On using Different Distance Measures for Fuzzy Numbers in Fuzzy Linear Regression Models

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Outline

1. Introduction
2. Preliminaries
3. Fuzzy Regression with Monte Carlo Method
4. Distance Measure for Fuzzy Numbers
5. Application
   - Application for Second Category
   - Application for Third Category
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6. Conclusion
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One of these error measures depends on the error measure defined by Kim and Bishu (1998). In this error measure, distance of two fuzzy numbers has to be calculated. Therefore, distance measure between two fuzzy numbers plays an important role in fuzzy regression with Monte Carlo method.
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Highlight the utility of distance measures
Calculate different distance measures in fuzzy linear regression with Monte Carlo method.
Estimate the parameters of fuzzy linear regression with Monte Carlo method according to the different distance measures.
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Definition 2.1. \( \mu_A(x) \) is the membership function of an element \( x \) belonging to a fuzzy set \( \tilde{A} \), where \( 0 \leq \mu_A(x) \leq 1 \).

Definition 2.2. A general fuzzy number \( \tilde{A} \) is a normal convex fuzzy set of \( \mathbb{R} \) with a piecewise continuous membership function. The left and right sides of fuzzy numbers are \( L(x) = a_2 - x a_2 - a_1 \) and \( R(x) = x - a_3 a_4 - a_3 \) respectively.

Definition 2.3. The \( \alpha \)-cut of a fuzzy number \( \tilde{A} \) is a non-fuzzy set defined as \( \tilde{A}(\alpha) = \{ x \in \mathbb{R}, \mu_A(\alpha) \geq \alpha \} \).

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then the first triangular fuzzy numbers is $\tilde{V}_{0k} = (x_{3k}/x_{1k}/x_{2k})$. 
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Choi and Buckley (2008) classified fuzzy regression models in three categories:

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- Input and output data are both fuzzy (Third Category)
### Fuzzy linear regression model (Second Category)

\[
\tilde{Y}_l = \tilde{A}_0 + \tilde{A}_1 x_{1l} + \tilde{A}_2 x_{2l} + \ldots + \tilde{A}_m x_{ml} \quad l = 1, 2, \ldots, n
\]  

(1)

### Fuzzy linear regression model (Third Category)

\[
\tilde{Y}_l = a_0 + a_1 \tilde{X}_{1l} + a_2 \tilde{X}_{2l} + \ldots + a_m \tilde{X}_{ml} \quad l = 1, 2, \ldots, n
\]  

(2)
Predicted values
Fuzzy linear regression model (Second Category)

\[ \tilde{Y}_{lk}^* = \tilde{V}_0k + \tilde{V}_{1k}x_{1l} + \tilde{V}_{2k}x_{2l} + \ldots + \tilde{V}_{mk}x_{ml} \quad l = 1, 2, \ldots, n \]
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(3)

Fuzzy linear regression model (Third Category)

\[ \tilde{Y}_{lk}^* = v_{0k} + v_{1k} \tilde{x}_{1l} + v_{2k} \tilde{x}_{2l} + \ldots + v_{mk} \tilde{x}_{ml}; \quad l = 1, 2, \ldots, n \]  

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\[ D = \int |\mu_{\tilde{Y}}(x) - \mu_{\tilde{Y}_k^*}(x)| \, dx \]

\[ E = \frac{\int_{S_{\tilde{Y} \cup \tilde{Y}_k^*}} |\mu_{\tilde{Y}}(x) - \mu_{\tilde{Y}_k^*}(x)| \, dx}{\int_{S_{\tilde{Y}}} \mu_{\tilde{Y}}(x) \, dx} \]
Error Measure (Abdalla & Buckley (2007))

\[ E_1 = \sum_{l=1}^{n} \left[ \int_{-\infty}^{\infty} |\tilde{Y}_l(x) - \tilde{Y}_\ast_{lk}(x)| \, dx \right] \]

