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Generation of N-Scroll Attractors via Sine Function

Wallace K. S. Tang, G. Q. Zhong, G. Chen, and K. F. Man

***Abstract*—A new approach for generating n-scroll attractors is introduced. It is demonstrated that n-scroll attractors can be generated using a simple sine or cosine function. A guideline is given so that a different number of scrolls can be designed easily by modifying two variables in the function. An electronic circuit is also designed for the implementation and the observation of a 9-scroll attractor is reported for the first time.**

***Index Terms*—Chaos generator, Chua's circuit.**

I. Introduction

Complex attractors with n-double scrolls were reported in [1], [2] by introducing additional break points in the nonlinear element for a Chua's circuit or using cellular neural networks with a piecewise-linear output function [3]. It is also demonstrated in [4]–[6], that odd number scrolls can be observed by similar modification. Up to now, only piecewise-linear functions are adopted for the generation of n-scroll attractors. Due to the constraint of the input dynamic range of the Op-amp, only a maximum of 6 scrolls is reported [5].

In this paper, a new family of continuous functions for generating n-scroll attractors is proposed. It is shown that n-scroll attractors can be obtained with a simple sine or cosine function. The approach provides the ease of design for n-scroll attractors by modifying only two parameters in the function. With the use of a commercial trigonometric function chip, an electronic circuit is designed and implemented. The input dynamic range of the trigonometric function chip can be increased by simple Op-amp operations and a 9-scroll attractor is observed for the first time.

The organization of the paper is as follows. The state equation of the modified Chua's circuit is given in Section II, where simulation is also given to illustrate the existence of the n-scroll attractors. In Section III, the designed electronic circuit for implementation is explained in details, and the attractors with different number of scrolls are observed with simple parameter adjustments. Finally, concluding remarks are given in Section IV.

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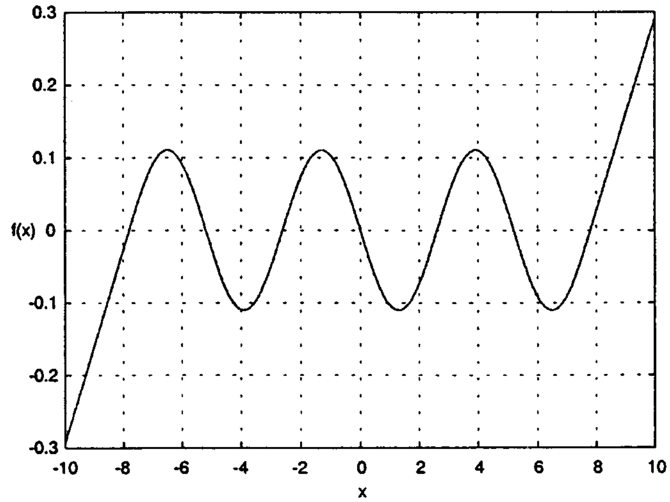

Fig. 1. Proposed sine function $f(x)$ with $a = 1.3, b = 0.11, c = 3$ and $d = 0$.

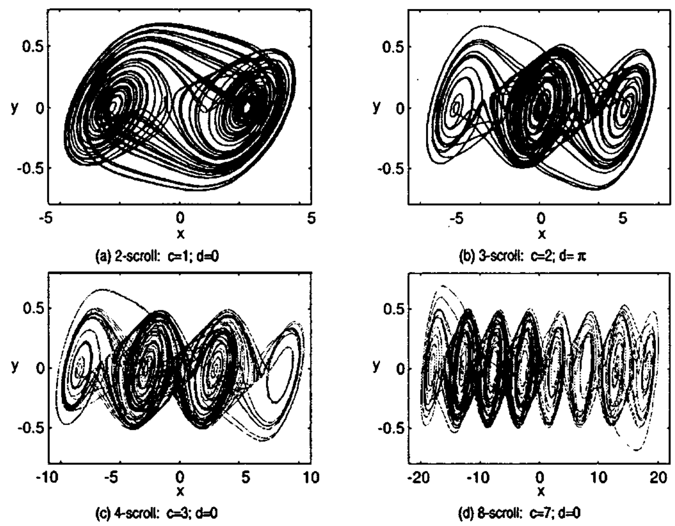

Fig. 2. N-scroll generated by the dimensionless state equation.

