates
3−Class, 3−way Interaction EM,convex

Figure 17. Estimates vs Truth
Cumulative False Nonmatch Classification Rates
3−Class, ja Interaction EM

Figure 18. Estimates vs Truth
Cumulative False Nonmatch Classification Rates
3−Class, ja Interaction EM, convex