Integral Modeling and Simulation in Some Thermal Problems

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Abstract: - In this paper the method of integral equations is proposed for some thermal problems of engineering (radiative heat transfer, heat conduction). Presented models lead to a system of Fredholm integral equations and Volterra-Fredholm integral equations, respectively. We propose various numerical methods (discretization method and projection-iterative method) providing to a system algebraic equations. In some cases simulation methods can be used. Computational results for integral modeling are given.

Key-Words: Integral equations method, Fredholm integral equation, radiative heat transfer, radiosity, Volterra-Fredholm integral equation, heat conduction problems.

1. Introduction

The aim of this paper is to present the advantages of the integral equations method (IEM), and the possibilities of its application to various branches of engineering, particularly to the problems arising in power engineering [4]. It is an analytical-numerical method and requires great efforts from highly skilled specialists (mathematicians, computer scientists, engineers). IEM seems to be a natural method, especially in the field of electrodynamics. This is a case of electromagnetic field being described with integral equations, the kernels of which are searched for by integral transformations in the domain of space variables, while in the time domain the expected system response has a form of integral formulas. It confines the expected solution valid for the whole domain to a predefined part of the space subject to analysis. This enables a considerable reduction of the size of the system of equations. In such a case, the minimization of the size of the systems of equations and the reduction of computation time, while maintaining the accuracy, becomes highly important.

Integral equations, or rather their systems, are often matched with mathematical models describing the current density distribution at the cross-section of a working conductor or in the cartridge of an induction heater. The knowledge of the current density distribution may be a base for determining some electrodynamic values such as magnetic induction or distribution of electrodynamic forces acted at selected points of the conductors. Moreover, some problems of the radiative heat transfer are reducible to a system of Fredholm integral equations.

We restrict to the following mathematical models in electrical engineering: radiative heat transfer and Fourier's problems. Presented models lead to a system of Fredholm integral equations, and Volterra-Fredholm integral equations, respectively. We propose various computational methods (discretization and projection-iterative methods) providing to a system algebraic equations.

2. Integral equations in the radiosity problems

2.1. Fundamental theory

The papers [1,7] present theoretical foundations the modelling of phenomena related to visualisation [1] performed by means of computer graphics software and for the modelling of radiative heat transfer [2]. Since the equations describing both of these processes are very similar, there is a possibility of applying certain computer graphics programmes to resolve problems related to radiative heat transfer. It is explores all necessary supplements making it possible to perform such calculations. Thermokinetics, describing radiative heat transfer; lighting engineering, investigating problems of determination of surface illumination and computer-generated graphics, resolving issues connected with visualisation (it is the creation of seemingly three-dimensional representations of virtual reality on a two-dimensional screen, based on mathematical descriptions of the scene) – they all examine, to a greater or smaller extent, the same phenomena of emission, transmission and absorption of optical radiation. The similarity of phenomena occurring in all of these cases additionally offers the possibility to use similar research tools to investigate them. Only the simplest tasks involving radiative heat transfer or lighting engineering can be solved in [1] using analytical methods. Practically, all more demanding problems in these fields are currently solved using numerical methods or by means of modelling and simulation: (see [1,5,7]). It thus seems interesting to adapt such sophisticated computer graphics software to solve very complex problems involved in radiative heat transfer.

Contemporary advanced computer graphics software and interior visualisation applications are based on the visualisation equation given below:

$$L(x^0,x^1)=g(x^0,x^1)\cdot\left[L_e(x^0,x^1)+\int_{\Omega}\rho(x^0,x^2,x^1)L(x^2,x^1)dx^2\right] \quad (1)$$

where $L(x^0,x^1)$ is luminance of point x^0: the total of luminance of radiation emitted $L_e(x^0, x^1)$ and reflected (integral value) in the direction of point x^1; $g(x^0, x^1)$ - factor dependent on the geometry of the system, defining the "visibility" of point x^1 from x^0; $\rho(x^0,x^0,x^1)$ – specular reflectance of radiation for point x^0, with radiation propagating from the direction of point x^2 and reflected in the direction of x^1. Integration is performed along the whole hemisphere Ω surrounding x^0. This is illustrated by fig.1.

