# A Relational Model of Cognitive Maps

Chaib-draa, B and J. Desharnais

Département d'informatique, Faculté des Sciences

Université Laval, Sainte-Foy, QC, G1K 7P4, Canada

e-mail: `chaib@ift.ulaval.ca`

**Abstract**

A useful tool for causal reasoning is the language of cognitive maps developed by political scientists to analyze, predict and understand decisions. Although, this language is based on simple inference rules and its semantics is ad-hoc, it has many attractive aspects and has been found useful in many applications: administrative sciences, game theory, information analysis, popular political developments, electrical circuits analyze, cooperation man-machines, distributed group decision support and adaptation and learning, etc. In this paper, we show how cognitive maps can be viewed in the context of relation algebra, and how this algebra provides a semantic foundation that helps to develop a computational tool using the language of cognitive maps.

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## 1 Introduction

Causal knowledge generally involves many interacting concepts that make them difficult to deal with, and for which analytical techniques are inadequate (Park, 1995). In this case, other techniques, and particularly techniques stemmed from qualitative reasoning, can be used to cope with this kind of knowledge. A cognitive map (*CM*) is based on those techniques and is adequate for dealing with interacting concepts.

Generally, the basic elements of a *CM* are simple. The concepts an individual uses are represented as *points*, and the causal relationships between these concepts are represented as *arrows* between these points. This representation gives a graph of points and arrows, called a *cognitive map*. The strategic alternatives, all of the various causes and effects, goals, and the ultimate utility of the decision-making agent can all be considered as concept variables, and represented as points in the causal map. Causal relationships can take on basic values + (such as: promotes, enhances, helps, is benefit to, etc.), − (such as: retards, hurts, prevents, is harmful to, etc.) and 0 (such as: has no effect on, does not matter for, etc.). With this representation, it is then relatively easy to see how concepts and causal relationships are related to each other and to see the overall causal relationships of one concept with another. For instance, the *CM* of Figure 1, studied by Wellman (Wellman, 1994) and taken from (Levi and Tetlock, 1980), explains how the Japanese made the decision to attack Peal Harbor. Indeed, this portion of a *CM* states that "remaining idle *promotes* the attrition of Japanese strength while *enhancing* the defensive preparedness of the US, both of which *decrease* Japanese prospects for success in war". This shows that a *CM* is a set of concepts as "Japan remains idle", "Japanese attrition", etc. and a set of signed edges representing causal relations like "promote(s)", "enhance(s)", "decrease(s)", etc.

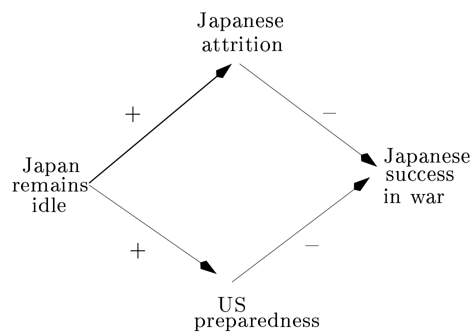


Figure 1: A Cognitive Map (from [Levi and Tetlock, 1980])

Note that the concepts' domains are not necessarily defined precisely since there are no obvious scales for measuring "US preparedness", "success in war", etc. Nevertheless, it seems easy to catch the intended meaning of the signed relationships in this model (Wellman, 1994). As any cognitive map, the $CM$ of Figure 1 can be transformed in a matrix called an adjacency or valency matrix. A valency matrix is a square matrix with one row and one column for each concept in a $CM$. For instance, if we note the concepts "Japan remains idle", "Japanese attrition", "Japanese success in war", "US preparedness" by $a, b, c, d$ respectively, then the valency matrix of the $CM$ represented in Figure 1 is the following:

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{pmatrix} 0 & + & 0 & + \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & - & 0 \end{pmatrix} \end{array}$$

Inferences that we can draw from this *CM* are based on a qualitative reasoning similar to " the friends of my friends are friends". Thus, in the case of Figure 1, remaining idle decreases the prospects for Japanese success in a war along two causal paths. Notice that the relationship between idleness and war prospects is negative since both paths agree. In these conditions, Japan has interest to start war as soon as possible if "he believes" that war is inevitable. Thus, a *CM* provides an intuitive framework in which to form decisions.

Cognitive maps (*CMs*) may be more complex than that given in this paper (see for example (Buede and

Ferrell, 1993) for larger examples). Furthermore, they are often cyclic since cyclicness or feedback represent interesting dynamic systems (Kosko, 1992). For instance, the *CM* of Figure 2 represents view on research in advanced countries which is a dynamic system reflected by many cycles: (1), (2), (3), etc. Such cycles are hard to represent using trees, as for instance Markov or Bayesian trees, which are acyclic representations by design.

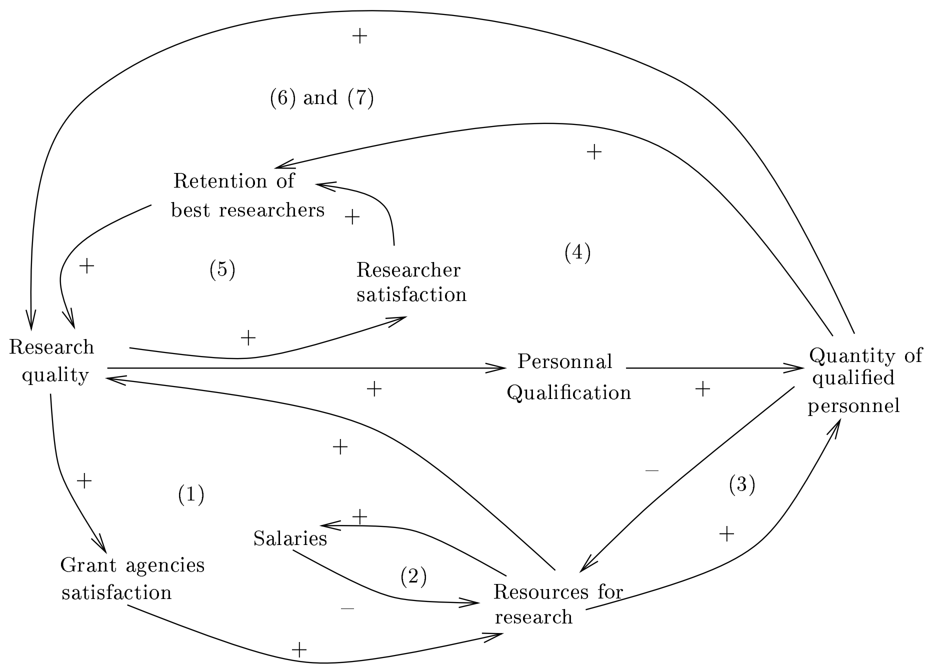


Figure 2: A cognitive map representing an organization as loops.

In summary, *CMs* are a power tool which allows users to represent and reason on causal relationships as reflected in realistic dynamic systems. Cyclic *CMs* have been considered as ad-hoc representations with simple inference rules and no semantics. In this paper, we propose a relational model of cognitive maps as a semantic. With this model, we have developed a computational model that we are using in the context of multiagent environments.

This paper is organized as follows. The next section relates previous works about cognitive maps. Section 3 sketches the classical calculus of relations and details our relational model of cognitive maps. Finally, section 5 presents the implementation of the proposed model and its applications in the context of multiagent systems.