\[ \tilde{Y}_l = \left( y_{l1}/y_{l2}/y_{l3} \right) \text{ and } \tilde{Y}_\ast_{lk} = \left( y_{lk1}/y_{lk2}/y_{lk3} \right) \]

\[ \tilde{V}_k \in \{ \tilde{V}_1, ..., \tilde{V}_N \} \text{ and } v_k \in \{ v_1, ..., v_N \} \]
Error Measure (Abdalla & Buckley (2007))

\[
E_1 = \frac{\sum_{i=1}^{n} \left[ \int_{-\infty}^{\infty} |\tilde{Y}_i(x) - \tilde{Y}_{ik}^*(x)| \, dx \right]}{\left[ \int_{-\infty}^{\infty} \tilde{Y}_i(x) \, dx \right]}
\] (5)
Error Measure (Abdalla & Buckley (2007))

\[ E_1 = \sum_{i=1}^{n} \frac{\int_{-\infty}^{\infty} \left| \tilde{Y}_i(x) - \tilde{Y}_{ik}^*(x) \right| dx}{\int_{-\infty}^{\infty} \tilde{Y}_i(x) dx} \] (5)

- \( \tilde{Y}_i = (y_{i1}/y_{i2}/y_{i3}) \) and \( \tilde{Y}_{ik}^* = (y_{ik1}/y_{ik2}/y_{ik3}) \)
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E_1 = \frac{\sum_{l=1}^{n} \left[ \int_{-\infty}^{\infty} |\tilde{Y}_l(x) - \tilde{Y}_{lk}^{*}(x)| dx \right]}{\int_{-\infty}^{\infty} \tilde{Y}_l(x) dx} \tag{5}
\]

- \(\tilde{Y}_l = (y_{l1}/y_{l2}/y_{y3})\) and \(\tilde{Y}_{lk}^{*} = (y_{lk1}/y_{lk2}/y_{lk3})\)

\(\tilde{V}_k \in \{\tilde{V}_1, ..., \tilde{V}_N\}\) and \(v_k \in \{v_1, ..., v_N\}\)
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**Kaufmann (1991)**

\[
d(\tilde{A}, \tilde{B}) = \int_0^1 (|A^L(\alpha) - B^L(\alpha)| + |A^U(\alpha) - B^U(\alpha)|) \, d\alpha
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\[ d(\tilde{A}, \tilde{B}) = \int_0^1 (|A^L(\alpha) - B^L(\alpha)| + |A^U(\alpha) - B^U(\alpha)|) \, d\alpha \]

- \([A^L(\alpha), A^U(\alpha)]\) and \([B^L(\alpha), B^U(\alpha)]\) are the closed intervals of \(\alpha\)-cuts
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- \[ E^*(\widetilde{A}) = a_2 - (a_2 - a_1) \int_0^\infty L(x)dx \]
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\[ EV(\tilde{A}) = \frac{1}{2} \left[ E^*(\tilde{A}) - E^*(\tilde{A}) \right] \]
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\[ EV(\tilde{A}) = \frac{1}{2} \left[ E^*(\tilde{A}) - E^*(\tilde{A}) \right] \]

\[ \sigma(\tilde{A}, \tilde{B}) = |EV(\tilde{A}) - EV(\tilde{B})| \]
Heilpern-2 (1997)

\[ d_p(\tilde{A}, \tilde{B}) = \int_0^1 dp(\tilde{A}(\alpha), \tilde{B}(\alpha)) d\alpha \]

\[ \tilde{A}(\alpha) = [A_L(\alpha), A_U(\alpha)] \quad \text{and} \quad \tilde{B}(\alpha) = [B_L(\alpha), B_U(\alpha)] \]

\[ dp(\tilde{A}(\alpha), \tilde{B}(\alpha)) = \begin{cases} \frac{1}{p}, & 1 \leq p \leq \infty; \\ \max |A_L(\alpha) - B_L(\alpha)|, |A_U(\alpha) - B_U(\alpha)|, & p = \infty. \end{cases} \]

