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**JEL classification:** C22, C53, C63

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Abstract

Exponential smoothing is one of the most popular forecasting methods. We present a method for bootstrap aggregation (bagging) of exponential smoothing methods. The bagging uses a Box-Cox transformation followed by an STL decomposition to separate the time series into trend, seasonal part, and remainder. The remainder is then bootstrapped using a moving block bootstrap, and a new series is assembled using this bootstrapped remainder. On the bootstrapped series, an ensemble of exponential smoothing models is estimated. The resulting point forecasts are averaged using the mean. We evaluate this new method on the M3 data set, showing that it consistently outperforms the original exponential smoothing models. On the monthly data, we achieve better results than any of the original M3 participants. We also perform statistical testing to explore significance of the results. Using the MASE, our method is significantly better than all the M3 participants on the monthly data.

**Keywords:** bagging, bootstrapping, exponential smoothing, STL decomposition.

## 1 Introduction

After more than 50 years of widespread use, exponential smoothing is still one of the most practically relevant forecasting methods available (Goodwin, 2010). Reasons for this are its simplicity and transparency, as well as its ability to adapt to many situations. It also has a solid theoretical foundation in ETS state-space models (Hyndman et al, 2002, 2008; Hyndman and Athanasopoulos, 2013).

In the M3 forecasting competition (Makridakis and Hibon, 2000; Koning et al, 2005), exponential smoothing methods obtained competitive results, and with the `forecast` package (Hyndman and Khandakar, 2008) in the programming language R (R Core Team, 2014), a fully automatic software for fitting ETS models is available. So, ETS models are usable, highly relevant in practice, and have a solid theoretical foundation, which makes any attempts to improve their forecast accuracy a worthwhile endeavour.

Bootstrap aggregating (bagging), as proposed by Breiman (1996), is a popular method in Machine Learning to improve the accuracy of predictors (Hastie et al, 2009; Zhang and Zhang, 2009). An ensemble of predictors is estimated on bootstrapped versions of the input data, and the output of the ensemble is calculated by averaging (mean, median, trimmed mean, weighted averaging, etc.), often yielding better point predictions. In this work, we propose a bagging methodology for exponential smoothing methods, and evaluate it on the M3 data. As our input data are non-stationary time series, both serial dependence and non-stationarity have to be taken into account. We resolve these issues by applying STL decomposition (Cleveland et al, 1990), and a moving block bootstrap (MBB, see, e.g., Lahiri, 2003) on the residuals of the decomposition.

Specifically, our proposed method of bagging is as follows. After applying a Box-Cox transformation to the data, the series is decomposed into trend, seasonal and remainder components. The remainder component is then bootstrapped using the MBB, the trend and seasonal components are added back, and the Box-Cox transformation is inverted. In this way, we generate a random pool of similar bootstrapped time series. For each one of these bootstrapped time series, we choose a model among several exponential smoothing models, using the bias-corrected AIC. Then, point forecasts are calculated using all the different models, and the resulting forecasts are averaged.

The only related work we are aware of is the work of Cordeiro and Neves (2009) who use a sieve bootstrap to perform bagging with ETS models. They fit an ETS model to the data, then fit an AR model to the residuals, and generate new residuals from this AR process. They also test their method on the M3 dataset. They have some success on quarterly and monthly data, but the overall results are not promising. In fact, the bagged forecasts are often not as good as the original forecasts applied to the original time series. Our bootstrapping procedure works differently, and yields better results. In particular, we are able to outperform the original M3 methods in monthly data.

The rest of the paper is organized as follows. In Section 2, we discuss the proposed methodology in detail. Section 3 presents the experimental setup and the results, and Section 4 concludes the paper.

# 2 Methods

In this section, we describe in detail the different parts of our proposed methodology, which are exponential smoothing, and the novel bootstrapping procedure involving a Box-Cox transformation, STL decomposition, and the MBB. We illustrate the steps using series M495 from the M3 dataset, which is a monthly series.

## 2.1 Exponential smoothing

Exponential smoothing divides the time series into seasonal, trend, and error component. For example, the Holt-Winters purely additive model is defined by the following recursive equations:

$$
\begin{aligned}
\ell_t &= \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1}) \\
b_t &= \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1} \\
s_t &= \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m} \\
\hat{y}_{t+h|t} &= \ell_t + hb_t + s_{t-m+h_m^+}
\end{aligned}
$$

Here, $\ell_t$ denotes the series level at time $t$, $b_t$ denotes the slope at time $t$, $s_t$ denotes the seasonal component of the series at time $t$, and $m$ denotes the number of seasons in a year. The constants $\alpha$, $\beta^*$, and $\gamma$ are smoothing parameters in the $[0,1]$-interval, $h$ is the forecast horizon, and $h_m^+ = [(h-1) \bmod m] + 1$.

Nowadays, there is a whole family of ETS models, with a solid state space foundation. The models can be distinguished by the type of error, trend, and seasonality they use. Distinguishing between additive and multiplicative error only has consequences for the prediction intervals, not for the point forecasts. In the example above, trend and seasonality are additive. In general, the trend can be non-existent, additive, multiplicative, or damped additive/multiplicative. The seasonality can be non-existent, additive, or multiplicative. So, in total there are 30 models with different combinations of error, trend and seasonality. For more detailed descriptions, we refer to Hyndman et al (2002) and Hyndman et al (2008).

