

# Buses for Anonymous Message Delivery

## (Preliminary Version)

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### Abstract

This work develops a novel approach to hide the senders and the receivers of messages. The intuition is taken from an everyday activity that hides the “communication pattern” – the public transportation system. To describe our protocols, busses are used as a metaphor: Busses, i.e., messages, are traveling on the network, each piece of information is allocated a seat within the bus. Routes are chosen and buses are scheduled to traverse these routes. Deterministic and randomized protocols are presented, the protocols differ in the number of buses in the system, the worst case traveling time, and the required buffer size in a “station.” In particular, a protocol that is based on cluster partition of the network is presented; in this protocol there is one bus traversing each cluster. The clusters’ size in the partition gives time and communication tradeoffs. One advantage of our protocols over previous works is that they are not based on statistical properties for the communication pattern. Another advantage is that they do not require that all the processors in the communication network are busy.

## 1 Introduction

Throughout the history encryption was used to hide the *contents* of transmitted data. The rapid growth in the use of the Internet only increased the necessity of encryption. However, encryption does not hide all the relevant information, for example, it does not hide the identity of the communicating parties. In this work we deal with the problem of anonymous communication – communication that not disclose the identity of the sender and receiver.

We develop a novel approach to hide the senders and the receivers of messages. The intuition is taken from an everyday activity that hides the “communication pattern” – the public transportation system. For example, a traveler that takes buses from one place to another remains anonymous, and it is hard to trace

him. Metaphorically, we consider the pieces of information that senders send to receivers as passengers. There are “buses,” i.e., messages, traveling on the network, and each piece of information is allocated a seat within a bus. The sender and receiver are modeled as bus stations. Our aim is to simulate this metaphor in the digital world while keeping the anonymity of the sender and receiver. We will also try to hide the information that a message is sent, that is, hide the number of passengers on each bus.

**Previous work.** One of the first works to consider the problem of hiding the communication pattern in the network is the work of Chaum [5] where the concept of a *mix* is introduced. A single processor in the network, called a mix, serves as a relay. Each processor  $P$  that wants to send a message  $m$  to a processor  $Q$  encrypts  $m$  using  $Q$ 's public key to obtain  $m'$ . Then  $P$  encrypts the pair  $(m', q)$  using the public key of the mix. The mix decrypts the message and forwards  $m'$  to  $q$ . This scheme has been extended in [13, 14, 16, 17] where several mixes are used to cope with the possibility of compromising the single mix. The mix schemes operate under some statistical assumption on the pattern of communication. If a single message is sent then an adversary that monitors the communication channels can observe the sender and the receiver of the particular message. Another example for a problematic case is when all the processors send a message to the same destination – in this case the identity of the receiver is revealed.

Another approach is to use generic secure multi-party function evaluation protocols (see, for example, [12, 2, 6, 4, 3]). However, all these generic protocols are complicated and require many rounds of communication; we suggest more efficient protocols for hiding the communication pattern. Furthermore, the above solutions assume that there is a communication link between any two processors; there are solutions without this assumption, e.g., [8, 15, 9].

An approach based on “xor-trees” has been presented in [11]. The scheme presented in [11] fits long communication sessions during which the data xored with pseudo-random bits (that cancel each other) is transferred towards the root which in turn broadcasts the arriving information to the nodes in the tree. The solution presented in [11] is an extension of the DC-net approach suggested in [6].

**Our contribution.** We present solutions that do not rely on statistical properties for the communication pattern. In our solution, unlike the solution presented in [11], not all the processors in the communication network are busy in the transmission (when communication between two parties takes place). Moreover, there is no need to store information (such as the result of xoring arriving bits) in memory, thus it is more suitable for a fault-tolerant environment. For example, the scheme may be a base for a robust anonymous message delivery by retransmitting a new *bus* upon a time-out.

