Improved Instrumented Compliant Wrist Design

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Improved Instrumented Compliant Wrist Design

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GRASP LAB 334

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Improved Instrumented Compliant Wrist Design†

Thomas Lindsay and Richard P. Paul

Abstract

Interaction between robot and environment is an extremely important aspect of robotic research. Compliance helps reduce the impact effects of robot/environment interaction. Hybrid position/force control is important in most robotic tasks; accurate position control is needed in unconstrained directions, and accurate force control is needed in constrained directions. Force control can be more responsive with a compliant force/torque sensor, but positional accuracy is reduced with compliance. An instrumented compliant wrist device can be used to achieve both responsive force control and accurate position control.

The wrist is connected in series between the end of the robot and the tool, and is designed to partially surround the tool, thus reducing the distance between the end of the robot and the end of the tool. The wrist device uses rubber elements for compliance and damping, and a serial linkage, with potentiometers at each joint, is used for sensing the deflections produced in the wrist.

This document describes the newest version of the instrumented compliant wrist, including modifications and improvements to the wrist described in “Design of a Tool Surrounding Compliant Instrumented Wrist”, available as tech report MS-CIS-91-30, GRASP LAB 258 from the University of Pennsylvania. Changes include a more protective sensing linkage structure and improved electronics. The compliance, kinematics, and accuracy of the wrist are presented. Also, software for determining the wrist transform, and plans for the wrist are given.

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<td>Sensing Linkage - Exploded View</td>
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<td>18</td>
<td>Linkage Piece 1</td>
<td>24</td>
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<td>Linkage Piece 2</td>
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<td>Linkage Piece 3</td>
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<td>Linkage Piece 4</td>
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<td>22</td>
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<td>34</td>
</tr>
</tbody>
</table>
1 Introduction

The wrist outlined herein is a solution to a complex problem: compliance in a robot wrist is desired to reduce the effect of impacts between the robot and the environment, and to create a more responsive force control. However, a compliant wrist by itself limits the effective stiffness of the manipulator in position control, and the exact position of the end of the wrist (and thus the environment, when in contact) is lost [7]. By instrumenting the wrist as described here, both problems are overcome: active control can be used to increase the stiffness of the system, and the position transform of the wrist is sensed. Using the instrumented wrist as a compliant force/torque sensor leads to more responsive force control than with a stiff sensor [5], and more accurate position control than with a compliant wrist [8].

The wrist is overall 4.25 x 4.25 x 3.0 inches high (108 x 108 x 76 mm). A 1.75 x 1.75 inch (44.5 x 44.5 mm) tool can be mounted inside the wrist to a depth of 2.5 inches (63.5 mm) maximum, depending on the desired flexure of the wrist.

This report is organized as follows: section 2 outlines the compliance of the wrist, and how it can be modified with different compliant elements, section 3 describes the sensing linkage kinematics, section 4 is a short analysis of the accuracy of the wrist, section 5 contains a simple software routine to compute the wrist transform, section 6 contains the mechanical and electrical plans for the wrist, and section 7 gives a conclusion including current research using the wrist and future work on the wrist.
2 Compliant Structure

The compliant structure of the wrist is composed of 12 rubber elements, which provide compliance and a small degree of damping. Figure 2 shows the compliant structure design, with the bottom plate (attached to the robot) fixed to the four aluminum blocks at the corners, and the top plate (where the tool is attached) fixed to the four compliant elements (cylinders) at the top. The tool can be partially enclosed in the center of this structure.

The stiffness in each direction can be approximated as follows:

\[ K_x = K_y = \left( \frac{1}{4K_a} + \frac{1}{8K_r} \right)^{-1} \]  
\[ K_z = \left( \frac{1}{4K_a} + \frac{1}{4K_a + 4K_r} \right)^{-1} \]  
\[ K_{\psi} = \left( \frac{1}{4K_r L_1^2} + \frac{1}{8K_a L_1^2} \right)^{-1} \]  
\[ K_{\phi} = K_{\theta} = \left( \frac{1}{2K_a L_1^2} + \frac{1}{4K_r L_1^2 + 4K_r L_2^2} \right)^{-1} \]

where \( K_a \) and \( K_r \) are the axial and shear stiffnesses of a single element, and \( L_1 \) and \( L_2 \) are shown in figure 3. This approximation uses the axial and shear stiffness of the rubber elements as supplied by the manufacturer, but ignores any bending stiffness. Age and
wear for the rubber will also change the stiffness parameters, but this effect has been
ignored. For the rubber elements used, the axial stiffness $K_a = 66.7$ lb/in. (2.63 N/mm)
and the shear stiffness $K_s = 12.0$ lb/in. (472 N/mm). The approximate compliance of
the wrist is tabulated below.