\[ (8) \]
Heilpern-2 (1997)

\[ d_p(\tilde{A}, \tilde{B}) = \int_0^1 d_p(\tilde{A}(\alpha), \tilde{B}(\alpha) d\alpha) \] (7)
Heilpern-2 (1997)

\[ d_p(\tilde{A}, \tilde{B}) = \int_0^1 d_p(\tilde{A}(\alpha), \tilde{B}(\alpha)) d\alpha \] (7)

\[ \tilde{A}(\alpha) = [A^L(\alpha), A^U(\alpha)] \text{ and } \tilde{B}(\alpha) = [B^L(\alpha), B^U(\alpha)] \]
Heilpern-2 (1997)

\[
d_p(\tilde{A}, \tilde{B}) = \int_0^1 d_p(\tilde{A}(\alpha), \tilde{B}(\alpha)) d\alpha
\]

\[
\tilde{A}(\alpha) = [A^L(\alpha), A^U(\alpha)] \text{ and } \tilde{B}(\alpha) = [B^L(\alpha), B^U(\alpha)]
\]

\[
d_p\left(\tilde{A}(\alpha), \tilde{B}(\alpha)\right) =
\begin{cases}
(0.5)(|A^L(\alpha) - B^L(\alpha)|^p + |A^U(\alpha) - B^U(\alpha)|^p)^{1/p}, & 1 \leq p \leq \infty; \\
\max|A^L(\alpha) - B^L(\alpha)|, |A^U(\alpha) - B^U(\alpha)|, & p = \infty.
\end{cases}
\]
Heilpern-3 (1997)

\[ \tilde{A} = (a_1, a_2, a_3, a_4) \]
\[ \tilde{B} = (b_1, b_2, b_3, b_4) \]
Heilpern-3 (1997)

\[ \tilde{A} = (a_1, a_2, a_3, a_4) \]
\[ \tilde{B} = (b_1, b_2, b_3, b_4) \]

\[
\delta_p(\tilde{A}, \tilde{B}) = \begin{cases} 
0.25 \left( \sum_{i=1}^{4} |a_i - b_i|^p \right)^{1/p}, & 1 \leq p < \infty; \\
\max(|a_i - b_i|), & p = \infty.
\end{cases}
\]
Chen & Hsieh (1998)

\[ P(A) = \frac{\int_0^w \alpha \left( \frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_0^w \alpha d\alpha} \]
Chen & Hsieh (1998)

\[
P(A) = \frac{\int_0^w \alpha \left( \frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_0^w \alpha d\alpha}
\]

\[\tilde{A} = (a_1, a_2, a_3, a_4)\]
Chen & Hsieh (1998)

\[ P(A) = \frac{\int_0^w \alpha \left( \frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_0^w \alpha d\alpha} \]

\[ \tilde{A} = (a_1, a_2, a_3, a_4) \]

\[ P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} \]
Chen & Hsieh (1998)

\[
P(A) = \frac{\int_0^w \alpha \left( \frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_0^w \alpha d\alpha}
\]

\[
\tilde{A} = (a_1, a_2, a_3, a_4)
\]

\[
P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}
\]

\[
P(A) = \frac{a_1 + 4a_2 + a_4}{6}
\]
Chen & Hsieh (1998)

\[ P(A) = \frac{\int_0^w \alpha \left( \frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_0^w \alpha d\alpha} \]

\[ \tilde{A} = (a_1, a_2, a_3, a_4) \]

\[ P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} \]

\[ P(A) = \frac{a_1 + 4a_2 + a_4}{6} \]
Chen & Hsieh (1998)

\[
P(A) = \frac{\int_0^w \alpha \left( \frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_0^w \alpha d\alpha}
\]