In R, exponential smoothing is implemented in the `ets` function from the `forecast` package (Hyndman and Khandakar, 2008). All the different models are fitted automatically to the data; i.e., the smoothing parameters and initial conditions are optimized using maximum likelihood with a simplex optimizer (Nelder and Mead, 1965). Then, the best model from the model family is chosen using the bias-corrected AIC.

## 2.2 The Box-Cox transformation

This is a popular transformation to stabilize the variance of a time series, originally proposed by Box and Cox (1964). It is defined as follows:

$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (y_t^\lambda - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

Depending on a parameter $\lambda$, the transformation is essentially the identity ($\lambda = 1$), the logarithm, ($\lambda = 0$), or a transformation somewhere between. A difficulty is the way to choose the parameter $\lambda$. In this work, we restrict it to lie in the $[0,1]$-interval, and then use the method of Guerrero (1993) to choose its value. For the example time series M495, this method gives $\lambda = 6.61 \times 10^{-5}$. Figure 1 shows the original series and the Box-Cox transformed version using this $\lambda$.

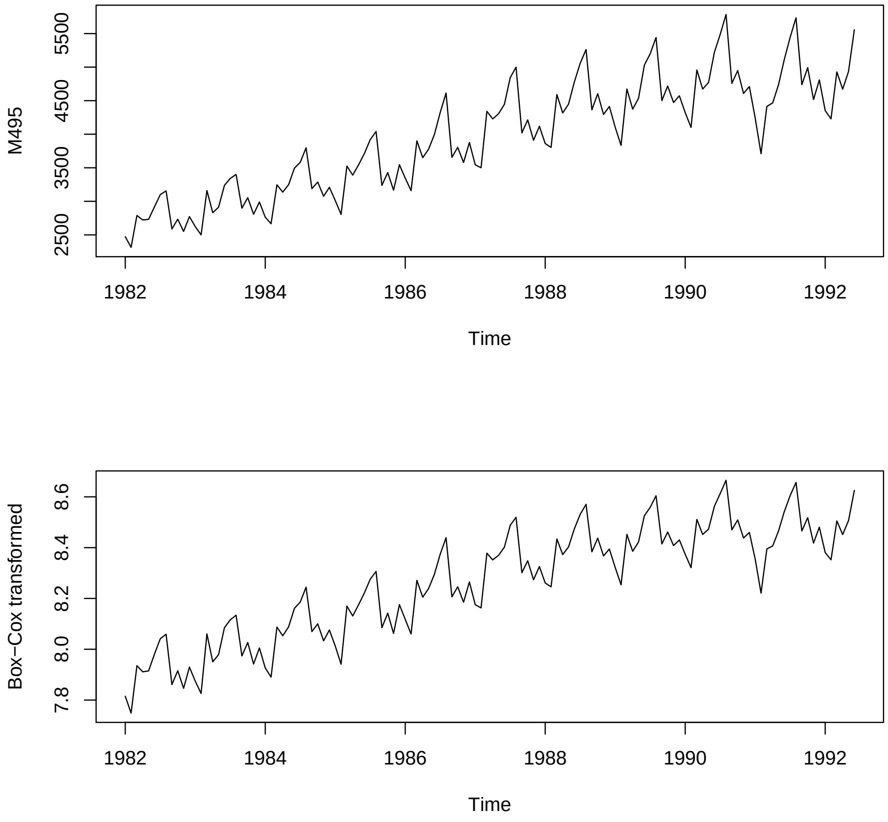

**Figure 1:** *Series M495 of the M3 dataset, which is a monthly time series. Above is the original series, below the Box-Cox transformed version, with* $\lambda = 6.61 \times 10^{-5}$.

## 2.3 Time series decomposition

For non-seasonal time series, we use the loess method (Cleveland et al, 1992) (a smoothing method based on local regressions) to decompose the time series into trend and remainder components. For seasonal time series, we use STL decomposition (seasonal-trend decomposition based on loess), as presented by Cleveland et al (1990), to obtain trend, seasonal and remainder components.

In loess, for each data point a neighborhood is defined. The points in that neighborhood are then weighted (using so-called *neighborhood weights*) according to their distance from the respective data point. Finally, a polynomial of degree $d$ is fitted to these points. Usually, $d = 1$ and $d = 2$ are used, i.e., linear or quadratic curves are fitted. The trend component is equal to the value of the polynomial at each data point. In R, loess smoothing is available by the function `loess`. For the non-seasonal data of our experiments, i.e., the yearly data of the M3 competition, we use the function with a degree of $d = 1$. The neighborhood size is defined in this function by a parameter $\alpha$, which is the proportion of overall points to include in the neighborhood, with tricubic weighting. To get a constant neighborhood of 6 data points, we define this parameter to be 6 divided by the length of the time series in consideration.