<table>
<thead>
<tr>
<th>$K_x$</th>
<th>$K_y$</th>
<th>$K_z$</th>
<th>$K_\phi$</th>
<th>$K_\theta$</th>
<th>$K_\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lb/in</td>
<td>lb/in</td>
<td>lb/in</td>
<td>in-lb</td>
<td>in-lb</td>
<td>in-lb</td>
</tr>
<tr>
<td>(N/mm)</td>
<td>(N/mm)</td>
<td>(N/mm)</td>
<td>(N-m)</td>
<td>(N-m)</td>
<td>(N-m)</td>
</tr>
<tr>
<td>41.65</td>
<td>41.65</td>
<td>70.59</td>
<td>61.37</td>
<td>61.37</td>
<td>68.81</td>
</tr>
<tr>
<td>(7.29)</td>
<td>(7.29)</td>
<td>(12.36)</td>
<td>(6.93)</td>
<td>(6.93)</td>
<td>(7.77)</td>
</tr>
</tbody>
</table>

The compliant structure is extremely modular. By exchanging the rubber elements
for ones of different properties, the stiffness of the wrist can be changed. For example,
the elements can be replaced with any of three similar elements produced by the same
manufacturer, or taken away entirely. If the stiffness approximation equations shown
above are broken down into individual element stiffnesses, the following equations apply
(see figure 4 for element placement numbers):

$$K_z = \left( \frac{1}{(K_{1a} + K_{2a} + K_{3a} + K_{4a})} \right) + \left( \frac{1}{(K_{5r} + K_{6r} + K_{7r} + K_{8r} + K_{9r} + K_{10r} + K_{11r} + K_{12r})} \right)^{-1} \tag{5}$$

$$K_x = \left( \frac{1}{(K_{1r} + K_{2r} + K_{3r} + K_{4r})} \right) + \left( \frac{1}{(K_{7a} + K_{8a} + K_{11a} + K_{12a} + K_{5r} + K_{6r} + K_{9r} + K_{10r})} \right)^{-1} \tag{6}$$
Below is a table of axial and shear stiffness for sample compliant elements. In the current design, mount A is used for all positions on the wrist. Mount n occurs when no mount element is used for a site.

Below are some examples of using different elements for the compliant structure.

<table>
<thead>
<tr>
<th>Mount [1]</th>
<th>$K_a$</th>
<th>$K_r$</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lb/in</td>
<td>lb/in</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>66.7</td>
<td>12.0</td>
<td>mount used</td>
</tr>
<tr>
<td></td>
<td>(2.63)</td>
<td>(.472)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>92.3</td>
<td>17.6</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>175.0</td>
<td>37.5</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>228.6</td>
<td>50.0</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>0.0</td>
<td>0.0</td>
<td>no mount</td>
</tr>
</tbody>
</table>
Figure 4: Rubber Element Placement

<table>
<thead>
<tr>
<th>Element:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
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<td>Properties:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_x$</td>
<td>$K_y$</td>
<td>$K_z$</td>
<td>$K_\theta$</td>
<td>$K_\phi$</td>
<td>$K_\psi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comments</td>
<td>lb/in</td>
<td>lb/in</td>
<td>lb/in</td>
<td>lb-in</td>
<td>lb-in</td>
<td>lb-in</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>current design</td>
<td>A</td>
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<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>stiffest</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>169.57</td>
<td>169.57</td>
<td>278.27</td>
<td>240.47</td>
<td>240.47</td>
<td>281.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>most compliant</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>n</td>
<td>n</td>
<td>A</td>
<td>A</td>
<td>n</td>
<td>n</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>stiff in $K_z$, $K_\theta$, $K_\phi$</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>46.02</td>
<td>46.02</td>
<td>160.05</td>
<td>132.34</td>
<td>132.34</td>
<td>73.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stiff in $K_x$, $K_\psi$</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>122.30</td>
<td>122.30</td>
<td>86.88</td>
<td>77.55</td>
<td>77.55</td>
<td>227.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>compliant in $K_x$, $K_\theta$</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>n</td>
<td>n</td>
<td>A</td>
<td>A</td>
<td>n</td>
<td>n</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>24.00</td>
<td>40.68</td>
<td>40.68</td>
<td>11.35</td>
<td>66.15</td>
<td>63.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>even in trans. dirs.</td>
<td>B</td>
<td>C</td>
<td>C</td>
<td>B</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>81.63</td>
<td>81.63</td>
<td>81.39</td>
<td>72.00</td>
<td>72.00</td>
<td>142.71</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>even in rot. dirs.</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>42.58</td>
<td>42.58</td>
<td>82.01</td>
<td>70.84</td>
<td>70.84</td>
<td>69.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
3 Linkage Kinematics