Let us note two additional important properties of our scheme. First note that,

since the buses traverse the network in fixed routes and fixed schedule, the adversary cannot learn whether there is any communication between the processors or not. In addition, our scheme can cope with an adversary that monitors any number of processors.

We also extend the scheme to cope with two extensions of the model: (1) protocols which work even if the topology of the network is unknown, (2) protocols which tolerate Byzantine processors.

**Organization.** The rest of the paper is organized as follows. The problem statement appears in Section 2. Two simple solutions that achieve minimum communication and minimum time, respectively, are presented in Section 3. A solution that introduces a tradeoff between time and communication is presented in Section 4. Lower bounds on the possible tradeoffs between time and communication are presented in Section 5. Extensions for the case of Byzantine processors and unknown topology appear in Section 6.

## 2 The System and Threat Models

We consider a network of  $n$  processors, denoted  $p_1, \dots, p_n$ , connected by  $m$  communication links. We use the communication graph  $G(V, E)$  to represent our network,  $V$  is the set of processors and  $E$  is the set of communication links connecting the processors (that is  $n = |V|$  and  $m = |E|$ ). We assume that  $G$  is connected. Processors communicate by sending and receiving messages. The system is synchronous – there is a common global *pulse* (possibly implemented by synchronized distributed clocks) that triggers (whenever the clock reaches an integer value) the processors to send messages, messages sent in a certain pulse, arrive to the neighboring processor before the next pulse.

Some processor,  $p_i$ , called *sender*, may decide to communicate with another (not necessarily neighboring) processor  $p_j$ , called *receiver*. Our objective is to hide the fact that  $p_i$  communicates with  $p_j$ . That is, we want to hide the identities of  $p_i$  and  $p_j$ . Furthermore, some of our protocols even hide the fact that a message was sent. A protocol that achieves these goals is called an *anonymous message delivery protocol*. We note that the vast majority of known cryptographic techniques focus on hiding the *contents* of the transmitted data, but not the fact that data has been transmitted.

We consider two types of adversaries *listening adversary* and *Byzantine adversary*. The listening adversary, also known as a honest-but-curious adversary, can monitor all the communication links and also monitor the internal contents of the processors of the network. We do not allow the adversary to monitor the internal contents of the sender or the receiver. (Otherwise, the adversary can easily identify the sender or the receiver.) This adversary is honest, i.e., it cannot change any messages, delete messages, add any messages, or change the state of

any processor.

The *Byzantine adversary* is more powerful than the listening adversary. The Byzantine adversary can monitor the communication links and the internal contents of the processors of the network (except the memory of the sender and the receiver). In addition, for some parameter  $t$ , it can control up to  $t$  processors in the network. These processors can insert messages, delete messages, or arbitrarily change messages that they receive (before forwarding the messages). That is, these processors can deviate from the pre-defined protocol.

We evaluate a solution by its *time complexity*, its *communication complexity*, and its *buffer complexity*: the time complexity is the worst case time required to transmit a message from a sender to a receiver, the communication complexity is the maximal number of messages that are sent simultaneously by the processors in the network, and the buffer complexity is the buffer size required for each processor to store incoming and outgoing messages in each time step.

## 2.1 Cryptographic Primitives

The first cryptographic primitive that we use is *encryption* which guarantees the secrecy of messages. That is, a sender can send an encrypted message such that only the intended recipient can decrypt it. We will require *semantic security* – the encryption is randomized and an eavesdropper cannot distinguish in polynomial time between encryptions of any pair of messages. We consider two types of encryption:

**Symmetric key encryption.** Both sender and receiver have a common secret key, which is used for both encryption and decryption.

**Public key encryption.** The receiver has a secret private key which it uses for decryption. Furthermore, the receiver publishes a public key which is used for encryption by any sender; an eavesdropper cannot distinguish between messages even if it has the public key.

Typical symmetric key encryption schemes are faster than public key encryption schemes; however they require every pair of processors in the network to have a common secret key.