In STL, loess is used to divide the time series into trend, seasonal, and remainder components. The division is additive, i.e., summing the parts gives the original series again. In detail, the steps performed during STL decomposition are: (i) detrending; (ii) cycle-subseries smoothing — series are built for each seasonal component, and smoothed separately; (iii) low-pass filtering of smoothed cycle-subseries — the sub-series are put together again, and smoothed; (iv) detrending of the seasonal series; (v) deseasonalizing the original series, using the seasonal component calculated in the previous steps; and (vi) smoothing the deseasonalized series to get the trend component. In R, the STL algorithm is available through the `stl` function. We use it with its default parameters. The degrees for the loess fitting in steps (iii) and (iv) are $d = 1$, and $d = 0$ in step (ii). Figure 2 shows the STL decomposition of series M495 of the M3 dataset, as an example.

## 2.4 Bootstrapping the remainder

As time series data are typically autocorrelated, adapted versions of the bootstrap exist (see Lahiri, 2003; Gonçalves and Politis, 2011). A prerequisite is stationarity of the series, which we achieve by bootstrapping the remainder of the STL (or loess) decomposition.

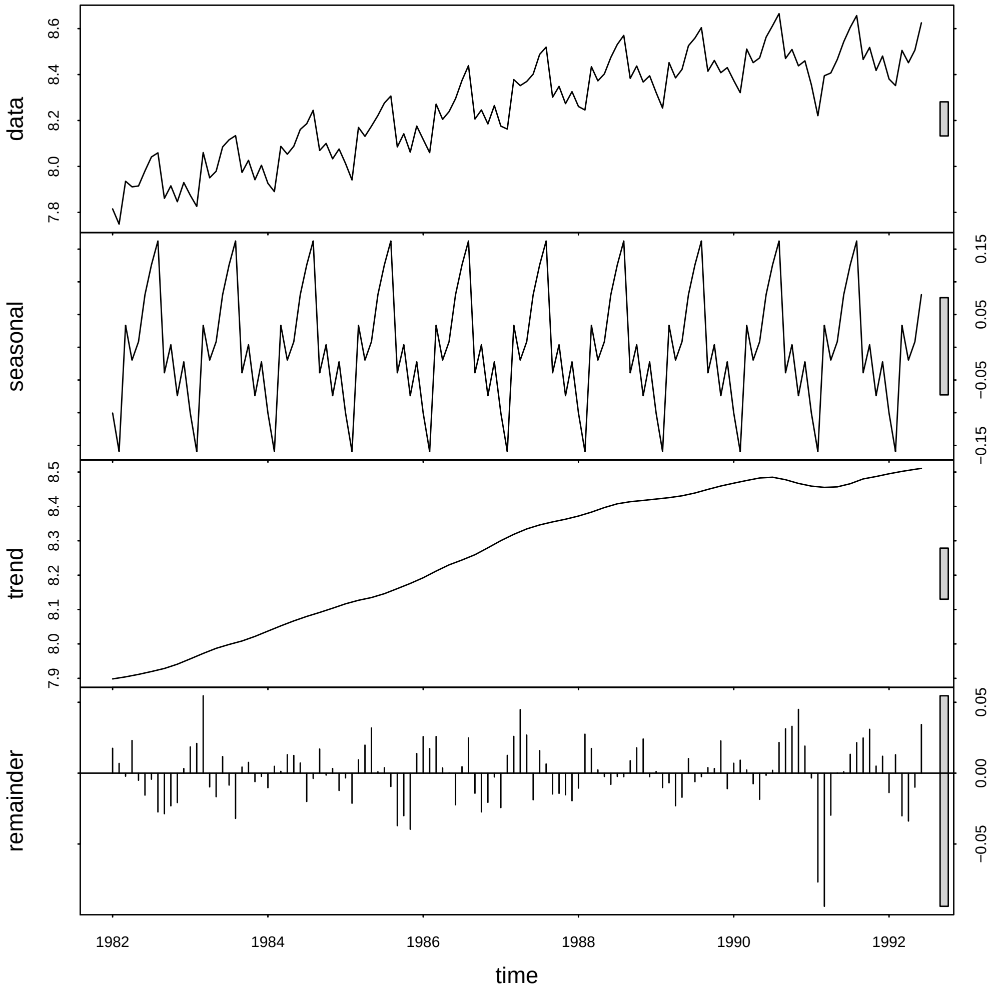

**Figure 2:** *STL decomposition into trend, seasonal part, and remainder, of the Box-Cox transformed version of series M495 of the M3 dataset.*

In the MBB as originally proposed by Künsch (1989), data blocks of equal size are drawn from the series, until the desired series length is achieved. For a series of length $n$, with a block size of $l$, $n-l+1$ (overlapping) possible blocks exist.

We use block sizes of $l = 8$ for yearly and quarterly data, and $l = 24$ for monthly data, i.e., at least two full years, to ensure any remaining seasonality is captured. As the shortest series we use have $n = 14$ observations in total for the yearly data, care has to be taken that every value from the original series can possibly be placed anywhere in the bootstrapped series. To achieve this, we draw $\lfloor n/l \rfloor + 2$ blocks from the remainder series. Then, we discard from the beginning of the bootstrapped series a random amount between zero and $l-1$ values. Finally, to obtain a series with the same length as the original series, we discard the amount of values necessary to obtain the required length. This processing ensures that the bootstrapped series does not necessarily begin or end on a block boundary.