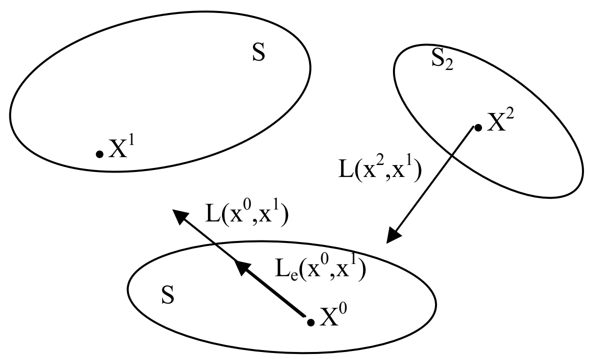

Fig. 1. Illustration of equation (1).

The equation (1) was written in the terminology used in lighting engineering or computer graphics, where the concept of luminance L_v [lm/m^2/sr] is used, referring to visible radiation. Thermokinetics, however, uses the concept of radiance L [W/m^2/sr] referring to all optical radiation (including thermal radiation).

The solution of equation (1) for every point of surfaces $S_0...S_n$ under consideration consists of determination of luminance of each of these points. This is the basic information, necessary for further construction of visual images of surfaces examined. Unfortunately, the equation (1) cannot be solved analytically. Only simulation methods can be applied. One of a commonly used method is backward ray tracing.

The equation describing heat balance of point x^0 in Siegel and Howell [8] or Modest [6] has a form that is similar to (1), as given below:

$$p_{eff}(x^0,T,\theta,\phi) = p_e(x^0,T,\theta,\phi) + \int_{\Omega}\rho(x^0,\theta,\phi,\theta_{in},\phi_{in})p_{in}(x^2,T_j,\theta_{in},\phi_{in})\cos\theta d\omega \quad (2)$$

where, in radiative heat transfer terminology: p_{eff} stands for surface density of effective radiant intensity (radiance) of point x^0 in the direction of x^1, defined by angles (θ, ϕ); T represents temperature and the index 'in' concerns incident radiation.

Usually, to solve system (2), it is necessary to add boundary conditions determining the energy-heat transfer between boundary surfaces and ambient environment. For radiative heat transfer systems, these conditions are defined as Dirichlet conditions or Neumann conditions. In the first case, temperature distribution *t* functions on the examined boundary surface S_i should be determined [6,9]

$$\underset{\mathbf{r_0}\in S_i}{t(\vec{\mathbf{r}}_0)} = t(x_0,y_0,z_0) \quad (3)$$

And in the second, however, surface-specific power density *p* proportional to the derivative of the searched function *t*, penetrating from outside into the system through the boundary surface, is given [5,7]

$$\underset{\mathbf{r_0}\in S_i}{p(\vec{\mathbf{r}}_0)} = p(x_0,y_0,z_0) = -\lambda \left.\frac{\partial t}{\partial r}\right|_{S_i} \quad (4)$$

The system of (2) equations together with boundary conditions (3)-(4) is usually nonsolving by typical methods (number of integral equations is too big).

But when the equation (2) with boundary conditions (3)-(4), refer only to diffuse radiation, it is simplified the system of (2)-(4) equations and obtains a simple linear equations which describe transfer of radiant flux between *n* surfaces [1,8]:

$$\sum_{i=1}^{n}\left(\frac{\delta_{k,i}}{\varepsilon_i}-\varphi_{k,i}\frac{1-\varepsilon_i}{\varepsilon_i}\right)\cdot\frac{P_{out,i}}{S_i}=\sum_{i=1}^{n}\left(\delta_{k,i}+\varphi_{k,i}\right)\cdot\sigma T_i^4 \quad (5)$$

where: P_{zi} - radiant power at surface S_i; T_i – temperature of surface S_i; σ - Stefan's constant; ε_i - total emissivity of surface S_i and φij – form factor between surface S_i and S_j.
In this case where it is impossible, the most perspective seems to be a stochastic modelling, using (in heat radiative problems), a stochastic methods. A classic Monte-Carlo method of simulation is wildly used but it needs a big amount of computer time and large memory resources when tasks with complex geometry or non diffuse radiation are calculated. Back rays tracing method, which is widely used in computer graphics programs seems to be more effective also in radiative transfer simulations.