The second cryptographic primitive that we use is *authentication* which guarantees that if a sender sends a message to a receiver and a third party alters this message, then the receiver can detect this fact. Again we can consider two types: (1) symmetric key authentication in which the sender uses the common key to authenticate a message and the receiver uses the common key to verify the authenticity of the message, and (2) public key authentication, known as signatures, in which the sender uses its private key to sign a message and the receiver uses the public key to verify the validity of the signature.

### 3 Simple Solutions

In this section we present two simple protocols, one with optimal communication complexity and another with optimal time complexity. In Section 4 we will generalize the ideas of these protocols, and present protocols that exhibit tradeoffs between time and communication. In all our protocols we metaphorically view each message as a *bus*. The protocols vary according to the number of buses in the system, and the way they travel in the communication graph.

#### 3.1 Communication Optimal Protocol

We start with a solution with message complexity 1, i.e., in each time unit only one processor sends a message to one other processor. Using our metaphor, there is only one bus traveling in the system. We next define how the bus travels in the communication graph. First, fix any spanning tree in the graph. Next, use an Euler tour (that is, a DFS tour) of the spanning tree to define a ring. The bus is rotating through the ring, and has  $n^2$  seats. Seat  $s_{i,j}$  is used to communicate an encrypted message from processor  $p_i$  to  $p_j$ ; this message is encrypted either using the symmetric key of  $p_i$  and  $p_j$ , or using the public key of  $p_j$  (depending which encryption infra-structure exists). Each time the bus gets to processor  $p_i$  it changes each message in the *row* of seats  $s_{i,j}$  either to a message it wants to send to  $p_j$ , or to some detectable garbage. Furthermore,  $p_i$  checks what messages were sent to it, by decrypting the  $n$  messages located in the  $i$ -th *column* and ignoring the ones containing garbage.

By the semantic security of the encryption, a listening adversary cannot tell whether a seat contains garbage or a real message, i.e., it cannot tell if two processors are communicating. Next, we state the communication and time complexities of our solution. The communication complexity of this solution is optimal – there is single bus. However, the time complexity is quite bad: it can take at most  $2n - 1$  time units until the bus reaches the sender, and at most  $2n - 1$  additional time units until the bus reaches the destination. The buffer complexity of this protocol is  $n^2$ . We summarize the properties of this protocol below.

**Theorem 1** *There is an anonymous message delivery protocol with communication complexity 1, time complexity  $O(n)$ , and buffer complexity  $O(n^2)$ .*

We emphasize again that since there is one bus, most of the time each processor is not involved in executing this protocol and does not need to store any information between two visits of the bus. Furthermore, by Theorem 4, the time complexity in any protocol with communication complexity 1 is  $\Omega(n)$ .

### 3.1.1 Reducing the Number of Seats

In this section we present a protocol that reduces the number of seats in each bus assuming that not too many messages are sent simultaneously. We modify the above protocol where instead of assigning a seat for any source/destination pair, the sender writes its message in a randomly chosen seat (deleting the previous contents of the seat). However, the sender wants to hide the fact that it wrote a message to some seat, thus it will change the contents of all the seats in the bus. To achieve this goal, the sender encrypts the message using the public keys, in reverse order, of all the processors in the Euler tour between the sender and receiver. When the bus gets to some processor, it replaces the contents of each seat by the decryption of the previous contents under its private key. Next, if any message makes sense, then the processor knows that this is a message sent to it, and it changes it to a random contents. To determine the size of the bus that may serve well under this policy we use the so-called birthday problem (or birthday paradox). As an example, if you have a class of 23 students, then with probability  $1/2$  there will be two students with the same birthday. More formally,

**Claim 1** *Suppose  $k$  balls are randomly and independently assigned to  $m$  bins. The probability that all balls fell into distinct bins is  $\approx e^{-k(k-1)/2m}$ .*

Assume that we have an upper bound  $k$  on the number of messages that will be sent anonymously. Thus, if we take the size of the bus to be  $m = O(k^2)$  then the probability that two processors will randomly choose the same seat is less than  $1/4$ . If we take the size of the bus to be  $m = O(sk^2)$ , for some security parameter  $s$ , then the probability of a collision drops to  $1 - e^{-1/s} \approx 1 - (1 - 1/s) = 1/s$ .