\[\tilde{A} = (a_1, a_2, a_3, a_4)\]

\[
P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}
\]

\[
P(A) = \frac{a_1 + 4a_2 + a_4}{6}
\]

\[
|P(A) - P(B)|
\] (10)
Outline

1. Introduction
2. Preliminaries
3. Fuzzy Regression with Monte Carlo Method
4. Distance Measure for Fuzzy Numbers
5. Application
   - Application for Second Category
   - Application for Third Category
   - Solutions
6. Conclusion
In this section, there are two different applications.
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We consider different distance measures for fuzzy numbers given in Section 4 in the error measure \( E_1 \) for fuzzy linear regression models with Monte Carlo approach.
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First application is for the second fuzzy regression model category and the other one is for the third fuzzy regression model category.

We consider different distance measures for fuzzy numbers given in Section 4 in the error measure ($E_1$) for fuzzy linear regression models with Monte Carlo approach.
Table: Data for the application (Second category)

<table>
<thead>
<tr>
<th>Fuzzy Output</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.27/5.83/9.39)</td>
<td>2.00</td>
<td>0.00</td>
<td>15.25</td>
</tr>
<tr>
<td>(0.33/0.85/1.37)</td>
<td>0.00</td>
<td>5.00</td>
<td>14.13</td>
</tr>
<tr>
<td>(5.43/13.93/22.43)</td>
<td>1.13</td>
<td>1.50</td>
<td>14.13</td>
</tr>
<tr>
<td>(1.56/4.00/6.44)</td>
<td>2.00</td>
<td>1.25</td>
<td>13.63</td>
</tr>
<tr>
<td>(0.64/1.65/2.66)</td>
<td>2.19</td>
<td>3.75</td>
<td>14.75</td>
</tr>
<tr>
<td>(0.62/1.58/2.54)</td>
<td>0.25</td>
<td>3.50</td>
<td>13.75</td>
</tr>
<tr>
<td>(3.19/8.18/13.17)</td>
<td>0.75</td>
<td>5.25</td>
<td>15.25</td>
</tr>
<tr>
<td>(0.72/1.85/2.98)</td>
<td>4.25</td>
<td>2.00</td>
<td>13.50</td>
</tr>
</tbody>
</table>
Before the application we have to decide the intervals for $I_i, i = 0, 1, 2, 3$ to obtain the model coefficients as explained in Definition 2.5.
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We use same intervals in order to compare the results we have with the results from Abdalla and Buckley (2007) in the literature.
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We use same intervals in order to compare the results we have with the results from Abdalla and Buckley (2007) in the literature.

Four separate intervals ($MCI, MCII, MCIII, MCIV$) that they studied are given with Table 2.
Before the application we have to decide the intervals for $I_i$, $i = 0, 1, 2, 3$ to obtain the model coefficients as explained in Definition 2.5.

We use same intervals in order to compare the results we have with the results from Abdalla and Buckley (2007) in the literature.

Four separate intervals ($MCI$, $MCII$, $MCIII$, $MCIV$) that they studied are given with Table 2.
Table: Intervals for $l_i$, $i = 0, 1, 2, 3$ for second category

<table>
<thead>
<tr>
<th>Interval</th>
<th>$MC_I$</th>
<th>$MC_{II}$</th>
<th>$MC_{III}$</th>
<th>$MC_{IV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_0$</td>
<td>[-1,0]</td>
<td>[0,1]</td>
<td>[-18.174,-18.174]</td>
<td>[28.000,47.916]</td>
</tr>
<tr>
<td>$l_1$</td>
<td>[-1,0]</td>
<td>[-1,0]</td>
<td>[-1.083,-1.083]</td>
<td>[-2.542,-2.542]</td>
</tr>
<tr>
<td>$l_2$</td>
<td>[-1.5,-0.5]</td>
<td>[-1.5,-0.5]</td>
<td>[-1.150,-1.150]</td>
<td>[-2.323,-2.323]</td>
</tr>
<tr>
<td>$l_3$</td>
<td>[0,1]</td>
<td>[0,1]</td>
<td>[1.733,2.149]</td>
<td>[-1.354,-1.354]</td>
</tr>
</tbody>
</table>
Results for using different definitions of distance measures in fuzzy linear regression with MC method for minimizing $E_1$