## 2 Related works

*CMs* follow *personal construct theory* first put forward by Kelly (Kelly, 1955). His theory provides a basis for representing an individual's multiple perspectives. Kelly suggests that understanding how individuals organize their environments requires that subjects themselves define the relevant dimensions of that environment. He proposed a set of techniques, known collectively as *repertory grid*, in order to facilitate empirical research guided by the theory. Personal construct theory has spawned many fields and has been used in the fields of international relations (Axelrod, 1976; Buede and Ferrell, 1993), administrative sciences (Ross and Hall, 1980), management sciences (Eden, 1979; Diffenbach, 1993; Smithin and Sims, 1982), game theory (Klein and Cooper, 1982), information analysis (Montazemi and Conrath, 1986), popular political developments (Taber and Siegel, 1987; Taber, 1991), electrical circuits analyze (Styblinski and Meyer, 1988) distributed group decision support (Zhang and Chen, 1988; Zhang et al., 1992; Zhang, 1996) and adaptation and learning (Kosko, 1986; Kosko, 1988; Kosko, 1992). Some of those studies were based on crisp *CMs* (Axelrod, 1976; Eden, 1979; Diffenbach, 1993; Smithin and Sims, 1982; Klein and Cooper, 1982; Montazemi and Conrath, 1986; Buede and Ferrell, 1993) and others on fuzzy *CMs* (Kosko, 1986; Kosko, 1988; Kosko, 1992; Zhang and Chen, 1988; Zhang et al., 1992; Zhang, 1996). In the case of crisp *CMs*, what matters is whether, the causal effects are $+$, negative $-$ or 0 and the relative strengths of those causal relations are ignored. The idea of Fuzzy *CMs* was introduced by Kosko (Kosko, 1986) who introduced causal algebra operating in the range of $[0, 1]$ for propagating causality on a fuzzy cognitive map. Due to the limited range of fuzzy numbers, Kosko proposed to convert negative influences into positive ones by using the idea of *dis-concepts*.In this context, Kosko developed a fuzzy causal map and introduced a fuzzy causal algebra operating in the range of $[0, 1]$ for propagating causality. However, due the limited range of fuzzy number, negative influences were converted to positive ones, with the same absolute values, by using dis-concepts or dis-factors. This is based on the following fact:

$$\text{Replace every } v_i \xrightarrow{-} v_j \text{ with } v_i \xrightarrow{+} \sim v_j$$

Although this solution is attractive, doubling the size of the concept set may increase computation time and space to unacceptable levels, particularly for the large *CMs*. Moreover, the author did not give a semantics for the fuzzy causal values such as *none*, *some*, etc.

In the same context, Zhang and his colleagues proposed a system called POOL2 (Zhang and Chen, 1988) which is a generic system for fuzzy cognitive map development and decision analysis. This system uses an approach in which both negative and positive assertions are weighted and kept separately based on the negative-positive-neutral (NPN) interval $[-1, 1]$ instead of values in $[0, 1]$. Later, the same team proposed the D-POOL system (Zhang et al., 1992). This system is based on NPN logics and NPN relations and strives for a cooperative or compromised solution between cognitive maps (from relevant agents) through coherent communication and perspective sharing. Finally, the NPN causal inference has been also used by Park (Park, 1995) to study a fuzzy time cognitive map with time lag on each arrow. The author developed a method of translating the fuzzy *CM* that has different time lags into a fuzzy cognitive map having the same unit-time lag.

Notice that the NPN approach is a particular technique for associating numbers or intervals with edges on directed graphs. Consequently, fuzzy *CMs* stemmed from this approach are not really qualitative models, but rather quantitative models where quantities are combined by propagation along paths. In other words, the interpretation adopted by Zhang and his colleagues is based on fuzzy interval calculus which has no semantic account in terms of fundamental concept (Wellman, 1994). In fact, the definition of a precise semantic interpretation of qualitative causality has received very little attention since all approaches to *CMs* were based on simple inference mechanisms in order to give rise to a qualitative calculus about the consequences of a *CM*. The only work that we are aware of in this context is Wellman's approach (Wellman, 1994). This author used an approach based on graphical dependency models (the Bayesian Networks), for probabilistic reasoning, and sign algebras, for qualitative reasoning. This type of approach is usually used in AI and is only applicable in the acyclic case (i.e., a graph with no cycles) (Wellman, 1994). As stated previously however, the acyclic case does not reflect realistic systems since these systems contain often cycles and feedbacks.

In this paper, we propose an alternative approach based on relation algebra and which takes into account the cyclic case. Precisely, we use propagation-based inference procedures, based on relation algebra, to derive relations among arbitrary connected concepts.

## 3 A Relational Theory of Causal Maps

The following description of classical cognitive maps comes mostly from (Axelrod, 1976) and (Nakumara et al., 1982). Generally, causal links (causal relations) between two concepts $v_i$ and $v_j$ have one of the eight values indicated in Table 1.

TABLE 1

CAUSAL LINKS IN A CAUSAL MAP

| Causal Relations | Descriptions |
|---|---|
| $v_i \xrightarrow{+} v_j$ | $v_i$ facilitates $v_j$, $v_i$ helps $v_j$, $v_i$ promotes $v_j$, etc. |
| $v_i \xrightarrow{-} v_j$ | $v_i$ hinders $v_j$, $v_i$ hurts $v_j$, $v_i$ prevents $v_j$, $v_i$ is harmful to $v_j$, etc. |
| $v_i \xrightarrow{0} v_j$ | $v_i$ has no effect on $v_j$, $v_i$ does not matter or is neutral to $v_j$, etc. Generally, this relation is not depicted in cognitive maps. |
| $v_i \xrightarrow{\oplus} v_j$ | $v_i$ does not retard $v_j$, $v_i$ does not hurt $v_j$, $v_i$ does not prevent $v_j$, etc. |
| $v_i \xrightarrow{\ominus} v_j$ | $v_i$ does not promote $v_j$, $v_i$ does not help $v_j$, $v_i$ is of no benefit to $v_j$, etc. |
| $v_i \xrightarrow{\pm} v_j$ | $v_i$ affects in some non-zero way $v_j$, etc. |
| $v_i \xrightarrow{?} v_j$ | $+$, $-$, and 0 can exist between variables $v_i$ and $v_j$. |
| $v_i \xrightarrow{a} v_j$ | Conflicting assertions about the same relation have been made, this relation is called *ambivalent*. |

These causal relations are used to build cognitive maps, defined as follows.

**Definition 1** A *causal map* $CM := (C, A)$ is a directed graph that represents an individual's (i.e., an agent, a group of agents or an organization) assertions about its beliefs with respect to its environment. The components of this graph are a set of points $C$ (the vertices) and a set of arrows $A$ (the edges) between these points. The arrows are labeled by elements of the set $\mathcal{C} := \{a, +, -, 0, \oplus, \ominus, \pm, ?\}$. A point represents a *concept* (also called *concept variable* in the sequel), which may be a goal or an action option of any agent. It can also represent the utility of any agent or the utility of a group or an organization, or any other concept appropriate to multiagent reasoning. An arrow represents a causal relation between concepts, that is, it represents a causal assertion of how one concept variable affects another. The concept variable at the origin of an arrow is called a *cause variable* and that at the end point of the arrow is called the *effect variable*. A *path* from variable $v_1$ to variable $v_n$ is a sequence of points $v_1, v_2, \ldots, v_n$, together with the non-zero arrows (i.e., arrows labeled by a relation different from 0) $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n$. A path is *trivial* if it consists of a single point. The *valency matrix* $V$ of a causal map $M$ is a square matrix of size $n$, where $n$ is the number of concepts in $M$. The entry $V_{ij}$ is the label of the arrow from $v_i$ to $v_j$ in $M$. If there is no such arrow, then of the arrow from $v_i$ to $v_j$ in $M$. If there is no such arrow, then $V_{ij} = 0$. ∎

Before detailing our model, we present a summary of the classical theory of *CMs*.