Of course, if there is a collision then the first message gets lost. A possible way to overcome this problem is that the recipient sends an acknowledgment to the sender using the same seat. If the sender does not get the message, then the sender resends the message. (The expected number of times that a message will be sent is slightly above 1.) If a sender  $p_i$  sees that it resends a message too many times, then  $p_i$  can decide to double the size of the bus. However,  $p_i$  does not want to reveal that it is trying to send a message, thus it can send an anonymous message to any other (random or fixed) processor to double the size of the bus. Similarly, a processor that receives acknowledgments for several messages in a row can send an anonymous message to reduce the size of the bus.

Another way to reduce the number of seats is to assume that each time the bus gets to  $p_i$  it will send only one message.<sup>1</sup> In this case, we can use a bus with only  $n$  seats: Each processor has a single seat  $s_i$  in the bus that can be used for sending a message to another processor in the ring. The message  $m$  is encrypted by the sender in a way that ensures that only the receiver can decrypt  $m$ . That is, when

<sup>1</sup>Alternatively, if  $p_i$  has more than one processor with whom it wants to communicate, then it will use a buffer to store these messages; this will increase the delivery time to  $n^2$ .

the bus gets to a processor  $p_j$  it tries to decrypt the messages in the  $n - 1$  seats  $s_i$ , where  $i \neq j$ , and receives the messages that it can verify their authenticity.

### 3.2 Full Communication – Time Optimal Protocol

We next present a protocol with optimal time complexity, however with bad communication complexity. In this protocol two buses travel through every link – a bus in each direction. The nodes transfer seats from one bus to another according to the shortest path criteria. A seat  $s_{i,j}$  that arrives at a node  $p_k$  is assigned to a bus that traverses the link attached to  $p_k$  that is on a shortest path to  $p_j$ . The seats that are transferred use the routing information, and may be transferred together with the routing messages that are repeatedly exchanged. That is, the communication in this protocol is “swallowed” by the communication of the routing-update protocol.

As in the previous protocol all messages are encrypted before they are assigned to seats, and garbage messages are sent if there is no real message. Thus, privacy is guaranteed. Next we state the communication and time complexities of this protocol. The communication complexity of this protocol is the number of buses, i.e.,  $2m$  (where  $m$  is the number of edges in the graph). This protocol has optimal time for message arrival, which is the number of links in the shortest path between the receiver and sender. The buffer complexity of a node is the number of shortest paths that contain this node. This number can be small or big depending on the communication graph. For example, if the graph is a complete graph, each bus contains one seat, and the buffer complexity of a node is the number of its neighbors, i.e.,  $n - 1$ , however, the number of buses is  $O(n^2)$ . The other extreme is a star. In this case the buffer complexity of the center is  $O(n^2)$  and the number of buses is  $n$ .

**Theorem 2** *There is an anonymous message delivery protocol with communication complexity  $2m$  and buffer complexity  $O(n^2)$ . The time complexity between two nodes is the distance between the nodes in the communication graph.*

## 4 Time and Communication Tradeoff

In this section we will examine more sophisticated protocols that can be tuned up to trade time and communication. The first observation is that the full communication protocol presented in Section 3.2 already presents tradeoffs between time and communication: the protocol can use any connected spanning sub-graph of the communication graph with two buses on each edge of the subgraph. This reduces the communication complexity but might increase the time complexity since the distance between two nodes in the sub-graph might be bigger. To obtain the minimum number of buses, the protocol uses a spanning tree; in this case the communication complexity is  $O(n)$ .