Example 1

As an example we consider the heating coil element (see fig.2)

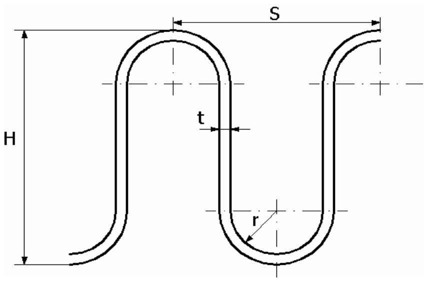

Fig.2 Heating coil element

The simulation of radiative heat transfer, basic on equation (2) and back trace method was done for several size of heating coil. The real size and diffuse radiation (own and multi reflected one) was taken into consideration. The uniform of irradiance distribution on a wall of furnace was tested. The partial result is presented on fig.3.

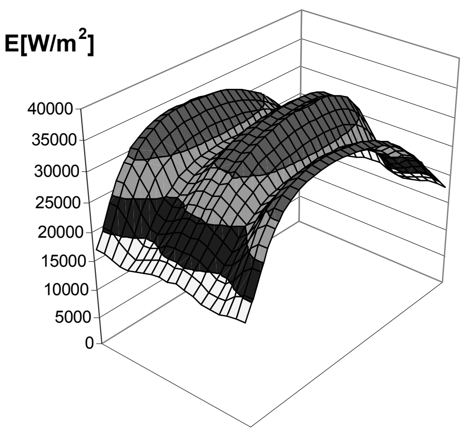

Fig.3. Irradiance distribution E [W/m^2] on a furnace wall for heat coil (H=60mm and r=8mm)

Two local extreme are indicated, and a zone of relative uniform irradiation can be determinate. It is useful for setting of working zone in resistance furnaces.

Remark2. Presented problems (without simplification) lead to a system of integral equations of the Fredholm type,

$$y(\mathrm{r}) = f(\mathrm{r}) + \int_{\mathrm{S}} k(\mathrm{r},\mathrm{r}')y(\mathrm{r}')dS', \tag{6}$$

where S is a certain surface or domain, and y is unknown function. It by the discretization method can be reduced to a system of algebraic equations [1,5,7].
Moreover, Fredholm integral equations are particular case of integral equations arising in the Fourier's theory [3,4,5].
Below the approximate methods for integral equations (6) of radiosity problems will be presented.

2.2. Discretization method

By a partition a domain S for N elements $\Delta S_1, \Delta S_2,$, ΔS_N we get

$$y(r) + \sum_{n=1}^{N} \int_{\Delta S_n} k(r,r')y(r')dS' = f(r). \tag{7}$$

Assuming, that a function $y(r)$ is constant in every element ΔS_n and it is equal y_n we obtain

$$y(r) + \sum_{n=1}^{N} y_n \int_{\Delta S_n} k(r,r')dS' = f(r). \tag{8}$$

This equation is satisfied in points $r_m \in \Delta S_m$, $m=1, 2, \ldots, N$. Introduce notation $f(r_m) = f_m$ the equation (**8**)leads to the following system of algebraic equations:

$$y_m + \sum_{n=1}^{N} y_n \int_{\Delta S_n} k(r,r')dS' = f_m, \quad m = 1, 2, \ldots, N. \tag{9}$$

Calculating $y_1, y_2, \ldots, y_N$ being values of the function $y(r)$ at elements $\Delta S_1, \Delta S_2, \ldots, \Delta S_N$, the solution of equation (**6**) is determined in domain S by (8).