## 3.1 Classical Theory of *CMs*

Four operators are defined on the set $\mathcal{C}$ of causal relations. They are *union* ($\cup$), *intersection* ($\cap$), *sum* ($|$) and *multiplication* ($*$). The laws of union and intersection are obtained by considering $a, +, -, 0, \oplus, \ominus, \pm$ and ? as shorthands for $\{\}, \{+\}, \{-\}, \{0\}, \{0,+\}, \{0,-\}, \{+,-\}$ and $\{+,0,-\}$, respectively. Thus, one has (Axelrod, 1976; Nakumara et al., 1982)

$$\begin{aligned} \oplus &= 0 \cup +, & \ominus &= 0 \cup -, \\ \pm &= + \cup -, & ? &= 0 \cup + \cup -, \\ a &= + \cap 0 = + \cap - = 0 \cap -. \end{aligned} \tag{1}$$

It can be seen that $a$ denotes conflicting assertions about a given link. Although not stated explicitly in (Nakumara et al., 1982), the law

$$\forall x : a \cup x = x \tag{2}$$

follows from the author's considerations.

The laws of sum are given below on the left (with "do" meaning "distributes over"). From these laws, one can deduce a table giving the result of the application of sum to any pair of causal relations.

$$\begin{aligned} &\forall\, x, y \in \mathcal{C}, \\ &\text{(a)} \quad 0|y = y, \\ &\text{(b)} \quad a|y = a, \\ &\text{(c)} \quad y|y = y, \\ &\text{(d)} \quad +|- = ?, \\ &\text{(e)} \quad | \text{ do } \cup, \\ &\text{(f)} \quad x|y = y|x. \end{aligned} \tag{3}$$

| $\mid$ | $a$ | $0$ | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | ? |
|---|---|---|---|---|---|---|---|---|
| $a$ | $a$ | $a$ | $a$ | $a$ | $a$ | $a$ | $a$ | $a$ |
| $0$ | $a$ | $0$ | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | ? |
| $+$ | $a$ | $+$ | $+$ | ? | $+$ | ? | ? | ? |
| $-$ | $a$ | $-$ | ? | $-$ | ? | $-$ | ? | ? |
| $\oplus$ | $a$ | $\oplus$ | $+$ | ? | $\oplus$ | ? | ? | ? |
| $\ominus$ | $a$ | $\ominus$ | ? | $-$ | ? | $\ominus$ | ? | ? |
| $\pm$ | $a$ | $\pm$ | ? | ? | ? | ? | $\pm$ | ? |
| ? | $a$ | ? | ? | ? | ? | ? | ? | ? |

The laws of multiplication are given below on the left. On the right is the table deduced from these laws.

For any $x, y \in \mathcal{C}$,

(a) $+ * y = y$,

(b) $0 * y = 0$,

(c) $a * y = a$ if $y \neq 0$,

(d) $- * - = +$,

(e) $*$ do $\cup$,

(f) $x * y = y * x$.

| $*$ | $a$ | $0$ | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | $?$ |
|---|---|---|---|---|---|---|---|---|
| $a$ | $a$ | $0$ | $a$ | $a$ | | | $a$ | |
| $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $+$ | $a$ | $0$ | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | $?$ |
| $-$ | $a$ | $0$ | $-$ | $+$ | $\ominus$ | $\oplus$ | $\pm$ | $?$ |
| $\oplus$ | | $0$ | $\oplus$ | $\ominus$ | $\oplus$ | $\ominus$ | $?$ | $?$ |
| $\ominus$ | | $0$ | $\ominus$ | $\oplus$ | $\ominus$ | $\oplus$ | $?$ | $?$ |
| $\pm$ | $a$ | $0$ | $\pm$ | $\pm$ | $?$ | $?$ | $\pm$ | $?$ |
| $?$ | | $0$ | $?$ | $?$ | $?$ | $?$ | $?$ | $?$ |

(4)

Some entries are not given, because the laws of $*$ lead to contradictory results. For instance, according to law (4c), $a * \oplus = a$. But, using laws (4b,4c,4e,4f) and Equations (1) and (2), one gets $a * \oplus \;=\; a * (0 \cup +) \;=\; a * 0 \cup a * + \;=\; 0 \cup a \;=\; 0$. In other words, the system is not consistent.

Despite this major drawback, let us explain what these operations are intended for. Multiplication is used to calculate *indirect effects*. For instance, from $v_i \xrightarrow{-} v_j \xrightarrow{-} v_k$, there is an indirect effect $v_i \xrightarrow{+} v_k$ (Equation (4d) is $- * - = +$). Given the set $\mathcal{C}$ of causal relations and their interpretation given in Table 1, the six rules of multiplication seem rather reasonable. For example, rule (4b) says that, if $v_i$ has no effect on $v_j$, it is natural that $v_i$ has no indirect effect on $v_k$ through $v_j$ ($0 * y = 0$), no matter what the effect $y$ of $v_j$ on $v_k$ is. Rule (4c) says that, if the effect from $v_i$ to $v_j$ is ambivalent, the indirect effect from $v_i$ to $v_k$ through $v_j$ is also ambivalent ($a * y = a$), even if the effect from $v_j$ to $v_k$ is not ambivalent. Rule (4f) states that $*$ is commutative.

The sum operator is used to accumulate indirect effects from different paths. For example, if there is a path from $v_i$ to $v_j$ with indirect effect $+$ and another path with indirect effect $-$, the net effect is $?$, according to law (3d). Nakumara *et al.* (Nakumara et al., 1982) consider all the rules of Equation (3) easily acceptable, except for rule (3b); they do not justify this statement, though.

The operators $*$ and $|$ can be lifted to matrices. Assume that $V$ and $W$ are square matrices of size $n$. Addition and multiplication of matrices are defined as follows:

$$(V|W)_{ij} \;=\; V_{ij}|W_{ij}, \tag{5}$$

$$(V * W)_{ij} \;=\; (V_{i1} * W_{1j})|\cdots|(V_{in} * W_{nj}). \tag{6}$$

The $n^{th}$ *power* of a square matrix $V$, for $n > 0$, is then naturally defined by

$$V^1 := V \text{ and } V^n := V * V^{n-1}. \tag{7}$$

The total effect of one concept on another is calculated according to the following definition.

**Definition 2** The *total effect* of variable $v_i$ on variable $v_j$ is the sum of the indirect effects of all paths from $v_i$ to $v_j$. Let $V$ be the valency matrix of a causal map. The *total effect matrix* $V_t$ is the matrix that has as its $ij^{th}$ entry the total effect of $v_i$ on $v_j$. That is, $V_t = V|V^2|V^3|\ldots$.

It is easy to check that the sum operator is $\subseteq$-monotonic. This implies that there is a $k$ such that $V_t = V|V^2|\cdots|V^k$. ■

In summary, it is important to notice that the classical view of cognitive maps is an intuitive view with ad-hoc rules to calculate direct and indirect effects. Furthermore, there is no a precise meaning of the primitive concepts, neither a sound formal treatment of relations between concepts. Finally, as we previously shown, the proposed model is not consistent. These considerations brought us to develop the formal model presented in the next section.