<table>
<thead>
<tr>
<th>Definitions</th>
<th>Parameters</th>
<th>MCI</th>
<th>MCII</th>
<th>MCIII</th>
<th>MCIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaufmann (1991)</td>
<td>$A_0$</td>
<td>-0.8530</td>
<td>-0.5900</td>
<td>-0.2935</td>
<td>-18.1740</td>
</tr>
<tr>
<td></td>
<td>$A_1$</td>
<td>-0.6934</td>
<td>-0.6033</td>
<td>-0.3096</td>
<td>-0.2712</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>-1.4064</td>
<td>-1.3966</td>
<td>-1.3162</td>
<td>-0.8220</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>0.5474</td>
<td>0.5727</td>
<td>0.5923</td>
<td>0.2591</td>
</tr>
<tr>
<td>Heilpern-1 (1997)</td>
<td>$A_0$</td>
<td>-0.8472</td>
<td>-0.7690</td>
<td>-0.1782</td>
<td>0.0653</td>
</tr>
<tr>
<td></td>
<td>$A_1$</td>
<td>-0.8527</td>
<td>-0.3696</td>
<td>-0.0810</td>
<td>-0.8627</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>-1.4198</td>
<td>-1.1616</td>
<td>-0.5778</td>
<td>-1.4075</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>0.0251</td>
<td>0.6431</td>
<td>0.7575</td>
<td>0.1453</td>
</tr>
<tr>
<td>Heilpern-2 (1997)</td>
<td>$A_0$</td>
<td>-0.8530</td>
<td>-0.5900</td>
<td>-0.2935</td>
<td>0.0607</td>
</tr>
<tr>
<td></td>
<td>$A_1$</td>
<td>-0.6934</td>
<td>-0.6033</td>
<td>-0.3096</td>
<td>-0.2712</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>-1.4064</td>
<td>-1.3966</td>
<td>-1.3162</td>
<td>-0.8220</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>0.5474</td>
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<tr>
<td>Heilpern-3 (1997)</td>
<td>$A_0$</td>
<td>-0.8530</td>
<td>-0.5900</td>
<td>-0.2935</td>
<td>0.0607</td>
</tr>
<tr>
<td></td>
<td>$A_1$</td>
<td>-0.6934</td>
<td>-0.6033</td>
<td>-0.3096</td>
<td>-0.2712</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>-1.4064</td>
<td>-1.3966</td>
<td>-1.3162</td>
<td>-0.8220</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>0.5474</td>
<td>0.5727</td>
<td>0.5923</td>
<td>0.2591</td>
</tr>
<tr>
<td>Chen and Hsieh (1998)</td>
<td>$A_0$</td>
<td>-0.7617</td>
<td>-0.7454</td>
<td>-0.5821</td>
<td>0.0716</td>
</tr>
<tr>
<td></td>
<td>$A_1$</td>
<td>-0.6857</td>
<td>-0.4093</td>
<td>-0.3824</td>
<td>-0.9107</td>
</tr>
<tr>
<td></td>
<td>$A_2$</td>
<td>-1.3294</td>
<td>-1.1576</td>
<td>-0.5469</td>
<td>-1.3458</td>
</tr>
<tr>
<td></td>
<td>$A_3$</td>
<td>0.2521</td>
<td>0.4794</td>
<td>0.8036</td>
<td>0.2596</td>
</tr>
</tbody>
</table>
### Table: Data for the application (Third category)