II. Modified Chua's Circuit

A. Dimensionless State Equation

Chua's circuit is adopted as the vehicle for the investigation. The dimensionless state equation of Chua's circuit modified with a sine function, is given by

$$\begin{aligned} \dot{x} &= \alpha[y - f(x)] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \tag{1}$$

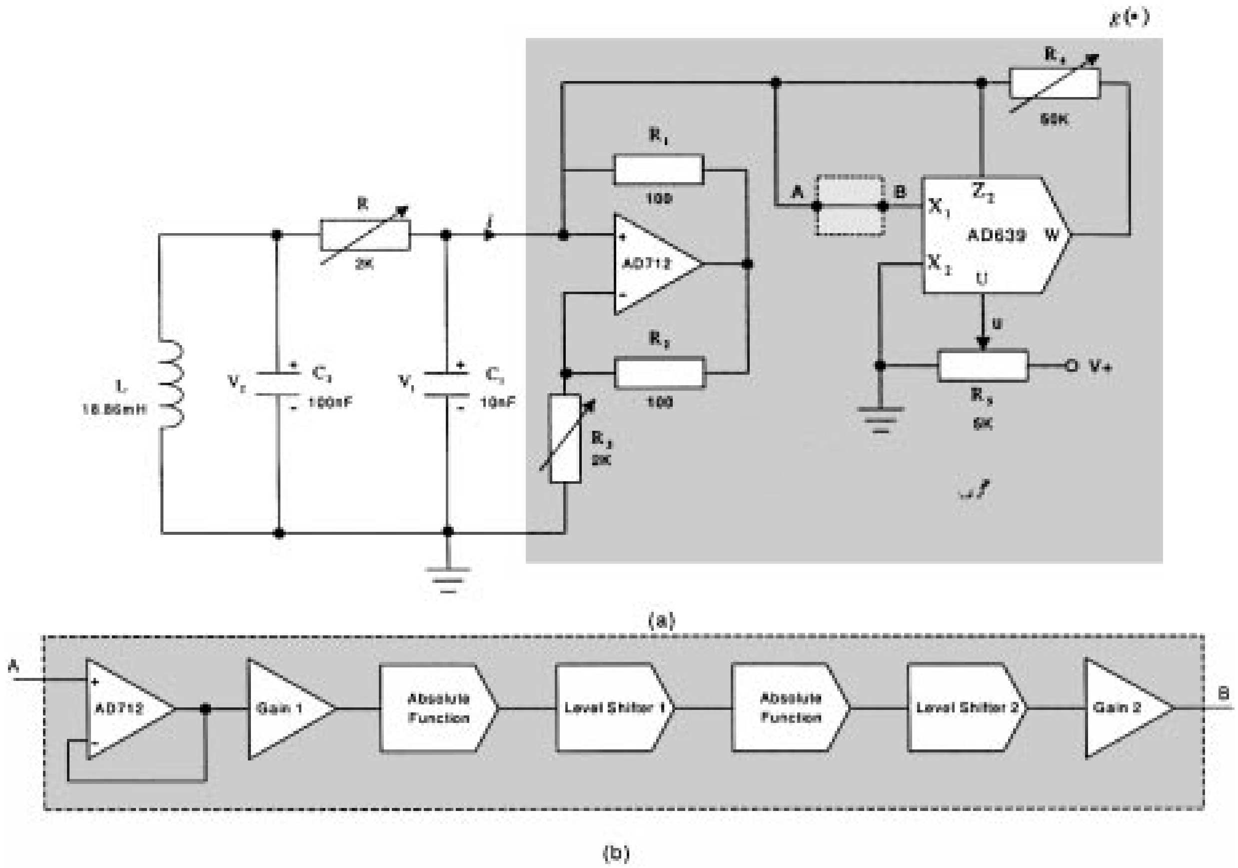

Fig. 3. (a) Modified Chua's circuit with $g(\cdot)$. (b) Circuit to be inserted into the dashed block A–B for increasing dynamic range of the sine or cosine function.

where

$$f(x) = \begin{cases} \frac{b\pi}{2a}(x - 2ac), & \text{if } x \geq 2ac \\ -b\sin\left(\frac{\pi x}{2a} + d\right), & \text{if } -2ac < x < 2ac \\ \frac{b\pi}{2a}(x + 2ac), & \text{if } x \leq -2ac \end{cases} \quad (2)$$

Here, in (1) and (2), $\alpha, \beta, a, b, c, d$ are suitable constants, to be specified below for different applications.

An n-scroll attractor is generated with the following relationship:

$$n = c + 1 \quad (3)$$

and

$$d = \begin{cases} \pi, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases} \quad (4)$$

The function in (2) is depicted in Fig. 1. It can be easily verified that c governs the number of periods existing in the function, and hence the number of equilibrium points of (1). The equilibrium points are $(x_{\rm eq}, 0, -x_{\rm eq})$ with $x_{\rm eq} = 2ak$ and $k = 0, \pm 1, \cdots, \pm c$.