### 3.2 A Relational Model of Causal Maps

### 3.3 Relations and Relation Algebra

The mathematical definition of a relation in terms of set theory is the following.

**Definition 3** A relation $R$ on a set $F$ is a subset of the Cartesian product $F \times F$. Elements $x, y \in F$ are said to be in relation $R$ when $(x, y) \in R$. ■

We conventionally employ the symbols $(\vee, \wedge, \rightarrow)$ for *conjunction*, *disjunction* and *implication* between predicates and truth values. We use $(\cup, \cap, \subseteq)$ for the *union*, *intersection* and *inclusion* of sets. Finally, we use $(\sqcup, \sqcap, \sqsubseteq)$ to denote *union*, *intersection* and inclusion of relations. Other symbols used in this text are: $\Longrightarrow, \Longleftrightarrow$ are metalevel implication and equivalence, respectively, $:=$ is definitional equality and $:\Longleftrightarrow$ is definitional equivalence.

Notice that relations are sets, and consequently we can consider their intersection, union, complementation and inclusion. What follows is a definition of some of the usual operations on relations.

**Definition 4** Let $R, S \sqsubseteq F \times F$. The basic operations on relations are:

1. union $\quad R \sqcup S := \{(x, y) \mid (x, y) \in R \vee (x, y) \in S\}$,
2. intersection $\quad R \sqcap S := \{(x, y) \mid (x, y) \in R \wedge (x, y) \in S\}$,
3. complement $\quad \overline{R} := \{(x, y) \mid (x, y) \notin R\}$,
4. product, composition $\quad R \circ S := \{(x, z) \mid \exists y \in F : (x, y) \in R \wedge (y, z) \in S\}$,

5. converse, transposition $\quad R^\top := \{(x,y) \mid (y,x) \in R\}$,
6. empty $\quad O := \{(x,y) \mid \text{false}\} \sqsubseteq F \times F$,
7. universal $\quad L := \{(x,y) \mid \text{true}\} = F \times F$,
8. identity $\quad I := \{(x,y) \mid x = y\} \sqsubseteq F \times F$,
9. power $\quad R^0 := I$, and $R^n = R \circ R^{n-1}$ if $n > 0$,
10. inclusion $\quad R \sqsubseteq S :\Longleftrightarrow \forall x, y : [(x,y) \in R \to (x,y) \in S]$. ■

*Priority of operations:* The unary operations ($^\top$, $^-$) are performed first, followed by the binary operation ($\circ$), and finally by the binary operations ($\sqcup$, $\sqcap$).

A finite relation $R$ can be represented by a Boolean matrix, using the convention $R_{xy} = 1 \Longleftrightarrow (x,y) \in R$ and $R_{xy} = 0 \Longleftrightarrow (x,y) \notin R$. The definition of the relational operators for Boolean matrices follows: we use $\wedge$ and $\vee$ as operators on the set $\{0,1\}$, considered as a set of truth values in the usual way.

$$\begin{array}{lll} (V \sqcup V')_{ik} := V_{ik} \vee V'_{ik}, & (\overline{V})_{ik} := \neg V_{ik} \text{ (negation)}, & (V \circ V')_{ik} := \bigvee_{j=1}^{n} V_{ij} \wedge V'_{jk}. \\ (V \sqcap V')_{ik} := V_{ik} \wedge V'_{ik}, & (V^\top)_{ik} := V_{ki}, & \end{array} \tag{8}$$

Thus, for example, if $F = \{a, b\}$, we have

$$O = \begin{array}{c c} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} a \\ b \end{array} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{array}, \quad L = \begin{array}{c c} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} a \\ b \end{array} & \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^\top = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

(this example also shows that the labels of rows and columns may be explicit or implicit).

Relations over a set, Boolean matrices and some types of graphs are instances of a more abstract concept, called *relation algebra*. A relation algebra is itself an extension of a Boolean algebra, which is a more familiar concept.

**Definition 5** A *Boolean algebra* is an algebra of the form $(\mathcal{B}, \sqcup, \sqcap, \overline{\ }, O, L)$, which satisfies the identities (Ladkin and Maddux, 1994; Schmidt and Ströhlein, 1993):

$$\begin{array}{lll} (Q \sqcup R) \sqcup S = Q \sqcup (R \sqcup S), & (Q \sqcap R) \sqcup Q = Q, & R \sqcup \overline{R} = L, \\ (Q \sqcap R) \sqcap S = Q \sqcap (R \sqcap S), & (Q \sqcup R) \sqcap Q = Q, & R \sqcap \overline{R} = O. \\ R \sqcup S = S \sqcup R, & (Q \sqcap R) \sqcup S = (Q \sqcup S) \sqcap (R \sqcup S), & \\ R \sqcap S = S \sqcap R, & (Q \sqcup R) \sqcap S = (Q \sqcap S) \sqcup (R \sqcap S), & \end{array}$$

■

From this definition, other familiar laws can be derived (see (Ladkin and Maddux, 1994) for details) The partial ordering $\sqsubseteq$ on a Boolean algebra is defined by $R \sqsubseteq S \Longleftrightarrow R \sqcap S = R$.

**Definition 6** A structure $\mathfrak{A}$ is a *relation algebra* iff (Ladkin and Maddux, 1994; Schmidt and Ströhlein, 1993)

$$\mathfrak{A} = (A, \sqcup, \sqcap, \overline{\phantom{x}}, O, L, \circ, {}^\top, I),$$

where $(A, \sqcup, \sqcap, ^-, O, L)$ is a Boolean algebra (called the *reduct* of $\mathcal{R}$), $\circ$ is a binary operation, $^\top$ is a unary operation, $I \in A$, and the following identities hold:

$$(Q \circ R) \circ S = Q \circ (R \circ S), \qquad (R \sqcup S)^\top = R^\top \sqcup S^\top,$$
$$(Q \sqcup R) \circ S = Q \circ S \sqcup R \circ S, \qquad (R \circ S)^\top = S^\top \circ R^\top,$$
$$R \circ I = R = I \circ R, \qquad R^\top \circ \overline{R \circ S} \sqcap S = O.$$
$$R^{\top^\top} = R,$$

■

We refer to $L$ as the *Boolean unit* of $\mathfrak{A}$, and to $I$ as its *identity element*. Here are some theorems that can be derived from these axioms:

$$\overline{R}^\top = \overline{R^\top}, \qquad (R \sqcap S)^\top = R^\top \sqcap S^\top,$$
$$I^\top = I, \qquad Q \circ (R \sqcap S) \sqsubseteq Q \circ R \sqcap Q \circ S,$$
$$O^\top = O, \qquad Q \sqsubseteq R \Longrightarrow Q^\top \sqsubseteq R^\top,$$
$$L^\top = L, \qquad Q \sqsubseteq R \Longrightarrow Q \circ S \sqsubseteq R \circ S,$$
$$O \circ R = R \circ O = O, \qquad Q \sqsubseteq R \Longrightarrow S \circ Q \sqsubseteq S \circ R.$$

Schmidt and Ströhlein (Schmidt and Ströhlein, 1993) mention that the set of all $n \times n$ matrices with coefficients from a homogeneous relation algebra again form a relation algebra, with the relational operators on these matrices defined as follows:

$$\begin{array}{lll} (V \sqcup V')_{ik} := V_{ik} \sqcup V'_{ik}, & (\overline{V})_{ik} := \overline{V_{ik}}, & (V \circ V')_{ik} := \bigsqcup_{j=1}^{n} V_{ij} \circ V'_{jk}. \\ (V \sqcap V')_{ik} := V_{ik} \sqcap V'_{ik}, & (V^\top)_{ik} := (V_{ki})^\top, & \end{array} \tag{9}$$