We next present protocols which reduce the number of buses to less than  $n$ . In these protocols we divide the graph into clusters and construct bus routes within each cluster. For concreteness, we choose specific partitions to clusters that are based on [10], however similar partitions can be used as well (see the related work in [10]).

The partition scheme of [10] uses a spanning tree of the communication graph, and partitions its nodes and edges to clusters. One way to partition the tree is a node partition which results in clusters with at least  $x$  nodes and no more than  $\delta x$  nodes, where  $x$  can be chosen to be any value in the range  $1, \dots, n$  and  $\delta$  is the maximum degree of a node in the tree. In this partition neighboring clusters are connected by a single link. The partition scheme that we will use is edge partition, that is, each edge is contained in exactly one cluster. In this case each cluster contains at least  $x$  edges and no more than  $3x$  edges. (In fact at most one cluster is of size  $3x$  and all the rest are of at most  $2x$ .) Each cluster is a connected sub-graph of the spanning tree, i.e., it is a tree that contains  $O(x)$  nodes. In this partition two neighboring clusters are connected by a single node.

Once we have the partition to clusters, we have one bus in each cluster which performs an Euler tour on the spanning tree of the cluster. There are at most  $\lceil n/x \rceil$  clusters in the graph, thus the number of buses, i.e., the communication complexity, is no more than  $\lceil n/x \rceil$ . If a message is sent from a node in one cluster to a node in another cluster then this message should move from one bus to another until it reaches the cluster of the receiver. That is, when a bus reaches a node that is part of more than one cluster (recall that we use an edge partition), then seats are transferred from one bus to another. The bus in  $Cluster_\ell$  has a seat  $s_{i,j}$  for every  $p_i$  and  $p_j$  such that the simple path connecting them in the spanning tree passes through an edge of  $Cluster_\ell$ . We next analyze the buffer complexity: For a given node and a given seat  $s_{i,j}$ , there can be at most two clusters containing the node such that the path from  $p_i$  to  $p_j$  in the spanning tree uses an edge from the cluster. Thus, the buffer size of each node is at most twice the number of simple paths in the tree passing through the node. This number is at most  $O(n^2)$ .

#### 4.1 Bus Scheduling

We would like to minimize the time required for a message to arrive to its destination. To achieve this goal, buses in clusters with a common node, should reach the common node simultaneously in order to transfer seats. We show how to schedule the buses to satisfy this condition. Recall that we consider a synchronous settings, where the bus traverses an edge in a single time unit. Furthermore, we use the fact that clusters have similar sizes. Let us first consider an ideal case, where the clusters have identical size. In this case, we can start in the cluster that contains the root of the spanning tree, schedule its bus, and whenever the bus reaches a node shared with another cluster, we start scheduling the bus of the neighboring cluster. Since we consider a spanning tree then there are no cycles and this scheduling is

possible.

If the clusters have different number of nodes, we first schedule the bus in a cluster  $Cluster_\ell$  with the maximum number of edges  $m_{\max}$ . Recall that an Euler tour in this cluster will take  $2m_{\max}$  time units. Then whenever the bus reaches a node that is part of other clusters, the buses of the other clusters are scheduled. It is possible that a neighboring cluster  $Cluster_j$  has  $m' < m_{\max}$  nodes, in such a case the bus of  $Cluster_j$  will wait,  $O(m_{\max} - m')$  time units, for the bus of  $Cluster_\ell$ , whenever it reaches the node that is common to  $Cluster_\ell$  and  $Cluster_j$ . The procedure continues in a fashion similar to the case of identical size clusters.

We next analyze the time complexity of this protocol. If the distance between node  $p_i$  and node  $p_j$  in the spanning tree is  $d$ , then the path can pass through at most  $d$  clusters, and in each cluster it would take less than  $2m_{\max}$  steps until the message would pass to the next cluster. Thus, the delivery time from  $p_i$  to  $p_j$  is  $O(dx)$  (since  $m_{\max} < 3x$ , where  $x$  is the parameter chosen in the edge partition scheme). In the worst case the message will pass through each edge of the spanning tree at most twice and the delivery time would be  $O(n)$ .