The sensing mechanism is composed of a serial linkage chain with potentiometers at each joint (see figure 5). Voltage across the potentiometers is measured to determine the joint angles. Using a simple forward kinematic formulation, the transformation between the robot wrist and the end of the tool can be calculated. The kinematic skeleton of the wrist is shown in figure 6.

The D-H parameters for the sensing linkage mechanism, shown in figures 7 and 8 are:
Also needed is the transform between the end of the robot (Frame b) and the first link frame:

\[
\begin{array}{cccc}
\text{joint} & a & d & \alpha & \theta \\
& \text{mm} & \text{mm} & \text{deg.} & \text{deg.} \\
1 & 0 & -25.00 & -90 & \theta_1 \\
2 & 0 & 98.82 & -90 & \theta_2 \\
3 & 0 & 17.86 & 90 & -90 + \theta_3 \\
4 & 0 & 98.42 & 90 & -90 + \theta_4 \\
5 & 0 & 49.61 & 90 & 90 + \theta_5 \\
6 & 49.21 & -25.00 & 180 & 180 + \theta_6 \\
\end{array}
\]

Also needed is the transform between the end of the robot (Frame b) and the first link frame:

\[
\mathbf{^b}_A^0 = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]  \hspace{1cm} (11)

where \( l_1 = l_2 = 49.21 \text{mm} \).

With this information, a transform from the end of the robot to the end of the wrist can be formed. A further transform from the end of the wrist to the end of the tool will complete the transformation from the end of the robot to the tip of the tool.

Relating the \((i - 1)\)th link frame to the \(i\)th link frame is a transform matrix of the form:
Figure 7: Side view of linkage structure

Figure 8: Top view of linkage structure
\[
  i^{-1} A_i = \begin{bmatrix}
  \cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i \\
  \sin \theta_i & \cos \alpha_i \cos \theta_i & -\sin \alpha_i \cos \theta_i & a_i \sin \theta_i \\
  0 & \sin \alpha_i & \cos \alpha_i & d_i \\
  0 & 0 & 0 & 1
  \end{bmatrix}
\] (12)

The link transforms for the wrist are:

\[
  ^0 A_1 = \begin{bmatrix}
  \cos \theta_1 & 0 & -\sin \theta_1 & 0 \\
  \sin \theta_1 & 0 & \cos \theta_1 & 0 \\
  0 & -1 & 0 & d_1 \\
  0 & 0 & 0 & 1
  \end{bmatrix}
\] (13)

\[
  ^1 A_2 = \begin{bmatrix}
  \cos \theta_2 & 0 & -\sin \theta_2 & 0 \\
  \sin \theta_2 & 0 & \cos \theta_2 & 0 \\
  0 & -1 & 0 & d_2 \\
  0 & 0 & 0 & 1
  \end{bmatrix}
\] (14)

\[
  ^2 A_3 = \begin{bmatrix}
  \sin \theta_3 & 0 & -\cos \theta_3 & 0 \\
  -\cos \theta_3 & 0 & -\sin \theta_3 & 0 \\
  0 & 1 & 0 & d_3 \\
  0 & 0 & 0 & 1
  \end{bmatrix}
\] (15)

\[
  ^3 A_4 = \begin{bmatrix}
  \sin \theta_4 & 0 & -\cos \theta_4 & 0 \\
  -\cos \theta_4 & 0 & -\sin \theta_4 & 0 \\
  0 & 1 & 0 & d_4 \\
  0 & 0 & 0 & 1
  \end{bmatrix}
\] (16)

\[
  ^4 A_5 = \begin{bmatrix}
  -\sin \theta_5 & 0 & \cos \theta_5 & 0 \\
  \cos \theta_5 & 0 & \sin \theta_5 & 0 \\
  0 & 1 & 0 & d_5 \\
  0 & 0 & 0 & 1
  \end{bmatrix}
\] (17)

\[
  ^5 A_6 = \begin{bmatrix}
  -\cos \theta_6 & -\sin \theta_6 & 0 & -a_6 \cos \theta_6 \\
  -\sin \theta_6 & \cos \theta_6 & 0 & -a_6 \sin \theta_6 \\
  0 & 0 & -1 & d_6 \\
  0 & 0 & 0 & 1
  \end{bmatrix}
\] (18)