Let us show intuitively why it works. Consider the matrices below. The left one is a $4 \times 4$ Boolean matrix. The middle one is the same matrix, but with groupings of rows and columns; the result is a $2 \times 2$ matrix whose entries are $2 \times 2$ Boolean matrices, that is, a matrix with four entries that are relations. The right one corresponds to the middle one: each submatrix is simply replaced by a relation identifier in the obvious way. It is easy to see that applying the operations described in Equation (8) to $4 \times 4$ matrices such as the left one gives the same result as applying the operations of Equation (9) to matrices such as the right one.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \left(\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \qquad \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

Let us call a matrix whose entries are relations a *relational matrix*. In the same manner than a valency matrix can be associated to a causal map (see Definition 1), one can associate to a relational matrix a graph with arrows labeled by relations (we call such a map a *relational causal map*). Thus, if we had relations modeling the classical causal relations ($+, -, \ominus$, etc.), and relational operations modeling the operations on classical causal relations ($\cup, \cap, |, *$), we would have a relational model of cognitive maps. Now, in view of the contradiction brought to light in Section 3.1, this is an impossible task. Hence, our goal is rather to find an alternative relational description of cognitive maps, while trying to keep as much as possible of the flavor of classical causal maps. Another goal is to introduce a flexible model that can easily be extended if additional precision is required, rather than being stuck with a small set of causal relations. This model is presented in the following section.

### 3.3.1 The Relation Algebra of Causal Maps

Let $\Delta := \{-1, 0, 1\}$. Numbers in $\Delta$ are intended to represent changes (variations) in a concept variable, with $-1, 0, 1$ denoting decrease, stability and increase, respectively. How these variations are measured and what exactly is varying does not concern us here; it could be, e.g., the utility of a variable, an amount of something, etc. Next, we define the relations $+, 0, -$ on the set $\Delta$.

These are

$$
+ := \begin{array}{c} \\ 1 \\ 0 \\ -1 \end{array}
\begin{array}{c} \begin{array}{ccc} 1 & 0 & -1 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}
\qquad
0 := \begin{array}{c} \\ 1 \\ 0 \\ -1 \end{array}
\begin{array}{c} \begin{array}{ccc} 1 & 0 & -1 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \end{array}
\qquad
- := \begin{array}{c} \\ 1 \\ 0 \\ -1 \end{array}
\begin{array}{c} \begin{array}{ccc} 1 & 0 & -1 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array}
$$

Recall that these relations are used to label arrows of a relational causal map, thus linking cause variables to effect variables. Consider relation $+$. It is interpreted as follows: an increase in the cause variable causes an increase in the effect variable, a decrease in the cause variable causes a decrease in the effect variable, and stability of the cause variable promotes stability of the effect variable (note that $+$ is just the identity relation and could be written $I$). Relation 0 says that the cause variable promotes stability of the effect variable, no matter how it changes. Relation $-$ is interpreted similarly.

We now use the primary relations $+, -$ and 0 to define $\oplus, \ominus, \pm, ?$ and $!$.

$$\oplus := 0 \sqcup +, \quad \ominus := 0 \sqcup -, \quad \pm := + \sqcup -, \quad ? := + \sqcup - \sqcup 0, \quad ! := + \sqcap -.$$

Before discussing further the interpretation of the various relations (in particular $O, 0$ and $!$), we present the tables showing the result of the application of the relational operators $\sqcup, \sqcap, \circ$ to the above relations.

These tables are constructed by using Equation (8). Thus, for instance, it is easy to verify that $- \circ - = +$ by multiplying the matrix representing $-$ by itself.

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \circ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

TABLE 2

Table for $\sqcup$

| $\sqcup$ | $O$ | 0 | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | ? | ! |
|---|---|---|---|---|---|---|---|---|---|
| $O$ | $O$ | 0 | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | ? | ! |
| 0 | 0 | 0 | $\oplus$ | $\ominus$ | $\oplus$ | $\ominus$ | ? | ? | 0 |
| $+$ | $+$ | $\oplus$ | $+$ | $\pm$ | $\oplus$ | ? | $\pm$ | ? | $+$ |
| $-$ | $-$ | $\ominus$ | $\pm$ | $-$ | ? | $\ominus$ | $\pm$ | ? | $-$ |
| $\oplus$ | $\oplus$ | $\oplus$ | $\oplus$ | ? | $\oplus$ | ? | ? | ? | $\oplus$ |
| $\ominus$ | $\ominus$ | $\ominus$ | ? | $\ominus$ | ? | $\ominus$ | ? | ? | $\ominus$ |
| $\pm$ | $\pm$ | ? | $\pm$ | $\pm$ | ? | ? | $\pm$ | ? | $\pm$ |
| ? | ? | ? | ? | ? | ? | ? | ? | ? | ? |
| ! | ! | 0 | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | ? | ! |

TABLE 3

Table for $\circ$

| $\circ$ | $O$ | 0 | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | ? | ! |
|---|---|---|---|---|---|---|---|---|---|
| $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ |
| 0 | $O$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $+$ | $O$ | 0 | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | ? | ! |
| $-$ | $O$ | 0 | $-$ | $+$ | $\ominus$ | $\oplus$ | $\pm$ | ? | ! |
| $\oplus$ | $O$ | 0 | $\oplus$ | $\ominus$ | $\oplus$ | $\ominus$ | ? | ? | 0 |
| $\ominus$ | $O$ | 0 | $\ominus$ | $\oplus$ | $\ominus$ | $\oplus$ | ? | ? | 0 |
| $\pm$ | $O$ | 0 | $\pm$ | $\pm$ | ? | ? | $\pm$ | ? | ! |
| ? | $O$ | 0 | ? | ? | ? | ? | ? | ? | 0 |
| ! | $O$ | ! | ! | ! | ! | ! | ! | ! | ! |

TABLE 4

TABLE FOR $\sqcap$

| $\sqcap$ | $O$ | $0$ | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | $?$ | $!$ |
|---|---|---|---|---|---|---|---|---|---|
| $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ | $O$ |
| $0$ | $O$ | $0$ | $!$ | $!$ | $0$ | $0$ | $!$ | $0$ | $!$ |
| $+$ | $O$ | $!$ | $+$ | $!$ | $+$ | $!$ | $+$ | $+$ | $!$ |
| $-$ | $O$ | $!$ | $!$ | $-$ | $!$ | $-$ | $-$ | $-$ | $!$ |
| $\oplus$ | $O$ | $0$ | $+$ | $!$ | $\oplus$ | $0$ | $+$ | $\oplus$ | $!$ |
| $\ominus$ | $O$ | $0$ | $!$ | $-$ | $0$ | $\ominus$ | $-$ | $\ominus$ | $!$ |
| $\pm$ | $O$ | $!$ | $+$ | $-$ | $+$ | $-$ | $\pm$ | $\pm$ | $!$ |
| $?$ | $O$ | $0$ | $+$ | $-$ | $\oplus$ | $\ominus$ | $\pm$ | $?$ | $!$ |
| $!$ | $O$ | $!$ | $!$ | $!$ | $!$ | $!$ | $!$ | $!$ | $!$ |

The relational composition operation ($\circ$) corresponds to the multiplication operation ($*$) of classical cognitive maps. Comparing Table 3 and the table in Equation (4), we see that that, with the exception of the classical $a$ and the relational !, there is an exact correspondence. Also, the classical 0 corresponds to both the relational $O$ and the relational 0. Although the status of the classical $a$ is not clear, the fact that it is interpreted by the empty set (see Section 3.1) leads us to compare it to the relation $O$; the match is not too bad.