**Theorem 3** *For every  $x$ , where  $1 \leq x \leq n$ , there is an anonymous message delivery protocol with communication complexity  $O(n/x)$ , buffer complexity  $O(n^2)$ , and time complexity between two nodes is  $O(\min(dx, n))$ , where  $d$  is the distance between the nodes in the spanning tree.*

## 4.2 Reducing the Number of Seats

We can reduce the number of seats in a bus, i.e., reduce the buffer complexity. We use a bus with  $O(n^2/x^2)$  seats, a seat  $s_{k,\ell}$  for a message that should be transferred from the  $k$ 'th cluster to the  $\ell$ 'th cluster. In this case only one message can be sent at a time from a particular cluster to another cluster. It is possible that more than one processor in  $Cluster_k$  will try to transmit a message to  $Cluster_\ell$ . We use a probabilistic approach, where each processor in  $Cluster_k$  that would like to send a message to  $Cluster_\ell$  uses a random function to decide whether to overwrite the seat  $s_{k,\ell}$ . To ensure that overwrites are not observed each message is changed at every node. To do so, every message is encrypted in a nested fashion, using all the keys of the processors in the route to the bus exchange node.

## 5 Lower Bounds

In this section we prove lower bounds on the time/communication tradeoffs. As a warm-up we start with the simple case where there is one bus traversing the communication tree according to some Euler tour. This tour, whose length is  $O(n)$ , traverses each leaf of the tree once and there are at least two leaves. Thus, for any two leaves  $u$  and  $v$  the distance between  $u$  and  $v$  or  $v$  and  $u$  in the tour is at least

$n/2$ , and it takes at least  $n/2$  time units to send a message from  $u$  to  $v$  or from  $v$  to  $u$ . The next lemma generalized the above simple scenario.

**Lemma 1** *In any protocol with communication complexity 1, there are two nodes in the graph such that the average delivery time from one node to the second is  $\Omega(n)$ .*

**Proof.** A necessary condition for sending a message from a node  $u$  to a node  $v$  is that  $u$  sends some message on one of its outgoing edges. In each time unit there is at most one node sending a message. For any  $t$ , consider the sequence of nodes that send messages in the first  $t$  time units. (We do not assume anything about this sequence other than that it contains at most  $t$  nodes.) There is at least one node  $u$  that appears at most  $t/n$  times in this sequence. In other words, the average distance from two occurrences of  $u$  in the sequence is  $\Omega(n)$ . Fix such  $u$  and pick any node  $v$ . Assume that  $u$  wants to send a message to  $v$  one time unit after each time that it appears in the sequence. It takes  $\Omega(n)$  time units for  $u$  to send a message to  $v$ .  $\square$

Note that the above proof does not use the anonymity requirement of the delivery protocol, but only relies on the message complexity. Every protocol with communication complexity  $c$  and time complexity  $t$  can be transformed into a protocol with communication complexity 1 and time complexity  $tc$ . Thus,

**Theorem 4** *In any protocol with communication complexity  $c$ , there are two nodes in the graph such that the average delivery time from one node to the second is  $\Omega(n/c)$ .*

The above theorem implies that the tradeoff in Theorem 3 cannot be improved by a factor bigger than  $O(d)$  where  $d$  is the distance between the two nodes in the spanning tree. For example, if we consider a complete binary tree with  $n$  nodes, then  $d = O(\log n)$ . We next show that if we consider a “natural” partition of the complete binary tree into clusters then we will obtain message complexity  $n/x$  and time complexity  $\Theta(x \log n / \log x)$ . More formally, we consider a complete binary tree of height  $\ell d$  for some parameters  $\ell$  and  $d$ , and partition the tree into clusters of size  $x \stackrel{\text{def}}{=} 2^d - 1$  where each cluster is a complete binary tree of depth  $d$ . The  $\log x$  factor improvement in the upper bound for this case follows from the observation that any simple path in the tree passes through at most  $2 \log n / \log x$  clusters.