<table>
<thead>
<tr>
<th>Fuzzy output</th>
<th>$X_{1/}$</th>
<th>$X_{2/}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(55.4/61.6/64.7)</td>
<td>(5.7/6.0/6.9)</td>
<td>(5.4/6.3/7.1)</td>
</tr>
<tr>
<td>(50.5/53.2/58.5)</td>
<td>(4.0/4.4/5.1)</td>
<td>(4.7/5.5/5.8)</td>
</tr>
<tr>
<td>(55.7/65.5/75.3)</td>
<td>(8.6/9.1/9.8)</td>
<td>(3.4/3.6/4.0)</td>
</tr>
<tr>
<td>(61.7/64.9/74.7)</td>
<td>(6.9/8.1/9.3)</td>
<td>(5.0/5.8/6.7)</td>
</tr>
<tr>
<td>(69.1/71.7/80.0)</td>
<td>(8.7/9.4/11.2)</td>
<td>(6.5/6.8/7.1)</td>
</tr>
<tr>
<td>(49.6/52.2/57.4)</td>
<td>(4.6/4.8/5.5)</td>
<td>(6.7/7.9/8.7)</td>
</tr>
<tr>
<td>(47.7/50.2/55.2)</td>
<td>(7.2/7.6/8.7)</td>
<td>(4.0/4.2/4.8)</td>
</tr>
<tr>
<td>(41.8/44.0/48.4)</td>
<td>(4.2/4.4/4.8)</td>
<td>(5.4/6.0/6.3)</td>
</tr>
<tr>
<td>(45.7/53.8/61.9)</td>
<td>(8.2/9.1/10.0)</td>
<td>(2.7/2.8/3.2)</td>
</tr>
<tr>
<td>(45.4/53.5/58.9)</td>
<td>(6.0/6.7/7.4)</td>
<td>(5.7/6.7/7.7)</td>
</tr>
</tbody>
</table>
Before the application we have to decide the intervals for $I_i, i = 0, 1, 2$ to obtain the model coefficients as explained in Definition 2.4.
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We use same intervals in order to compare the results we have with the results from Abdalla and Buckley (2008) in the literature.
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We use same intervals in order to compare the results we have with the results from Abdalla and Buckley (2008) in the literature.

Four separate intervals ($MCI, MCII, MCIII, MCIV$) that they studied are given with Table 5.
Before the application we have to decide the intervals for $I_i, i = 0, 1, 2$ to obtain the model coefficients as explained in Definition 2.4.

We use same intervals in order to compare the results we have with the results from Abdalla and Buckley (2008) in the literature.

Four separate intervals ($MC_{I}, MC_{II}, MC_{III}, MC_{IV}$) that they studied are given with Table 5.
Table: Intervals for $I_i$, $i = 0, 1, 2$ for third category

<table>
<thead>
<tr>
<th>Interval</th>
<th>$MCI$</th>
<th>$MCII$</th>
<th>$MCIII$</th>
<th>$MCIV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_0$</td>
<td>[0,5]</td>
<td>[0,37]</td>
<td>[16.528,16.528]</td>
<td>[33.808,36.601]</td>
</tr>
<tr>
<td>$I_1$</td>
<td>[0,6]</td>
<td>[0,6]</td>
<td>[3.558,3.982]</td>
<td>[1.294,3.756]</td>
</tr>
<tr>
<td>$I_2$</td>
<td>[0,4]</td>
<td>[0,6]</td>
<td>[2.575,2.575]</td>
<td>[0.423,0.473]</td>
</tr>
</tbody>
</table>
Table: Results for using different definitions of distance measures in fuzzy linear regression with MC method for minimizing $E_1$.