The relational union operation ($\sqcup$) has similarities with both classical union ($\cup$) and classical sum ($|$). For example, assuming that the classical 0 corresponds to the relational 0, we have the classical law $0 \cup + = \oplus$ and the relational law $0 \sqcup + = \oplus$. Assuming that the classical 0 corresponds to the relational $O$, we have the classical law $+|0 = +$ and the relational law $+ \sqcup O = +$. The most conspicuous divergence concerns the classical $a$. The law $a|y = a$ means that it is not possible to weaken any contradiction; contradictions propagate in the calculation of the total effect, because of laws (3b) and (4c). As we indicated in Section 3.1, Nakumara *et al.* (Nakumara et al., 1982) find it difficult to accept $a|y = a$. In our case, no relation plays the role of $a$.

In our approach, the empty relation $O$ is used to denote "unrelatedness" or "ambivalence". Asserting that there is no relationship between a cause variable and an effect variable is just the same as making a contradictory assertion about this relationship. Another way to realize that $O$ corresponds to ambivalence is to "move" from $L$ to $O$ by adding information. The universal relation $L$ indicates that a variation of the cause variable can cause any variation of the effect variable (increase, decrease or no variation); this is complete uncertainty. Adding information, one goes from $L$ through, e.g., $?, \pm, +$. The relation $+$ represents perfect information: any variation of the cause variable is related to a single variation of the effect variable. Adding more information (too much information, contradictory information), one then goes through ! to reach $O$. With respect to the representation of relational maps by matrices, the absence of an arrow between two concepts in the graph is represented by $O$ in the corresponding relational matrix.

Note that we distinguish between the two relationships $O$ and 0 since, in our model, 0 indicates that the relationship between two concepts *exists* and is "neutral" Also, the relation $! = + \sqcap - = + \sqcap 0 = - \sqcap 0$ is partially ambivalent (somewhat less than $O$); it is a weak ambivalent relation.

There are at least two ways to obtain a relation algebra $\mathfrak{A} = (A, \sqcup, \sqcap, \bar{\ }, O, L, \circ, ^\top, +)$ from the set of relations $\{+, 0, -\}$. One way is to take for $A$ the full set of relations over the set $\Delta$ (the full set of $3 \times 3$ matrices). This gives $2^9 = 512$ relations. The other way is to take for $A$ the closure of $\{+, 0, -\}$ under the five relational operations; the result is a set of 32 relations, whose atoms (minimal non-$O$ relations) are

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(these could be taken as primitive relations instead of $+, 0, -$).

A causal map $CM$ can then be built just as in Section 3.1, using relations in $A$ to label the arrows. Alternatively, one can construct the associated (relational) valency matrix $V$. Indirect effects of length $k$ are given by the $k$-th power of $V$, $V^k$. Indirect effects are added by means of $\sqcup$. The total effect matrix is the transitive closure of $V$, which is

$$V^+ := \bigsqcup_{k>0} V^k \tag{10}$$

In fact, $V^+$ corresponds to the matrix $V_t$ of Definition 2 (we use $V^+$ rather than $V_t$ because it is the standard notation for transitive closure). A $n \times n$ matrix whose entries are $3 \times 3$ matrices is a $3n \times 3n$ matrix. This implies that $V^+ = \bigsqcup_{k=0}^{3n-1} V^k$ (see (Schmidt and Ströhlein, 1993)). It also means that $V^+$ can be computed in $O((3n)^3) = O(n^3)$ steps using the Roy-Warshall algorithm; better algorithms also exist (Schmidt and Ströhlein, 1993). Furthermore, large systems with few interconnections may use space-efficient representations (the implementation need not use the matrix view). Hence the approach developed here can be used beyond a few agents and a few concepts.

### 3.3.2 Discussion

Causal maps were originally proposed to capture the qualitative causal relationships that exist between concepts in a structure of decision. Some *intuitive* inference mechanisms, based on reasoning from cause to effect, were proposed (Axelrod, 1976; Nakumara et al., 1982). In this section, we have defined a precise semantic interpretation of qualitative causality in terms of relation algebra, to justify these intuitive inference mechanisms. Indeed, as discussed in the previous subsection, our model justifies most of the rules proposed by Nakumara *et al.* As we have pointed out, the main difference concerns ambivalence. Another difference is our law $+ \sqcup - = \pm$ versus $+|- = ?$. According to (Nakumara et al., 1982), $+|- = ?$ says that the sum of $+$ and $-$ depends on which of the indirect effects $+$ or $-$ is stronger; the rule expresses that the total effect is plus $(+)$ if the indirect effect $+$ is stronger than the indirect effect $-$, is minus $(-)$ if $+$ is weaker than $-$, and is zero $(0)$ if $+$ is as strong as $-$. It may be argued, however, that in a qualitative approach it is difficult to know how a relation can be as strong or stronger than another. It seems more reasonable to retain $+$ and $-$ for further reasoning. Our model takes this point of view into account by having $+ \sqcup - = \pm$.

It is also important to point out that, contrary to other models of cognitive maps, our model distinguishes between the "unrelated" relation (i.e., $O$) and the "neutral" relation (i.e., $0$). The first relation expresses that there is no relation between two concepts, whereas the second relation indicates that one concept has a neutral relation to another.

Our model also takes into account nonreversible causation, contrary to classical models of cognitive maps. An example of nonreversible causation is "an increase in $v_i$ causes an increase in $v_j$, but a decrease in $v_i$ does not cause a decrease in $v_j$". For instance, the normal interpretation of "smoking causes illnesses" involves nonreversible causation, because stopping smoking does not put out illnesses. In classical *CM* theory, only reversible causation is allowed, because, e.g., $v_i \xrightarrow{+} v_j$ is taken to mean *both* that "an increase in $v_i$ causes an increase in $v_j$" *and* "a decrease in $v_i$ causes a decrease in $v_j$". In our model, reversible causation can be expressed by choosing the appropriate relation among the set of 512 possible relations; for example, $\{(1,1),(0,0),(-1,0),(-1,1)\}$ expresses that an increase in the cause variable causes an increase in the effect variable; it also says that a decrease in the cause variable causes anything but a decrease in the effect variable.

The $\sqcap$ operation is used to combine relations when they are asserted together. Suppose, for instance, that an agent $A$ wants to produce a matrix $V$ by combining the matrices $V_1$ and $V_2$ transmitted by two other agents $A_1$ and $A_2$. If $A$ considers the information sent by $A_1$ and $A_2$ to be reliable, then she should define $V := V_1 \sqcap V_2$; the result might be that some concepts become related by ambivalent relations $(O, !)$. On the other hand, if $A$ considers both $V_1$ and $V_2$ to be possible (e.g., they represent a range of opinions), then she should define $V := V_1 \sqcup V_2$; the result is fuzzier information than that of either $V_1$ or $V_2$.

We have just mentioned that many more relations can be asserted than the few causal relations of the classical theory. There are also two new operations, complementation and converse, which still have to be exploited. Complementation allows to say that the relationship between two concepts is, e.g., anything but 0 (expressed by $\overline{0}$). Converse allows talking about "backward causality" (consequence).