We next show that this upper bound is tight for this partition. To prove this claim consider an Euler tour in a complete binary tree with  $x$  nodes starting from the root, and let  $v$  be the first leaf visited in the tour. The distance in the tour between  $v$  and the root is  $\Omega(x)$ . Now we consider the complete binary tree, and define a sequence of  $\log n / \log x$  nodes  $v_0, v_1, \dots, v_{\log n / \log x}$ , where  $v_0$  is the root of the tree, and  $v_i$  is a leaf in the cluster of  $v_{i-1}$  whose distance from  $v_{i-1}$  in the Euler tour of the cluster is  $\Omega(x)$ . Thus, the delivery time of a message from  $v_{\log n / \log x}$  to the root is  $\Omega(x \log n / \log x)$  no matter how the buses are scheduled.

## 6 Extensions

In this section we show how simple modifications to the idea of the buses can cope with two extensions to the model. The first extension is when the topology of the communication graph is unknown, and the second is to a Byzantine adversary, that is, an adversary that can cause processors to behave maliciously.

We start with the scenario where the processors in the network do not know the topology of the network (for example, the network can change periodically). The solution we propose for this problems is to use a random walk on the communication graph. More precisely, there is one bus traversing the graph, and in each step the processor holding the bus chooses uniformly one of its neighbors, and sends the bus to the chosen neighbor. Aleliunas et al. [1] proved that the expected time of a random walk that visits all the nodes of a graph with  $n$  nodes and  $m$  edges is  $O(nm)$ . Thus, the expected delivery time of a message using a random walk (in an unknown graph) is  $O(nm)$ . This bound on the delivery time is tight for some graphs, e.g. the so called lollipop graph. However, it is too pessimistic for some graphs, e.g., for a clique the cover time is  $O(n \log n)$  (and not  $O(n^3)$ ).

We now turn to the case in which processors are Byzantine, that is, they may try to add/delete or change messages in a malicious way. First note that the communication graph must be  $t + 1$  connected in order to tolerate  $t$  faults. Otherwise, there is a cut of  $t$  or less Byzantine processors that can partition the graph into two isolated connected components. We therefore assume that the communication graph is  $t + 1$  connected, thus, by Menger's theorem, for every two nodes there are  $t + 1$  paths connecting them such that there is no internal node common to two of these paths. We describe a protocol in which there are two buses on each link, one in each direction. When  $p_i$  wants to anonymously send a message to  $p_j$ , then  $p_i$  authenticates this message using a private key common to  $p_i$  and  $p_j$ . This ensures that a Byzantine processor can not generate/change a message originating from some sender in a way that is not identified by the receiver. Thus, the Byzantine processor can only drop messages. Processor  $p_i$  sends the message over  $t + 1$  disjoint paths, therefore the message will reach the destination through at least one path with no Byzantine processor. To achieve anonymity we use the mechanism of the full communication protocol described in Section 3.2. The number of seats in a bus equals to the number of paths that use this link in the bus traveling direction.

We next discuss how to reduce the number of buses. Given a communication graph that is at least  $t + 1$  connected, we will find a spanning sub-graph that is  $t + 1$  connected and contains fewer edges. Finding a  $t + 1$ -connected spanning subgraphs that has the minimum number of edges is NP-hard. However, there are good approximation algorithms for this problem. A recent result [7] describe an efficient algorithm that returns a graph whose number of edges is no more than  $1 + 1/(t + 2)$  times the optimal number of edges. In particular, the number of edges is no more than  $(t + 1)n$ . This, however, might increase the delivery time

since the length of the  $t$  disjoint paths might be longer.

