Joint Optimization of Asymmetry and Diversity of Bit-Interleaved Space-Time code

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Abstract—The joint optimization scheme that combined asymmetric modulation with modulation diversity in bit-interleaved space-time coded modulation system (BI-STCM) was proposed. The upper pairwise error possibility was analyzed in BI-STCM under Rayleigh fading channels. The method of achieving modulation diversity and the optimal rotated angle of the constellation to achieve the maximum diversity gain were analyzed. The minimum Euclidean distance between symbols with hamming distance one can be increased in asymmetric constellation for several ordinary symbol mapping schemes. So the coding and diversity gain can be obtained and the upper bound of pairwise error possibility is decreased. Theoretical analysis and simulation results show that the proposed scheme can greatly improve the performance of BI-STCM under Rayleigh fading channels.

Index Terms—Bit Interleaved Coded Modulation; Asymmetric Modulation; Modulation Diversity

I. INTRODUCTION

In the last decades, most of works on bit interleaved space-time coded modulation have been proposed to show its high capacity [1-5]. Different kinds of space-time codes have been investigated including space-time trellis codes (STTC), STBC, and layered STBC for power efficient communications over MIMO channels. However, these coding schemes were still relatively far away from the promised high power efficiency by referring to their capacity limits. Space-time coded modulation have an excellent performance due to it jointly considered transmit diversity, coding and modulation. In recent years, many scholars proposed bit interleaved space time coded modulation (BI-STCM) that combined bit interleaved coded modulation with space time transmit diversity. The diversity gain and coding gain of the communication system and anti-fading performance were improved greatly. Reference [1] investigated the performance bounds. Reference [2] investigated the capacity of the system. Reference [3] analyzes the theoretical bit error threshold and takes conclusion of adopting set partition (SP) mapping rather than Gray mapping in iterative decoding. Reference [4] and reference [5] analyzed the performance bounds of bit interleaved space-time coded modulation and investigated the optimal constellation mapping.

In wireless communication systems, the transmission quality seriously impacted by the channel fading. The performance of coded system is determined by the diversity order [11]. The key issue to realizing modulation diversity is rotating the constellation with a certain angle, which makes maximum difference at in phase and quadrature components (I, Q) between any two modulation signal points. So the I, Q components were respectively influenced by independent fading. The dependency of them is decreased under fading channels. Reference [6] investigated the influence of modulation diversity on bit interleaved space time coded modulation.

Motivated by constellation rotation and modulation diversity [6, 9, 10], we investigate the joint optimization of combined asymmetric modulation with modulation diversity so as to improve the performance of BI-STCM under Rayleigh fading channels. On the basis of analyzing the parameters of pairing bit error probability and the influence on Euclidean distance of several common mapping schemes, we analyze the asymmetric modulation improved the distance performance and as a result, it improved the both coding gain and diversity gain of the system and lower the upper bound of pairing error probability.

The rest of this paper is organized as follows: in the next section, we give the system model of the proposed scheme. The method of achieving modulation diversity is introduced in Section 3. Constellation rotation is introduced in Section 4. Simulation results and analysis are shown in Section 5. Finally, conclusions and our future work are presented in Section 6.

II. SYSTEM MODEL

The asymmetric modulation system of bit-interleaved space-time coded with $N_t$ transmit antenna and $N_r$ receive antenna, as shown in figure 1. In the transmitter, the information bit stream $b_i$ first enter into the convolutional encoder, generating bit stream $d_i$ by interleave. Every $m$ bits of the group were mapped to the high-order phase modulation asymmetric constellation formed by rotation, then interleaved by I/Q components and transmitted by space-time modulation. Spectral efficiency of the system is $R = R_c N_c \log_2 M$ bits/s/Hz, of which $R_c$ is the bit rate of the convolutional encoder, $M$ is the modulation order, $m = \log_2 M$. By reasonable selection of the convolutional encoder rate, the number of the transmit antennas and modulation order, the spectral
efficiency of the system can conveniently be adjusted. In the k symbol period, \(d_k = [d_k^1,...,d_k^{N_t}]\) were mapped to a communication channel on the \(N_t\) antenna. So the transmitting vector of the k symbol period can also be expressed as \(x_k = [x_k^1,...,x_k^{N_t}]^\top = \mu(d_k)\).