2.3 Method of special kernels

If $k(r,r') = \sum_{i=1}^{m} a_i(r)b_i(r')$, where $\{a_i(r)\}$ and $\{b_i(r')\}$ are linear independent systems of functions on S, then equation (6) is of the form

$$y(\mathrm{r}) = f(\mathrm{r}) + \sum_{i=1}^{m} a_i(r)\int_{\mathrm{S}} b_i(r')y(\mathrm{r}')dS'.$$

Introducing the notation

$$\int_S b_i(r')y(r')dS' = C_i$$

we get the following solution of equation (6)

$$y(r) = f(r) + \sum_{i=1}^{m} C_i a_i(r).$$

Numbers C_i we get multiplied the above by $b_i(r)$ and integrating on S. Then we obtain

$$\int_S y(r)b_i(r)ds = \int_S f(r)b_i(r)ds + \sum_{j=1}^{m} C_j \int_S a_j(r)b_i(r)dS \cdot$$

By notations

$$\int_S f(r)b_i(r)ds = f_i,$$

and

$$\int_S a_j(r)b_i(r)dS = d_{ij}$$

we obtain a system of algebraic equations:

$$\sum_{j=1}^{m} [\delta_{ij} - d_{ij}]C_j = f_i \quad \text{for } i=1,2,...,m.$$

to calculate numbers C_i .

3. Integral equations in the heat conduction problems

We consider the integral equations of the mixed type

$$u(x,t) = f(x,t) + \int_0^t \int_M K(x,t,y,s)u(y,s)dyds \qquad (10)$$

which generalize Volterra and Fredholm integral equations. The presented equations play a very important role in electromagnetics and heat conduction problems. Some initial-boundary problems for some differential partial equations in physics are reducible to the above integral equation. Consider this equation in space-time, where f is a given function in the domain $D = M \times [0,T]$ (M is a compact subset of m-dimensional Euclidean space) and u is an unknown function in D. The kernel K is defined in the domain

$\Omega = \{(x,t,y,s) : x, y \in M, 0 \le s \le t \le T\}$.

In this paper we will present new numerical method and give computational results. It is based on iterative method with corrections defined by orthogonal polynomials of the Lagrange type.

3.1. Iterative-collocation method

For integral equation (10) with $M=[a, b]$ we propose new the following method

$$u_k(x,t) = f(x,t) + \int_0^t \int_a^b K(x,t,y,s) \cdot [u_{k-1}(y,s) + p_k^n(y,s)]dyds \qquad (11)$$

The corrections p_k^n can be defined by

$$\Delta u_k(x,t) = u_k(x,t) - u_{k-1}(x,t)$$

in points $x_p \in [a,b]$ and $t_q \in [0,T]$ for $p,q = 0,1,...,n$, being zeros of some orthogonal polynomials such that

$$p_k^n(x_p,t_q) = \Delta u_k(x_p,t_q), \qquad (12)$$

where

$$p_k^n(x,t) = \sum_{i=0}^{n}\sum_{j=0}^{n} c_{ijk} \cdot \varphi_i(x) \cdot \psi_j(t), \qquad (13)$$

and

$$\varphi_i(x_p) = \delta_{ip}, \qquad \psi_j(t_q) = \delta_{jq} \qquad (14)$$

Because the polynomials of the Lagrange type satisfy the above condition we can choose the fundamental Lagrange polynomials of the form

$$l_i(x) = \frac{(x-x_0)\cdot(x-x_1)...(x-x_{i-1})\cdot(x-x_{i+1})...(x-x_n)}{(x_i-x_0)\cdot(x_i-x_1)...(x_i-x_{i-1})\cdot(x_i-x_{i+1})...(x_i-x_n)}$$

$$i = 0,1,...,n. \qquad (15)$$

From here it follows

$$c_{pqk} = p_k^n(x_p,t_q) = \Delta u_k(x_p,t_q). \qquad (16)$$

Hence we get the following system of algebraic equations

$$c_{pqk} = g_{pqk} + \sum_{i=0}^{n}\sum_{j=0}^{n} c_{ijk} \cdot A_{ij}(p,q) \qquad (17)$$

where

$$A_{ij}(p,q) = \int_0^{t_q}\int_a^b K(x_p,t_q,y,s) \cdot \varphi_i(y) \cdot \psi_j(t)dyds \qquad (18)$$

$$g_{pqk} = \int_0^{t_q}\int_a^b K(x_p,t_q,y,s) \cdot [u_{k-1}(y,s) - u_{k-2}(y,s) - p_{k-1}^n]dyds \qquad (19)$$