B. Simulations

With the dimensionless state (1)–(2), when $\alpha = 10.814$, $\beta = 14.0$, $a = 1.3$, $b = 0.11$, 2-scroll, 3-scroll, 4-scroll, and 8-scroll attractors are generated with $c = 1, 2, 3$ and 7, respectively, as depicted in Fig. 2(a)–(d). Similar n-scroll attractor can be observed by replacing the sine function with a cosine function.

III. Circuit Implementation

A. Dimensional State Equation

In order to construct an electronic circuit for the proposed system, (1) is converted back to the dimensional state equation and modified as follows:

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{RC_1}(v_2 - v_1) - \frac{1}{C_1}g(v_1) \\ \frac{dv_2}{dt} &= \frac{1}{RC_2}(v_1 - v_2) + \frac{1}{C_2}i_L \\ \frac{di_L}{dt} &= -\frac{1}{L}v_2 \end{aligned} \quad (5)$$

where $g(v_1) = ((b\pi)/(2a) - (1/R))v_1 - (b\pi)/(4a)(|v_1 + 2ac| - |v_1 - 2ac|) - b\sin((\pi v_1)/(2a)) + d)$. Here, all notations are defined in the circuitry shown in Fig. 3(a).

B. Basic Circuit Design

A sine function is utilized to obtain the nonlinearity needed for generating chaos in the circuit. The modified Chua's circuit is shown in Fig. 3(a). In this set-up, the negative resistor, $g(\cdot)$, consists of two parts connected in parallel: a one-port described by a $v - i$ characteristic with linear negative slope [7], and a one-port with a $v - i$ characteristic described by a sine function shown in Fig. 1.

A commercial trigonometric function chip AD639 is chosen for the circuitry implementation. The resistor R_4 in Fig. 3(a) is used to convert the transfer characteristic to the driving-point characteristic of AD639 [8]. The amplitude (u) of the sine function can be adjusted by tuning the resistor R_5.

The original angular input range of AD639 is $\pm 500^{\circ}$. A maximum of four scrolls can be observed, although incomplete, as shown in Fig. 4.

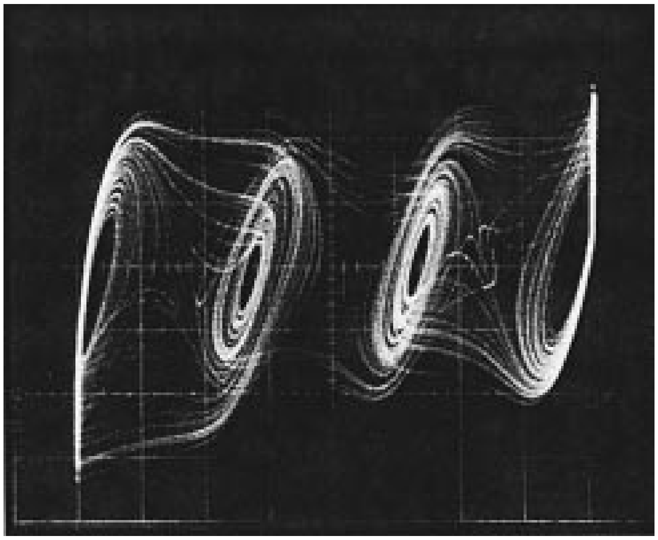

Fig. 4. Phase portrait in $v_{C1} - v_{C2}$ plane of the 4-scroll attractor. (Horizontal scale 2.5 V/div., vertical scale 0.5 V/div.).

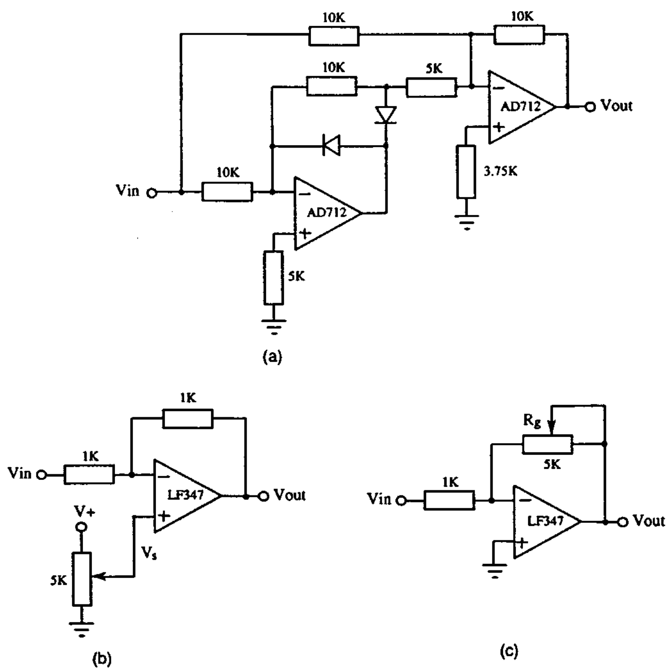

Fig. 5. Circuits for (a) Absolute operation. (b) Level shifter. (c) Gain.