In the literature, there exist various other methods for bootstrapping time series, such as the tapered block bootstrap (Paparoditis and Politis, 2001), the dependent wild bootstrap (DWB) (Shao, 2010a), and the extended tapered block bootstrap (Shao, 2010b). However, Shao (2010a) concludes that "for regularly spaced time series, the DWB is not as widely applicable as the MBB, and the DWB lacks the higher order accuracy property of the MBB." So that "the DWB is a complement to, but not a competitor of, existing block-based bootstrap methods." We performed preliminary experiments (which are not reported here), using the tapered block bootstrap and the DWB, but finally decided to use the MBB only, as the other procedures where not giving substantial advantages.

Another type of bootstrap is the sieve bootstrap as proposed by Bühlmann (1997), and used by Cordeiro and Neves (2009) in an approach similar to ours. Here, the dependence in the data is tackled by fitting a model and later bootstrapping the residuals, assuming they are uncorrelated. This bootstrapping procedure has the disadvantage that it has to be assumed that the model captures all relevant information in the time series. We performed experiments (also unreported) using an ETS model instead of the STL procedure, and using an AR process for bootstrapping instead of the MBB. However, the results were consistently worse than with the procedure presented here.

After bootstrapping the remainder, the trend and seasonality are combined with the bootstrapped remainder, and the Box-Cox transformation is inverted, to get the final bootstrapped sample. Figure 3 gives an illustration of bootstrapped versions of the example series M495.

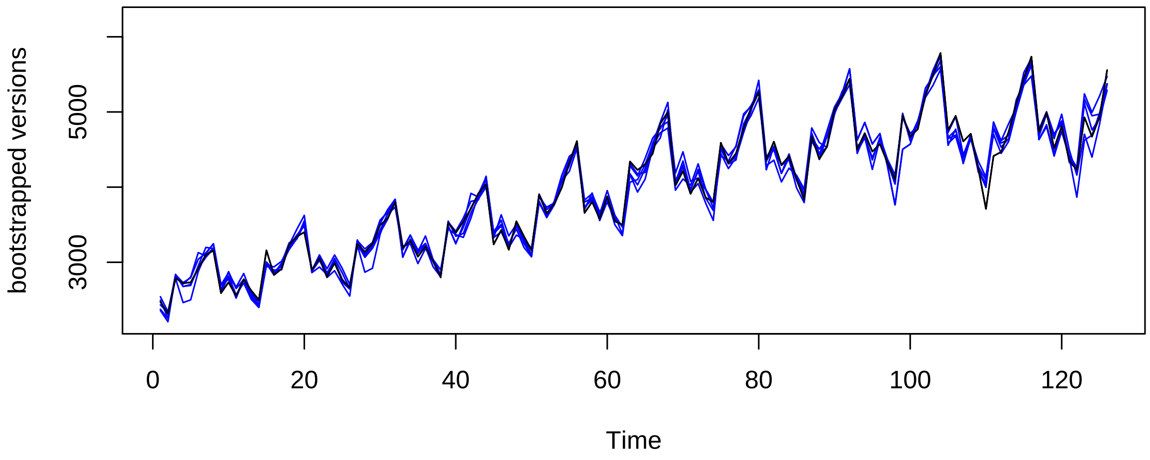

**Figure 3:** *Bootstrapped versions (blue) of the original series M495 (black). Five bootstrapped series are shown. It can be seen that the bootstrapped series resemble the behavior of the original series quite well.*

### 2.5 The overall procedure

To summarize, a scheme of the bootstrapping procedure is given in Algorithm 1. At first, $\lambda$ is calculated according to Guerrero (1993). Then, the Box-Cox transformation is applied to the series, and the series is decomposed into trend, seasonal part, and remainder, using STL or loess. The remainder is then bootstrapped using the MBB, the components are added together again, and the Box-Cox transformation is inverted.

**Algorithm 1** Generating bootstrapped series

```
procedure BOOTSTRAP(ts, num.boot)
    $\lambda \leftarrow$ BoxCox.lambda(ts)                ▷ use automatic procedure to determine $\lambda$
    ts.bc $\leftarrow$ BoxCox(ts, $\lambda$)
    if ts is seasonal then
        [trend, seasonal, remainder] $\leftarrow$ stl(ts.bc)
    else
        seasonal $\leftarrow$ 0
        [trend, remainder] $\leftarrow$ loess(ts.bc)
    end if
    recon.series[1] $\leftarrow$ ts                        ▷ add the original series as a sample
    for i in 2 to num.boot do
        boot.sample[i] $\leftarrow$ MBB(remainder)
        recon.series.bc[i] $\leftarrow$ trend + seasonal + boot.sample[i]
        recon.series[i] $\leftarrow$ InvBoxCox(recon.series.bc[i], $\lambda$)
    end for
    return recon.series
end procedure
```

Then, to every bootstrapped version of the time series, an ETS model is fitted and used for forecasting. For each horizon, the final resulting forecast is calculated as the mean over the forecasts from the single models.

## 3 Experimental study

In this section, we describe the forecasting methods, error measures, and statistical tests that were used in the experiments, and the results obtained on the M3 dataset, separately for yearly, quarterly, and monthly data.

### 3.1 Compared methods

We use the `ets` function from the `forecast` package (Hyndman and Khandakar, 2008) for model building. Bootstrapped versions of the series are generated as discussed in Section 2.