The method of iterative decoding was used in the receiver. The received signal from the MIMO channel \(y_k = H_k x_k + n_k\) was firstly demapping, in which \(H_k\) is the channel fading coefficients matrix, \(n\) is the plural adding Gauss white noise vector. Soft information value was calculated using the soft input and soft output posterior probability of MIMO [7]. The posteriori log-likelihood ratio was acquired for each bit on transmit antenna. Then it was de-interleaved and feed into convolutional decoder. The decoder calculates the bit log-likelihood ratio \(L(b)\) according to the maximum posteriori probability (MAP) algorithm. At the same time, the newly log-likelihood ratio pass through the bit interleaver, feedback to the demapping module of the MIMO system and as the external information of it to conduct the next iteration. In the final round of iteration, the decoding sequence information can be obtained by the hard decision of \(L(b)\).

As the interleaved signal was directly mapped onto the rotated constellation points, the asymmetric modulation method does not increase the complexity of the system.

III. MODULATION DIVERSITY

In the two-dimensional constellation, any QAM (Quadrature Amplitude Modulation) or PSK (Phase Shift Keying) signal can be decomposed into I and Q branch signal vector. I/Q components of the same signal have the same fading coefficient as it transmitted through the same channel at the same time. However, if the tributary signals of I/Q are interleaved (as shown in Figure 2), and interleaving depth more than the coherence time of the channel, so that the I/Q components are transmitted independently in the non-coherent time of the channel. They would have irrelevant fading coefficients in the channel. Thus the correlation of the I/Q components can be broken. The diversity order of the system can be doubled. It can be seen that this diversity will not have an impact on the system’s power and spectrum efficiency except for the increase of a certain complexity. However, in order to break this correlation for the slow fading channel, the interleaving depth may be larger, which result in a larger delay as well as a large amount of memory. A method for reducing the complexity is replacing interleave with delay of one of the tributary signals (as shown in Figure 3). As delay exceeds the average duration of fading, the diversity order has been improved.

For a set of transmitted modulation symbol sequence, assume its length is \(N_r \times N_t\) (of which \(N_t\) is integer). Because in each transmission symbol period, there are \(N_t\) symbols are transmitted simultaneously, the transmitted sequence can be expressed as matrix \(X = [x_1,...,x_{N_t}]\) with \(N_r \times N_t\). If \(X\) is judged as \(\hat{X}\) at the receiver, the pairwise error probability expressed as \(P(\hat{X} \rightarrow X)\), the average upper bound of bit error rate and frame error rate of the system can be obtained from the weighted pairwise error probability \(P(X \rightarrow \hat{X})\). The pairwise error probability depends on not only the fading of channel, but also the corresponding decoding strategy. If \(c\) and \(\hat{c}\) expressed as bit sequence that corresponding to \(X\) and \(\hat{X}\), in that way \(P(c \rightarrow \hat{c}) = P(X \rightarrow \hat{X})\). For the \(N_r \times N_t\) modulation symbol, if the convolutional encoder (assumed rate of input information bits is \(R_c = 1/n\)) corresponding to the length of \(N_k\), then the length of the convolutional encoder output is \(N_c = nN_c = mN_rN_t\).

In the first round of demapping iteration in MIMO system, no decoding bit prior information. The initial value is set to zero. At each iteration course after that, the output of MAP decoder served as a priori information and feed back to the MIMO solutions to improve the quality of metric. Thus each transmitted bit was separated. The external information of the given bits has to do with prior probability of other transmitted bits simultaneously. If the feedback information from the decoder completely accurate, that is, all other bits are completely known, the demapping becomes a calculation of log-likelihood between two symbols in the t antenna that the code distance is one in the same subnet. If the \(d_k^{N_t}\) is the code bit in the k symbol period and mapped to the t
transmission antenna on the modulation symbol \( N \) bit position, of which \( N = 1, \ldots, m \), \( i = 1, \ldots, N_1 \). The upper bound of pairwise error conditional probability is [8]:

\[
P(c \rightarrow \hat{c} | H_k) \leq e^{-\frac{E_s}{N_0} \sum_{i=1}^{N} \sum_{j=1}^{m} (H_{c_i}^N - \mu d_{c_i}^N)^2}
\] (1)

If \( d_{H}^{c,N} \) represents the Hamming distance between real transmission bits sequence \( d_{c}^N \) and the bits of error events sequence \( d_{\hat{c}}^N \), which \( d_{H}^{c,N} = |\mu(d_{c_i}^N) - \mu(d_{\hat{c}_i}^N)| \) is the squared Euclidean distance corresponding to the two series, it obviously meets the next equation:

\[
d_H^{c,N} \geq d_E^{c,N} d_{e_{\min}}
\] (2)

Of which, \( d_{E_{\min}} \) is the minimum square Euclidean distance between all symbols that hamming distance is one.