Multiplying the A matrices yields \( ^6 b T_6 \), the transformation of the wrist.

\[
  ^6 b T_6 = ^6 b A_0 \: ^0 A_1 \: ^1 A_2 \: ^2 A_3 \: ^3 A_4 \: ^4 A_5 \: ^5 A_6
\] (19)

4 Accuracy

Accuracy of the wrist can be broken into two areas: positional accuracy and hysteresis effects. The positional accuracy is a function of smallest change in position that can be sensed, and sensor noise. The hysteresis effects deal with the ability of the wrist to return to a given position after it has been moved.
4.1 Positional Accuracy

The positional accuracy of the wrist is much improved with the addition of a signal conditioner. The potentiometer voltages are differenced from a mid-range reference voltage, and amplified in the signal conditioner board, which is located in close proximity to the wrist itself in order to reduce the effects of line transmission noise. In order to assess the positional accuracy, data from a stationary wrist, free-space motion, and sliding motion (wrist sliding along a flat surface) is presented.

Figure 9 shows sensor data collected when the manipulator is stationary. The fluctuations in sensor data here are caused solely by electrical noise. The actual data is shown in figure 9(a), and the distribution of this data, in histogram format, is shown in 9(b).

Figure 10 shows sensor data from a free-space constant velocity motion, with data collected in the direction of motion. The data fluctuations here are caused by both the electrical noise, as above, and the motion vibrations of the wrist/robot system.
Figure 11: Sensor Data from Sliding Motion

Figure 11 shows sensor data from a sliding motion, while the manipulator is in contact with a surface. Data again is collected in the direction of motion. The fluctuations present in this data result from the electrical and mechanical noise, as above, as well as additional noise associated with the sliding motion. This sliding noise results from:

- Non-ideal control laws which cause the normal force with the surface to fluctuate slightly.
- Non-homogeneous surface friction.
- Coupling of orthogonal forces.

Although data for the sliding motion is highly dependent upon the control laws used for such a motion, it has been presented here as an example of the positional accuracy that may be obtained in a typical application.

Tabulated below are the statistical parameters from the three motions described above.

<table>
<thead>
<tr>
<th>Motion</th>
<th>mean</th>
<th>median</th>
<th>$\sigma$</th>
</tr>
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<td>0.0040</td>
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<td>slide</td>
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### 4.2 Hysteresis Effects

The non-zero mean in the no motion data is an example of the hysteresis effects (data from the wrist in motion is not expected to have a zero mean). Hysteresis effects account for a large portion of the wrist inaccuracy. Factors that contribute to the hysteresis are the natural hysteresis of the rubber compliant elements, which is so small as to be barely noticeable, and effects from friction of the potentiometers coupled with a small amount of bending in the sensing linkage structure.

Tests show that the worst-case inaccuracy due to hysteresis is approximately .6 mm (.025 in.) for translation and .0099 radians (.57 degrees) for rotation. It is clear that this far outweighs the positional inaccuracy.
5  Wrist Software

Below is a listing (in C) for a subroutine to find the wrist transform. Note that in the code, the following are defined:

\[
\begin{align*}
\mathbf{u} &= bA_0^0A_1^1A_2^2A_3^3A_4 \\
\mathbf{v} &= u^4 A_5 \\
bT_6 &= v^5 A_6
\end{align*}
\]

/*------------------------------------------------------------* /
/* WristUpdate(car_diffs) */
/* * Reads current angles of wrist sensor and computes the wrist * transform, also puts the difference of cartesian deflection (from * home position) in the array car_diffs[6] */
/*------------------------------------------------------------* /