C. Higher Number of Scrolls

In order to increase the range of the angular input for generating n-scroll attractors $(n > 4)$, some properties of the trigonometric function are utilized. Considering $x \in [-(2m+1)\pi, (2m+1)\pi]$, where m is a positive integer, we have

$$\begin{aligned}\cos(x) &= \cos(|x|)\\ &= \sin\left(\frac{\pi}{2} - |x|\right)\\ &= \sin\left(m\pi + \frac{\pi}{2} - |x|\right)\\ &= \sin(y)\end{aligned} \tag{6}$$

where $y = m\pi + (\pi/2) - |x|$.

It is observed that $y \in [-(m + 1/2)\pi, (m + 1/2)\pi]$, implying that only half of the angular range of x is needed. Hence, the number of scrolls generated with the original input range of AD639 can be doubled by the use of an absolute operation and a voltage shift in the circuit.

Fig. 3(b) depicts the circuit to be inserted into the dashed block A–B in Fig. 3(a) for the generation of higher number of scrolls. Detailed circuits for the gain, level shifter and absolute operation can be found in Fig. 5. By adjusting the voltage shifters and the gains of the circuit in

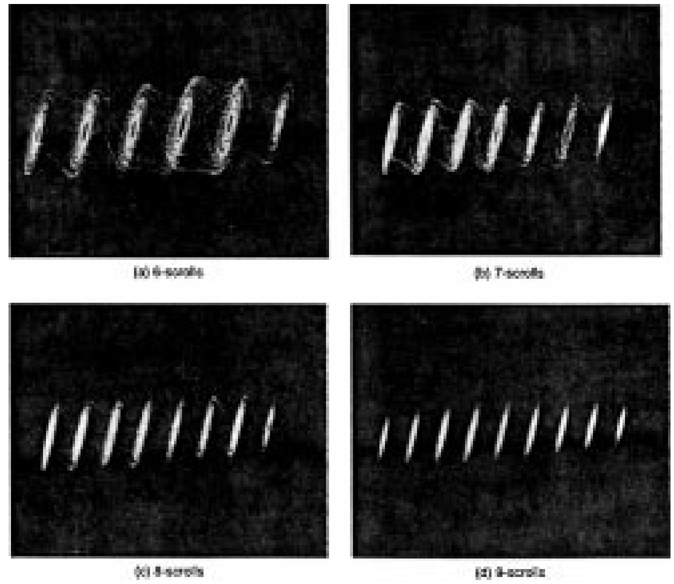

Fig. 6. Phase portrait in $v_{C1} - v_{C2}$ plane of n-scroll attractors. (a) $n = 6$. (b) $n = 7$. (c) $n = 8$. (d) $n = 9$. Parameter values are selected as listed in Table I.

TABLE I
PARAMETER VALUES SELECTED FOR THE OBSERVATIONS SHOWN IN FIG. 6

n	$R(k\Omega)$	$R_3(k\Omega)$	$R_4(k\Omega)$	$R_{g1}(k\Omega)$	$R_{g2}(k\Omega)$	$u(V)$	$v_{s1}(V)$	$v_{s2}(V)$
6	1.396	1.414	12.07	0.545	6.37	2.47	0.07	0.69
7	1.392	1.410	12.07	0.661	6.37	2.47	0.551	0.7
8	1.392	1.408	12.07	0.757	6.37	2.47	0.852	0.7
9	1.343	1.361	12.07	0.757	6.37	2.47	1.127	0.701

- R_{g1} and R_{g2} refer to the variable resistor in Fig. 5(c) for Gain 1 and Gain 2, respectively.
- u is the voltage adjustment for the amplitude of sine function in AD639.
- v_{s1} and v_{s2} refer to the voltage level tuned in Fig. 5(b) for Level shifter 1 and 2, respectively.