In \( c \) and \( \hat{c} \), the Hamming distances between two sequences can be expressed as \( d_H^{c,N} = \sum_{k=1}^{N} d_{H}^{c,k,N} \). If \( d_H^{c,N} \) represents the total Hamming distances between sequences \( c \) and \( \hat{c} \), from formula (1) we can obtained:

\[
P(X \rightarrow \hat{X}) \leq (\frac{E_s}{4N_0})^{-N_1} d_H^{c,N} \prod_{i=1}^{N} (d_{E_{\min}})^{-N_1} d_H^{c,N} (c, \hat{c})
\] (3)

Form (3) we can see that under Rayleigh fading channels, the diversity gain \( G_s = N_1 d_H^{c,N} \), the coding gain \( G_c = \prod_{i=1}^{N} (d_{E_{\min}})^{d_{H}^{c,N} (c, \hat{c})} \). In order to reduce the upper bound of pairwise error probability of this system, the diversity and the coding gain must be made maximization. Hamming distance determines the diversity gain, and to increase the coding gain, the product distance of sequences must be made maximum. That is, the \( d_{E_{\min}} \) must be made maximum. In mean that the minimum squared Euclidean distances are the maximum among vectors of symbols that hamming distances is one.

When the signal noise ratio is higher, equation (3) can be further simplified as [8]:

\[
P(X \rightarrow \hat{X}) \leq (\frac{E_s}{4N_0})^{-N_1} \prod_{i=1}^{N} d_H^{c,N} (c, \hat{c}) d_{E_{\min}}^{-1} N_1
\] (4)

From equation (4) we can see that in Rayleigh fading channels, the maximum diversity gain of the system is \( N_1 N_1 \). In order to increase the coding gain and reduce the upper bound of pairwise error probability of the system \( d_{E_{\min}} \) must be made maximization.

IV. CONSTELLATION ROTATION

The asymmetric modulation constellation can be made by all or part points of the constellation rotation of MPSK. As stated above, in the errorless feedback conditions, to reduce the upper bound of pairwise error probability of bit-interleaved space-time coded modulation with iterative decoding system under the Rayleigh fading channels, the diversity gain and coding gain of the system must be maximized. In addition to the Hamming distance between sequences be designed maximum, increase the minimum squared Euclidean distance between sequences which hamming distances is one, that is \( d_{E_{\min}} \) be made maximum, the higher coding gain can be obtained. The impacts of asymmetric modulation constellation on \( d_{E_{\min}} \) discussed as follows.

Fig. 4 gives asymmetric modulation constellation of 8PSK with three mapping (Gray, SP, Mixed). Asymmetry of the constellation is represented by the rotation angle of \( \theta \). For 8PSK, when \( \theta \) equals to \( \pi/4 \), asymmetric modulation constellation becomes symmetric constellation.

![Figure 4. Asymmetric modulation constellation](image)

The \( \theta \) angle increased, it means that asymmetric modulation, the Euclidean distance can be increased between symbols that Hamming distance is one and the bit is different at the third bit position. As can be seen, in the range of \( \theta \in (\pi/4, \pi/2) \), for the Gray mapping, the \( \theta \) angle increased, although the Euclidean distance increased between symbols that the Hamming distance is one which the third bit position is deferent, but the Euclidean distance will be made decrease between symbols that the first or second bit position is deferent. Their Euclidean distances are shorter than them in circumstances of symmetrical modulation constellation. Therefore, the performance of the system is degradation in asymmetric modulation for Gray mapping.