WristUpdate(car_diffs,tw)
float car_diffs[N];
struct transform tw;
{
    float c1,c2,c3,c4,c5,c6,    /* angle cosines */
         s1,s2,s3,s4,s5,s6,    /* angle sines */
         u11,u12,u13,u14,u21,u22,u23,u24,u31,u32,u33,u34,
    float ang;
    /* Link parameter values */
    float l1 = 49.21, l2 = 49.21;
    float d1 = -25.00,d2 = 98.82,d3 = 17.86,d4 = 98.42,d5=49.61,d6= -25.00;
    float a6 = 49.21;

    ang=rw_jang(0);    /* read joint angle from A/D board */
    c1 = cos(ang);
    s1 = sin(ang);

    ang=rw_jang(1);
    c2 = cos(ang);
    s2 = sin(ang);

    ang=rw_jang(2);
    c3 = cos(ang);
    s3 = sin(ang);
ang=rw_jang(3); 
c4 = cos(ang); 
s4 = sin(ang);

ang=rw_jang(4); 
c5 = cos(ang); 
s5 = sin(ang);

ang=rw_jang(5); 
c6 = cos(ang); 
s6 = sin(ang);

u11 = c1 * c2 * s3 * s4 - s1 * c3 * s4 + c1 * s2 * c4;
u12 = -c1 * c2 * c3 - s1 * s3;
u13 = -c1 * c2 * s3 * c4 + s1 * c3 * c4 + c1 * s2 * s4;
u14 = -d4 * c1 * c2 * c3 - d4 * s1 * s3 
- d3 * c1 * s2 - d2 * s1 + 11;

u21 = -s1 * c2 * s3 * s4 - c1 * c3 * s4 - s1 * s2 * c4;
u22 = s1 * c2 * c3 - c1 * s3;
u23 = s1 * c2 * s3 * c4 + c1 * c3 * c4 - s1 * s2 * s4;
u24 = d4 * s1 * c2 * c3 - d4 * c1 * s3 
+ d3 * s1 * s2 - d2 * c1 + 12;

u31 = s2 * s3 * s4 - c2 * c4;
u32 = -s2 * c3;
u33 = -s2 * s3 * c4 - c2 * s4;
u34 = -d4 * s2 * c3 + d3 * c2 - d1;

v11 = -u11 * s5 + u12 * c5;
v12 = u13;
v13 = u11 * c5 + u12 * s5;
v14 = d5 * u13 + u14;

v21 = -u21 * s5 + u22 * c5;
v22 = u23;
\v23 = u21 * c5 + u22 * s5;
v24 = d5 * u23 + u24;

v31 = -u31 * s5 + u32 * c5;
v32 = u33;
v33 = u31 * c5 + u32 * s5;
v34 = d5 * u33 + u34;
/* Wrist transform */
tw.n.x = -v11 * c6 - v12 * s6;
tw.o.x = -v11 * s6 + v12 * c6;
tw.a.x = -v13;
tw.p.x = -a6 * v11 * c6 - a6 * v12 * s6 + d6 * v13 + v14;

tw.n.y = -v21 * c6 - v22 * s6;
tw.o.y = -v21 * s6 + v22 * c6;
tw.a.y = -v23;
tw.p.y = -a6 * v21 * c6 - a6 * v22 * s6 + d6 * v23 + v24;

tw.n.z = -v31 * c6 - v32 * s6;
tw.o.z = -v31 * s6 + v32 * c6;
tw.a.z = -v33;
tw.p.z = -a6 * v31 * c6 - a6 * v32 * s6 + d6 * v33 + v34;

/* Compute roll, pitch, and yaw angles from wrist transform */
noatorpy(&car_diffs[5],&car_diffs[4],&car_diffs[3],&tw);
car_diffs[0] = tw.p.x;
car_diffs[1] = tw.p.y;
car_diffs[2] = tw.p.z - 67.86; /* total wrist thickness */
}

/----------------------------------------------------------------------------------------------------------*/

6 Wrist Plans

6.1 General

The compliant structure and the sensing linkage are sandwiched between the top and bottom plate. The compliant structure is connected to the bottom plate with four 8-32 x 1/2” countersunk machine screws, and to the top plate by the compliant elements. The sensing linkage is attached to the top and bottom plates by two 8-32 x 1/8” countersunk machine screws. The wrist is connected to the robot via a quick mount mechanism (Lord Corporation, not shown), which bolts into the four 8-32 threaded holes in the bottom plate. A 20-pin connector is also attached to the bottom plate.
Figure 12: Top Plate

Top Plate
Needed: 1
Material: 1/8" Aluminum Plate
Note: All holes tapped for 8-32 threads except where noted
Units = Inches
Figure 13: Bottom Plate

Bottom Plate
Needed: 1
Material: 1/8" Aluminum Plate
Note: All holes 5/32 diameter and countersunk except where noted
Units = Inches
### 6.2 Compliant Structure

The 12 compliant elements are part number 10Z2-302A from Stock Drive Products\(^1\). These elements have 8-32 x 3/8" threaded studs. Four of these must have one stud shortened to 3/16", one each attached to compliant structure piece 2. All elements connected to compliant structure piece 1 are attached with 8-32 hex nuts.