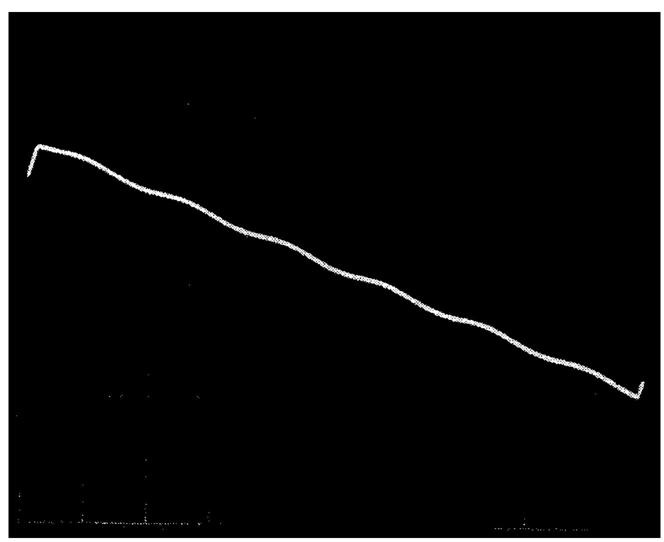

Fig. 7. Measured $v - i$ characteristic $g(v_1)$ with a sine function nonlinearity for 6-scroll attractor. (Horizontal scale 2.5 V/div., vertical scale 2.5 mA/div.).

Fig. 3(b), a number of n-scroll $(n = 6, 7, 8, 9)$ attractors are experimentally observed, as shown in Fig. 6(a)–(d). The chosen parameter values are listed in Table I. The measured $v - i$ characteristic $g(v_1)$ of the negative resistor synthesized for the 6-scroll attractor is also shown in Fig. 7, for illustration.

IV. Conclusion

In this work, a new approach for generating n-scroll attractors is proposed. Based on the proposed technique, 2-, 3-, and 4-scroll attractors can easily be generated by using the modified Chua's circuit implemented by a commercial trigonometric function chip. Furthermore, with the use of the voltage-shift circuit and the absolute functional circuit, the attractors with higher number of scrolls, even or odd, can also be obtained. The observations obtained in the experimental model validate the proposed approach, and an attractor with maximum of nine scrolls is generated and observed experimentally for the first time.

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Differential-Voltage Attenuator Based on Floating-Gate MOS Transistors and Its Applications

S. Vlassis and S. Siskos

***Abstract*—In this brief, a very simple differential voltage attenuator based on floating-gate MOS transistors (FGMOS) is proposed. The attenuator constructed by only two stacked identical FGMOS in saturation region, provides a voltage output proportional to the difference of the two input voltages. The advantages of this attenuator are the low supply operation, the rail-to-rail input range with small linearity error and the single-ended input processing. A very efficient technique to transform any circuit that requires only balanced inputs into the single-ended counterpart based on the attenuator, is proposed. Using this technique, a number of single-ended computational circuits are produced such as voltage squarer, four-quadrant multiplier, and vector summation circuit. The circuits can be fabricated in standard double-poly, double-metal CMOS technology and they are suitable for analogue signal processing and neural networks applications. SPICE simulation results using 2-μm MIETEC CMOS process parameters demonstrate the feasibility and the accuracy of the circuits.**

***Index Terms*—Differntial-to-single-ended converter, floatin-gate MOSFETs, voltage attenuator, voltage squarer and multiplier.**

I. Introduction

Voltage attenuators are useful building blocks in analog monolithic applications such as in feedback amplifiers and transconductors as input stages [1]. Also, a voltage attenuator can be used in high-performance-finite-gain amplifiers which can be used in the feedback path of an operational amplifier instead of resistor [2]. This paper proposes a differential voltage attenuator built with two stacked identical double-gate-floating-gate MOS (FGMOS) transistors operating in saturation [3].

In the last few years, FGMOS transistors have found many applications in electronic programming [4], Op-amp offset compensation [5], D/A and A/D converters [6], neural-networks applications [7], low-power operation [4], inverters, and amplifiers [7]–[9]. Recently, an increased number of publications on the use of the FGMOS in analog computational circuits have been reported such voltage squarers and multipliers [10]–[12]. The FGMOS drain current is proportional to the square of the weighted sum of the input signals [5]–[9].

A very efficient technique, which uses only two attenuators as input stages is proposed, in order to transform any circuit that requires only balance inputs into its single-ended counterpart. This technique can be applied to analog computational circuits. The analog computational circuits are very useful nonlinear functional circuits finding many applications in analog signal processing, fuzzy systems and neural networks. Most of these circuits require balanced inputs for proper operation and they have small linear input range.

Using the proposed technique can be designed computational circuits that can manipulate single-ended input signals with rail-to-rail dynamic input range and relatively small nonlinearity. A voltage squarer, a four-quadrant voltage multiplier and vector summation circuit are presented.

The paper is organized as follows. In Section II, the basic structure of the FGMOS transistor is described. The principle of operation of

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