Here:

$$u_0(x,t) = 0,$$

$$u_1(x,t) = f(x,t), \qquad (20)$$

$$p_1^n(x,t) = 0.$$

If

$$C_k = [c_{00k}, c_{01k}, ..., c_{0nk}, c_{10k}, c_{11k}, ..., c_{1nk}, ..., c_{n0k}, c_{n1k}, ..., c_{nnk}]^T$$

$$G_k = [g_{00k}, g_{01k}, ..., g_{0nk}, g_{10k}, g_{11k}, ..., g_{1nk}, ..., g_{n0k}, g_{n1k}, ..., g_{nnk}]^T$$

and $A = [A_{ik}(m,n)]$

for $i,k = 0,1,...,n; \quad m = 1,...,n.$ (21)

system (17) can be written in the form

$$C_k = G_k + A \cdot C_k. \tag{22}$$

In this iterative process the matrix A is constant and G_k is determined for every k.

Introduce notations:

$$V_k(y,s) = u(y,s) - u_{k-1}(y,s) - p_k^n(y,s),$$

$$K_n(x,t,y,s) = \sum_{i=0}^{n}\sum_{j=0}^{n} K(x_p, t_q, y, s)\cdot \varphi_i(x)\cdot \psi_j(t),$$

$$C = \sup_{(x,t)\in D} \int_0^t \int_a^b |K(x,t,y,s)|^2\, dyds, \tag{23}$$

$$q_n = \frac{\|K_n - K\|_{L^2_\Omega}\cdot\left(1+\|r\|_{L^2_\Omega}\right)}{1-\|K_n - K\|_{L^2_\Omega}\cdot\left(1+\|r\|_{L^2_\Omega}\right)} \xrightarrow{n\to\infty} 0,$$

where r is a resolving kernel defined as a series [3]

$$r(x,t,y,s) = \sum_{i=1}^{\infty} K^{(n)}(x,t,y,s) \tag{24}$$

with iterated kernels $K^{(n)}$ determined by formulas

$$K^{(n)}(x,t,y,s) = \int_s^t \int_a^b K(x,t,z,w)\cdot K^{(n-1)}(z,w,y,s)\,dzdw$$

for $n = 2,3,...$ (25)

where $K^{(1)}(x,t,y,s) = K(x,t,y,s).$ (26)

Let R be a space of the Riemann integrable functions on D with a norm

$$\|u\|_R = \sup_{(x,t)\in D}\{|u(x,t)|\} \tag{27}$$

and L_D^2 be a space with the norm

$$\|u\|_{L^2_D} = \left(\int_0^T \int_a^b |u(x,t)|^2\, dxdt\right)^{1/2} \tag{28}$$

Theorem

If $f \in R(D)$ and $k \in C(\Omega)$, then the sequence $\{u_k\}$ defined by formulas (11)-(12) tends for $k \to \infty$ to a unique solution $u \in R(D)$ of equation (11) and the following estimates

$$\|u_k - u\|_{R_D} \le C\cdot q_n^{k-1}\cdot \|V_1\|_{L^2_D} \tag{29}$$

and

$$\|u_k - u\|_{L^2_D} \le q_n^{k-1}\cdot \|K\|_{L^2_\Omega}\cdot \|V_1\|_{L^2_D} \tag{30}$$

hold.

Proof. By some transformations we get

$$p_k^n(x,t) = \int_0^t \int_a^b K_n(x,t,y,s)\cdot[u_{k-1}(y,s) + p_k^n(y,s) - p_{k-1}^n(y,s)]dyds$$

and

$$V_k(x,t) = g_k(x,t) + \int_0^t \int_a^b K(x,t,y,s)\cdot V_k(y,s)\,dyds, \tag{31}$$

where

$$g_k(x,t) = \int_0^t \int_a^b [K_n(x,t,y,s) - K(x,t,y,s)]\cdot[V_k(y,s) - V_{k-1}(y,s)]dyds. \tag{32}$$

It is clear that a solution of the mixed integral equation (31) can be presented in the form [2]

$$V_k(x,t) = g_k(x,t) + \int_0^t \int_a^b r(x,t,y,s)\cdot g_k(y,s)\,dyds. \tag{33}$$