An important advantage of the model is that it can be easily contracted or extended, by starting with a different set $\Delta$. Choosing $\Delta := \{-1, 1\}$ results in a smaller model. Choosing $\Delta := \{-2, -1, 0, 1, 2\}$ gives a larger model, in which finer distinctions can be made; for example, it becomes possible to say that a large increase (2) in the cause variable causes a small decrease ($-1$) in the effect variable.

# 4 Implementation and Application to Multiagent Environments

## 4.1 The $\mathcal{SR}\circ\Psi lab$ Tool

The crisp causal reasoning model presented in this paper has been implemented in a system used as a computational tool supporting the relational manipulations. This tool is called $\mathcal{SR}\circ\Psi lab$ (Caron, 1996) and is built over the Ψlab software[1], a freeware package developed by INRIA, France. This tool enables users 1) to edit matrices about relations, 2) store matrices in the working memory, 3) execute algebraic operations on matrices and 4) calculate the total effect matrix as precised by Equation 10. Any session begins by presenting a matrix called "working copy" which is displayed on the screen for editing. Using this matrix allows users to represent relations like $+$, $-$, etc. A whole set of matrices can be kept in the working session to allow any combination of relations.

With this tool, we are investigating the causal reasoning in multiagent environments (Chaib-draa, 1997). Causal reasoning is important in multiagent environments because it allows to model interrelationships or causalities among a set of individual and social concepts. This provides a foundation to 1) test a model about the prediction of how agents will respond to expected (or not) events; 2) explain how agents have done specific actions; 3) make a decision in a distributed environment; 4) analyze and compare the agents' causal representations. All these aspects are important for coordination, conflict solving and the emergence of cooperation between agents.

## 4.2 Reasoning about Changes in an Organization of Agents

Weick (Weick, 1969) suggested to change the prevalent static view of an organization of agents to a dynamic view which is sustained by *change*. Precisely, he proposed that organization and change were two sides of the same social phenomena. His reasoning was that an organization is a process of co-evolution of agents'

[1] This software can be obtained by anonymous ftp from "ftp.inria.fr:/INRIA/Scilab".

perceptions, cognitions and actions. In this context, Weick proposed a theory of organization and change based on the graphs of loops in evolving social systems. Recently, additional investigation guided by this approach (Bougon and Komocar, 1990) tried to articulate how cognitive maps provide a way to identify the loops that produce and control an organization.

In multiagent systems, the study of an organization of agents has generally focused on some structural models such as (Moulin and Chaib-draa, 1996): 1) centralized and hierarchical organizations, 2) organizations as authority structure, 3) market-like organizations, 4) organizations as communities with rules of behavior. All these structures missed dynamic aspects and influences that exist in an organization of agents. Generally, dynamic aspects and influences evolve through paths that close on themselves and form loops. We have realized that such loops are important for an organization of agents for two main reasons: i) a change in an organization is the result of deviation amplifying loops, ii) the stability of an organization is the result of deviation countering loops (Bougon and Komocar, 1990).

As an example, consider the organization that binds researchers, grant agencies and qualified personnel and for which, we only consider the three basic relationships $(+, 0^2, -)$ for the sake of simplicity and readability. The causal map representing this organization is shown in Figure 2. The meaning of this *CM* is clear and we shall explain it no more. In this causal map, concepts link together to form loops, some of which are numbered (1) to (7). Loops as (1), (4)–(7), etc., containing an *even* number of "$-$" relations, are *deviation-amplifying* loops. Change in the organization is the result of such loops, because any initial increase (or decrease) in any concept loops back to that concept as an additional increase (or decrease) which, in turn, leads to more increase (or decrease). Thus, in loop (5), an increase in "research quality" improves "researcher satisfaction". Increase in "satisfaction of researchers" allows, in turn, to improve the "retention of the best researchers". Finally, the improvement of "retention of the best researchers" improves "research quality".

Loops as (2) and (3), containing an *odd* number of "$-$" relations, are *deviation-countering* loops (Bougon and Komocar, 1990). The stability of the organization is the result of such loops. In the case of loop (2), for instance, an increase of "resources for research" can lead to an increase of "salaries" which, in turn, reduces the resources allowed to research. If this reduction is not enough to compensate the initial increase of resources, then a residual increase of salaries takes place which, in turn, reduces the resources, and so on, until a balance between the initial increase of resources and salaries is reached. Thus, deviation-countering loops are useful for stabilizing the growth generated in an organization.

Thus we can conceptualize an organization of agents as a "whole" composed of loops of influences. This is a wholistic approach in which the "whole" constrains the concepts and the relationships between them.

[2] 0 represents here the "unrelated" and the "neutral".

By achieving this, we obtain a dynamic system in which deviation-amplifying loops are responsible for change and deviation-countering loops are responsible for stability of the organization. Using these loops, an individual strategist can direct strategic change in the desired direction. This can be done by 1) choosing and changing a loop or 2) choosing and changing a set of loops (Bougon and Komocar, 1990). Our $\mathcal{SR}\circ\Psi lab$ tool can be used in this context to 1) identify the type of loops (deviation-amplifying or deviation countering) and 2) develop a strategic plan to change a wholistic system by changing its loops. Notice that an organization considered as a whole of loops, is represented by its valency matrix in the context of $\mathcal{SR}\circ\Psi lab$. The study of this valency matrix allows one to identify the type of loops. Firstly, if $V_{ii}^{+} = +$, then there exists at least one deviation-amplifying loop through node $i$. Secondly, if $V_{ii}^{+} = -$, then there exists at least one deviation-countering loop through $i$. Strategic changes to a wholistic system can be made by changing a loop or a set of loops (Bougon and Komocar, 1990). Of course, the loop to be changed should be a weak loop which is loosely coupled to the system. Changing a loop (from deviation-amplifying to deviation-countering, or vice-versa), can be done by 1) adding, removing, or replacing a node; 2) changing the label of a link. All those changes can be done by users of $\mathcal{SR}\circ\Psi lab$ tool in an easly way, by editing and manipulating the valency and the total effect matrices.

### 4.3 Disparities between Agents

Another approach that we are investigating concerns the reduction of disparities between agents. In this context, we have considered every individual agent as seeing a situation through an unique set of perceptual filters that reflects its capabilities and its experience, as suggested by the personal construct theory of Kelly (Kelly, 1955). Precisely, we have used causal maps at different levels to represent the subjective views. Thus, first order cognitive maps show the views of individuals (or group of individuals) such as $I$, $J$, $K$, etc. Second order cognitive maps show what agent $X$ thinks agent $Y$ is thinking and vice versa. Third order maps show what agent $X$ thinks agent $Y$ thinks agent $Z$ is thinking and vice versa. Similarly, higher order cognitive maps can be constructed. Here also, our $\mathcal{SR}\circ\Psi lab$ tool provides an inference procedure that allows individuals to reason on others in the context of negotiation, coordination and cooperation between agents. This reasoning can bear on 1) predicting what others can do (this helps in negotiation and coordination between agents), 2) explaining what others have done; 3) trying to demonstrate to others the importance of some area of causal relationships between concepts (this helps in negotiation and mediation between agents), 4) analyze and compare the agents' causal representations. Readers interested by this approach can be refer to (Chaib-draa, 1997).