We use an ensemble size of 30, so that we estimate models on the original time series and on 29 bootstrapped series. We compare two different model fitting approaches. The first one fits both additive and multiplicative models, the second one uses the Box-Cox transformation with the same value for $\lambda$ as in the bagging procedure to handle non-additive aspects of the time series, and then the models can be restricted to additive models only. For comparison, both approaches are used in bagged and non-bagged versions. So, in total, the following procedures are employed:

**ETS** the original exponential smoothing method choosing among all possible models using the bias-corrected AIC.

**AdditiveETS.BC** exponential smoothing, choosing only from the additive models, and using a Box-Cox transformation with the same value for $\lambda$ as in the bootstrapping.

**BaggedETS** for each of the 30 series, a model is chosen from all exponential smoothing models using the bias-corrected AIC. Then, for the forecasts, the outcome of all methods is averaged using the mean.

**BaggedETS.BC** the Box-Cox transformation with the same value for $\lambda$ as in the bootstrapping is used during model estimation, and the models are restricted to additive models only.

### 3.2 Evaluation methodology

We use the yearly, quarterly, and monthly series from the M3 competition. There are 645 yearly, 756 quarterly, and 1428 monthly series, so that in total 2829 series are used. As forecast horizons, we follow the M3 methodology, so that we forecast 6 values for yearly series, 8 values for quarterly series, and 18 values for monthly series. The original data, as well as the forecasts of the methods that participated in the competition, are available in the R package `Mcomp` (Hyndman, 2013).

We use the symmetric MAPE (sMAPE) to measure the errors. The sMAPE is defined as

$$\text{sMAPE} = \text{mean}\left(200\frac{|y_t - \hat{y}_t|}{|y_t| + |\hat{y}_t|}\right),$$

where $y_t$ is the true value of the time series $y$ at time $t$, and $\hat{y}_t$ is the respective forecast. This definition is slightly different from the definition given by Makridakis and Hibon (2000), as they do not use absolute values in the denominator. However, as the series in the M3 have all strictly positive values, this difference in the definition should not have any effect in practice (only if a method forecasts negative values).

Furthermore, we use also the mean absolute scaled error (MASE) as proposed by Hyndman and Koehler (2006). It is defined as the mean absolute error on the test set, scaled by the mean absolute error of a benchmark method on the training set. As benchmark, the naïve forecast is used, taking into account the seasonality of the data. So, the MASE is defined as:

$$\text{MASE} = \frac{\text{mean}\left(\left|y_t - \hat{y}_t\right|\right)}{\text{mean}\left(\left|y_i - y_{i-m}\right|\right)},$$

where $m$ is the periodicity, which is 1 for yearly data, 4 for quarterly data, and 12 for monthly data. The variable $i$ runs over the training data, $t$ over the test data.

We calculate the sMAPE and MASE as averages over all horizons per series. Then, we calculate overall means of these measures across series, and also rank the forecasting methods for each series and calculate averages of the ranks across series. Calculating the average ranks has the advantage of being more robust to outliers than the overall means.

## 3.3 Statistical tests of the results

We use the aligned Friedman rank-sum test for multiple comparisons to detect statistically significant differences within the methods, and the post-hoc procedure of Hochberg and Rom (1995) to further analyze those differences (García et al, 2010)[1]. The statistical testing is done using the sMAPE measure.

At first, we use the testing framework to determine if the differences among the proposed models and the basis models are statistically significant. Then, in a second step, we use the bagged exponential smoothing method and compare it with the testing framework to the methods that originally participated in the M3 competition. A significance level of $\alpha = 0.05$ is used.

## 3.4 Results on the yearly data

Table 1 shows the results for all methods on the yearly data. The results are ordered by average rank of sMAPE. It can be seen that the BaggedETS method performs better than the other

[1] More information can be found on the thematic web site of SCI2S about *Statistical Inference in Computational Intelligence and Data Mining* `http://sci2s.ugr.es/sicidm`

| | Rank sMAPE | Mean sMAPE | Rank MASE | Mean MASE |
|---|---|---|---|---|
| ForcX | 12.022 | 16.480 | 12.007 | 2.769 |
| AutoBox2 | 12.370 | 16.593 | 12.388 | 2.754 |
| RBF | 12.381 | 16.424 | 12.389 | 2.720 |
| Flors.Pearc1 | 12.471 | 17.205 | 12.476 | 2.938 |
| THETA | 12.530 | 16.974 | 12.557 | 2.806 |
| ForecastPro | 12.615 | 17.271 | 12.636 | 3.026 |
| ROBUST.Trend | 12.769 | 17.033 | 12.809 | 2.625 |
| PP.Autocast | 12.800 | 17.128 | 12.788 | 3.016 |
| DAMPEN | 12.841 | 17.360 | 12.828 | 3.032 |
| **BaggedETS** | 12.853 | 17.492 | 12.840 | 2.973 |
| COMB.S.H.D | 12.960 | 17.072 | 12.910 | 2.876 |
| SMARTFCS | 13.310 | 17.706 | 13.333 | 2.996 |
| **AdditiveETS.BC** | 13.333 | 18.357 | 13.389 | 3.557 |
| **BaggedETS.BC** | 13.408 | 18.258 | 13.411 | 3.478 |
| HOLT | 13.554 | 20.021 | 13.565 | 3.182 |
| WINTER | 13.554 | 20.021 | 13.565 | 3.182 |
| **ETS** | 13.884 | 18.655 | 13.963 | 3.532 |
| ARARMA | 14.025 | 18.356 | 14.129 | 3.481 |
| Flors.Pearc2 | 14.029 | 17.843 | 14.054 | 3.016 |
| B.J.auto | 14.068 | 17.726 | 14.041 | 3.165 |
| Auto.ANN | 14.420 | 18.565 | 14.394 | 3.058 |
| AutoBox3 | 14.602 | 20.877 | 14.592 | 3.177 |
| THETAsm | 14.684 | 17.922 | 14.598 | 3.006 |
| AutoBox1 | 14.931 | 21.588 | 14.930 | 3.679 |
| NAIVE2 | 15.270 | 17.880 | 15.195 | 3.172 |
| SINGLE | 15.316 | 17.817 | 15.212 | 3.171 |