## References

- [1] R. Aleliunas, R. M. Karp, R. J. Lipton, L. Lovász, and C. Rackoff. Random walks, universal traversal sequences, and the complexity of maze problems. In *Proc. of the 11th Annu. ACM Symp. on the Theory of Computing*, pp. 218–223, 1979.
- [2] M. Ben-Or, S. Goldwasser, and A. Wigderson. Completeness theorems for noncryptographic fault-tolerant distributed computations. In *Proc. of the 20th Annu. ACM Symp. on the Theory of Computing*, pp. 1–10, 1988.
- [3] R. Canetti, U. Feige, O. Goldreich, and M. Naor. Adaptively secure multiparty computation. In *Proc. of the 28th Annu. ACM Symp. on the Theory of Computing*, pp. 639 – 648, 1996.
- [4] D. Chaum, C. Crépeau, and I. Damgård. Multiparty unconditionally secure protocols. In *Proc. of the 20th Annu. ACM Symp. on the Theory of Computing*, pp. 11–19, 1988.
- [5] D. Chaum. Untraceable electronic mail, return addresses, and digital pseudonyms. *Communication of the ACM*, vol. 24, no. 2, pp. 84–88, 1981.
- [6] D. Chaum. The dining cryptographers problem: unconditional sender and recipient untraceability. *Journal of Cryptology*, vol. 1, pp. 65–75, 1988.
- [7] J. Cheriyan and R. Thurimella. Approximating minimum-size  $k$ -connected spanning subgraphs via matching. *SIAM J. on Computing*, vol. 30, no. 2, pp. 528–560, 2000.
- [8] D. Dolev. The Byzantine generals strike again. *J. of Algorithms*, vol. 3, pp. 14–30, 1982.
- [9] D. Dolev, C. Dwork, O. Waarts, and M. Yung. Perfectly secure message transmission. *J. of the ACM*, vol. 40, no. 1, pp. 17–47, 1993.
- [10] S. Dolev, E. Kranakis, D. Krizanc, and D. Peleg. Bubbles: adaptive routing scheme for high-speed dynamic networks. *SIAM J. on Computing*, vol. 29, no. 3, pp. 804–833, 1999.
- [11] S. Dolev and R. Ostrovsky. Xor-trees for efficient anonymous multicast and reception. *ACM Transactions on Information and System Security*, vol. 3, no. 2, pp. 63–84, 2000.

- [12] O. Goldreich, S. Micali, and A. Wigderson. How to play any mental game. In *Proc. of the 19th Annu. ACM Symp. on the Theory of Computing*, pp. 218–229, 1987.
- [13] A. Pfitzmann. How to implement ISDNs without user observability – some remarks. TR 14/85, Fakultät für Informatik, Universität Karlsruhe, 1985.
- [14] A. Pfitzmann, B. Pfitzmann, and M. Waidner. ISDN-MIXes – Untraceable communication with very small bandwidth overhead. In *Proc. Kommunikation in verteilten Systemen*, pp. 451–463, 1991.
- [15] T. Rabin and M. Ben-Or. Verifiable secret sharing and multiparty protocols with honest majority. In *Proc. of the 21st ACM Symp. on the Theory of Computing*, pp. 73–85, 1989.
- [16] C. Rackoff and D. Simon. Cryptographic defense against traffic analysis. In *Proc. of the 25th Annu. ACM Symp. on the Theory of Computing*, pp. 672–681, 1993.
- [17] P. F. Syverson, D. M. Goldschlag, and M. G. Reed. Anonymous connections and onion routing. In *Proc. of IEEE Symposium on Security and Privacy*, pp. 44–54, 1997.
- [18] M. Waidner and B. Pfitzmann. The dining cryptographers in the disco: unconditional sender and recipient untraceability with computationally secure serviceability. In *Advances in Cryptology – EUROCRYPT 89*, vol. 434 of LNCS, pp. 690, Springer-Verlag, 1990.

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