From (31) we obtain

$$\|V_k\|_{L^2_D} \le \left(1+\|r\|_{L^2_D}\right)\|g_k\|_{L^2_D}. \tag{34}$$

Similarly, by (32) we get

$$\|g_k\|_{L^2_D} \le \|k_n - k\|_{L^2_\Omega}\|V_k - V_{k-1}\|_{L^2_D} \le\le \|K_n - K\|_{L^2_\Omega}\left(\|V_k\|_{L^2_D} + \|V_{k-1}\|_{L^2_D}\right). \tag{35}$$

Hence

$$\|g_k\|_{L^2_D} \le \left(\|V_k\|_{L^2_D} + \|V_{k-1}\|_{L^2_D}\right)\|K_n - K\|_{L^2_\Omega}\left(1+\|r\|_{L^2_\Omega}\right) \tag{36}$$

Using (36) in (34) we get

$$\|V_k\|_{L^2_D} \le q_n \|V_{k-1}\|_{L^2_D}. \tag{37}$$

By the induction we have

$$\|V_k\|_{L^2_D} \le q_n^{k-1}\|V_1\|_{L^2_D} \tag{38}$$

We can get

$$\|K_n - K\|_{L^2_\Omega} \xrightarrow{n\to\infty} 0. \tag{39}$$

Hence $q_n \xrightarrow{n\to\infty} 0$ and a convergence of studied method is proved. For an error estimates let us notice, that

$$\|u_k - u\|_{R_D} \le C\|V_k\|_{L_D^2} \tag{40}$$

and

$$\|u_k - u\|_{L_D^2} \le \|K\|_{L_\Omega^2} \|V_k\|_{L_D^2} . \tag{41}$$

3.2 Numerical experiments

In this method the corrections $p^n{}_k$ defined by (13) have coefficients calculated from algebraic system (17) . Other numerical method analyzed in paper [5] led to the system Volterra equations. Presented method is easier to calculate.

Below tables contain relative errors δ in points t_j and x_i:

$$\delta = \left| \frac{u_k(x_i, t_j) - u(x_i, t_j)}{u(x_i, t_j)} \right| . \tag{42}$$

Here: $n+1$ means a number of basis functions and k – a number of iterations. Fundamental Lagrange polynomials form basis functions.

Example 2

$$u(x,t) = \sin\left(\frac{t\pi}{2}\right)\left[e^x - \sin\left(\frac{t\pi}{2}\right)\right] + \int_0^t \int_{-1}^1 \frac{\pi}{2} e^{-y} \cos\left(\frac{s\pi}{2}\right) u(y,s)\,dyds$$

Table 1 Relative errors for *n=5, k=10*

t/x	-1	-0.4	-0.2	0.2	0.4	1
0.1	$3.2\cdot10^{-5}$	$2.1\cdot10^{-5}$	$1.4\cdot10^{-5}$	$9.5\cdot10^{-6}$	$6.4\cdot10^{-6}$	$4.3\cdot10^{-6}$
0.2	$2.6\cdot10^{-4}$	$1.7\cdot10^{-4}$	$1.1\cdot10^{-4}$	$7.7\cdot10^{-4}$	$5.2\cdot10^{-5}$	$3.5\cdot10^{-5}$
0.3	$3.9\cdot10^{-4}$	$2.6\cdot10^{-4}$	$1.8\cdot10^{-4}$	$1.2\cdot10^{-2}$	$7.9\cdot10^{-5}$	$5.3\cdot10^{-5}$
0.4	$4.0\cdot10^{-4}$	$2.7\cdot10^{-4}$	$1.8\cdot10^{-4}$	$1.2\cdot10^{-4}$	$8.1\cdot10^{-5}$	$5.5\cdot10^{-5}$
0.5	$3.2\cdot10^{-4}$	$2.2\cdot10^{-4}$	$1.5\cdot10^{-4}$	$9.8\cdot10^{-5}$	$6.5\cdot10^{-5}$	$4.4\cdot10^{-5}$
0.6	$2.1\cdot10^{-4}$	$1.4\cdot10^{-4}$	$9.5\cdot10^{-5}$	$6.3\cdot10^{-5}$	$4.3\cdot10^{-5}$	$2.9\cdot10^{-5}$
0.7	$1.1\cdot10^{-4}$	$7.2\cdot10^{-5}$	$4.8\cdot10^{-5}$	$3.2\cdot10^{-5}$	$2.2\cdot10^{-5}$	$1.5\cdot10^{-5}$
0.8	$3.6\cdot10^{-5}$	$2.4\cdot10^{-5}$	$1.6\cdot10^{-5}$	$1.7\cdot10^{-5}$	$7.2\cdot10^{-6}$	$4.9\cdot10^{-6}$
0.9	$3.4\cdot10^{-6}$	$2.6\cdot10^{-6}$	$1.5\cdot10^{-6}$	$1.0\cdot10^{-6}$	$6.9\cdot10^{-7}$	$4.6\cdot10^{-7}$
1	$1.5\cdot10^{-5}$	$1.0\cdot10^{-5}$	$7.0\cdot10^{-6}$	$4.7\cdot10^{-6}$	$3.1\cdot10^{-6}$	$2.1\cdot10^{-6}$