**Table 1:** *Results for the yearly series, ordered by the first column, which is the average rank of sMAPE. The other columns show mean sMAPE, average rank of MASE, and mean of MASE.*

methods, consistently outperforming them in all measures. The original ETS method does not perform particularly well on this dataset.

Table 2 shows the results of the first case of statistical testing, where we compare the ETS methods among themselves. The table shows the $p$-values adjusted by the post-hoc procedure. The aligned Friedman test has an overall $p$-value of $2.17 \times 10^{-10}$, which is highly significant. The method with the best ranking, BaggedETS in this case, is chosen as the control method. We can see then from the table that differences against all comparison methods are significant on the chosen significance level.

Table 3 shows the results of the second case of statistical testing, where we choose BaggedETS as the best method and compare it against the methods from the M3 competition. The overall result of the Friedman aligned rank sum test is a $p$-value of $2.46 \times 10^{-10}$, which is highly significant.

| Method | $p_{Hoch}$ |
|---|---|
| BaggedETS | – |
| AdditiveETS.BC | 0.023 |
| BaggedETS.BC | 0.023 |
| ETS | 0.002 |

**Table 2:** *Results of statistical testing for yearly data, using only exponential smoothing methods. Adjusted p-values calculated from the aligned Friedman test with Hochberg's post-hoc procedure are shown. A horizontal line separates the methods that perform significantly worse than the best method from the ones that do not.*

| Method | $p_{Hoch}$ |
|---|---|
| ForcX | – |
| AutoBox2 | 0.598 |
| RBF | 0.598 |
| ROBUST.Trend | 0.598 |
| THETA | 0.598 |
| ForecastPro | 0.598 |
| Flors.Pearc1 | 0.598 |
| PP.Autocast | 0.574 |
| DAMPEN | 0.344 |
| **BaggedETS** | 0.344 |
| COMB.S.H.D | 0.085 |
| SMARTFCS | 0.056 |
| ARARMA | 0.001 |
| B.J.auto | $5.20 \times 10^{-4}$ |
| WINTER | $5.16 \times 10^{-5}$ |
| HOLT | $5.16 \times 10^{-5}$ |
| Flors.Pearc2 | $5.16 \times 10^{-5}$ |
| Auto.ANN | $1.37 \times 10^{-5}$ |
| AutoBox3 | $2.81 \times 10^{-7}$ |
| SINGLE | $7.79 \times 10^{-8}$ |
| THETAsm | $6.94 \times 10^{-8}$ |
| NAIVE2 | $5.69 \times 10^{-8}$ |
| AutoBox1 | $2.62 \times 10^{-11}$ |

**Table 3:** *Results of statistical testing for yearly data, using BaggedETS and the original results of the M3. Adjusted p-values calculated from the aligned Friedman test with Hochberg's post-hoc procedure are shown. A horizontal line separates the methods that perform significantly worse than the best method from the ones that do not.*

| | Rank sMAPE | Mean sMAPE | Rank MASE | Mean MASE |
|---:|---:|---:|---:|---:|
| THETA | 11.493 | 8.956 | 11.483 | 1.087 |
| COMB.S.H.D | 12.196 | 9.216 | 12.187 | 1.105 |
| ROBUST.Trend | 12.489 | 9.789 | 12.500 | 1.152 |
| DAMPEN | 12.692 | 9.361 | 12.697 | 1.126 |
| ForcX | 12.843 | 9.537 | 12.847 | 1.155 |
| PP.Autocast | 12.847 | 9.395 | 12.866 | 1.128 |
| B.J.auto | 13.161 | 10.260 | 13.169 | 1.188 |
| ForecastPro | 13.192 | 9.815 | 13.208 | 1.204 |
| HOLT | 13.236 | 10.938 | 13.171 | 1.225 |
| RBF | 13.267 | 9.565 | 13.251 | 1.173 |
| WINTER | 13.413 | 10.840 | 13.355 | 1.217 |
| **BaggedETS** | 13.425 | 10.030 | 13.438 | 1.192 |
| ARARMA | 13.426 | 10.186 | 13.393 | 1.185 |
| **BaggedETS.BC** | 13.493 | 10.059 | 13.523 | 1.209 |
| Flors.Pearc1 | 13.500 | 9.954 | 13.511 | 1.184 |
| AutoBox2 | 13.513 | 10.004 | 13.552 | 1.185 |
| **AdditiveETS.BC** | 13.671 | 9.895 | 13.704 | 1.224 |
| **ETS** | 13.713 | 9.838 | 13.697 | 1.214 |
| Auto.ANN | 13.961 | 10.199 | 13.976 | 1.241 |
| THETAsm | 14.238 | 9.821 | 14.217 | 1.211 |
| SMARTFCS | 14.249 | 10.153 | 14.296 | 1.226 |
| Flors.Pearc2 | 14.335 | 10.431 | 14.415 | 1.255 |
| AutoBox3 | 14.458 | 11.192 | 14.386 | 1.272 |
| SINGLE | 14.637 | 9.717 | 14.610 | 1.229 |
| AutoBox1 | 14.729 | 10.961 | 14.735 | 1.331 |
| NAIVE2 | 14.823 | 9.951 | 14.814 | 1.238 |