---

\(^1\)New Hyde Park, NY, (516) 328-0200. Stiffer elements are part numbers 10Z2-302B, 10Z2-302C, and 10Z2-302D.
Compliant Structure Piece 1

Needed: 4
Material: Aluminum
Units = Inches
Figure 16: Compliant Structure Piece 2

Needed: 4
Material: Aluminum
Units = Inches

Compliant Structure Piece 2

8-32 Threads
0.2500

8-32 Threads
0.2500

0.2500

1 59/64

.5000

8-32 Threads (Approx .5 in. deep). Continue minor diameter (.1360) through piece
6.3 Sensing Linkage

Potentiometers used at joints 0, 2, and 5 are part number 381 N 1000 S; joints 1, 3, and 4 are part number RV6 NAYSD 10 2 A from Clarostat Mfg. Co., Inc.\textsuperscript{2}. Potentiometers are attached to linkage pieces with 4-40 x 1/4" hex head machine screws and 4-40 flat washers. Each potentiometer has an additional shaft bearing, part number K-FBB-2/4 from Small Parts, Inc.\textsuperscript{3}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{sensing_linkage_exploded_view.png}
\caption{Sensing Linkage - Exploded View}
\end{figure}

\textsuperscript{2}Dover, NH, (800) 872-0042.
\textsuperscript{3}Miami Lakes, FL, (305) 557-8222.
Figure 18: Linkage Piece 1

Linkage Piece 1
Needed: 2
Material: Aluminum
Units = Inches
Linkage Piece 2
Needed: 2
Material: Aluminum
Units = Inches

Figure 19: Linkage Piece 2
Figure 20: Linkage Piece 3
Needed: 1
Material: Aluminum
Units = Inches

Linkage Piece 3
Needed: 1
Material: Aluminum
Units = Inches
Figure 21: Linkage Piece 4

Linkage Piece 4
Needed: 1
Material: Aluminum
Units = Inches

Dimensions:
- φ0.2500
- 0.1250
- 0.1250
- 0.1250
- 0.1875
- 2.5000
- 0.6875
- 0.1875
- 0.6250
- 0.1875
- 1.1250
- 0.3750
- 0.6250
- 0.6875
- 0.6875
- 0.1250
- 0.1250
- 0.1250
Figure 22: Linkage Piece 5

Linkage Piece 5
Needed: 1
Material: Aluminum
Units = Inches
### 6.4 Electronics

#### WRIST CONNECTOR JW1

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Figure 24: Wrist Electronics

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Figure 25: Signal Conditioning Board
7 Conclusion and Future Work

The wrist outlined here is currently in use in the GRASP lab. Projects using the wrist include “Teleprogramming: Towards Delay-Invariant Remote Manipulation” [2], “Robotic Exploration of Surfaces with a Compliant Wrist Sensor” [6], and “Simplifying Tool Usage in Teleoperative Tasks” [4]. Figure 26 illustrates the previous wrist design in use exploring the environment\(^4\). Figure 27 shows the robot/wrist holding an impact wrench, undoing a bolt. Figure 28 shows the Penn Hand attached to the wrist, using artificial intelligence to learn object grasping techniques. The wrist has been shown to be a useful force/torque sensor for hybrid position/force control implementations[3].

Two major improvements to the wrist would improve the usefulness and accuracy. First, determination of the complete 6x6 stiffness matrix for the wrist would both improve the accuracy of force control algorithms and characterize the effects of the coupling of orthogonal forces. This would lead to more accurate control. Second, the accuracy of the sensing linkage could be improved by using resolvers instead of potentiometers at the linkage joints, or possibly substituting a parallel sensing linkage structure using LVDTs as position transducers.

\(^4\)Note that application pictures shown in this section are actual implementations of the previous wrist design
Figure 27: Impact wrench attached to wrist

Figure 28: Penn Hand attached to wrist (courtesy, M. Salganicoff)
References