## 4.4 *CMs* for Representing Qualitative Distributed Decision Making

*CMs* can also help an agent or a group of agents considered as a whole to make a decision. Given a causal map with one or more decision variables and a utility variable, which decision should be taken and which should be rejected? To achieve this, the concerned entity should calculate the total effect (as precised by Equation 10) of each decision on the utility variable. Those decisions that have a positive total effect on utility should be chosen, and decisions that have a negative total effect should be rejected. Decisions with a nonnegative total effect on utility should not be rejected, decisions with a nonpositive total effect should not be accepted. No advice can be given about decisions with an universal, a non-zero, or an ambivalent total effect on utility.

To illustrate the decision-making process in the context of multiagent environments, consider, for example, the causal map of the Professor $P_1$ (considered as an agent) shown in Figure 3. This professor has to choose between two courses $D_1$ and $D_2$ ($D_1$ and $D_2$ are decisions variables). Furthermore, $P_1$ works with a colleague $P_2$ in the same research group (this group is called here $G_{12}$) and shares with her some students. $P_1$'s causal map, shown in Figure 3, includes the following beliefs. $D_1$ favors the theoretical knowledge of $G_{12}$'s students. Greater theoretical knowledge gives a greater motivation to students. Greater motivation of students gives a better quality of research for group $G_{12}$, which gives, in turn, a greater utility of $G_{12}$. $P_2$ gives a course $C_1$ that improves, as $D_1$, the theoretical knowledge of $G_{12}$'s students. This course, however, has the disadvantage to be very hard and this makes $G_{12}$'s students lose their motivation. Finally, the second decision variable $D_2$ is an easy course that decreases the workload of $P_1$. Obviously, decreasing $P_1$'s workload increases her utility.

In this case, how can $P_1$ make her choice between the two courses $D_1$ and $D_2$? Notice that in the context of our example, $P_1$ should reason about other agents (i.e., $P_2$ and $G_{12}$) to make her decision. Under some circumstances, she can also collaborate with them to develop her decision. In this sense, the decision-making process considered here is a multiagent process. To run this process, it might be useful to convert the causal map being analyzed to the form of a valency matrix $V$. With the valency matrix, $P_1$ can calculate indirect paths of length 2 (i.e. $V^2$), 3 (i.e. $V^3$), etc., and the total effect matrix $V^+$ (see Equation 10). In fact, $V^+$ tells $P_1$ how the decision variables $D_1$ and $D_2$ affect her utility and $G_{12}$'s utility. Here also our $\mathcal{SR}\circ\Psi lab$ tool allows one to calculate direct and indirect effects and consequently allows agents to make decisions. As explained previously, each concerned agent should calculate the total effect of each decision on the utility variable. Those decisions that have a positive total effect on utility should be chosen, and decisions that have a negative effect should be rejected. Advice on other total effects can be based on heuristics. Adopting this procedure for the example of Section 2.3 gives the following matrix of size $2 \times 2$ (keeping only the relevant entries) involving two decision concepts ($DC$), $D_1$ and $D_2$, and two utilities considered as value concepts

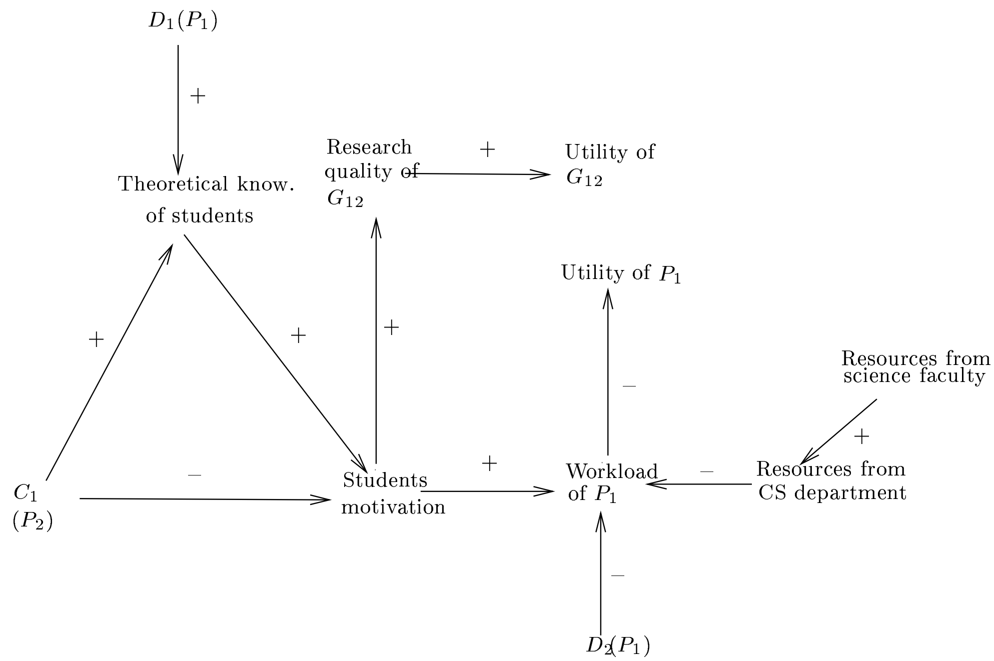


Figure 3: An Illustrative Example for Decision-Making in a Multiagent Environment.

($VC$), namely, Utility of $G_{12}$ and Utility of $P_1$.

| $DC \backslash VC$ | Utility of $G_{12}$ | Utility of $P_1$ |
|---|---|---|
| $D_1$ | $+$ | $-$ |
| $D_2$ | $-$ | $+$ |

Thus, $P_1$ perceives 1) decision $D_1$ as having a positive effect on Utility of $G_{12}$ and a negative effect on her utility; 2) decision $D_2$ as having a negative effect on Utility of $G_{12}$ and a positive effect on her utility. In these conditions, it depends on how $P_1$ and $P_2$ want to cooperate and how do they rank Utility of $G_{12}$ and Utility of $P_1$. If we assume, for example, that Utility of $G_{12}$ is more important than utility of $P_1$, then decision $D_1$ would be preferred. Conversely, $D_2$ would be the preferred decision if utility of $P_1$ is more important than utility of $G_{12}$.

# 5 Conclusion and Future Work

We have explained that the classical model of cognitive maps is an intuitive model with ad-hoc rules to calculate direct and indirect effects. It is also a model which has no a precise meaning of the primitive concepts, neither a sound formal treatment of relations between concepts. Finally, we have shown that this model is not consistent. These considerations brought us to develop a cognitive map representation based on relation algebra. This representation, (1) defines a precise semantic interpretation of qualitative causalities; (2) justifies most of the classical intuitive inference laws for reasoning from cause to effect; (3) provides users with formulae to determine certain quantitative and qualitative features of cognitive maps.

There are many directions in which the proposal made here can be extended.

- The full possibilities of relation algebra have yet to be exploited. In particular, it allows equation solving, which would certainly be useful. Also, as we have indicated in the text, the relational operations of complementation and conversion offer ways of expressing relationships between concepts that are not available in the classical theory of cognitive maps. Another option is to study "fuzzy relations" between agents' concepts (Chaib-draa, 1994; Kosko, 1992; Zhang et al., 1992; Zhang, 1996). Our approach might be extended in this direction to take into account many degrees and vague degrees of influence between agents such as: none, very little, sometimes, a lot, usually, more or less, etc.
- Applications such as the following ones must be investigated in greater depth: 1) negotiation and mediation between agents reasoning about their subjective views, 2) knowledge available to or necessary to agents in the case of nested cognitive maps, 3) reasoning about the wholistic approach, 4) reasoning on social laws, particularly for qualitative decision making and coordination.

**Acknowledgments:** This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under grants OGP–0121634 and OGP–0089769.

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