**Table 4:** *Results for the quarterly series, ordered by the first column, which is the average rank of sMAPE.*

We see that the ForcX method obtains the best ranking and is used as the control method. Differences to the BaggedETS method are not significant.

### 3.5 Results on the quarterly data

Table 4 shows the results for all methods on the quarterly data, ordered by average rank of sMAPE. It can be seen that the two bagged method versions outperform the comparison methods in terms of average rank of sMAPE, and average rank and mean MASE, but not in mean sMAPE. This may indicate that in general the bagged versions perform better, but there are some single series where they yield worse sMAPE results.

Table 5 shows the results of statistical testing considering only the ETS methods. The aligned Friedman test for multiple comparisons results in a $p$-value of $2.04 \times 10^{-10}$, which is highly

| Method | $p_{Hoch}$ |
|---|---|
| BaggedETS.BC | – |
| BaggedETS | 0.777 |
| ETS | 0.777 |
| AdditiveETS.BC | 0.777 |

**Table 5:** *Results of statistical testing for quarterly data, using only exponential smoothing methods. None of the results is statistically significant (so no horizontal line is drawn).*

| Method | $p_{Hoch}$ |
|---|---|
| THETA | – |
| COMB.S.H.D | 0.148 |
| DAMPEN | 0.031 |
| PP.Autocast | 0.031 |
| ROBUST.Trend | 0.031 |
| ForcX | 0.018 |
| RBF | 0.002 |
| ForecastPro | $7.66 \times 10^{-4}$ |
| ARARMA | $1.19 \times 10^{-4}$ |
| Flors.Pearc1 | $1.19 \times 10^{-4}$ |
| B.J.auto | $1.07 \times 10^{-4}$ |
| AutoBox2 | $1.07 \times 10^{-4}$ |
| **BaggedETS** | $1.60 \times 10^{-5}$ |
| HOLT | $1.56 \times 10^{-5}$ |
| WINTER | $6.46 \times 10^{-6}$ |
| THETAsm | $3.54 \times 10^{-6}$ |
| SINGLE | $1.63 \times 10^{-7}$ |
| Auto.ANN | $1.16 \times 10^{-7}$ |
| NAIVE2 | $9.64 \times 10^{-9}$ |
| SMARTFCS | $8.33 \times 10^{-9}$ |
| Flors.Pearc2 | $3.12 \times 10^{-10}$ |
| AutoBox3 | $5.43 \times 10^{-11}$ |
| AutoBox1 | $1.91 \times 10^{-11}$ |

**Table 6:** *Results of statistical testing for quarterly data, using BaggedETS.BC and the original results of the M3. A horizontal line separates the methods that perform significantly worse than the best method from the ones that do not. We see that only the COMB.S.H.D is not worse with statistical significance than the THETA method.*

significant. The method with the best ranking is BaggedETS.BC. However, we can see from the table that no differences to the other methods are statistically significant.

Table 6 shows the results for statistical testing of the BaggedETS method against the methods from the original M3 competition (though BaggedETS.BC was chosen in the last test as the control method, we use BaggedETS here, as from Table 4 we can see that it has slightly better performance. Anyway, the methods achieve very similar results). The overall result of the

Friedman aligned rank sum test is a $p$-value of $2.62 \times 10^{-10}$, which is highly significant. We see from the table that the THETA method performs best and is chosen as the control method. It outperforms all methods but COMB.S.H.D with statistical significance.

### 3.6 Results on the monthly data

Table 7 shows the results for all methods on the monthly data, ordered by average rank of sMAPE. The bagged versions outperform again the other methods. Furthermore, BaggedETS.BC performs particularly well, also outperforming all the methods from the M3. Only ForecastPro achieves a slightly better mean MASE.