<table>
<thead>
<tr>
<th>Distance Measures</th>
<th>Parameters</th>
<th>$MCI$</th>
<th>$MCII$</th>
<th>$MCIII$</th>
<th>$MCIV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaufmann (1991)</td>
<td>$a_0$</td>
<td>1.9138</td>
<td>1.8114</td>
<td>16.5280</td>
<td>33.8108</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>4.7655</td>
<td>4.7820</td>
<td>3.5733</td>
<td>3.1333</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>3.6687</td>
<td>3.6775</td>
<td>2.5750</td>
<td>0.4730</td>
</tr>
<tr>
<td>Heilpern-1 (1997)</td>
<td>$a_0$</td>
<td>2.4841</td>
<td>0.3650</td>
<td>16.5280</td>
<td>33.8106</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>4.9058</td>
<td>4.8024</td>
<td>3.5580</td>
<td>2.7181</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>3.4424</td>
<td>3.9099</td>
<td>2.5750</td>
<td>0.7430</td>
</tr>
<tr>
<td>Heilpern-2 (1997)</td>
<td>$a_0$</td>
<td>1.9138</td>
<td>1.8114</td>
<td>16.5280</td>
<td>33.8108</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>4.7655</td>
<td>4.7820</td>
<td>3.5733</td>
<td>3.1333</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>3.6687</td>
<td>3.6775</td>
<td>2.5750</td>
<td>0.4730</td>
</tr>
<tr>
<td>Heilpern-3 (1997)</td>
<td>$a_0$</td>
<td>4.8121</td>
<td>5.3534</td>
<td>16.5280</td>
<td>33.8111</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>4.5835</td>
<td>4.5590</td>
<td>3.5580</td>
<td>3.0608</td>
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<tr>
<td></td>
<td>$a_2$</td>
<td>3.4776</td>
<td>3.3425</td>
<td>2.5750</td>
<td>0.4730</td>
</tr>
<tr>
<td>Chen and Hsieh (1998)</td>
<td>$a_0$</td>
<td>2.1047</td>
<td>0.5538</td>
<td>16.5280</td>
<td>33.8086</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>5.0605</td>
<td>5.0276</td>
<td>3.5580</td>
<td>3.0994</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>3.3305</td>
<td>3.6148</td>
<td>2.5750</td>
<td>0.4730</td>
</tr>
</tbody>
</table>
**Table:** Error measures for application (second category)

<table>
<thead>
<tr>
<th></th>
<th>MCI</th>
<th>MCII</th>
<th>MCIII</th>
<th>MCIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abdalla and Buckley (2008)</td>
<td>6.169</td>
<td>5.812</td>
<td>7.125</td>
<td>8.201</td>
</tr>
<tr>
<td>Kaufmann (1991)</td>
<td>32.63132</td>
<td>31.0182</td>
<td>24.1279</td>
<td>110.6466</td>
</tr>
<tr>
<td>Heilpern-3 (1997)</td>
<td>16.3649</td>
<td>15.104</td>
<td>9.2622</td>
<td>40.2581</td>
</tr>
</tbody>
</table>
Table: Error measures for application (third category)

<table>
<thead>
<tr>
<th>E_1</th>
<th>MCI</th>
<th>MCI\text{II}</th>
<th>MCI\text{III}</th>
<th>MCI\text{IV}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaufmann (1991)</td>
<td>52.7943</td>
<td>83.9582</td>
<td>19.0558</td>
<td>24.3161</td>
</tr>
</tbody>
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Outline

1. Introduction
2. Preliminaries
3. Fuzzy Regression with Monte Carlo Method
4. Distance Measure for Fuzzy Numbers
5. Application
   - Application for Second Category
   - Application for Third Category
   - Solutions
6. Conclusion
Why we did this study!!!

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Reason

- Only one definition of distance measure has been used in fuzzy regression with Monte Carlo method until now.
- Hence, we investigate using different definitions of distance measure between fuzzy numbers in estimating the parameters of fuzzy regression with Monte Carlo method.
Future Works !!!
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- Making a simulation above the intervals according to the distance measures. For deciding which distance measure is the best for estimating the parameters.
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