For the SP mapping and Mixed mapping, if \( \theta \) increased, the Euclidean distance would be increased simultaneously between symbols which Hamming distance is one and in the third bit position is deferent. At the same time, the Euclidean distances are unchanged between symbols which Hamming distance is one and in the first or second bit position is deferent. Therefore, the minimum squared Euclidean distance \( d_{E_{\min}} \) can be increased through asymmetric modulation in these two kinds of mapping methods. The coding gain of the system can be achieved higher. Thereby the upper bound of pairwise error probability of the system can be reduced.

The minimum squared product distance is a main parameter of anti-fading performance under Rayleigh fading channels. The purpose of constellation rotation is to make it maximum. That is making I/Q components the greatest differences so that obtaining the diversity gain.

The minimum square product distance is given by the following formula [12]:

...
\[ d_p^\prime = \min \{ \prod_{i=1}^{M} |x_i - x'_i|^2 \}, \forall (x, x')_{x \neq x'} \] (5)

Of which, \( x_i \) and \( x_i \) are respectively components of two constellation points along \( i \) dimension. The optimum rotation angle is a rotated angle of constellation that makes equation (5) maximum.

V. SIMULATION RESULTS

The performance of proposed scheme is simulated and compared with the symmetrical one. The simulation parameters are set as follows: the length of the input data block frame is 260 bits; the convolutional code with code rate 1/3 and constraint length 3. The generating polynomial \((g_1, g_2, g_3) = (5, 7, 7)_8\), the length of random interleave is 780 bits; modulation constellation is 8PSK, with rotation angle \( \theta \) are 45° (symmetric modulation), 55°, 65°, 75° respectively; four transmit antenna and two receive antenna; iteration number is 8.

Fig. 5 shows the error frame rate performance comparison of BI-STCM system under Rayleigh fading channels through Gray, SP and Mixed mappings with symmetric and asymmetric modulation. The asymmetry rotation angle \( \theta \) is 55°. We can see that the Gray mapping has poor performance using asymmetric modulation. This is because the minimum Euclidean distance becomes shorter between symbols. Compared to the symmetrical modulation, nearly 1 dB of SNR can be obtained in asymmetric modulation of SP and mixed mapping at circumstances of \( 10^{-3} \) of the frame error rate. Furthermore, the performance of SP mapping is superior to mixed mapping. This is due to better distance properties of the SP mapping, as stated above.

Fig. 6 shows the comparison between the proposed modulation diversity scheme (BI-STC-MD) and without one (BI-STC) under AWGN and Rayleigh fading channels. The new scheme using the same constellation rotation angle 30°, interleaving depth of I/Q is 100 modulation symbols. The simulation results show that modulation diversity can obtain diversity gain more than 1 dB at the bit error rate of \( 10^{-4} \) under Rayleigh fading channels. Compared to AWGN channel, both of the performance are almost the same. This is due to the modulation diversity did not change the Euclidean distance between the constellation points, thus modulation diversity without sacrificing system performance in AWGN channels.

In addition, from the simulation results we can also see that error frame rate is higher under Rayleigh fading channels. This is due to that the interleaver length is chosen as 780 bits. That is, there are no interleaver among frames. The interleaver was restricted to the internal of the frame, the analysis can be made simplified. The correlation of channels was not considered under quasi-static fading channels. But the effect of asymmetric modulation on the performance of BI-STCM system is consistent under fast Rayleighn fading channels or quasi-static fading channels.

VI. CONCLUSION

By analyzing the upper bound of pairwise error probability of space-time bit-interleaved coded modulation system, we can see that under Rayleigh fading channels, the coding gain and diversity gain of the system mainly determined by the minimum Euclidean distance between symbols that Hamming distance is one. By means of asymmetric modulation, the Euclidean distance can be improved between symbols that Hamming distance is one for the SP mapping and mixed mapping. So the coding gain of the system can be improved and the upper bound of pairwise error probability can be reduced.

In order to improve the diversity order that mainly determines the bit error performance under fading channels, the joint optimization scheme was proposed that combined modulation diversity with asymmetric modulation in bit-interleaved space-time coded modulation. The method of realizing modulation diversity and the optimal rotated angle of the constellation to achieve maximum diversity gain were analyzed. The impact on the performance of BI-STCM-ID was depicted. Theoretical analysis and simulation results show that modulation diversity can greatly improve the performance of BI-STCM-ID under Rayleigh fading channels. Furthermore, modulation diversity combined with space-time diversity possesses the complementary effects on improving the performance of fading channels.
REFERENCES


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