Table 8 shows the results of statistical testing considering only the ETS methods. The aligned Friedman test gives a $p$-value of $p < 10^{-10}$, so that differences are highly significant. The method

| | Rank sMAPE | Mean sMAPE | Rank MASE | Mean MASE |
|---|---|---|---|---|
| **BaggedETS.BC** | 11.178 | 13.739 | 11.148 | 0.852 |
| THETA | 11.474 | 13.892 | 11.405 | 0.858 |
| ForecastPro | 11.578 | 13.898 | 11.597 | 0.848 |
| **BaggedETS** | 12.047 | 14.347 | 11.925 | 0.854 |
| **AdditiveETS.BC** | 12.424 | 14.224 | 12.373 | 0.891 |
| COMB.S.H.D | 12.527 | 14.466 | 12.584 | 0.896 |
| **ETS** | 12.648 | 14.375 | 12.718 | 0.895 |
| HOLT | 12.834 | 15.795 | 12.794 | 0.909 |
| ForcX | 12.844 | 14.466 | 12.894 | 0.894 |
| WINTER | 13.117 | 15.926 | 13.100 | 1.165 |
| RBF | 13.302 | 14.760 | 13.323 | 0.910 |
| DAMPEN | 13.578 | 14.576 | 13.631 | 0.908 |
| AutoBox2 | 13.677 | 15.731 | 13.738 | 1.082 |
| B.J.auto | 13.746 | 14.796 | 13.744 | 0.914 |
| AutoBox1 | 13.790 | 15.811 | 13.800 | 0.924 |
| Flors.Pearc2 | 13.915 | 15.186 | 13.947 | 0.950 |
| SMARTFCS | 13.933 | 15.007 | 13.833 | 0.919 |
| Auto.ANN | 13.975 | 15.031 | 13.992 | 0.928 |
| ARARMA | 14.155 | 15.826 | 14.164 | 0.907 |
| PP.Autocast | 14.256 | 15.328 | 14.340 | 0.994 |
| AutoBox3 | 14.339 | 16.590 | 14.242 | 0.962 |
| Flors.Pearc1 | 14.627 | 15.986 | 14.636 | 1.008 |
| THETAsm | 14.674 | 15.380 | 14.687 | 0.950 |
| ROBUST.Trend | 14.874 | 18.931 | 14.781 | 1.039 |
| SINGLE | 15.329 | 15.300 | 15.412 | 0.974 |
| NAIVE2 | 16.157 | 16.891 | 16.190 | 1.037 |

**Table 7:** *Results for the monthly series, ordered by the first column, which is the average rank of sMAPE.*

| Method | $p_{Hoch}$ |
|---|---|
| BaggedETS.BC | – |
| BaggedETS | $3.12 \times 10^{-7}$ |
| AdditiveETS.BC | $8.09 \times 10^{-12}$ |
| ETS | $1.04 \times 10^{-17}$ |

**Table 8:** *Results of statistical testing for monthly data, using only exponential smoothing methods. The BaggedETS.BC method clearly outperforms all other methods.*

| Method | $p_{Hoch}$ |
|---|---|
| **BaggedETS.BC** | – |
| THETA | 0.087 |
| ForecastPro | 0.087 |
| COMB.S.H.D | $2.27 \times 10^{-7}$ |
| ForcX | $1.59 \times 10^{-8}$ |
| HOLT | $1.69 \times 10^{-10}$ |
| RBF | $1.36 \times 10^{-10}$ |
| DAMPEN | $9.02 \times 10^{-12}$ |
| Auto.ANN | $6.64 \times 10^{-14}$ |
| WINTER | $5.13 \times 10^{-14}$ |
| B.J.auto | $1.09 \times 10^{-14}$ |
| Flors.Pearc2 | $6.43 \times 10^{-15}$ |
| SMARTFCS | $4.01 \times 10^{-16}$ |
| AutoBox1 | $1.61 \times 10^{-17}$ |
| AutoBox2 | $5.00 \times 10^{-18}$ |
| PP.Autocast | $2.12 \times 10^{-18}$ |
| AutoBox3 | $3.46 \times 10^{-24}$ |
| ARARMA | $2.30 \times 10^{-24}$ |
| THETAsm | $6.77 \times 10^{-25}$ |
| Flors.Pearc1 | $1.95 \times 10^{-25}$ |
| SINGLE | $9.72 \times 10^{-35}$ |
| ROBUST.Trend | $4.50 \times 10^{-49}$ |
| NAIVE2 | $4.17 \times 10^{-62}$ |

**Table 9:** *Results of statistical testing for monthly data, using BaggedETS.BC and the original results of the M3. BaggedETS.BC performs best, and only the THETA and ForecastPro methods do not perform significantly worse. However, their p-values are close to 0.05 so that they are nearly significant.*

with the best ranking is BaggedETS.BC, and we can see from the table that it outperforms all the other methods with statistical significance.

Table 9 shows the results for statistical testing of the BaggedETS.BC method against the methods from the original M3 competition. The overall result of the Friedman aligned rank sum test is again a $p$-value of $p < 10^{-10}$, so that it is highly significant. We see from the table that BaggedETS.BC is the best method, and that only the THETA method and ForecastPro are not significantly worse on the chosen 5% significance level. However, their $p$-values are close to this level. Using MASE for the significance tests (results are not reported here), all methods perform significantly worse than BaggedETS.BC.

## 4 Conclusions

In this work, we have presented a novel method of bagging for exponential smoothing methods, using Box-Cox transformation, STL decomposition, and the moving block bootstrap. The method is able to consistently outperform the basic exponential smoothing methods. On the monthly data of the M3 competition, the bagged exponential smoothing method with Box-Cox transformation is able to outperform, with statistical significance, all methods that took part in the competition. So, especially for monthly data this method can be recommended to be used in practice.

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