Example 3

$$u(x,t) = x^2 \cdot \left(e^{-t} - \frac{2}{3}t^3\right) + \int_0^t \int_{-1}^1 x^2 t^2 e^s u(y,s)\,dyds$$

Table 2. Relative errors for *n=5, k=7*

t/x	-1	-0,4	-0,2	0,2	0,4	1
0.1	$2.21\cdot10^{-10}$	$6.13\cdot10^{-10}$	$3.03\cdot10^{-10}$	$2.21\cdot10^{-10}$	$6.13\cdot10^{-10}$	$3.04\cdot10^{-10}$
0.2	$3.61\cdot10^{-9}$	$3.41\cdot10^{-9}$	$3.44\cdot10^{-9}$	$3.61\cdot10^{-9}$	$3.41\cdot10^{-9}$	$3.44\cdot10^{-9}$
0.3	$1.37\cdot10^{-8}$	$1.40\cdot10^{-8}$	$1.37\cdot10^{-8}$	$1.37\cdot10^{-8}$	$1.40\cdot10^{-8}$	$1.38\cdot10^{-8}$
0.4	$2.83\cdot10^{-8}$	$2{,}80\cdot10^{-8}$	$2.73\cdot10^{-8}$	$2.83\cdot10^{-8}$	$2.80\cdot10^{-8}$	$2.73\cdot10^{-8}$
0.5	$4.19\cdot10^{-8}$	$4.19\cdot10^{-8}$	$4.36\cdot10^{-8}$	$4.19\cdot10^{-8}$	$4.19\cdot10^{-8}$	$4.37\cdot10^{-8}$
0.6	$5.37\cdot10^{-8}$	$6.78\cdot10^{-8}$	$4.62\cdot10^{-8}$	$5.37\cdot10^{-8}$	$6.78\cdot10^{-8}$	$4.62\cdot10^{-8}$
0.7	$7.49\cdot10^{-8}$	$9.01\cdot10^{-8}$	$4.99\cdot10^{-8}$	$7.49\cdot10^{-8}$	$9.01\cdot10^{-8}$	$5.00\cdot10^{-8}$
0.8	$1.52\cdot10^{-8}$	$1.35\cdot10^{-8}$	$1.64\cdot10^{-8}$	$1.52\cdot10^{-8}$	$1.35\cdot10^{-8}$	$1.64\cdot10^{-8}$
0.9	$1.71\cdot10^{-7}$	$4.28\cdot10^{-7}$	$6.00\cdot10^{-7}$	$1.71\cdot10^{-8}$	$4.28\cdot10^{-7}$	$6.00\cdot10^{-7}$
1	$2.90\cdot10^{-6}$	$1.64\cdot10^{-6}$	$1.64\cdot10^{-6}$	$2.90\cdot10^{-6}$	$1.64\cdot10^{-6}$	$1.65\cdot10^{-6}$

4. Conclusion

In this paper we restrict to the following mathematical models in electrical engineering: radiative heat transfer and Fourier's problems. Presented models lead to a system of Fredholm integral equations and Volterra-Fredholm integral equations, respectively. We propose discretization method and projection-method) providing to a system algebraic equations.

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